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A double sampling scheme for process variability monitoring

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Abstract

Control charts are effective tools for signal detection in both manufacturing processes and service processes. Much of the data in service industries come from processes exhibiting nonnormal or unknown distributions. The commonly used Shewhart variable control charts, which depend heavily on the normality assumption, are not appropriately used here. This paper thus proposes a standardized asymmetric exponentially weighted moving average (EWMA) variance chart with a double sampling scheme (SDS EWMA-AV chart) for monitoring process variability. We further explore the sampling properties of the new monitoring statistics and calculate the average run lengths when using the proposed SDS EWMA-AV chart. The performance of the SDS EWMA-AV chart and that of the single sampling EWMA variance (SS EWMA-V) chart are then compared, with the former showing superior out-of-control detection performance versus the latter. We also compare the out-of-control variance detection performance of the proposed chart with those of nonparametric variance charts, the nonparametric Mood variance chart (NP-M chart) with runs rules, and the nonparametric likelihood ratio-based distribution-free EWMA (NLE) chart and the combination of traditional EWMA (CEW) and the SS EWMA-V control charts by considering cases in which the critical quality characteristic presents normal, double exponential, uniform, chi-square, and exponential distributions. Comparison results show that the proposed chart always outperforms the NP-M with runs rules, the NLE, CEW, and the SS EWMA-V control charts. We hence recommend employing the SDS EWMA-AV chart. Finally, a numerical example of a service system for a bank branch in Taiwan is used to illustrate the application of the proposed variability control chart.

KEYWORDS
average run length, binomial distribution, control chart, free distribution, process variability

1 | INTRODUCTION

Control charts are commonly used tools for detecting out-of-control process dispersion to improve the quality of manufacturing processes and service processes. In the past few years, more and more statistical process control techniques have been applied to the service industry, with control charts also becoming an effective tool in improving service quality; see MacCarthy and Wasusri,[1] Tsung et al,[2] and Ning et al.[3] The commonly used Shewhart variables control charts, which depend on a normality assumption, are not suitable for monitoring most service process data when variables exhibit nonnormal or unknown distributions. Hence, some research has been conducted to deal with process monitoring of variables having nonnormal or distribution-free data. Some related research has also looked at nonparametric approaches to deal with process location monitoring; see, for example, Ferrell,[4] Bakir and Reynolds,[5] Amin et al,[6] Altukife,[7,8] Bakir,[9,10] Chakraborti and Eryilmaz,[11] Chakraborti and Graham[12], Chakraborti et al,[13] Li et al,[14] Zou and Tsung,[15] Graham et al,[16,17] and Grahama et al.[18] However, little research has been done to investigate process variability monitoring; see, for example, Das and Bhattacharya,[19] Jones-Farmer and Champ,[20] and Zombade and Ghute.[21]

For practitioners who want to easily implement the scheme of control charts with distribution-free statistics, Yang et al[22] proposed a new sign chart for variables data to monitor the deviation of process measurement between target without the assumption of a normal process distribution or a distribution of known form. Yang[23] set up an improved mean chart for variables data to monitor the process mean for distribution-free quality variables. Their approach is quite easy to use, and even easier than some of the above published nonparametric approaches. As Yang et al[22] and Yang[23] did not address the variance chart, Yang and Arnold[24] subsequently offered an asymmetric exponential weighted moving average (EWMA) variance chart, and Yang and Arnold[25] initiated a new arcsine-transformed symmetric EWMA variance chart for variables data exhibiting free distribution so as to quickly monitor shifts of the process variance. Their approaches are also quite easy to use and illustrated better detection ability than the existing variance or standard deviation (SD) charts, the nonparametric Mood variance chart (NP-M chart) with runs rules developed by Zombade and Ghute,[21] and the nonparametric likelihood ratio-based distribution-free EWMA (NLE) chart and the combination of traditional EWMA (CEW) proposed by Chou and Tsung[15] by considering cases in which the critical quality characteristic has a normal, a double exponential, and a uniform distribution. Furthermore, for variables data with a double exponential, a uniform or an exponential distribution, their proposed variance chart always performs better than the improved Shewhart R charts (IRCs) developed by Khoo and Lim.[26] However, their proposed variance chart performs worse than those of the IRCs developed by Khoo and Lim[26] and Zhang[27] for a small sample size under variables data with a normal distribution.

Daudin[28] first proposed a type of process monitoring scheme called doubles sampling (DS) to improve the detection ability of the Shewhart $\overline{X}$ control chart for the small shifts in the in-control process mean. He and Grigoryan[29] extended Daudin DS $\overline{X}$ control chart to DS $\overline{X}-S$ control charts to simultaneously detect small shifts in process mean and variance. They showed that the former chart has better detection ability compared to traditional $\overline{X}-S$ control charts. However, the DS scheme has not been discussed in nonparametric location and/or dispersion control charts. Herein, we shall develop the standardized DS asymmetric EWMA variance (SDS EWMA-AV) chart and investigate its process detection performance for small-to-medium shifts in process variability. The paper is organized as follows. Section 2 discusses the construction of a newly proposed SDS EWMA-AV chart and analyzes its out-of-control detection performance. Section 3 compares the detection performances among the proposed SDS EWMA-AV chart and some existing variance charts. Section 4 gives a numerical example. Section 5 summarizes the findings and provides a recommendation.

2 | THE SDS EWMA-AV CHART

Daudin[28] presented that the double sampling scheme is a counterpart to double sampling plans. This procedure offers better statistical efficiency, in average run length (ARL), than the single sampling scheme without the increased sampling. Alternatively, the procedure can be used to reduce sampling without reducing statistical efficiency. The double sampling procedure proposed allows one to assume that two successive samples can be taken without any intervening time, therefore coming from the same probability distribution.

