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A Double Sampling Scheme for Process Mean Monitoring

Su-Fen Yang, and Sin-Hong Wu

***Abstract*—Mean control charts are effective tools for detecting mean shifts of an interesting quality variable in both manufacturing processes and service processes. Much of the data in service industries come from processes exhibiting non-normal or unknown distributions. The commonly used Shewhart mean control charts, which depend heavily on the normality assumption, are not appropriately used here. This paper thus proposes an asymmetric EWMA mean chart with a double sampling scheme (DS EWMA-AM chart) for monitoring mean shifts of a process with variables data. Furthermore, we explore the sampling properties of the new mean monitoring statistics, and investigate the out-of-control detection performance of the proposed DS EWMA-AM chart using average run lengths. The detection performance of the DS EWMA-AM chart and that of the single sampling EWMA mean (SS EWMA-AM) chart are then compared, with the former showing superior out-of-control detection performance versus the latter. We also compare the out-of-control mean detection performance of the proposed chart with those of non-parametric mean control charts, like the likelihood ratio-based distribution-free NLE, CWE, SS EWMA-AM, the SL, the SU, and the VSS and DSVSI $\overline{X}$ control charts by considering cases in which the critical quality characteristic presents normal, double exponential, uniform, chi-square and exponential distributions, respectively. Comparison results show that the proposed control chart always outperforms the existing mean control charts. We hence recommend employing the DS EWMA-AM chart. A numerical example of a service system for a bank branch in Taiwan is used to illustrate the application of the proposed mean control chart. Finally, we give a discussion for future study.**

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***Index Terms*—average run length, control chart, free-distribution, process mean.**

I. INTRODUCTION

Control charts are commonly used tools for detecting out-of-control process mean in order to improve the quality of manufacturing processes and service processes. In the past few years, more and more statistical process control techniques have been applied to the service industry. Control charts are becoming an effective tool in improving service quality; see MacCarthy and Wasusri [1], Tsung et al. [2], Ning et al. [3], Yang and Yang [4], and Yang and Arnold [5]. The commonly used Shewhart variables mean control charts, which depend on a normality assumption, are not suitable for monitoring most service process data when quality variables are not normal or unknown distributions. Hence, some research has been conducted to deal with process monitoring of variables having non-normal or distribution-free data. Some related research has also looked at non-parametric approaches to deal with process location monitoring; see, for example, Ferrell [6], Bakir and Reynolds [7], Amin et al. [8] , Altukife [9, 10], Bakir [11, 12] Chakraborti and Eryilmaz [13], Chakraborti and Graham [14], Chakraborti et al. [15], Li et al. [16], Zou and Tsung [17], Graham et al. [18,19], Grahama et al. [20]. For quickly monitoring shifts of the process variance, Yang and Arnold [5] offered an asymmetric EWMA variance chart and Yang and Arnold [21] initiated a new arcsine transformed symmetric EWMA variance chart for variables data exhibiting free distribution.

For practitioners who want to easily implement the scheme of control charts with distribution-free statistics, Yang et al. [22] proposed a new sign chart for data having distribution-free variables data in order to monitor the deviation of the process measurement and target. Yang [23] set up an improved Mean Chart for variables data to monitor the process mean of quality variables with no-normal or unknown distributions. Both approaches (Yang et al. [22] and Yang [23]) are quite easy to use, and even simpler than some of the above published non-parametric approaches. However, they do not illustrate a detection ability comparison with existing mean charts, like the likelihood ratio-based distribution-free NLE and CWE control charts proposed by Zhou and Tsung [17], the SL control chart based on the Lepage statistic developed by Mukherjee and Chakraborti [24], the SC chart based on the Cucconi statistic proposed by Chowdhury et al. [25], and the double sampling and variable sampling intervals (DSVSI) $\overline{X}$ chart addressed by Carot et al. [26]. Furthermore, the detection ability of the sign chart [22] and Mean Chart [23] do not seem efficient at detecting small shifts in mean.

Daudin [27] first proposed the double sampling (DS) scheme to improve the detection ability in small mean shifts of the Shewhart $\overline{X}$ control chart and variable sampling intervals (VSI) $\overline{X}$ control chart. This procedure offers better statistical efficiency, in terms of average run length (ARL), versus the single sampling scheme without increased sampling. Alternatively, the procedure can be used to reduce sampling

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without degrading statistical efficiency. Carot et al. [26] demonstrated that the detection ability of the DSVSI $\overline{X}$ control chart is faster than both the VSS $\overline{X}$ control chart and the Shewhart $\overline{X}$ control chart. However, all the above papers assumed that the quality variable follows normal distribution. The DS scheme has not yet discussed mean and/or dispersion control charts for critical quality variable with free distributions. To allow the distribution-free mean chart to be more efficient in detecting small mean shifts without any increased sampling, we herein develop the DS asymmetric EWMA mean (DS EWMA-AM) chart, investigate its process detection performance in process mean shifts, and compare its detection performance with some existing mean control charts. The paper is organized as follows. Section 2 discusses the construction of a newly proposed DS EWMA-AM chart, and analyzes its out-of-control mean detection performance. Section 3 compares the mean detection performances among the proposed DS EWMA-AM chart and some existing charts. Section 4 gives a numerical example. Section 5 summarizes the findings and provides a recommendation. Section 6 discusses some future studies.

II. THE DS EWMA-AM CHART

Daudin [27] presented that the double sampling scheme is a counterpart to double sampling plans. This procedure offers better statistical efficiency, in terms of average run length (ARL), than the single sampling scheme without the increased sampling. Alternatively, the procedure can be used to reduce sampling without reducing statistical efficiency. The double sampling procedure proposed allows one to assume that two successive samples can be taken without any intervening time, therefore coming from the same probability distribution.

