

考試科目	微積分(-)	系別	應用數學	考試時間	7月9日上午第2節 星期六
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For credit, show all work.

1. (10%) What value should be assigned to a to make the function

$$f(x) = \begin{cases} ax & , \quad x > \frac{\pi}{2} \\ \sqrt{|\sin x + 3 \cos x|} & , \quad x \leq \frac{\pi}{2} \end{cases}$$

continuous at $x = \frac{\pi}{2}$?

2. (10%) Find points on the curve $2x^2 + xy + 2y^2 = 30$ (a) where the tangent is parallel to the x-axis and (b) where the tangent is parallel to the y-axis.
3. (10%) Find the linearization of $f(x) = \sqrt{x^2 + 16}$ at $x = 3$.
4. (20%) A person 5 ft tall walks at the rate of 3 ft/sec away from a streetlight that is 20 ft above the ground. (a) At what rate is the tip of this person's shadow moving? (b) At what rate is the length of this person's shadow changing?
5. (10%) Find the derivative of $f(x)$ with respect to x , if $f(x) = x \cos(\log_3 x)$.
6. (10%) Graph the function $y = \frac{x^2 - 5}{2x - 6}$.
7. (10%) Find the point of the graph of the equation $y = x^2$ that is nearest the point (3,0).
8. (20%) Find the following limits, if they exist.

(a) $\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{\log(1+x)}$

(b) $\lim_{x \rightarrow 0^+} \sqrt{x} \cdot \log x$

(c) $\lim_{x \rightarrow 0^+} \frac{\csc x}{\cot x}$

(d) $\lim_{x \rightarrow 0^+} (e^x + x)^{\frac{1}{x}}$

考試科目	微積分(二)	系別	應用數學系	考試時間	7月9日 星期六 下午第四節
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(1) Evaluate the following indefinite integrals

(a) $\int x \ln x \, dx$

(b) $\int \tan x \, dx$

(c) $\int \frac{1}{\sqrt{1+x^2}} \, dx$ (15分)

(2) (a) Show that $\frac{\pi}{4}(1 - e^{-R^2}) \leq \int_0^R \int_0^R e^{-(x^2+y^2)} \, dx \, dy \leq \frac{\pi}{4}(1 - e^{-2R^2})$.

(b) Use (a) to evaluate the improper integral $\int_0^\infty e^{-x^2} \, dx$. (15分)

(3) Find the area of the region bounded by the graphs of $f(x) = \sin x$ and $g(x) = \cos x$ for $0 \leq x \leq \pi$. (15分)

(4) Show that the area of the surface generated by revolving the curve $y = x^2$, $0 \leq x \leq 1$ is given by $\frac{\pi}{4} \int_0^2 u^2 \sqrt{1+u^2} \, du$. (15分)

(5) Let R be the region in the plane bounded by a continuously differentiable simple closed curve C . Show that the area of R is given by $\frac{1}{2} \oint_C x \, dy - y \, dx$. (15分)

(6) Evaluate the sum of the following series if converges.

(a) $\sum_{n=0}^{\infty} \frac{(-1)^n 2^n}{n!}$

(b) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

(a) $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n}$ (15分)

(7) Let $f : [a, b] \rightarrow \mathbb{R}$ be a continuous function. Show that there exists $c \in [a, b]$ such that $\int_a^b f(x) \, dx = f(c)(b-a)$. (10分)