國立政治大學九十四學年度研究所博士班入學考試

第 全 頁

考試科目	數理統計	所別	統研所	考試時間	5月28日 上午第一節 星期六

1. Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from the pdf $f_X(x;\theta) = \frac{1}{\theta}e^{-\frac{x}{\theta}}, x > 0$.

   (a) (5pts). Please compute the moment-generating function of $X_1 + \ldots + X_n$.

   (b) (3pts). Is $\hat{\theta}_1 = \frac{1}{n}(X_1 + \ldots + X_n)$ an unbiased estimate of θ? Prove or disprove it.

   (c) (5pts). What is the cdf of $X_{(1)} = \min(X_1, \ldots, X_n)$? Namely, please find $\mathrm{P}(X_{(1)} \leq t)$.

   (d) (3pts). Is $\hat{\theta}_2 = nX_{(1)}$ an unbiased estimate of θ? Prove or disprove it.

   (e) (8pts). Suppose we are interested in estimating θ. Please find the corresponding Cramer-Rao lower bound.

   (f) (10pts). Which of $\hat{\theta}_1$ or $\hat{\theta}_2$ you will choose to estimate θ? Why.

2. Let $X \sim \mathrm{Bin}(n, p)$, and suppose the prior of p is $\pi(p) = \mathbf{1}_{0 \leq p \leq 1}$.

   (a) (8pts). Please find the posterior of p.

   (b) (12pts). What is the Bayes estimate of p using the loss function $L(p, \delta(X)) = \frac{(p-\delta(X))^2}{p(1-p)}$? Please show your work.

3. Let $X_1, \ldots, X_n$ be a sample from a population with finite mean μ and finite variance σ^2. Suppose that μ is unknown but σ^2 is known, and it is required to test $\mu = \mu_0$ against $\mu = \mu_1 (\mu_1 > \mu_0)$. Let n be sufficiently large so that the central limit theorem holds, and consider the test

$$\phi(X_1, \cdots, X_n; k) = \begin{cases} 1 & \text{if } \bar{X} > k, \\ 0 & \text{if } \bar{X} \leq k, \end{cases}$$

   where $\bar{X} = \frac{1}{n}\Sigma_{i=1}^{n} X_i$.

   (a) (8pts). Find k such that the test has (approximately) size α.

   (b) (8pts). What is the power of the test you obtain in (a) at $\mu = \mu_1$?

   (c) (10pts). If the type I and type II errors are fixed at α and β, respectively, find the smallest sample size needed.

4. (20pts). Let $X_1, X_2, \ldots$ be iid random variables with mean 0, variance 1, and $\mathrm{E}(X_1^4) < \infty$. Find the limiting distribution of $\sqrt{n}\left(\frac{X_1X_2 + X_3X_4 + \ldots + X_{2n-1}X_{2n}}{X_1^2 + X_2^2 + \ldots + X_{2n}^2}\right)$.

備考	試題隨卷繳交