A. The Control Limits of The EWMA-AM Chart

Following Yang [23], in order to monitor process mean, a random sample of size n, $X_1, X_2, \ldots, X_n$, is taken from the process. Denote the X_j, $j = 1, 2, \ldots, n$, have in-control mean μ_0 and in-control variance σ^2. Next, define

$$Y_j = X_j - \mu_0,$$

$$I_j = \begin{cases} 1, & \text{if } Y_j > 0 \\ 0, & \text{otherwise,} \end{cases}$$

$$p_0 = P(Y_j > 0).$$

Let M be the total number of $Y_j > 0$, and then, $M = \sum_{j=1}^{n} I_j$ has a binomial distribution with parameters (n, p_0), M~B(n, p_0). The value of p_0 depends on the distribution of the X_j's. Note that monitoring the shifts in process mean using M statistic is equivalent to monitoring the changes in process proportion, p_0. For example, if the X_j's are symmetric distributed, then $p_0 = P(Y_j > 0) = 0.5$. If the X_j's are exponential distributed, Exp(1), then the quantity p_0 is 0.368. If the X_j's are chi-square distributed, χ_3^2, then the quantity p_0 is 0.362. The value of p_0 can be arbitrarily small, but it usually ranges 0.30 to 0.50.

We propose the following procedure to construct the DS EWMA-AM chart.

First, take a sample of size n_1, compute M_1 statistic and the sample statistic $EWMA_{M_{1,t}}$ at time t, where $EWMA_{M_{1,t}} = \lambda M_{1,t} + (1-\lambda) EWMA_{M_{1,t-1}}$, $0 < \lambda < 1$, $t = 1, 2, \ldots, \infty$, and $M_1 \sim B(n_1, p_0)$. Hence, the in-control mean and variance of the statistic $EWMA_{M_{1,t}}$ are

$$E(EWMA_{M_{1,t}}) = n_1 p_0,$$

and

$$Var(EWMA_{M_{1,t}}) = \frac{\lambda[1-(1-\lambda)^{2t}]}{2-\lambda}(n_1 p_0 (1-p_0)).$$

At the first stage, we construct a DS EWMA-AM chart with upper control limit (UCL), upper warning limit (UWL), center line (CL), lower warning limit (LWL) and lower control limit (LCL) as follows.

$$\mathrm{UCL}_{M1} = n_1 p_0 + \mathrm{L}_1 \sqrt{\frac{\lambda[1-(1-\lambda)^{2t}]}{2-\lambda} n_1 p_0 (1-p_0)},$$

$$\mathrm{UWL}_{M1} = n_1 p_0 + \mathrm{W}_1 \sqrt{\frac{\lambda[1-(1-\lambda)^{2t}]}{2-\lambda} n_1 p_0 (1-p_0)},$$

$$\mathrm{CL}_{M1} = n_1 p_0,$$

$$\mathrm{LWL}_{M1} = n_1 p_0 - \mathrm{W}_2 \sqrt{\frac{\lambda[1-(1-\lambda)^{2t}]}{2-\lambda} n_1 p_0 (1-p_0)},$$

$$\mathrm{LCL}_{M1} = n_1 p_0 - \mathrm{L}_2 \sqrt{\frac{\lambda[1-(1-\lambda)^{2t}]}{2-\lambda} n_1 p_0 (1-p_0)},$$

where L_1, L_2, W_1, and W_2 are appropriately chosen coefficients to reach a request in-control average run length (ARL), $0 < W_1 \le \mathrm{L}_1$, and $0 < W_2 \le \mathrm{L}_2$.

To easily apply the proposed control chart, we standardize the statistic $EWMA_{M_{1,t}}$. Hence the new plotting statistic, $Z_{AM_{1,t}}$, at stage 1 is defined as follows:

$$Z_{AM_{1,t}} = \frac{EWMA_{M_{1,t}} - n_1 p_0}{\sqrt{\frac{\lambda[1-(1-\lambda)^{2t}]}{2-\lambda} n_1 p_0 (1-p_0)}}.$$

Thus, the mean and variance of the new standardized statistic, $Z_{AM_{1,t}}$, are zero and one, respectively.

We let the new standardized DS EWMA-AM chart be the SDS EWMA-AM chart with the following control limits.

$$\mathrm{UCL}_{M1} = \mathrm{L}_1 \qquad \mathrm{UWL}_{M1} = \mathrm{W}_1,$$

$$\mathrm{CL}_{M1} = 0, \qquad \mathrm{LWL}_{M1} = -\mathrm{W}_2,$$

$$\mathrm{LCL}_{M1} = -\mathrm{L}_2.$$

The region between UWL and LWL is called the central region (CR), and the region between UCL and UWL or LWL and LCL is called the warning region (WR). There are three following possibilities at this first stage.

(1) If the monitoring statistic, $Z_{AM_{1,t}}$, falls in CR, then conclude the process mean is in-control.

(2) If the monitoring statistic, $Z_{AM_{1,t}}$, falls outside of UCL or LCL, then conclude the process mean is out-of-control.

(3) If the monitoring statistic, $Z_{AM_{1,t}}$, falls in WR, then go to the second stage and take a second sample size, n_2.

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At the second stage one takes a sample of size n_2, which is always larger than n_1, and computes the statistic, M_2, where M_2 follows a binomial distribution with parameters (n_2 , p_0), M_2~B(n_2 , p_0). Decisions at the second stage are based on the combined statistic M_3=M_1+M_2, where M_3~B($n_1 + n_2, p_0$).

Hence the second-stage EWMA statistic, $EWMA_{M_{3,t}}$, is:

$$EWMA_{M_{3,t}} = \lambda M_{3,t} + (1-\lambda) EWMA_{M_{3,t-1}}.$$

The mean and variance of the statistic $EWMA_{M_{3,t}}$, $E(EWMA_{M_{3,t}})$ and $Var(EWMA_{M_{3,t}})$, are $(n_1+n_2)p_0$ and $Var(EWMA_{M_{3,t}}) = \frac{\lambda[1-(1-\lambda)^{2t}]}{2-\lambda}(n_1+n_2)p_0(1-p_0)$, respectively.

