國立政治大學九十七學年度研究所博碩士班入學考試命題紙 第1頁，共1頁

A811

考試科目	微積分	所別	數學系數學碩士班	考試時間	3月15日 星期六 第3節

1. (15%) Let $x_n = \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n}, n = 1, 2, \ldots$. Find $\lim_{n\to\infty} x_n$ if exists.

2. (15%) Evaluate the double integral

$$\iint_{x^2+y^2\le 1} e^{-x^2-y^2}\, dxdy.$$

3. (15%) Let $f: \mathbb{R} \to \mathbb{R}$ be a differentiable function satisfying $f'(x) > 0$ for all $x \in \mathbb{R}$. Show that f is an increasing function.

4. (15%) Find the sum of the following series if converges.

   (a) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$.

   (b) $\sum_{n=0}^{\infty} \frac{(\ln 2)^n}{n!}$.

5. (15%) Evaluate the line integral $\oint_C y\,dx - x\,dy$, where C is the unit circle $x^2 + y^2 = 1$ in the counterclockwise direction.

6. (15%) Let

$$f(x) = \begin{cases} 0 & \text{if } x \text{ is rational,} \\ 1 & \text{if } x \text{ is irrational.} \end{cases}$$

   Is f Riemann integrable on $[0, 1]$?

7. (10%) Let $f: \mathbb{R} \to \mathbb{R}$ be a continuous function and $p, q: \mathbb{R} \to \mathbb{R}$ be differentiable functions. Define

$$F(x) = \int_{p(x)}^{q(x)} f(t)\, dt, x \in \mathbb{R}.$$

   Show that F is differentiable and $F'(x) = f(q(x))q'(x) - f(p(x))p'(x)$ for all $x \in \mathbb{R}$.

備考	試題隨卷繳交

命題委員： （簽章） 97 年 3 月 3 日

國立政治大學九十七學年度研究所博碩士班入學考試命題紙

考試科目	線性代數	所別	數學系數學碩士班 A811	考試時間	3月15日 星期六 第四節

1. Let V denote the set of all solutions to the following linear system.

$$\begin{aligned} x_1 - x_2 \quad\quad +2x_4 &= 0 \\ 2x_1 - x_2 - x_3 + 3x_4 &= 0 \end{aligned}$$

(a) (10%) Find a basis S for the vector space V.

(b) (10%) Extend S to a basis for $\mathbb{R}^4$.

2. (20%) Prove or give a counterexample: "Let A be an $n \times n$ matrix whose reduced row echelon form is a diagonal matrix. Then A is diagonalizable."

3. Let $S = \{(1,1,0), (1,2,0), (0,1,2)\}$ be a subset in $\mathbb{R}^3$.

(a) (10%) Show that S is a basis for $\mathbb{R}^3$.

(b) (10%) Applying the Gram-Schmidt Process to S, find an orthonormal basis for $\mathbb{R}^3$.

4. Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be a linear operator such that $T(1,2) = (1,2)$ and $T(-2,1) = (2,-1)$.

(a) (6%) Find the matrix representation of T with respect to the basis $S = \{(1,2), (-2,1)\}$ of $\mathbb{R}^2$.

(b) (7%) Let S' be the standard basis of $\mathbb{R}^2$. Find the change-of-basis matrix from S to S'.

(c) (7%) Use part (a) and (b) to find the matrix representation of T with respect to the standard basis S' of $\mathbb{R}^2$.

5. Given the matrix

$$A = \begin{bmatrix} 1 & 1 & 2 & 1 \\ 1 & 0 & 3 & -3 \\ 2 & 0 & 1 & 1 \\ 3 & -1 & 1 & 5 \end{bmatrix}.$$

(a) (10%) Find the rank of A.

(b) (10%) Evaluate the determinant of A.

備考	試題隨卷繳交

命題委員：　　　　　（簽章）97年3月7日