2.1 | The control limits of the SDS EWMA-AV chart

Following Yang and Arnold,[25] to monitor process variance, a random sample of size n, $X_1, X_2, \dots, X_n$, is taken from the process X. Assume that the sample size n is even numbered for convenience (if not, delete one observation). Next, define

$$\begin{aligned} &Y_1 = (X_2 - X_1)^2/2, \\ &Y_2 = (X_4 - X_3)^2/2, .., Y_{\frac{n}{2}} = (X_n - X_{n-1})^2/2 \end{aligned} \tag{1}$$

$$E(Y_j) = \sigma^2, \; j = 1, 2, \dots, n/2. \tag{2}$$

and

$$I_j = \begin{cases} 1, & \text{if } Y_j > \sigma^2, \\ 0, & \text{otherwise,} \end{cases} \quad \text{for } j = 1, 2, \cdots, 0.5n. \tag{3}$$

Let V be the total number of $Y_j > \sigma^2$, and then, $V = \sum_{j=1}^{0.5n} I_j$ has a binomial distribution with parameters ($0.5n$, p_0), V ~ B($0.5n$, p_0), for an in-control process where $p_0 = P(Y_j > \sigma^2)$. The value of p_0 depends on the distribution of the X_i's. For example, if the X_i's are normally distributed, then $p_0 = P(Y_j > \sigma^2) = 0.3174$. If the distribution of $X_n - X_{n-1}$ is unimodal, then the quantity p_0 is bounded above by 4/9 using the Tchebychev inequality. The value of p_0 can be arbitrarily small, but it usually ranges from 0.25 to 0.50. Note that monitoring the shifts in process variance using V statistic is equivalent to monitoring the changes in process proportion, p_0.

We propose the following procedure to construct the SDS EWMA-AV chart.

First, take a sample of size n_1, compute V_1 statistic, where V_1 follows a binomial distribution with parameters $(0.5n_1,\ p_0)$, and the sample statistic $EWMA_{V_{1,t}}$ at time t, where $EWMA_{V_{1,t}} = \lambda V_{1,t} + (1-\lambda)EWMA_{V_{1,t-1}}$, $t = 1, 2, \ldots, \infty$. Let μ be the in-control mean and σ be the in-control SD of a critical process quality variable. Hence, the mean and variance of the statistic $EWMA_{V_{1,t}}$ are

$$E\left(EWMA_{V_{1,t}}\right) = 0.5n_1p_0, \tag{4}$$

and

$$Var\left(EWMA_{V_{1,t}}\right) = \frac{\lambda\left[1-(1-\lambda)^{2t}\right]}{2-\lambda}\left(0.5n_1p_0(1-p_0)\right). \tag{5}$$

At the first stage, we construct a DS EWMA-AV chart with upper control limit (UCL), upper warning limit (UWL), center line (CL), lower warning limit (LWL), and lower control limit (LCL) as follows.

$$\begin{aligned}
\mathrm{UCL_{v1}} &= 0.5n_1p_0 + L_{v1}\sqrt{\frac{\lambda\left[1-(1-\lambda)^{2t}\right]}{2-\lambda}0.5n_1p_0(1-p_0)},\\
\mathrm{UWL_{v1}} &= 0.5n_1p_0 + W_{v1}\sqrt{\frac{\lambda\left[1-(1-\lambda)^{2t}\right]}{2-\lambda}0.5n_1p_0(1-p_0)},\\
\mathrm{CL_{v1}} &= 0.5n_1p_0,\\
\mathrm{LWL_{v1}} &= 0.5n_1p_0 - W_{v2}\sqrt{\frac{\lambda\left[1-(1-\lambda)^{2t}\right]}{2-\lambda}0.5n_1p_0(1-p_0)},\\
\mathrm{LCL_{v1}} &= 0.5n_1p_0 - L_{v2}\sqrt{\frac{\lambda\left[1-(1-\lambda)^{2t}\right]}{2-\lambda}0.5n_1p_0(1-p_0)},
\end{aligned} \tag{6}$$

where L_{v1}, L_{v2}, W_{v1}, and W_{v2} are appropriately chosen coefficients to reach a preset in-control ARL, $0 < W_{v1} \le L_{v1}$, and $0 < W_{v2} \le L_{v2}$.

To easily apply the proposed control chart, we standardize the statistic $EWMA_{V_{1,t}}$. Hence the new plotting statistic, $Z_{AV_{1,t}}$, at stage 1 is defined as follows:

$$Z_{AV_{1,t}} = \frac{EWMA_{V_{1,t}} - 0.5n_1p_0}{\sqrt{\frac{\lambda\left[1-(1-\lambda)^{2t}\right]}{2-\lambda}0.5n_1p_0(1-p_0)}}. \tag{7}$$

Thus, the mean and variance of the new standardized statistic, $Z_{AV_{1,t}}$, are zero and one, respectively.

We let the new standardized DS EWMA-AV chart be the SDS EWMA-AV chart with the following control limits.

$$\begin{aligned}
\mathrm{UCL_{v1}} &= L_{v1}\\
\mathrm{UWL_{v1}} &= W_{v1},\\
\mathrm{CL_{v1}} &= 0,\\
\mathrm{LWL_{v1}} &= -W_{v2},\\
\mathrm{LCL_{v1}} &= -L_{v2}.
\end{aligned} \tag{8}$$

The region between UWL and LWL is called the central region (CR), and the region between UCL and UWL or LWL and LCL is called the warning region (WR). There are 3 following possibilities at this first stage.

1. If the monitoring statistic, $Z_{AV_{1,t}}$, falls in CR, then conclude the process variance is in control.
2. If the monitoring statistic, $Z_{AV_{1,t}}$, falls outside UCL or LCL, then conclude the process variance is out of control.
3. If the monitoring statistic, $Z_{AV_{1,t}}$, falls in WR, then go to the second stage and take a second sample size, n_2.

At the second stage one takes a sample of size n_2, which is always larger than n_1, and computes the statistic, V_2, where V_2 follows a binomial distribution with parameters $(0.5n_2,\ p_0)$, V_2~B$(0.5n_2,\ p_0)$. Decisions at the second stage are based on the combined statistic $V_3 = V_1 + V_2$, where V_3 ~ B$(\ 0.5(n_1+n_2),\ p_0)$. Hence the second-stage EWMA statistic, $EWMA_{V_{3,t}}$, is

$$EWMA_{V_{3,t}} = \lambda V_{3,t} + (1-\lambda)EWMA_{V_{3,t-1}}. \tag{9}$$

The mean and variance of the statistic $EWMA_{V_{3,t}}$, $E(EWMA_{V_{3,t}})$, and $Var(EWMA_{V_{3,t}})$, are $0.5(n_1+n_2)p_0$ and $Var(\ EWMA_{V_{3,t}}\ ) = \frac{\lambda\left[1-(1-\lambda)^{2t}\right]}{2-\lambda}0.5(n_1+n_2)p_0(1-p_0)$, respectively.