As in the first stage, we standardize the statistic $EWMA_{M_{3,t}}$. Hence, the new plotting statistic, $Z_{AM_{3,t}}$, at the second stage is defined as follows:

$$Z_{AM_{3,t}} = \frac{EWMA_{M_{3,t}} - (n_1+n_2)p_0}{\sqrt{\frac{\lambda[1-(1-\lambda)^{2t}]}{2-\lambda}(n_1+n_2)p_0(1-p_0)}}.$$

In the second stage, the control limits of the SDS EWMA-AM chart are as follows.

$$UCL_{M2} = L_3,$$
$$CL_{M2} = 0,$$
$$LCL_{M2} = -L_4.$$

At this second stage, there are only two of the following possibilities.

(1) If the monitoring statistic, $Z_{AM_{3,t}}$, falls between UCL and LCL, then conclude the process mean is in-control.
(2) If the monitoring statistic, $Z_{AM_{3,t}}$, falls outside of UCL or LCL, then conclude the process mean is out-of-control.

The properties of the SDS EWMA-AM chart depend on the eight parameters of L_1, L_2, L_3, L_4, W_1, W_2, n_1 and n_2. We may determine the values of the eight parameters to reach a request in-control ARL (ARL_0) with the constraint of average sample size $(E(N))$ is no larger than a single sample size (n_0), $E(N) \le n_0$.

We now adopt three combinations of n_1 and n_2, the small sample sizes (n_1=4, n_2=6), the medium sample sizes (n_1=6, n_2 =12), and the large sample sizes (n_1=8, n_2=16), and let n_0=5, 8, 10, $\lambda = 0.05$ and p_0=0.1,0.2, 0.3, 0.4, 0.5. The parameters of the control limits ($L_1, L_2, W_1, W_2, L_3, L_4$) are determined to satisfy the requested $ARL_0 \approx 370$ by the direct search computing method and Monte Carlo simulation using R program.

The control limits of the proposed chart are thus determined. Table 1 illustrates the corresponding values of the six parameters for fifteen combinations values of p_0, n_1, n_2, and n_0. It can be seen that when p_0 increases, L_1, W_1, and L_3 decrease, but L_2, W_2 and L_4 increase; when the sample size increases, L_2 increases, L_1 decreases, but no trends of change for W_1, W_2, L_3 and L_4 ; the in-control average sample size, E(N), is smaller than the fixed sample size, n_0.

B. Performance Measurement of the SDS EWMA-AM Chart

To measure the out-of-control mean detection performance of the SDS EWMA-AM chart, we calculate the out-of-control average run length (ARL_1) by considering the out-of-control proportion p_1 = 0.1, 0.2, 0.3,...,0.9 and the in-control proportion p_0 =0.1, 0.3, 0.5 for the fifteen combinations values of parameters in Table 1. The results are listed in Tables 2, 3 and 4. The results look reasonable, since ARL_1 decreases when p_1 is far away from p_0, or when the sample size increases. Furthermore, the SDS EWMA-AM chart performs better when n_1 and n_2 are larger.

Insert Tables I-IV

III. PERFORMANCE COMPARISON WITH EXISTING CONTROL CHARTS

We compare the out-of-control mean detection performance of our proposed SDS EWMA-AM control chart with that of SS EWMA-AM control chart proposed in Yang [23] by considering that $ARL_0 = 370$, $\lambda = 0.05$, $n_1 = 8$, $n_2 = 16$, and $n_0 = 10$ under p_0 = 0.1, 0.3, 0.5, respectively. Table 5 illustrates the results. We find that the proposed SDS EWMA-AM chart has a smaller average run length under the out-of-control process mean, that is it shows superior out-of-control detection performance than the SS EWMA-AM chart. Next, We compare the out-of-control mean detection performance of our proposed SDS EWMA-AM chart with those of existing non-parametric mean control charts, like the NLE and CWE control charts proposed by Zou and Tsung [17] , the SS EWMA-AM chart provided by Yang [23], the SC chart addressed by Chowdhury et al. [25], the SL control chart based on Lepage statistic developed by Mukherjee and Chakraborti [24], and DSVSI and VSS $\bar{X}$ charts proposed by Carot et al. [26], by considering that the critical quality characteristic has a normal distribution, a double exponential distribution, a uniform distribution, a chi-square distribution, and an exponential distribution, respectively.

First, we let the critical quality characteristic have a normal distribution, N(δ , 1). The out-of-control mean detection performance of our proposed SDS EWMA-AM chart with $n_1 = 8$, $n_2 = 16, \lambda = 0.05$, E(N)=9.81 and a request ARL_0=370 is compared with those of the NLE and the CEW charts (Zou and Tsung [17]) with historical observations, m=20000, and SS EWMA-AM chart with $n_0 = 10$. Table 6 lists the ARLs under various scale shift values ($0 \le \delta \le 3$). We find that the proposed SDS EWMA-AM chart has a smaller in-control average sample size and shows superior out-of-control detection performance than the NLE, CEW and SS EWMA-AM charts no matter whether the mean shift scale is small, medium, or large.

We also compare the out-of-control mean detection performance of our proposed SDS EWMA-AM chart with $n_1 = 4$, $n_2 = 6$, $\lambda = 0.05$, and $E(N) = 4.66$ and a request ARL_0=500 versus those of the SL charts (Mukherjee and Chakraborti [24]) and the SC chart (Chowdhury et al. [25]) with different historical observations, m=50,100, under a single sample size $n_0 = 5$ and the critical quality characteristic following a normal distribution, N(δ, 1), $0 \le \delta \le 1.5$. Table 7 lists the corresponding ARLs for various mean shift scales, δ. We find that the proposed SDS EWMA-AM chart shows

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superior out-of-control mean detection performance than the SC charts and SL charts with different historical observations.

