考試科目	微積分(一)	系別	應用數學系	考試時間	月 日 星期 上午/下午 第 節

1. Find an equation for the tangent line to the curve $y = F(x)$ at the point P where $x = 1$ if

$$F(x) = \int_1^{\sqrt{x}} \frac{2t+1}{t+2}\,dt \qquad (20\%)$$

2. Let $y = f(x)$ be a function that satisfies the differential equation

$$xy' = \sqrt{(\ln x)^2 - x^2}$$

Find the arc length of $y = f(x)$ between $x = \frac{1}{4}$ and $x = \frac{1}{2}$. (20%)

3. Suppose it is know that $f(0) = 3$ and

$$\int_0^{\pi} [f(x) + f''(x)] \sin x\, dx$$

What is $f(\pi)$? (20%)

4. For A and B positive constants, define

$$f(x) = (e^x + Ax)^{B/x}$$

a. Compute $L_1 = \lim_{x \to 0} f(x)$ and $L_2 = \lim_{x \to +\infty} f(x)$. (10%)

b. What is the largest value of A for which the equation $L_1 = BL_2$ has a solution? What are L_1 and L_2 in this case? (10%)

5. Suppose that $f(x)$, $f'(x)$, $f''(x)$ are all continuous on an interval I and that $x^* \in I$ is a critical point of $f(x)$. Using Taylor's Formula proves the following statement. (20%)

a. If $f''(x) \geq 0$ for all $x \in I$, then x^* is a global minimizer of $f(x)$ on I.

b. If $f''(x^*) > 0$, then x^* is a strict local minimizer of $f(x)$.

考試科目	微積分(二)	系別	應用數學系	考試時間	7月8日 星期六 上下午 第2節

1. (20%) Consider the sequence $\{a_n\}$ defined by

$$a_n = \int_1^n e^{-t^2}\,dt.$$

Show that $\{a_n\}$ converges.

2. (20%) Find the interval of convergence for the power series

$$\sum_{k=1}^{\infty} \frac{k!\,x^k}{k^k}.$$

3. (15%) Let

$$f(x,y) = \begin{cases} 1, & \text{if } x>0 \text{ and } y>0. \\ 0, & \text{otherwise}. \end{cases}$$

Show that the partial derivatives f_x and f_y exist at the origin, but f is not differentiable there.

4. (15%) Find the point on the intersection of the plane $x+2y+z=10$ and the paraboloid $z=x^2+y^2$ that is closest to the origin.

5. (15%) Evaluate $\iint_R \sin(x+2y)\cos(x-2y)\,dA$ over the region

$$R=\{(x,y)\mid x\ge 0,\ y\ge 0,\ x+2y\le 2\pi\}.$$

6. (15%) In an inverse square force field $\vec{F}=c\vec{r}/|\vec{r}|^3$, where c is a constant and $\vec{r}=x\vec{i}+y\vec{j}+z\vec{k}$, find the work done in moving a particle along the line from $(1,1,1)$ to $(3,3,3)$.