As in the first stage, we standardize the statistic $EWMA_{V_{3,t}}$. Hence, the new plotting statistic, $Z_{AV_{3,t}}$, at the second stage is defined as follows:

$$Z_{AV_{3,t}} = \frac{EWMA_{V_{3,t}} - 0.5(n_1+n_2)p_0}{\sqrt{\frac{\lambda\left[1-(1-\lambda)^{2t}\right]}{2-\lambda}0.5(n_1+n_2)p_0(1-p_0)}}. \tag{10}$$

In stage 2, the control limits of the SDS EWMA-AV chart are as follows.

$$\begin{aligned}
\mathrm{UCL_{v2}} &= L_{v3},\\
\mathrm{CL_{v2}} &= 0,\\
\mathrm{LCL_{v2}} &= -L_{v4}.
\end{aligned} \tag{11}$$

At this second stage, there are only two of the following possibilities.

1. If the monitoring statistic, $Z_{AV_{3,t}}$, falls between UCL and LCL, then conclude the process variance is in control.
2. If the monitoring statistic, $Z_{AV_{3,t}}$, falls outside UCL or LCL, then conclude the process variance is out of control.

The properties of the SDS EWMA-AV chart depend on the 8 parameters of Lv_1, Lv_2, Lv_3, Lv_4, Wv_1, Wv_2, n_1, and n_2. We may determine the values of the 8 parameters to reach a preset in-control ARL (ARL_0) with the constraint of average sample size is smaller than a single sample size, $\text{E(N)} \leq n_0$, where (E(N)) is the average sample size and n_0 is the single sample size.

We now adopt 3 combinations of n_1 and n_2, the small sample sizes ($n_1 = 4$, $n_2 = 6$), the moderate sample sizes ($n_1 = 6$, $n_2 = 12$), and the large sample sizes ($n_1 = 8$, $n_2 = 16$), and let $n_0 = 5, 8, 10$, $\lambda = 0.05$ and $p_0 = 0.1(0.1)0.4$. The parameters of the control limits (L_{v1}, L_{v2}, W_{v1}, W_{v2}, L_{v3}, L_{v4}) are determined to satisfy a preset $\text{ARL}_0 \approx 370$ through the grid search method, thus determine the control limits of the proposed chart. The procedure to determine the values of (L_{v1}, L_{v2}, W_{v1}, W_{v2}, L_{v3}, L_{v4}), given p_{v0}, n_1, n_2, n_0, λ, and $\text{ARL}_0 \approx 370$, is described as follows.

1. Let ARL_0 of the stage 1 control chart be 1280, where $L_{v1} < \infty$ and $L_{v2} \approx \infty$. We determine L_{v1} to satisfy a preset $\text{ARL}_0 \approx 1280$ using the grid search method. Given L_{v1} and letting $L_{v2} < \infty$, we search L_{v2} to satisfy a preset $\text{ARL}_0 \approx 740$. Consequently, k is the ratio of resulting L_{v1} and L_{v2}, $k = \frac{L_{v1}}{L_{v2}}$.
2. We determine W_{v1} and W_{v2} to satisfy $0 < W_{v1} \leq L_{v1}$, $0 < W_{v2} \leq L_{v2}$, $W_{v1}/W_{v2} = k$, and $E(n) \leq n_0$.
3. Given the resulting (L_{v1}, L_{v2}, W_{v1}, W_{v2}) and the specified upper bounds of L_{v3} and L_{v4} (UL_{v3} and UL_{v4}), for the stage 2 control chart, we determine L_{v3} and L_{v4} to satisfy $0 < L_{v3} < UL_{v3}$, $0 < L_{v4} < UL_{v4}$, $L_{v3}/L_{v4} = k$ and ARL_0 of the two-stage control charts$\approx$370.

Note that if ARL_0 of the two-stage control charts cannot approach 370, then (W_{v1}, W_{v2}, L_{v3}, L_{v4}) in steps 2 and 3 should be determined simultaneously to satisfy $\text{ARL}_0 \approx 740$ using the grid search method.

Table 1 illustrates the corresponding values of the 6 parameters for 12 combinations values of p_0, n_1, n_2, and n_0. It can be seen that when p_0 increases, L_{v1}, W_{v1}, and L_{v3} decrease, but L_{v2}, W_{v2}, and L_{v4} increase; when the sample size increases, L_{v2} and W_{v2} increase, but L_{v1} decreases; the in-control average sample size, E(N), is smaller than n_0.

2.2 | Performance measurement of the SDS EWMA-AV chart

To measure the out-of-control detection performance of the SDS EWMA-AV chart, we calculate the out-of-control ARL (ARL_1) by considering the out-of-control proportion $p_1 = 0.1(0.1)0.9$ and the in-control proportion $p_0 = 0.1(0.1)0.4$ for the 12 combination values of parameters in Table 1. The results are listed in Tables 2 and 3. The results look reasonable, because ARL_1 decreases when p_1 is far away from p_0, or when the sample size increases. Furthermore, compared to the single sampling EWMA-V (SS EWMA-V) chart with $n_0 = 10$ in Yang and Arnold,[24] we find that when $p_0 = 0.1$ the SDS EWMA-AV chart performs better than the SS EWMA-V chart under the 3 combinations of n_1 and n_2. When $p_0 = 0.4$ the SDS EWMA-AV chart with ($n_1 = 8$, $n_2 = 16$, E(N) = 9.88) and ($n_1 = 6$ and $n_2 = 12$, E(N) = 7.81) always performs better than the SS EWMA-V chart with $n_0 = 10$. However, when the SDS EWMA-AV chart has the combination of the small sample size ($n_1 = 4$, $n_2 = 6$, E(N) = 4.81), it performs better only when p_1 is far away from the in-control p_0, like $p_1 = 0.1, 0.6(0.1)0.9$. We note that the comparison seems not fair because of the SS EWMA-V chart with larger sample size. Furthermore, the SDS EWMA-AV chart performs better for larger n_1 and n_2. Hence, we conclude that the out-of-control detection performance of

TABLE 1 Parameters of the SDS EWMA-AV chart with $\text{ARL}_0 \approx 370$ and $\lambda = 0.05$ for various combinations of p_0, n_1, n_2, and n_0