We further compare the out-of-control mean detection performance of our proposed SDS EWMA-AM chart with $n_1 = 3,\ n_2 = 6,\ \lambda = 0.05,\ \mathrm{E(N)} = 3.81$, fixed sampling interval ($h_0 = 1$ time unit) and a request in-control adjusted average time to signal (AATS), AATS=370, versus those of the DSVSI chart with $n_1 = 3,\ n_2 = 15$ and variable sampling intervals ($h_1 = 1.70,\ h_2 = 0.05$), the VSS $\overline{\mathrm{X}}$ chart with $n_1 = 3,\ n_2 = 15$ and fixed sampling interval ($h_0 = 1$) (Carot et al. [26]), and the Shewhart $\overline{\mathrm{X}}$ chart with fixed sampling interval ($h_0 = 1$) under a single sampling of size $n_0 = 4$ and the critical quality characteristic following a normal distribution, N(δ , 1), $0 \le \delta \le 3.0$. Table 8 lists the corresponding ARLs for various mean shift scales, δ. We find that the proposed SDS EWMA-AM chart shows superior out-of-control detection performance than the DSVSI and VSS $\overline{\mathrm{X}}$ charts for small and medium mean shift scale, $0 \le \delta < 2.0$, but shows almost the same out-of-control mean detection performance for large shift scale, $2 \le \delta \le 3.0$. The proposed SDS EWMA-AM chart shows superior out-of-control mean detection performance than the Shewhart $\overline{\mathrm{X}}$ chart no matter whether the mean shift scale is small, medium or large.

Second, we let the critical quality characteristic have a double exponential distribution. The out-of-control mean detection performance of our proposed SDS EWMA-AM chart with $n_1 = 4,\ n_2 = 6, \lambda = 0.05,$ and $\mathrm{E(N)} = 4.66$ and a request ARL_0=500 is compared with those of the SS EWMA-AM chart and SL chart with different historical observations, m=30, 50, under a single sample of size $n_0 = 5$ when the critical quality characteristic has a double exponential distribution, DE(δ, $1/\sqrt{2}$), $0 \le \delta \le 1.5$. Table 9 illustrates the ARLs. We note that the proposed SDS EWMA-AM chart has superior out-of-control mean detection performance than those of the SS EWMA-AM and the SL charts no matter whether the shift scale is small or medium.

We also compare the out-of-control mean detection performance of our proposed SDS EWMA-AM chart with $n_1 = 4,\ n_2 = 6,\ \lambda = 0.05$ and $\mathrm{E(N)} = 4.66$ and a request ARL_0=500 with those of the SL and SC charts but with different historical observations, m=50,100, and under a single sampling of size $n_0 = 5$ and when critical quality characteristic has a double exponential distribution, DE($\sqrt{2}\delta$, 1), $0 \le \delta \le 2$. Table 10 lists the corresponding ARLs of various mean shift scales, δ. We find that the proposed SDS EWMA-AM chart shows superior out-of-control mean detection performance than the SL and SC charts.

Third, we let the critical quality characteristic have a Chi-square distribution. The out-of-control mean detection performance of our proposed SDS EWMA-AM chart with $n_1 = 8, n_2 = 16, \lambda = 0.05,$ and $\mathrm{E(N)} = 9.77$ and a request ARL_0=370 is compared with those of the NLE and CEW charts with historical observations, m=20000, (Zou and Tsung [17]) under a single sample of size $n_0 = 10$ and when critical quality characteristic has a Chi-square distribution, $(\chi_3^2 - 3)/\sqrt{6}$, $0 \le \delta \le 3$. Table 11 presents the ARLs. We discover that the proposed SDS EWMA-AM chart has superior out-of-control mean detection performance than those of the NLE and CEW charts, no matter whether the shift scale is small, medium, or large.

Fourth and finally, we let the critical quality characteristic have an exponential distribution and a uniform distribution, respectively. When critical quality characteristic has an exponential distribution, $\mathrm{Exp}(1) + \delta$, $0 \le \delta \le 3$, the out-of-control mean detection performance of our proposed SDS EWMA-AM chart with $n_1 = 8, n_2 = 16, \lambda = 0.05,$ $\mathrm{E(N)} = 9.81$ and a request ARL_0=370 is compared with that of the SS EWMA-AM chart under a single sample of size $n_0 = 10$. When critical quality characteristic has a uniform distribution, Unif $(-\sqrt{3}, \sqrt{3}) + \delta$, $0 \le \delta \le 3$, the out-of-control mean detection performance of our proposed SDS EWMA-AM chart is compared with that of the SS EWMA-AM chart under a single sample of size $n_0 = 10$. Table 12 shows their ARLs. We can state that the proposed SDS EWMA-AM chart has superior out-of-control mean detection performance than that of the SS EWMA-AM chart under the critical quality characteristic has an exponential or a uniform distribution, no matter the shift scale is small, medium, or large.

Insert Tables V-XII

IV. AN EXAMPLE

An in-control non-normal sampling service times $X_{1,\mathrm{t}}$–$X_{10,\mathrm{t}}$ (unit: minutes) measured from the first ten counters every day for 15 days (t=1, 2, ⋯, 15) in a bank branch (see Table 13) in Yang and Arnold [5] is used to illustrate the applications of the SDS EWMA-AM chart. To use the double sampling scheme to monitor the shifts in mean of service time, we adopt $n_1 = 4$ ($X_{1,\mathrm{t}}$–$X_{4,\mathrm{t}}$) and $n_2 = 6$ ($X_{5,\mathrm{t}}$–$X_{10,\mathrm{t}}$) and let $n_0 = 5$.

To construct the SDS EWMA-AM chart, let the given in-control mean, μ_0, be 5.77 with the corresponding in-control proportion p_0=0.4. Thus, the statistics $M_{1,\mathrm{t}}$ in stage 1 and $M_{3,\mathrm{t}}$ in stage 2 are calculated (see Table 13).

The SDS EWMA-AM chart with $\mathrm{ARL}_0 \approx 370$ and $\lambda = 0.05$ is constructed as follows.

The control limits of the proposed chart in Stage 1 are

$$\mathrm{UCL_{M1}} = 2.80,$$
$$\mathrm{UWL_{M1}} = 1.68,$$
$$\mathrm{LWL_{M1}} = -1.63,$$
$$\mathrm{LCL_{M1}} = -2.72.$$

The control limits of the proposed chart in Stage 2 are:

$$\mathrm{UCL}_{M2} = 2.49,$$
$$\mathrm{LCL}_{M2} = -2.42.$$

In Stage 1, all fifteen monitoring statistics, $Z_{AM_{1,\mathrm{t}}}$, fall in the CR of the Stage 1 chart (see Table 14). Hence, we do not need to go to Stage 2. This confirms the fifteen samples adopted from the service process with in-control (IC) mean of service time.

