國立政治大學103學年度博士班招生考試試題

考 試 科 目	數理統計	所 別	統 計 系	考 試 時 間	5 月 10 日(六) 第 一節

1. Let the joint pdf of X and Y be:

$$f(x,y)=\begin{cases} 2(y-x), & y-1<x<y, \quad 0<y<1 \\ 0, & \text{otherwise} \end{cases}.$$

(1) Find the marginal pdf for X. (8%)

(2) Find $E(Y\mid x)$. (12%)

(3) Let $U=X$ and $V=Y-X$, find the joint pdf of U and V. (10%)

2. Let $X_1,\cdots,X_n$ be a random sample from $f(x\mid\lambda,\beta)=\dfrac{\lambda\beta^{\lambda}}{x^{\lambda+1}}$, $x\geq\beta$; $\lambda,\beta>0$.

(1) Find the MLEs of λ and β. (10%)

(2) Let $\beta=1$.

(2a) Find the MVUE (minimum variance unbiased estimator) of λ. (10%)

(2b) Find the UMP size α test of $H_0:\lambda\leq 1$ versus $H_1:\lambda>1$. (10%)

3. Let $X_1,\cdots,X_n$ be a random sample from $f(x\mid\theta_1)=e^{-x/\theta_1}/\theta_1$, $x>0$; and $Y_1,\cdots,Y_m$ be another random sample from $f(y\mid\theta_2)=e^{-x/\theta_2}/\theta_2$, $y>0$. Find the LRT (likelihood ratio test) of $H_0:\theta_1=\theta_2$ versus $H_1:\theta_1\neq\theta_2$ and make it to have size α. (25%)

4. Let $X_i, i=1,2,\ldots,$ be independently from Bernoulli(p) and let $Y_n=\sum_{i=1}^{n}X_i/n$. (15%)

(1) For $p\neq 0.5$, what does $\sqrt{n}\left[Y_n(1-Y_n)-p(1-p)\right]$ converge in distribution to?

(2) What does $\sqrt{n}\left[\ln\left(\dfrac{Y_n}{1-Y_n}\right)-\ln\left(\dfrac{p}{1-p}\right)\right]$ converge in distribution to?

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