

考試科目 | 微積分(-) | 系別 | 應用數學系 | 考試時間 | 7月11日 上午 10:00 - 12:00

(10) 1 Find $y''(\frac{\pi}{2})$ if $y = \cos x + \sin x$.

(10) 2 Find $\frac{dz}{dx}$ when $z = \sin(\cos w)$ and $w = 2x - 5$.

(10) 3 Find the tangent and normal to the curve $x^2 - xy + y^2 = 7$ at the point $(-1, 2)$.

(15) 4 A man 6 feet tall walks at the rate of 5 feet per second toward a streetlight that is 16 feet above the ground.

(a) At what rate is the length of his shadow changing?

(b) At what rate is the tip of his shadow moving?

(10) 5 It took 14 seconds for a thermometer to rise from -19°C to 100°C when it was taken from a freezer and placed in boiling water. Show that somewhere along the way the mercury was rising at exactly $8.5^\circ\text{C}/\text{second}$.

(20) 6. Find the following limits:

(a) $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x + x^2}$

(b) $\lim_{x \rightarrow 0} \frac{3x - \sin x}{x}$

(c) $\lim_{x \rightarrow 0^+} x \cot x$

(d) $\lim_{x \rightarrow 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right)$

(10) 7 Find the extreme values of the function

$$f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4$$

(15) 8. Find the greatest and smallest values that the function

$$f(x, y) = xy$$

takes on the ellipse $\frac{x^2}{8} + \frac{y^2}{2} = 1$.

考試科目 | 微積分(二) | 系別 | 應用數學系 | 考試時間 | 7月11日 下午第4節
星期 下

- 1 (28%) Suppose that f has a negative derivative for all values of x and that $f(1) = 0$. Let

$$h(x) = \int_0^x f(t) dt$$

Which of the following statements must be true? Give reasons for your answers.

- a h is a twice-differentiable function of x .
- b h and $\frac{dh}{dx}$ are both continuous.
- c The graph of h has a horizontal tangent at $x = 1$.
- d h has a local maximum at $x = 1$.
- e h has a local minimum at $x = 1$.
- f The graph of h has an inflection point at $x = 1$.
- g The graph of $\frac{dh}{dx}$ crosses the axis at $x = 1$.

- 2 (12%) Evaluate

$$\lim_{n \rightarrow \infty} \frac{1}{n^{15}} (1^{15} + 2^{15} + 3^{15} + \dots + n^{15})$$

- 3 (12%) Find the area of the surface generated by revolving the curve $y = 2\sqrt{x}$, $1 \leq x \leq 2$, about the x -axis.
- 4 (12%) Is there a smooth curve $y = f(x)$ whose length over the interval $0 \leq x \leq a$ is always $\sqrt{2a}$? Give reason for your answer.
- 5 (12%) Evaluate the integral

$$\int_0^{\infty} \int_0^{\infty} \frac{1}{(1+x^2+y^2)^2} dx dy$$

- 6 (12%) Given the following equation:

$$\int_0^1 \int_0^{4-a-x^2} \int_a^{4-x^2-y} dz dy dx = \frac{4}{15}$$

Find the value of a .

- 7 (12%) Find the volume of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

where a, b and c are positive numbers.