For a new data set, (t=16, 17, ⋯, 25), the new automatic service system of the bank branch, from the first ten counters in a bank branch (see Table 15) in Yang and Arnold [5] is used. The new service times $X_{1,\mathrm{t}}$–$X_{4,\mathrm{t}}$ are measured from 4 counters at Stage 1, and the monitoring statistic $Z_{AM_{1,\mathrm{t}}}$ is calculated in Table 15. We find that the sample statistics $Z_{AM_{1,\mathrm{t}}}$ for t=16-18 are within CR of the Stage 1 chart , while the sample statistics

$Z_{AM_{1,t}}$, t=19-21, are within WR of the Stage 1 chart, and the sample statistics $Z_{AM_{1,t}}$, t=22-25, are outside of LCL_{M1} of the Stage 1 chart. This indicates the process mean is out-of-control (OC) on t=22-25, but the monitoring procedure goes to Stage 2 for t=19-21, and we calculate the monitoring statistic, $Z_{AM_{3,t}}$, using the second sample of service times $X_{5,t}$–$X_{10,t}$ from the other 6 counters. In Stage 2 we see that the sample statistics $Z_{AM_{3,t}}$, t=19, are within CR of the Stage 2 control chart, but samples t=20, 21 are outside of LCL_{M2} of the Stage 2 control chart. This indicates that the process mean is in-control at t=19, while the process mean is out-of-control at t=20, 21. Table 15 and Figure 1 illustrate the resulting statistics in the two stages.

Insert Tables 13-15, and Figure 1.

V. CONCLUSIONS

In this paper, we propose a mean control chart with distribution-free critical quality characteristic, the DS-EWMA-AM chart, based on a simple statistic and scheme in order to efficiently monitor the shifts in a process mean and without the increased sampling. The proposed DS EWMA-AM chart is easy for practitioners to apply even if they lack statistical training. A numerical example of a service system for a bank branch in Taiwan is used to illustrate the application of the proposed mean control chart. When compared with the existing control charts for the quality variable with a normal or a non-normal distribution—the NLE and CWE control charts, the SS EWMA-AM chart, the SL and SC charts, and the DSVSI and VSS $\bar{\mathbf{X}}$ charts - the proposed new DS EWMA-AM chart always performs better in detecting the out-of-control process with small, medium, and large mean scale shifts. We thus recommend using the new DS EWMA-AM chart.

VI. DISCUSSION

In this paper, we propose the DS EWMA-AM control chart with a distribution-free critical quality characteristic in order to efficiently monitor the shifts in a process mean. For a non-normal distribution, sometimes the process median is more important than the process mean, and hence monitoring the process median turns out to be quite important. The proposed DS EWMA-AM control chart with a distribution-free critical quality characteristic can become a DS EWMA median control chart by letting the proportion p_0=0.5. Furthermore, monitoring the process dispersion is always more important than monitoring the process location from the six sigma quality concept of the General Electric (GE) company. We extend the proposed approach of the DS EWMA-AM control chart to derive the DS EWMA variance/standard deviation control chart with a distribution-free critical quality characteristic so as to monitor any small shifts in variance or standard deviation. Many situations of industries nowadays, it is necessary to simultaneously monitor multiple related quality variables of a given process. The popular assumption of a multivariate normal distribution for related multiple quality variables is not always true in the real world. In order to detect mean/covariance shifts faster, we propose DS EWMA mean vector and/or covariance control charts for a process with distribution-free related quality variables in order to monitor the shifts in mean vector and/or covariance.

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TABLE I
Parameters of the SDS EWMA-AM chart with $\text{ARL}_0 \approx 370$ and $\lambda = 0.05$ for various combinations of p_0, n_1, n_2 and n_0.

No.	p_0	n_1	n_2	n_0	L_1	L_2	W_1	W_2	L_3	L_4	E(N)	ARL_0
1	0.1	4	6	5	3.10	2.44	1.86	1.46	2.71	2.14	4.76	371.89
2	0.1	6	12	8	3.00	2.54	1.68	1.42	2.63	2.23	7.74	371.24
3	0.1	8	16	10	3.00	2.54	1.83	1.55	2.52	2.13	9.83	370.65
4	0.2	4	6	5	2.95	2.57	1.77	1.54	2.60	2.26	4.73	368.31
5	0.2	6	12	8	2.92	2.61	1.64	1.46	2.62	2.34	7.76	367.30
6	0.2	8	16	10	2.89	2.67	1.76	1.63	2.42	2.24	9.76	374.84
7	0.3	4	6	5	2.88	2.70	1.73	1.62	2.47	2.32	4.71	373.44
8	0.3	6	12	8	2.86	2.71	1.60	1.52	2.53	2.40	7.83	368.89
9	0.3	8	16	10	2.84	2.71	1.73	1.65	2.39	2.28	9.95	370.10
10	0.4	4	6	5	2.80	2.72	1.68	1.63	2.49	2.42	4.78	369.50
11	0.4	6	12	8	2.81	2.74	1.57	1.53	2.50	2.44	7.76	368.81
12	0.4	8	16	10	2.81	2.75	1.71	1.68	2.35	2.30	9.87	371.41
13	0.5	4	6	5	2.76	2.76	1.66	1.66	2.51	2.51	4.79	373.77
14	0.5	6	12	8	2.76	2.76	1.55	1.55	2.48	2.48	7.83	371.62
15	0.5	8	16	10	2.76	2.76	1.68	1.68	2.35	2.35	9.81	371.36

