考試科目	數理統計	所別	統計所	考試時間	6月25日 星期二 上午第一節

(25%) 1. Suppose that an observation Y is equally likely to be taken from either of two populations P_1 & P_2. Suppose also that the following assumptions are made:

If the observation is taken from population P_1, then the p.d.f. (probability density function) of Y is

$$g_1(y) = \begin{cases} 1 & \text{if } 0 \le y \le 1, \\ 0 & \text{otherwise.} \end{cases}$$

If the observation is taken from population P_2, then the p.d.f. of Y is

$$g_2(y) = \begin{cases} 2y & \text{for } 0 \le y \le 1, \\ 0 & \text{otherwise.} \end{cases}$$

Suppose that the population from which Y is to be taken is not known. Find

(a) the predicted value of Y that minimizes the M.S.E. (mean squared error).

(b) the minimum value of the M.S.E.

If you are told the population from which Y is taken before predicting the value of Y. Find

(c) the minimum value of the overall M.S.E.

(25%) 2. Suppose that a nonnegative integer-valued random variable X satisfies $P(X \ge j+k \mid X \ge j) = P(X \ge k)$. Show that X has a geometric distribution.

(25%) 3. Let X_i, $i = 1, 2, \cdots, n$ be independent normally distributed with mean θ and variance θ^2 (i.e. $X_i \sim N(\theta, \theta^2)$). Does the above model have a complete sufficient statistic? Justify your answer.

(25%) 4. Let $X_1, X_2, \cdots, X_{49}$ be a random sample from a normal distribution with mean μ and variance σ^2 unknown. Find the most powerful size-0.05 test for testing the null hypothesis $H_0: \mu = \mu_0, \sigma^2 = \sigma_0^2$ against the alternative $H_a: \mu = \mu_1, \sigma^2 = \sigma_0^2$.

國立政治大學八十五學年度研究所博碩士班入學考試命題紙 第 頁

考試科目	線性模式	所別	統計研究所	考試時間	6月25日 星期 上下午第 節

(1) State and prove the Gauss-Markov theorem. Note that the matrices involved are not necessarily of full rank. Explain the importance of the theorem in linear statistical inference.

(2) Let X be an nxp matrix of constants of rank k. Partition X so that $X=[X_1,X_2]$, where X_1 is of size $n \times p_1$, $0<p_1<p$. Let the nx1 random vector Y be distributed $N_n(0,1)$.

$$Y'Y = Y'(I-XX^c)Y + Y'(XX^c-X_1X_1^c)Y + Y'(X_1X_1^c)Y = Q_1+Q_2+Q_3$$

Find the distributions of Q_1, Q_2, Q_3. (In the above expression, $X^c=(X'X)^-X'$, and $(X'X)^-$ is the generalized inverse of X'X.)

How to apply the result to the ANOVA of one-way layout, say.

(3) Evaluate

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(x_1^2 - 2x_1x_4)\exp\left\{-(1/2)Q\right\}dx_1dx_2dx_3dx_4$$

where $Q = 3x_1^2 + 2x_2^2 + 2x_3^2 + x_4^2 + 2x_1x_2 + 2x_3x_4 - 6x_1 - 2x_2 - 6x_3 - 2x_4 + 8$.

(1) 40% (2) 40% (3) 20%