

考試科目	微積分(-)	系別	應數系	考試時間	7月11日 星期五	第二節
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*無演算過程者不予計分。

1. Prove that $\lim_{x \rightarrow x_0} (f(x) + g(x)) = L_1 + L_2$, if $\lim_{x \rightarrow x_0} f(x) = L_1$ and $\lim_{x \rightarrow x_0} g(x) = L_2$.

(10 pts)

2. Graph the function $y = \frac{(x^2-3)}{(2x-4)}$.

① Find all the relative maxima and minima.

② Determine the intervals where the function is decreasing or increasing.

③ Determine the concavity.

④ Find all the asymptotes.

⑤ Sketch the graph. (4 pts each)

3. Find the local maxima, local minima, and saddle points of the function $f(x, y) = 4xy - x^4 - y^4$. (10 pts)

4. Find the derivative of $f(x, y) = xe^y + \cos(xy)$ at the point $(2, 0)$ in the direction of the vector $\vec{A} = 3\vec{i} - 4\vec{j}$. (10 pts)

5. Parametrize the line segment joining the points $P(-3, 2, -3)$ and $Q(1, -1, 4)$, and then find the distance from the point $S(1, 1, 5)$ to the line joining P, Q . (10 pts)

6. Find the limits.

① $\lim_{x \rightarrow \infty} (1+2x)^{1/(2 \ln x)}$ ② $\lim_{x \rightarrow 0} \frac{x^2}{\ln(\sec x)}$. (5 pts each)

7. Graph the curve $r^2 = 4 \cos \theta$. (10 pts)

8. Find the derivative of y with respect to the appropriate independent variable.

① $y = \left(\frac{(x+5)(x-1)}{(x-2)(x+3)} \right)^5, x > 2$. ③ $y = 2(\ln x)^{1/2}$

② $y = z \sec^{-1} z - \sqrt{z^2 - 1}, z > 1$. ④ $x + \sin y = xy$. (5 pts each)

考試科目	微積分(二)	系別	應用數學系	考試時間	7月11日 星期五	第四節
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* Show all your work.

1. Determine whether the following integral and power series are convergent or divergent.

(a) (10%) $\int_0^{\pi} \frac{1}{x \cos \sqrt{x}} dx$

(b) (10%) $\sum_{n=1}^{\infty} ne^{-n^2}$

2. (a) (10%) Find the Maclaurin series for $\arctan x$.

(b) (5%) Find the interval of convergence of the Maclaurin series in (a). (Be sure to include a check for convergence at the endpoints of the interval.)

(c) (5%) Using the results in (a) and (b), find a series to calculate π .

3. Evaluate the following integrals.

(a) (5%) $\int [\ln(x)]^2 dx$

(b) (5%) $\int \csc \theta d\theta$

4. (10%) Evaluate the limit

$$\lim_{n \rightarrow \infty} \sum_{n=1}^{\infty} \frac{1}{n} \sqrt{\frac{1}{n}}$$

5. (10%) Find the total length of the curve $r = 2(1 + \cos \theta)$.

6. (15%) Find the volume common to two solid cylinders, each with radius r , where the cylinders intersect at right angles.

7. (15%) Find a function f and a number a such that

$$\pi + \int_a^x \frac{f(t)}{te^{\sqrt{t}}} dt = 2\sqrt{x}$$

for all $x > 0$.