TABLE II
The ARL of the SDS EWMA-AM Chart for $\lambda = 0.05$ and $p_0 = 0.1$

p_1	SDS-EWMA-AM $n_1 = 4, n_2 = 6,$ $\text{E}(N) = 5.69$	SDS-EWMA-AM $n_1 = 6, n_2 = 12,$ $\text{E}(N) = 7.74$	SDS-EWMA-AM $n_1 = 8, n_2 = 16,$ $\text{E}(N) = 9.83$
	ARL	ARL	ARL
0.1	371.89	371.24	370.65
0.2	11.56	7.03	6.35
0.3	3.99	2.38	2.30
0.4	2.25	1.44	1.45
0.5	1.58	1.15	1.16
0.6	1.25	1.05	1.05
0.7	1.09	1.01	1.01
0.8	1.03	1.00	1.00
0.9	1.00	1.00	1.00

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TABLE III

The *ARL* of the SDS EWMA-AM Chart for λ = 0.05 and $p_0 = 0.3$

p_1	SDS-EWMA-AM $n_1 = 4, n_2 = 6,$ $\mathrm{E}(N) = 4.71$	SDS-EWMA-AM $n_1 = 6, n_2 = 12,$ $\mathrm{E}(N) = 7.83$	SDS-EWMA-AM $n_1 = 8, n_2 = 16,$ $\mathrm{E}(N) = 9.95$
	ARL	ARL	ARL
0.1	6.03	3.24	2.62
0.2	20.50	12.40	10.04
0.3	373.44	368.89	370.10
0.4	20.55	13.82	10.92
0.5	6.24	4.29	3.29
0.6	3.07	2.22	1.73
0.7	1.84	1.43	1.23
0.8	1.29	1.12	1.06
0.9	1.06	1.02	1.00

TABLE IV

The ARL of the SDS EWMA-AM Chart for λ = 0.05 and $p_0 = 0.5$

p_1	SDS-EWMA-AM $n_1 = 4, n_2 = 6,$ $\mathrm{E}(N) = 4.79$	SDS-EWMA-AM $n_1 = 6, n_2 = 12,$ $\mathrm{E}(N) = 7.83$	SDS-EWMA-AM $n_1 = 8, n_2 = 16,$ $\mathrm{E}(N) = 9.81$
	ARL	ARL	ARL
0.1	1.66	1.19	1.21
0.2	3.14	2.07	1.86
0.3	6.88	4.46	3.77
0.4	23.02	15.07	12.41
0.5	373.77	371.62	371.36
0.6	22.97	15.13	12.56
0.7	6.76	4.48	3.73
0.8	3.10	2.06	1.84
0.9	1.66	1.19	1.20

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TABLE V
Performance comparison of the SDS EWMA-AM chart and the SS EWMA-AM chart with $\text{ARL}_0 \approx 370$, $\lambda = 0.05$ and $n_0 = 10$ under $p_0 = 0.1, 0.3, 0.5$.

p_1	SDS-EWMA-AM $n_1 = 8, n_2 = 16$, $p_0 = 0.1$.	SDS-EWMA-AM $n_1 = 8, n_2 = 16$, $p_0 = 0.3$	SDS-EWMA-AM $n_1 = 8, n_2 = 16$, $p_0 = 0.5$	SS EWMA-AM $p_0 = 0.1$, $n_0 = 10$	SS EWMA-AM $p_0 = 0.3$, $n_0 = 10$	SS EWMA-AM $p_0 = 0.5$, $n_0 = 10$
	ARL	ARL	ARL	ARL	ARL	ARL
0.1	370.65	2.62	1.21	370.30	7.30	3.90
0.2	6.35	10.04	1.86	10.40	17.10	5.20
0.3	2.30	370.10	3.77	4.90	373.70	8.20
0.4	1.45	10.92	12.41	3.30	17.00	19.40
0.5	1.16	3.29	371.36	2.50	7.40	369.50
0.6	1.05	1.73	12.56	2.10	4.80	18.80
0.7	1.01	1.23	3.73	1.90	3.70	8.10
0.8	1.00	1.06	1.84	1.70	2.90	5.20
0.9	1.00	1.00	1.20	1.30	2.30	3.90

TABLE VI
Performance comparison of the SDS EWMA-AM, the EWMA-AM, the NLE and the CEW charts with $\text{ARL}_0 \approx 370$, $\lambda = 0.05$ and $n_0 = 10$ under N(δ , 1).

δ	SDS EWMA-AM chart $n_1 = 8, n_2 = 16$, E(N)=9.81	NLE chart m=20000	CEW chart m=20000	SS EWMA-AM chart
	ARL	ARL	ARL	ARL
0.00	371.36	369.00	370.00	371.03
0.25	12.89	98.00	84.90	18.73
0.50	4.03	36.10	29.30	8.29
0.75	2.14	20.10	16.20	5.62
1.00	1.51	14.10	11.00	4.45
1.50	1.10	7.65	6.28	3.44
2.00	1.01	4.57	4.04	3.07
3.00	1.00	2.08	2.11	3.00

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TABLE VII
Performance comparison of the SDS EWMA-AM, the SS EWMA-AM, the SL and the SC charts with $ARL_0 \approx 500$, $\lambda = 0.05$ and $n_0 = 5$ under N(δ , 1).

δ	SDS EWMA-AM chart $n_1 = 4, n_2 = 6$, E(N)=4.66	SL chart m=50	SL chart m=100	SC chart m=50	SC chart m=100
	ARL	ARL	ARL	ARL	ARL
0.00	507.85	499.62	513.00	497.30	509.40
0.25	27.23	292.69	257.60	288.60	253.60
0.50	8.37	94.69	66.50	92.20	68.60
1.00	2.88	9.09	7.70	8.50	7.70
1.50	1.69	2.32	2.10	2.20	2.10

TABLE VIII
Performance comparison of the SDS EWMA-AM, the Shewhart, the DSVSI and VSS $\overline{X}$ charts with $\lambda = 0.05$ and $n_0 = 4$ under N(δ , 1).