No.	p_0	n_1	n_2	n_0	L_{v1}	L_{v2}	W_{v1}	W_{v2}	L_{v3}	L_{v4}	E(N)	ARL_0
1	0.1	4	6	5	3.21	2.18	1.93	1.31	3.05	2.07	4.82	368.54
2	0.1	6	12	8	3.10	2.35	1.74	1.32	2.86	2.17	7.76	370.13
3	0.1	8	16	10	3.10	2.44	1.89	1.49	2.59	2.04	9.92	374.54
4	0.2	4	6	5	3.01	2.47	1.81	1.48	2.65	2.17	4.77	370.04
5	0.2	6	12	8	2.96	2.56	1.66	1.43	2.64	2.29	7.87	368.38
6	0.2	8	16	10	2.95	2.57	1.80	1.57	2.49	2.17	9.86	370.65
7	0.3	4	6	5	2.89	2.64	1.73	1.58	2.47	2.25	4.77	368.54
8	0.3	6	12	8	2.89	2.67	1.62	1.50	2.50	2.31	7.74	372.82
9	0.3	8	16	10	2.88	2.70	1.76	1.65	2.36	2.21	9.80	372.68
10	0.4	4	6	5	2.80	2.68	1.68	1.61	2.49	2.39	4.81	371.28
11	0.4	6	12	8	2.80	2.70	1.57	1.51	2.54	2.45	7.81	370.17
12	0.4	8	16	10	2.80	2.72	1.71	1.66	2.37	2.31	9.88	368.29

TABLE 2 The ARL of the SDS EWMA-AV chart for $\lambda = 0.05$ and $p_0 = 0.1$

p_1	SDS EWMA-AV $n_1 = 4$ and $n_2 = 6$	SDS EWMA-AV $n_1 = 6$ and $n_2 = 12$	SDS EWMA-AV $n_1 = 8$ and $n_2 = 16$	SS EWMA-V $n_0 = 10$
	ARL			
0.1	368.54	370.13	374.54	375.3
0.2	20.52	13.20	11.06	16.1
0.3	7.32	4.50	3.73	7.1
0.4	3.99	2.53	2.17	4.7
0.5	2.77	1.77	1.54	3.5
0.6	2.11	1.43	1.24	2.9
0.7	1.70	1.24	1.10	2.4
0.8	1.40	1.11	1.03	2.1
0.9	1.20	1.03	1.00	2.0

TABLE 3 The ARL of the SDS EWMA-AV chart for $\lambda = 0.05$ and $p_0 = 0.4$

p_1	SDS EWMA-AV $n_1 = 4$ and $n_2 = 6$	SDS EWMA-AV $n_1 = 6$ and $n_2 = 12$	SDS EWMA-AV $n_1 = 8$ and $n_2 = 16$	SS EWMA-V $n_0 = 10$
	ARL			
0.1	5.80	3.59	3.57	7.2
0.2	12.64	7.59	6.88	11.8
0.3	40.63	25.86	22.21	30.2
0.4	371.28	370.17	368.29	370.1
0.5	38.21	26.81	21.82	29.8
0.6	11.93	8.40	6.63	11.9
0.7	5.69	4.08	3.23	7.4
0.8	3.12	2.41	1.97	5.4
0.9	1.84	1.50	1.43	4.4

the SDS EWMA-AV chart is always better than those of the SS EWMA-V charts for medium and large sample of sizes.

3 | PERFORMANCE COMPARISON

In general, EWMA type control chart performs better than Shewhart-type chart for small shifts. The performance comparison study mixes two types of charts (EWMA and Shewhart) and two types of Shewhart charts [attribute like NP-M (Zombade and Ghute[21]) and variables].

We compare the out-of-control variance detection performance of our proposed SDS EWMA-AV chart with those of some existing nonparametric variance/dispersion control charts, like the SS EWMA-V, NLE, CEW, NP-M, and IRC charts in (Yang and Arnold[25]), by considering that the critical quality characteristic has a normal distribution, a double exponential distribution, a uniform distribution, a chi-square distribution, and an exponential distribution, respectively.

First, we let the critical quality characteristic have a normal distribution with mean 0 and variance $\sigma^2 = \delta^2$, N(0, δ^2). The in-control variance is 1, $\sigma = \delta = 1$. The out-of-control variance ($\delta > 1$) detection performance of our proposed SDS EWMA-AV chart with $n_1 = 8, n_2 = 16, \lambda = 0.05$, E(N) = 9.87 and a preset ARL_0 = 370 is compared with those of the NLE and the CEW charts (Zou and Tsung[15]) with historical observations, m = 20000, the NP-M chart, the SS EWMA-V chart (Yang and Arnold[24]), and the IRC charts (Khoo and Lim[26] and Zhang[27]) with a single sample size $n_0 = 10$. Table 4 lists the ARL_0 and ARL_1s under various scale shift values ($\delta \geq 1$) and their corresponding increased p_0 (or $p_1 - p_0$). In Table 4, we find that the proposed SDS EWMA-AV chart has a smaller average sample size under the in-control process and shows superior out-of-control detection performance than the NLE, CEW, NP-M, SS EWMA-V, and Khoo and Lim IRC charts no matter whether the shift scale is small, medium, or large. However, the proposed SDS EWMA-AV chart only performs better than the IRC chart in Zhang[27] for a very small shift scale in SD. To investigate the variance detection performance of the proposed SDS EWMA-AV chart with smaller sample sizes and larger ARL_0, we compare the SDS EWMA-AV chart with $n_1 = 4, n_2 = 6, \lambda = 0.05$, and E(N) = 4.7 and a preset

TABLE 4 Performance comparison among the SDS EWMA-AV, NP-M, NLE, CEW, SS EWMA-V, and IRC charts with $ARL_0 \approx 370$, $\lambda = 0.05$, $n_1 = 8$, $n_2 = 16$, and $n_0 = 10$ under N(0, δ^2).

