

考試科目	數理統計	所別	統計學系所	考試時間	5月24日(上) 下午第1節 星期六
<div style="float: right; border: 1px solid black; padding: 2px; writing-mode: vertical-rl;">國立政治大學圖書館</div> <p>1. Let X be a random variable with moment generating function $M_X(t)$, for $-h < t < h$.</p> <p>(a) (15pts). Please prove that $P(X \geq a) \leq e^{-at}M_X(t)$ for all a and $0 < t < h$.</p> <p>(b) (15pts). Let $X \sim N(0, 1)$. Please show that $P(X \geq a) \leq e^{-a^2/2}$ for all $a > 0$.</p> <p>2. Let $X_1, X_2,$ and X_3 be a random sample of size three from a uniform($\theta, 2\theta$) distribution, where $\theta > 0$.</p> <p>(a) (10pts). Please find the method of moments estimate of $\theta, \tilde{\theta}$, using the first moment. Is it unbiased?</p> <p>(b) (10pts). Find the MLE, $\hat{\theta}$.</p> <p>(c) (12pts). Find a constant k such that $E(k\hat{\theta}) = \theta$.</p> <p>(d) (8pts). Which of the two estimators, $\tilde{\theta}$ in (a) and $k\hat{\theta}$ in (c), can be improved in variance by using sufficiency? How?</p> <p>3. Let X_1, \dots, X_n be iid Poisson(λ) and their joint pdf be $f_\lambda(x_1, \dots, x_n)$.</p> <p>(a) (10pts). Please show that $\{f_\lambda(x_1, \dots, x_n)\}_{\lambda > 0}$ is a monotone likelihood ratio family in $T(x_1, \dots, x_n) = \sum_{i=1}^n x_i$. Namely, for every $\lambda_1 < \lambda_2$, $f_{\lambda_2}(x_1, \dots, x_n)/f_{\lambda_1}(x_1, \dots, x_n)$ is a nondecreasing function of $T(x_1, \dots, x_n)$.</p> <p>(b) (20pts). For any given λ_0, consider testing $H_0 : \lambda \leq \lambda_0$ versus $H_1 : \lambda > \lambda_0$. It has been proved that the test that rejects H_0 if and only if $T > t_0$ for some t_0 is a UMP test of size α where $\alpha = P_{\lambda_0}(T > t_0)$. Now consider the specific case $H_0 : \lambda \leq 1$ versus $H_1 : \lambda > 1$. Please use the Central Limit Theorem to determine t_0 and the sample size n so a UMP test satisfies $P_{\lambda=1}(\text{reject } H_0) = 0.05$, and $P_{\lambda=2}(\text{reject } H_0) = 0.95$. (95%-tile of $N(0, 1)$ is 1.645).</p>					
備 考	試 題 隨 卷 繳 交				
命 題 委 員 : -109- (簽 章) 年 5 月 12 日					