$\delta_1 \cdot \sqrt{n_0}$	SDDS EWMA-AM chart $n_1 = 3, n_2 = 6$	Shewhart $\overline{X}$ chart	DSVSI $\overline{X}$ chart	VSS $\overline{X}$ chart
	AATS	AATS	AATS	AATS
0.00	374.04	370.37	370.37	370.37
0.25	82.03	280.60	170.30	277.20
0.50	28.34	154.70	52.63	135.50
0.75	13.99	80.72	19.13	53.12
1.00	8.68	43.39	8.45	19.92
1.25	5.72	24.46	4.49	8.60
1.50	4.15	14.47	2.79	4.69
2.00	2.45	5.80	1.51	2.35
3.00	1.17	1.50	0.92	1.24

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TABLE VIII
Performance comparison of the SDS EWMA-AM, the SS EWMA-AM and the SL charts with $\text{ARL}_0 \approx 500, \lambda = 0.05$ and $n_0 = 5$ under DE(δ, $1/\sqrt{2}$).

δ	SDS EWMA-AM chart $n_1 = 4, n_2 = 6$, E(N)=4.66	SL chart m=30	SL chart m=50	SS EWMA-AM chart
	ARL	ARL	ARL	ARL
0.00	507.85	487.67	495.25	494.85
0.25	27.23	394.53	330.07	18.34
0.50	8.37	207.50	114.91	9.40
1.00	2.88	71.90	28.81	7.03
1.50	1.69	25.56	7.93	5.92

Table VIIII
Performance comparison of the SDS EWMA-AM, the SS EWMA-AM and the SL charts with $\text{ARL}_0 \approx 500, \lambda = 0.05$ and $n_0 = 5$ under DE(δ, $1/\sqrt{2}$).

δ	SDS EWMA-AM chart $n_1 = 4, n_2 = 6$, E(N)=4.66	SL chart m=30	SL chart m=50	SS EWMA-AM chart
	ARL	ARL	ARL	ARL
0.00	507.85	487.67	495.25	494.85
0.25	27.23	394.53	330.07	18.34
0.50	8.37	207.50	114.91	9.40
1.00	2.88	71.90	28.81	7.03
1.50	1.69	25.56	7.93	5.92

TABLE X
Performance comparison of the SDS EWMA-AM, the SL and the SC charts with $\text{ARL}_0 \approx 500$, $\lambda = 0.05$ and $n_0 = 5$ under DE($\sqrt{2}\delta$, 1).

$\sqrt{2}\delta$	SDS EWMA-AM chart $n_1 = 4, n_2 = 6$, E(N)=4.66	SL chart m=50	SL chart m=100	SC chart m=50	SC chart m=100
	ARL	ARL	ARL	ARL	ARL
0.00	507.85	493.2	508.3	492.7	509.6
0.25	22.13	403.1	366.9	419.0	381.6
0.50	8.09	235.2	159.2	240.4	191.0
1.00	3.37	36.1	19.9	41.4	26.5
1.50	2.22	5.93	4.1	7.2	4.8
2.00	1.69	2.0	1.7	2.1	1.8

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TABLE XI
Performance comparison of the SDS EWMA-AM, the NLE and the CEW charts with $\mathrm{ARL}_0 \approx 370$, $\lambda = 0.05$ and $n_0 = 10$ under $(\chi_3^2 - 3)/\sqrt{6} + \delta$

δ	SDS EWMA-AM chart $n_1=8, n_2=16$, E(N)=9.77	NLE chart m=20000	CEW chart m=20000
	ARL	ARL	ARL
0.00	373.64	373.00	120.00
0.25	11.23	54.40	65.00
0.50	2.97	30.70	29.40
0.75	1.39	22.70	17.10
1.00	1.03	17.90	11.80
1.50	1.00	12.30	7.03
2.00	1.00	8.91	4.65
3.00	1.00	5.11	2.32

TABLE 12
Performance comparison of the SDS EWMA-AM and the SS EWMA-AM chart with $\mathrm{ARL}_0 \approx 370$, $\lambda = 0.05$ and $n_0 = 10$ under $\mathrm{Unif}(-\sqrt{3}, \sqrt{3}) + \delta$ and $\mathrm{Exp}(1) + \delta$.

distribution	$\mathrm{Unif}(-\sqrt{3}, \sqrt{3}) + \delta$		$\mathrm{Exp}(1) + \delta$	
δ	SDS EWMA-AM chart $n_1=8, n_2=16$, E(N)=9.81	SS WMA-AM chart	SDS EWMA-AM chart $n_1=8, n_2=16$, E(N)=9.81	SS EWMA -AM chart
	ARL	ARL	ARL	ARL
0.00	371.36	371.03	368.09	369.56
0.25	87.31	105.80	54.90	69.63
0.50	31.19	39.96	16.61	24.00
0.75	21.58	28.97	11.20	17.17
1.00	15.96	22.64	7.96	13.24
1.50	9.77	15.22	4.43	8.79
2.00	6.64	11.59	2.84	6.43
3.00	4.80	9.38	1.96	4.98

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TABLE 13
The in-control service times from 10 counters in a bank branch

t	Stage 1					Stage 2							
	$X_{1,t}$	$X_{2,t}$	$X_{3,t}$	$X_{4,t}$	$M_{1,t}$	$X_{5,t}$	$X_{6,t}$	$X_{7,t}$	$X_{8,t}$	$X_{9,t}$	$X_{10,t}$	$M_{2,t}$	$M_{3,t}$
1	0.88	0.78	5.06	5.45	0	2.93	6.11	11.59	1.20	0.89	3.21	2	2
2	3.82	13.40	5.16	3.20	1	32.27	3.68	3.14	1.58	2.72	7.71	2	3
3	1.40	3.89	10.88	30.85	2	0.54	8.40	5.10	2.63	9.17	3.94	2	4
4	16.80	8.77	8.36	3.55	3	7.76	1.81	1.11	5.91	8.26	7.19	4	7
5	0.24	9.57	0.66	1.15	1	2.34	0.57	8.94	5.54	11.69	6.58	3	4
6	4.21	8.73	11.44	2.89	2	19.49	1.20	8.01	6.19	7.48	0.07	4	6
7	15.08	7.43	4.31	6.14	3	10.37	2.33	1.97	1.08	4.27	14.08	2	5
8	13.89	0.30	3.21	11.32	2	9.90	4.39	10.50	1.70	10.74	1.46	3	5
9	0.03	12.76	2.41	7.41	2	1.67	3.70	4.31	2.45	3.57	3.33	0	2
10	12.89	17.96	2.78	3.21	2	1.12	12.61	4.23	6.18	2.33	6.92	3	5
11	7.71	1.05	1.11	0.22	1	3.53	0.81	0.41	3.73	0.08	2.55	0	1
12	5.81	6.29	3.46	2.66	2	4.02	10.95	1.59	5.58	0.55	4.10	1	3
13	2.89	1.61	1.30	2.58	0	18.65	10.77	18.23	3.13	3.38	6.34	4	4
14	1.36	1.92	0.12	11.08	1	8.85	3.99	4.32	1.71	1.77	1.94	1	2
15	21.52	0.63	8.54	3.37	2	6.94	3.44	3.37	6.37	1.28	12.83	3	5