		SDS EWMA-AV Chart	NLE Chart, m = 20000	CEW Chart, m = 20000	NP-M Chart with Two-of-two Rule	SS EWMA-V Chart	Zhang IRC Chart	Khoo and Lim IRC Chart
p_1-p_0	$\sigma = \delta$	ARL						
0.	1.0	370.65	370.00	370.00	251.17	376.13	372.70	374.06
0.087	1.2	25.76	57.00	54.60	105.35	34.80	52.27	62.24
0.157	1.4	9.34	23.00	21.10	42.36	15.47	11.53	15.27
0.215	1.6	5.26	13.60	12.40	20.37	10.60	4.63	6.26
0.262	1.8	3.72	9.47	8.64	12.53	8.50	2.62	3.60
0.300	2.0	2.86	7.12	6.71	8.57	7.23	1.84	2.49
0.422	3.0	1.56	3.30	3.28	3.65	5.03	1.07	1.25

ARL_0 = 500 versus those of the SL charts (Mukherjee and Chakraborti[30]) and the SC chart (Chowdhury et al[31]) with different historical observations, m = 50, 100, under a single sample size $n_0 = 5$. Table 5 lists ARL_0 and ARL_1s under various scale shift values ($\delta \geq 1$) and their corresponding increased p_0 (or $p_1 - p_0$). In Table 5, we find that the proposed SDS EWMA-AV chart exhibits superior out-of-control detection performance over the SC and SL charts.

Second, we let the critical quality characteristic respectively have a double exponential distribution DE(0, $\delta/\sqrt{2}$) with variance $\sigma^2 = \delta^2$ and DE(0, δ) with variance $\sigma^2/2 = \delta^2$, $\delta \geq 1$. The in-control variance is 1, $\sigma = \delta = 1$, for DE(0, $\delta/\sqrt{2}$), and 2, $\sigma/\sqrt{2} = \delta = 1$, for DE(0, δ), respectively. The out-of-control variance detection performance of our proposed SDS EWMA-AV chart with $n_1 = 8$, $n_2 = 16$, $\lambda = 0.05$, and E(N) = 9.85, and a preset ARL_0 = 370 is compared with those of the NP-M chart, the SS EWMA-V chart, and the IRC chart in Khoo and Lim[26] with a single sample size $n_0 = 10$. Table 6 illustrates their ARLs for various shift scales of the process SD ($\delta \geq 1$) and the corresponding

TABLE 5 Performance comparison of the SDS EWMA-AV chart, SL charts, and SC charts with $ARL_0 \approx 500$, $\lambda = 0.05$, $n_1 = 4$, $n_2 = 6$, and $n_0 = 5$ under N(0, δ^2).

		SDDS EWMA-AV Chart	SL Chart, m = 50	SL Chart, m = 100	SC Chart, m = 50	SC Chart, m = 100
p_1-p_0	$\sigma = \delta$	ARL				
0.087	1.00	504.90	499.62	513.0	497.3	509.4
0.104	1.25	36.93	106.21	102.9	71.1	74.5
0.186	1.50	13.94	36.82	37.5	22.8	24.3
0.250	1.75	8.41	18.48	19.1	10.9	11.7
0.300	2.00	5.99	11.26	11.5	6.6	7.1

TABLE 6 Performance comparison among the SDS EWMA-AV, NP-M, SS EWMA-V, and Khoo and Lim IRC charts with $ARL_0 \approx 370$, $\lambda = 0.05$, $n_1 = 8$, $n_2 = 16$, and $n_0 = 10$ under DE(0, $\delta/\sqrt{2}$)

		SDS EWMA-AV Chart	NP-M Chart	SS EWMA-V Chart	Khoo and Lim IRC Chart
p_1-p_0	$\sigma = \delta$	ARL			
0.	1.0	372.50	253.42	370.51	370.37
0.077	1.2	29.20	136.49	39.61	132.45
0.139	1.4	10.34	67.19	16.90	46.20
0.196	1.6	5.93	37.02	11.34	20.64
0.241	1.8	4.05	23.24	8.84	11.40
0.281	2.0	3.14	15.74	7.42	7.22
0.316	2.2	2.62	11.85	6.57	5.10
0.345	2.4	2.23	9.23	5.95	3.86
0.371	2.6	1.99	7.62	5.53	3.10
0.393	2.8	1.80	6.41	5.20	2.60
0.414	3.0	1.71	5.60	4.92	2.26

TABLE 7 Performance comparison of the SDS EWMA-AV chart, the SL charts, and the SC charts with $\mathrm{ARL}_0 \approx 500$, $\lambda = 0.05$, $n_1 = 4$, $n_2 = 6$, and $n_0 = 5$ under DE(0, δ)

	$\sigma/\sqrt{2}$	SDS EWMA-AV Chart	SL Chart, m = 50	SL Chart, m = 100	SC Chart, m = 50	SC Chart, m = 100
p_1-p_0	$=\delta$			ARL		
0.	1.00	503.01	493.2	508.3	492.7	509.6
0.093	1.25	43.96	156.8	153.2	118.0	124.5
0.141	1.50	16.33	65.9	66.8	43.3	47.8
0.219	1.75	9.81	35.6	36.4	22.8	24.4
0.281	2.00	6.96	22.1	22.9	13.8	14.5

TABLE 8 Performance comparison of the SDS EWMA-AV, the NP-M, the SS EWMA-V, and the Khoo and Lim IRC charts with $\mathrm{ARL}_0 \approx 370$, $\lambda = 0.05$, and $n_1 = 8$, $n_2 = 16$, $n_0 = 10$ under δ•Unif$\left(-\sqrt{3}, \sqrt{3}\right)$.

		SDS EWMA-AV Chart	NP-M Chart	SS EWMA-V Chart	Khoo and Lim IRC
p_1-p_0	$\sigma = \delta$		ARL		
0.	1.0	369.57	255.79	373.68	366.43
0.090	1.2	26.59	54.28	36.88	26.37
0.154	1.4	10.75	17.30	16.99	24.73
0.208	1.6	6.58	8.98	11.75	23.75
0.243	1.8	4.83	6.07	9.37	21.90
0.288	2.0	3.94	4.58	8.03	18.63
0.312	2.2	3.42	3.91	7.16	15.72
0.343	2.4	3.03	3.47	6.60	13.13
0.364	2.6	2.72	3.20	6.13	11.10
0.383	2.8	2.52	3.01	5.83	9.50
0.400	3.0	2.36	2.87	5.56	8.28

