考試科目	微積分	所別	數學教學碩士在職專班 A811	考試時間	3月6日（六）第三節

1. (15%) Suppose f is differentiable on (a, b) and $f'(x) > 0$ for all $x \in (a, b)$. Show that f is strictly increasing on (a, b).

2. (30%) Find the following integrals.

   (a) $\int \csc(x)\, dx$

   (b) $\int e^x \cos(x)\, dx$

   (c) $\int_1^3 \frac{\ln x}{x^2}\, dx$

3. Let $f(x) = \ln(1 + x)$.

   (a) (8%) Find the Taylor series of f at $x = 0$.

   (b) (7%) Find the interval of convergence of the series you found in part (a).

4. (a) (5%) State Green's theorem.

   (b) (15%) Evalue the integral

$$\int_C xy\, dy - y^2\, dx$$

   where C is the square cut from the first quadrant by the lines $x = 1$ and $y = 1$.

5. Let $f(x, y) = x\sqrt{y}$ defined on $\mathbb{R}^2$.

   (a) (10%) Show that f is differentiable at the point $(1, 4)$.

   (b) (10%) Find the linearization of f at the point $(1, 4)$ and estimate $f(1.2, 4.1)$.

備註	試題隨卷繳交

考試科目	線性代數	所別	數學教學碩士在職專班 A811	考試時間	3月6日(六)第四節

∗ Show all your work.

1. (15 %) Let V and W be finite-dimensional vector spaces over a common field, and let β be a basis for V. Show that for any function $f : \beta \to W$ there exists exactly one linear transformation $T : V \to W$ such that $T(x) = f(x)$ for all $x \in \beta$.

2. (15 %) Let A and B be matrices such that product AB is defined. Prove that

   (a) $\mathrm{rank}(AB) \leq \mathrm{rank}(A)$;

   (b) $\mathrm{rank}(AB) \leq \mathrm{rank}(B)$.

3. (15 %) Prove that if $M \in \mathrm{M}_{n\times n}(F)$ can be written in the form

$$M = \begin{pmatrix} A & B \\ O & C \end{pmatrix},$$

where A and C are square matrices, then $\det(M) = \det(A) \cdot \det(C)$.

4. (20 %) Let $A = \begin{pmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ -1 & -1 & 1 \end{pmatrix}$. Find an invertible matrix Q and a diagonal matrix D such that $Q^{-1}AQ = D$.

5. (20 %) Apply the Gram-Schmidt process to the subset $S = \{(2, -1, -2, 4), (-1, 3, 7, 11), (-2, 1, -5, 5)\}$ of the inner product space $V = \mathbb{R}^4$ to obtain an orthogonal basis for span(S). Then normalize the vectors in this basis to obtain an orthonormal basis β for span(S).

6. (15 %) Find a Jordan canonical form J of $A = \begin{pmatrix} 11 & -4 & -5 \\ 21 & -8 & -11 \\ 3 & -1 & 0 \end{pmatrix}$.

備註	試題隨卷繳交