國立政治大學九十九學年度轉學生入學考試命題紙

考試科目	微積分(一)	系別	應用數學系	考試時間	7月9日 星期五 第二節

1. Use the definition of the limit to show that

   10% (a) $\lim_{x\to 2} x^2 = 4$.

   10% (b) $\lim_{x\to\infty} \frac{1}{x} = 0$.

2. Let $\lim_{n\to\infty} (1+\frac{1}{n})^n = e$.

   10% (a) Show that $\frac{d}{dx} \log_e x = \frac{1}{x}$.

   10% (b) Show that $\frac{d}{dx} e^x = e^x$.

3. 10% (a) Show that if $\frac{d^2}{dx^2} f(x) > 0$, then $f(x)$ is concave upward.

   10% (b) Show that if $\frac{d^2}{dx^2} f(x) < 0$, then $f(x)$ is concave downward.

4. Let $f(x) = \int_x^{x^3} e^{(t^2)}\, dt$.

   10% (a) Find $\frac{d}{dx} f(x)$.

   10% (b) Find an equation of the tangent line of $f(x)$ at $x=1$.

5. Find the following limits if possible:

   10% (a) $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x+y}$.

   10% (b) $\lim_{(x,y)\to(0,0)} \frac{x-y}{x+y}$.

備考	試題隨卷繳交

命題委員： （簽章） 年 月 日

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國立政治大學九十九學年度轉學生入學考試命題紙

考試科目	微積分（二）	系別	應用數學系	考試時間	99年07月09日 星期五第四節

★ Show *all* your work for credits!

1. Evaluate the following integrals:

   (a) [5%] $\int \frac{-2x+4}{(x^2+1)(x-1)^2}\,dx$

   (b) [5%] $\int \frac{1}{2+\sin x}\,dx$

2. (a) [10%] For a certain real number a, the following integral

$$\int_2^\infty \left(\frac{ax}{x^2+1} - \frac{1}{2x+1}\right) dx$$

   converges. Determine a and evaluate the integral.

   (b) [10%] For what values of the constants b and c will the following limit exist and be equal to 1?

$$\lim_{n\to\infty} \int_{-n}^{n} \frac{x^3+bx^2+cx}{x^2+x+1}\,dx.$$

3. [15%] If d is an arbitrary real number, let $s_n(d) = \sum_{k=1}^{n} k^d$. Determine the following limit:

$$\lim_{n\to\infty} \frac{s_n(d+1)}{n\,s_n(d)}.$$

4. [10%] Let $f(x)$ be a polynomial of degree m. Show that

$$\int e^x f(x)\,dx = e^x\left(f(x) - f'(x) + f''(x) - \cdots + (-1)^m f^{(m)}(x)\right).$$

5. [15%] Evaluate the following integral

$$\int_C \mathbf{F}\cdot\mathbf{r}'\,ds$$

   along the curve $C = \{(x,y,z) \in \mathbb{R}^3 \mid x^2+y^2 = 4,\ z = -3\}$, oriented counterclockwise as seen by a person standing at the origin and with respect to the right-handed Cartesian coordinates, where

$$\mathbf{F} = y\mathbf{i} + xz^3\mathbf{j} - zy^3\mathbf{k},$$

   $\mathbf{r}' = d\mathbf{r}/ds$ is the unit tangent vector, and s is the arc length of C.

6. [15%] Evaluate the following integral

$$I = \int_L 2xyz^2\,dx + (x^2z^2 + z\cos(yz))\,dy + (2x^2yz + y\cos(yz))\,dz$$

   along the line segment L from $P : (0,0,1)$ to $Q : (2,\pi/4,2)$.

7. [15%] Compute the area of the hemisphere of radius $t > 0$ and centered at the origin given by $S = \{(x,y,z) \in \mathbb{R}^3 \mid x^2+y^2+z^2 = t^2,\ z \geq 0\}$.

備考	試題隨卷繳交

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