($p_1 - p_0$). In Table 6, we note that the proposed SDS EWMA-AV chart has superior out-of-control detection performance than those of the NP-M, the SS EWMA-V, and the IRC charts no matter whether the shift scale is small, medium, or large. For smaller sample sizes and larger ARL_0, the detection performance of the SDS EWMA-AV chart with $n_1 = 4$, $n_2 = 6$, $\lambda = 0.05$, and E(N) = 4.72 and a preset $ARL_0 = 500$ is compared with those of the SL charts and the SC charts with different historical observations, m=50, 100, under a single sample size $n_0 = 5$. Table 7 illustrates their ARLs for various shift scales of the process SD ($\delta \geq 1$) and the corresponding ($p_1 - p_0$). In Table 7, we find that the proposed

TABLE 9 Performance comparison of the SDS EWMA-AV chart, the NLE chart, and the CEW chart and with $\mathrm{ARL}_0 \approx 370$, $\lambda = 0.05$, $n_1 = 8$, $n_2 = 16$, and $n_0 = 10$ under δ•$\left(\chi_3^2 - 3\right)/\sqrt{6}$

		SDS EWMA-AV Chart	NLE Chart m=20000	CEW Chart m=20000
p_1-p_0	$\sigma = \delta$		ARL	
0.	1.0	368.90	374.00	118.00
0.064	1.2	32.85	11.00	48.40
0.087	1.4	11.60	5.69	25.80
0.184	1.6	6.52	4.06	16.50
0.229	1.8	4.46	3.35	11.70
0.368	2.0	3.45	2.92	9.03
0.402	3.0	1.82	2.05	3.99

TABLE 10 Performance comparison of the SDS EWMA-AV chart, the SS EWMA-V, and the Khoo and Lim IRC chart with $\mathrm{ARL}_0 \approx 370$, $\lambda = 0.05$, $n_1 = 8$, $n_2 = 16$, and $n_0 = 10$ under Exp(δ)

		SDS EWMA-AV Chart	SS EWMA-V Chart	Khoo and Lim IRC Chart
p_1-p_0	$\sigma = \delta$		ARL	
0.	1.0	371.01	375.30	370.37
0.065	1.2	32.85	47.62	243.90
0.120	1.4	11.60	19.88	140.85
0.171	1.6	6.52	12.89	91.74
0.213	1.8	4.46	9.89	76.92
0.250	2.0	3.45	8.17	49.02
0.282	2.2	2.79	7.12	36.90
0.312	2.4	2.41	6.41	35.09
0.338	2.6	2.19	5.87	25.64
0.360	2.8	1.94	5.51	21.05
0.381	3.0	1.82	5.20	18.98

TABLE 11 The in-control service times from the first 10 counters in a bank branch

	Stage 1										Stage 2									
t	$X_{1,t}$	$X_{2,t}$	$X_{3,t}$	$X_{4,t}$	$Y_{1,t}$	$Y_{2,t}$	$V_{1,t}$	$EWMA_{V_{1,t}}$	$Z_{AV_{1,t}}$	Which region?	$X_{5,t}$	$X_{6,t}$	$X_{7,t}$	$X_{8,t}$	$X_{9,t}$	$X_{10,t}$	$Y_{3,t}$	$Y_{4,t}$	$Y_{5,t}$	$V_{3,t}$
1	0.88	0.78	5.06	5.45	0.01	0.08	0	0.57	−0.93	CR	2.93	6.11	11.59	1.20	0.89	3.21	5.06	53.98	2.69	1
2	3.82	13.40	5.16	3.20	45.89	1.92	1	0.59	−0.19	CR	32.27	3.68	3.14	1.58	2.72	7.71	408.70	1.22	12.45	2
3	1.40	3.89	10.88	30.85	3.10	199.40	1	0.61	0.22	CR	0.54	8.40	5.1	2.63	9.17	3.94	30.89	3.05	13.68	2
4	16.80	8.77	8.36	3.55	32.24	11.57	1	0.63	0.52	CR	7.76	1.81	1.11	5.91	8.26	7.19	17.70	11.52	0.57	1
5	0.24	9.57	0.66	1.15	43.52	0.12	1	0.65	0.76	CR	2.34	0.57	8.94	5.54	11.69	6.58	1.57	5.78	13.06	1
6	4.21	8.73	11.44	2.89	10.22	36.55	1	0.67	0.96	CR	19.49	1.20	8.01	6.19	7.48	0.07	167.26	1.66	27.45	2
7	15.08	7.43	4.31	6.14	29.26	1.67	0	0.63	0.46	CR	10.37	2.33	1.97	1.08	4.27	14.08	32.32	0.37	48.12	2
8	13.89	0.30	3.21	11.32	92.34	32.89	2	0.70	1.32	CR	9.90	4.39	10.5	1.70	10.74	1.46	15.18	38.72	43.06	4
9	0.03	12.76	2.41	7.41	81.03	12.50	1	0.72	1.45	CR	1.67	3.70	4.31	2.45	3.57	3.33	2.06	1.73	0.03	1
10	12.89	17.96	2.78	3.21	12.85	0.09	0	0.68	0.98	CR	1.12	12.61	4.23	6.18	2.33	6.92	66.01	1.90	10.53	1
11	7.71	1.05	1.11	0.22	22.18	0.40	0	0.65	0.55	CR	3.53	0.81	0.41	3.73	0.08	2.55	3.70	5.51	3.05	0
12	5.81	6.29	3.46	2.66	0.12	0.32	0	0.62	0.17	CR	4.02	10.95	1.59	5.58	0.55	4.10	24.04	7.96	6.30	0
13	2.89	1.61	1.30	2.58	0.82	0.82	0	0.58	−0.18	CR	18.65	10.77	18.23	3.13	3.38	6.34	31.05	114.01	4.38	2
14	1.36	1.92	0.12	11.08	0.16	60.06	1	0.61	0.05	CR	8.85	3.99	4.32	1.71	1.77	1.94	11.81	3.41	0.01	1
15	21.52	0.63	8.54	3.37	218.20	13.36	1	0.63	0.27	CR	6.94	3.44	3.37	6.37	1.28	12.83	6.13	4.50	66.70	2

Abbreviation: CR, central region.

SDS EWMA-AV chart shows superior out-of-control detection performance than the SL and SC charts.

