國立政治大學 100 學年度轉學生招生考試試題

考試科目	微積分(一)	系別	應用數學系	考試時間	7月8日(五)第 二 節

1. Use the definition of limit to show

10% (a) $\lim_{x\to 2} \frac{x^2-4}{x-2} = 4$.

10% (b) $\lim_{x\to\infty} \frac{1}{x} = 0$.

2. Suppose that $f''(x)$ is continuous.

10% (a) Show that if $f'(a)=0$ and $f''(a)<0$, then $f(a)$ is a relative maximum.

10% (b) Show that if $f'(a)=0$ and $f''(a)>0$, then $f(a)$ is a relative minimum.

3. Given that $f(x) = \int_{2x}^{x^2} e^{(t^2)}\,dt$

10% (a) Find $f'(x)$.

10% (b) Find an equation of the tangent line of $f(x)$ at $x=2$.

4. Let $f(x,y)$ be the objective function and $g(x,y)=0$ be the constrained equation.

10% (a) Show that if $f(a,b)$ is optimal, then $\frac{f_x(a,b)}{f_y(a,b)} = \frac{g_x(a,b)}{g_y(a,b)}$

10% (b) Show that $f_x(a,b)+\lambda g_x(a,b) = f_y(a,b)+\lambda g_y(a,b)$ where λ is a Lagrange multiplier.

5 (20%) Let $z=f(x,y)$ be a function and f_x and f_y exist and $z=P(x,y)$ be the tangent plane of $z=f(x,y)$ at $(a,b,f(a,b))$. Show that $P(a+\Delta x, b+\Delta y) - P(a,b) = f_x(a,b)\Delta x + f_y(a,b)\Delta y$ where Δx is the change in x and Δy is the change in y.

備註	試題隨卷繳交

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考試科目	微積分(二)	系別	應用數學系	考試時間	7月8日(五)第 4 節

1. Consider a segment of the curve described by the equation $x^{2/3} + y^{2/3} = 1$ in the first quadrant (i.e., when $0 \le x \le 1$ and $0 \le y \le 1$).
   (a) Find the length of the curve.
   (b) Find the area of the surface generated by revolving the curve about the x-axis.

   (30%)

2. Evaluate the integrals:
   (a) $\int_0^1 \frac{1}{(x^2+1)^2}\,dx$
   (b) $\int_0^{\ln 3} \frac{1}{e^x+2}\,dx$ (20%)

3. Evaluate the limits:
   (a) $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{\sqrt{n(n-k)}}$
   (b) $\lim_{x\to 0} \frac{1}{\sin x} \int_0^{\sin 2x} \cos 3t\,dt$ (20%)

4. Evaluate the integral $\iint_Q (x^2 - y^2)\,dxdy$, where Q is the region in the plane bounded by the lines $x - y = 0$, $x - y = 1$, $x + y = -2$, and $x + y = 2$.

   (15%)

5. What value for $p > 0$ makes $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n^p}\right)$ convergent? Prove your answer.

   (15%)

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