TABLE 14
The plotting statistics of the SDS EWMA-AM chart with $ARL_0 \approx 370$ for t=1, 2, ⋯, 15

t	Stage 1			
	$M_{1,t}$	$EWMA_{M_{1,t}}$	$Z_{AM_{1,t}}$	Detect result
1	0	1.520	-1.633	WR
2	1	1.494	-1.569	IC
3	2	1.519	-0.999	IC
4	3	1.593	-0.073	IC
5	1	1.564	-0.366	IC
6	2	1.585	-0.136	IC
7	3	1.656	0.501	IC
8	2	1.673	0.625	IC
9	2	1.690	0.737	IC
10	2	1.705	0.838	IC
11	1	1.670	0.542	IC
12	2	1.686	0.655	IC
13	0	1.602	0.016	IC
14	1	1.572	-0.204	IC
15	2	1.593	-0.047	IC

TABLE 15
The new service times on the SDS EWMA-AM chart with $ARL_0 \approx 370$

Stage	t	16	17	18	19	20	21	22	23	24	25
Stage 1	$X_{1,t}$	3.54	0.86	1.45	1.37	3.00	1.59	5.01	4.96	1.08	4.56
	$X_{2,t}$	0.01	1.61	0.19	0.14	2.46	3.88	1.85	0.55	0.65	0.44
	$X_{3,t}$	1.33	1.15	4.18	1.54	0.06	0.39	3.10	1.43	0.91	5.61
	$X_{4,t}$	7.27	0.96	0.18	1.58	1.80	0.54	1.00	4.12	0.88	2.79
	$M_{1,t}$	1	0	0	0	0	0	0	0	0	0
	$EWMA_{M_{1,t}}$	1.564	1.486	1.411	1.341	1.274	1.210	1.150	1.092	1.037	0.986
	$Z_{AM_{1,t}}$	-0.257	-0.803	-1.311	-1.784	-2.228	-2.644	-3.034	-3.402	-3.749	-4.076
	Detect result	IC	IC	IC	WR	WR	WR	OC	OC	OC	OC
Stage 2	$X_{5,t}$	-	-	-	0.45	3.25	1.58	-	-	-	-
	$X_{6,t}$	-	-	-	6.01	2.13	1.70	-	-	-	-
	$X_{7,t}$	-	-	-	4.59	2.22	0.68	-	-	-	-
	$X_{8,t}$	-	-	-	1.74	1.37	1.25	-	-	-	-
	$X_{9,t}$	-	-	-	3.92	2.13	6.83	-	-	-	-
	$X_{10,t}$	-	-	-	4.82	0.25	0.31	-	-	-	-
	$M_{2,t}$	-	-	-	1	0	1	-	-	-	-
	$M_{3,t}$	-	-	-	1	0	1	-	-	-	-
	$EWMA_{M_{3,t}}$	-	-	-	3.755	3.567	3.439	-	-	-	-
	$Z_{AM_{3,t}}$	-	-	-	-2.293	-3.389	-3.899	-	-	-	-
	Detect result	-	-	-	IC	OC	OC	-	-	-	-

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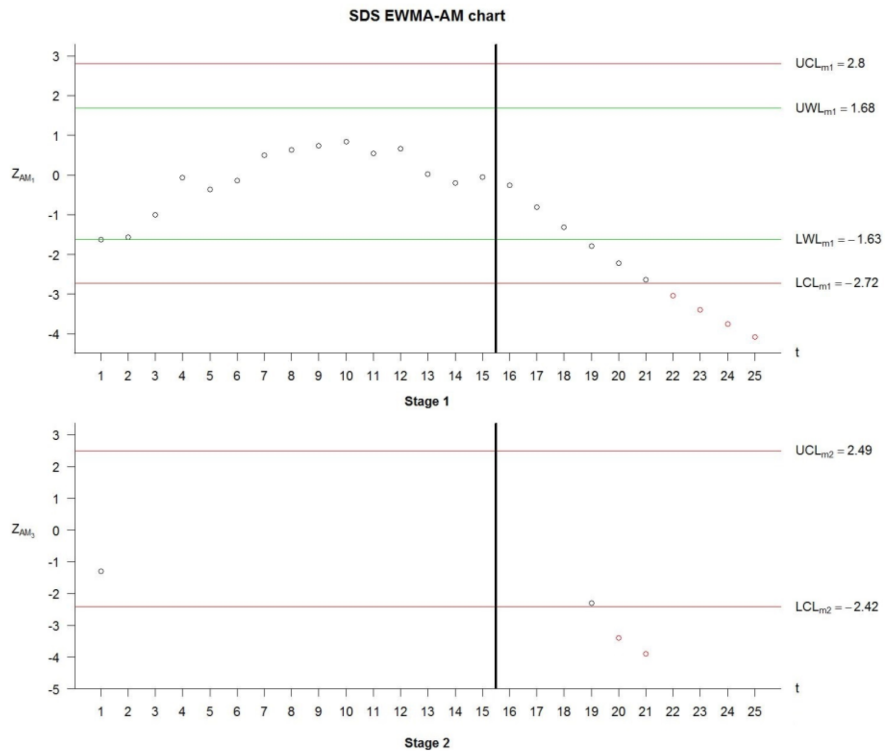

Figure 1: The SDS EWMA-AM chart for t=1, 2, ⋯, 25.