Third, we let the critical quality characteristic exhibit a uniform distribution, δ•Unif $\left(-\sqrt{3}, \sqrt{3}\right)$, with variance $\sigma^2=\delta^2$, $\delta \geq 1$. The in-control variance is 1, $\sigma=\delta=1$. The out-of-control variance detection performance of our proposed SDS EWMA-AV chart with $n_1=8, n_2=16, E(N)=9.72$, and $\lambda=0.05$ and a preset $\mathrm{ARL}_0 = 370$ is compared with those of the NP-M chart, the SS EWMA-V chart, and the IRC chart in Khoo and Lim[26] under a single sample size $n_0=10$. Table 8 illustrates the corresponding ARLs for various shift scales, $\delta \geq 1$, and their corresponding (p_1-p_0). We see that the proposed SDS EWMA-AV chart has superior out-of-control detection performance than those of the NP-M, the SS EWMA-V, and the IRC charts no matter whether the shift scale is small, medium, or large.

Fourth, we let the critical quality characteristic have a chi-square distribution, δ•$\left(\chi_3^2-3\right)/\sqrt{6}$, with variance $\sigma^2=\delta^2$, $\delta \geq 1$. The in-control variance is 1, $\sigma=\delta=1$. The out-of-control variance detection performance of our proposed SDS EWMA-AV chart with $n_1=8, n_2=16, \lambda=0.05$, and $\mathrm{E(N)}=9.85$ and a preset $ARL_0 = 370$ is compared with those of the NLE and CEW charts with historical observations, m=20, 000, (Zou and Tsung[15]) under a single sample size $n_0=10$. Table 9 presents the ARLs for various shift scales, $\delta \geq 1$, and their corresponding (p_1-p_0). We discover that the proposed SDS EWMA-AV chart has superior out-of-control detection performance than those of the NLE and CEW charts, no matter whether the shift scale is small, medium, or large.

Fifth and finally, we let the critical quality characteristic have an exponential distribution, Exp(δ), with variance $\sigma^2=\delta^2$, $\delta \geq 1$. The in-control variance is 1, $\sigma=\delta=1$. The out-of-control variance detection performance of our proposed SDS EWMA-AV chart with $n_1=8, n_2=16, \lambda=0.05$, and $\mathrm{E(N)}=9.74$ and a preset $ARL_0 = 370$ is compared with those of the SS EWMA-V chart and the IRC chart in Khoo and Lim[27] under a single sample size $n_0=10$ and $\delta \geq 1$. Table 10 shows the ARLs for various shift scales, $\delta \geq 1$, and their corresponding (p_1-p_0). We can state that the proposed SDS EWMA-AV chart has superior out-of-control detection performance than those of the SS EWMA-V chart, and the IRC chart, no matter the shift scale is small, medium, or large.

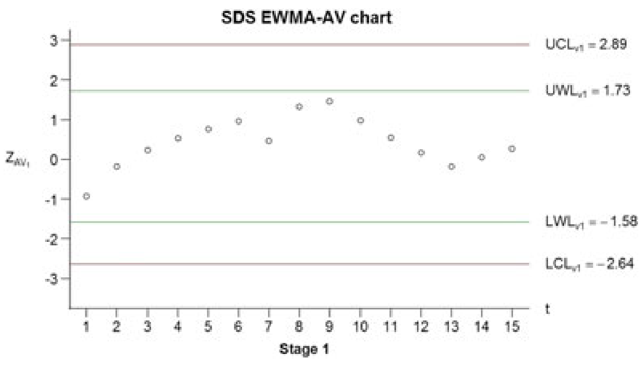

FIGURE 1 The SDS EWMA-AV chart for t = 1, 2, ⋯, 15 [Colour figure can be viewed at wileyonlinelibrary.com]

Note that for the critical quality characteristic having a uniform distribution, a chi-square distribution and an exponential distribution, we do not compare the detection performance of the SDS EWMA-AV chart with smaller sample sizes and larger ARL_0 with those of the SL, SC, NLE, CEW, NP-M, and IRC charts, because the latter did not exhibit these distributions and/or take the small sample size, $n_0=5$.

4 | AN EXAMPLE

An in-control nonnormal sampling service times $X_{1,t}-X_{10,t}$ (unit: min) measured from the first 10 counters every day for 15 days (t = 1, 2, ⋯, 15) in a bank branch (see Table 11) in Yang and Arnold[24] is used to illustrate the applications of the SDS EWMA-AV chart. To use the double sampling scheme to monitor the service time variability, we adopt $n_1=4$ ($X_{1,t}-X_{4,t}$) and $n_2=6$ ($X_{5,t}-X_{10,t}$) and let $n_0=5$.

To construct the SDS EWMA-AV chart, let the given in-control σ_0^2 be 30.097 and the in-control given proportion p_0 be 0.3. Thus, the statistics $V_{1,t}$ in stage 1 and $V_{3,t}$ in stage 2 are calculated (see Table 11).

The SDS EWMA-AV chart with a preset $\mathrm{ARL}_0=370$ and $\lambda=0.05$ is constructed as follows.

The control limits of the proposed chart in stage 1 are

$$\begin{aligned} \mathrm{UCL}_{v1} &= 2.89, \\ \mathrm{UWL}_{v1} &= 1.73, \\ \mathrm{LWL}_{v1} &= -1.58, \\ \mathrm{LCL}_{v1} &= -2.64. \end{aligned}$$

The control limits of the proposed chart in stage 2 are

$$\begin{aligned} \mathrm{UCL}_{v2} &= 2.47, \\ \mathrm{LCL}_{v2} &= -2.25. \end{aligned}$$

In stage 1, all 15 plotted statistics fall in the CR of the stage 1 chart (see Table 11 and Figure 1). Hence, we do not need to go to stage 2. This confirms the 15 samples adopted from the service process with in-control variance of service time.

For a new data set, (t = 16, 17, ⋯, 25), the new automatic service system of the bank branch, from the first 10 counters in a bank branch (see Table 12) in Yang and Arnold[24] is used. The new service times $X_{1,t}$–$X_{4,t}$ are measured from 4 counters at stage 1, and the monitoring statistic $Z_{AV_{1,t}}$ is calculated. We find that the sample statistics

TABLE 12 The plotting statistics of the SDS EWMA-AV chart with $ARL_0 \approx 370$ for the new samples of service times

Stage	t	16	17	18	19	20	21	22	23	24	25
Stage 1	$X_{1,t}$	3.54	0.86	1.45	1.37	3.00	1.59	5.01	4.96	1.08	4.56
	$X_{2,t}$	0.01	1.61	0.19	0.14	2.46	3.88	1.85	0.55	0.65	0.44
	$X_{3,t}$	1.33	1.15	4.18	1.54	0.06	0.39	3.10	1.43	0.91	5.61
	$X_{4,t}$	7.27	0.96	0.18	1.58	1.8	0.54	1.00	4.12	0.88	2.79
	$V_{1,t}$	0	0	0	0	0	0	0	0	0	0
	$EWMA_{V_{1,t}}$	0.593	0.564	0.536	0.509	0.483	0.459	0.436	0.414	0.394	0.374
	$Z_{AV_{1,t}}$	−0.071	−0.385	−0.677	−0.949	−1.204	−1.443	−1.668	−1.879	−2.079	−2.267
	Detect result	IC	IC	IC	IC	IC	IC	WR	WR	WR	WR
Stage 2	t	—	—	—	—	—	—	1	2	3	4
	$X_{5,t}$	—	—	—	—	—	—	0.09	4.06	2.02	1.73
	$X_{6,t}$	—	—	—	—	—	—	1.16	1.42	2.88	2.46
	$X_{7,t}$	—	—	—	—	—	—	2.69	1.43	1.76	0.53
	$X_{8,t}$	—	—	—	—	—	—	2.79	0.86	2.87	1.73
	$X_{9,t}$	—	—	—	—	—	—	1.84	0.67	1.97	7.02
	$X_{10,t}$	—	—	—	—	—	—	2.62	0.13	0.62	2.13
	$V_{3,t}$	—	—	—	—	—	—	0	0	0	0
	$EWMA_{V_{3,t}}$	—	—	—	—	—	—	1.425	1.354	1.286	1.222
	$Z_{AV_{3,t}}$	—	—	—	—	—	—	−1.464	−2.070	−2.533	−2.923
	Detect result	—	—	—	—	—	—	IC	IC	OC	OC

Abbreviations: IC, in control; OC, out of control.

$Z_{AV_{1,t}}$ for t = 16–21 are within CR of the stage 1 chart, while the sample statistics $Z_{AV_{1,t}}$, t = 22–25, are within WR of the stage 1 chart. Hence, the monitoring procedure goes to stage 2 for t = 22–25, and we calculate the monitoring statistic, $Z_{AV_{3,t}}$, using the second sample of service times $X_{5,t}$–$X_{10,t}$ from the other 6 counters. In stage 2 we see that the sample statistics $Z_{AV_{3,t}}$, t = 22, 23, are within CR of the stage 2 control chart, but samples t = 24, 25 are outside LCL of the stage 2 control chart. This indicates that the process variance is in-control at t = 22, 23, but the process variance is out-of-control at t = 24, 25. Table 12 and Figure 2 illustrate the resulting statistics in the two stages.

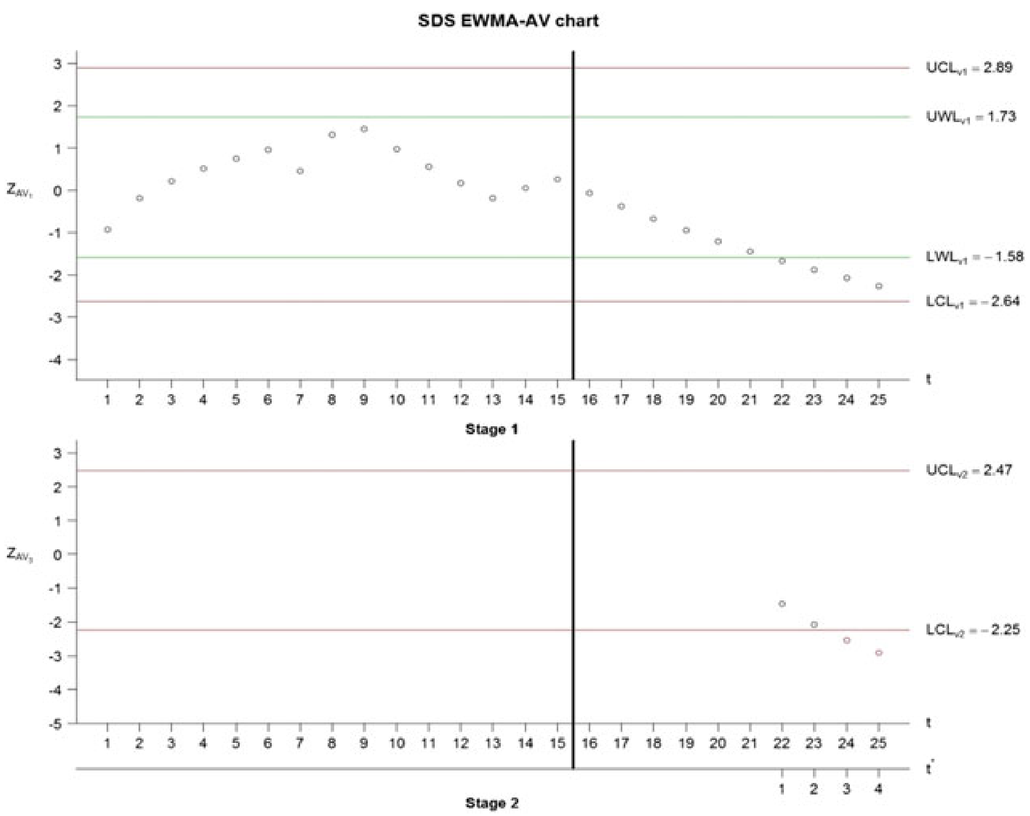

FIGURE 2 The SDS EWMA-AV chart for t = 1, 2, ⋯, 25 [Colour figure can be viewed at wileyonlinelibrary.com]

5 | CONCLUSIONS

In this paper, we propose a new control chart, the DS EWMA-AV chart, on the basis of a simple statistic and scheme to monitor the variance shifts in a process when the distribution of the critical quality characteristic is unknown or not normal. The proposed DS EWMA-AV chart is easy for practitioners to apply even if they lack statistical training. A numerical example of a service system for a bank branch in Taiwan is used to illustrate the application of the proposed variability control chart. When compared with the existing control charts for the quality variable with a normal or nonnormal distribution—the NP-M chart, the NLE and CWE control charts, the SS EWMA-V chart, and the IRC charts—the proposed new DS EWMA-AV chart always performs better in detecting the out-of-control variance with small, medium, and large scale shifts. We thus recommend using the new DS EWMA-AV chart.

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