A VG-NGARCH Model for Impacts of Extreme Events on Stock Returns

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Contents

Abstract

This article compares two types of GARCH models, namely, the VG-NGARCH and the GARCH-jump model with autoregressive conditional jump intensity, i.e., the GARJI model, to make inferences on the log of stock returns when there

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are irregular substantial price fluctuations. The VG-NGARCH model imposes a nonlinear asymmetric structure on the conditional shape parameters in a variance-gamma process, which describes the arrival rates for news with different degrees of influence on price movements and provides an ex ante probability for the occurrence of large price movements. On the other hand, the GARJI model, a mixed GARCH-jump model proposed by Chan and Maheu (Journal of Business & Economic Statistics 20:377–389, 2002), adopts two independent autoregressive processes to model the variances corresponding to moderate and large price movements, respectively. An empirical study using daily stock prices of four major banks, namely, Bank of America, J.P. Morgan Chase, Citigroup, and Wells Fargo, from 2006 to 2009 is performed to compare the two models. The goodness of fit of the VG-NGARCH model vs. the GARJI model is demonstrated.

Keywords
VG-NGARCH model • GARCH-jump model • Autoregressive conditional jump intensity • GARJI model • Substantial price fluctuations • Shape parameter • Variance-gamma process • Ex ante probability • Daily stock price • Goodness of fit

82.1 Introduction

To model asset returns, the following two frequently observed circumstances must be recognized: the volatility clustering and the leverage effect (Nelson 1991; Campbell and Hentschel 1992; Engle and Ng 1993). The two phenomena have led to the development of the family of nonlinear asymmetric GARCH models in financial forecasting and derivatives pricing (Nelson 1991; Engle and Ng 1993; Glosten et al. 1993; Ding et al. 1993). Nevertheless, Gaussian distributed return innovations in conventional ARCH-/GARCH-type models are unable to capture irregular substantial price fluctuations resulting from extreme news reports, even when the heteroskedasticity in the conventional ARCH-/GARCH-type models has been taken care.

To account for both normal and large price movements, a mixed GARCH-jump model that combines a GARCH-type model with a Poisson jump process for the dynamics of log-returns was first proposed by Jorion (1988). Later, complicated mixed GARCH-jump models that consider jumps in both log-returns and volatilities were developed by Duffie et al. (2000), Pan (2002), Eraker et al. (2003), and Eraker (2004). The mixed GARCH-jump model with autoregressive jump intensity (GARJI), proposed by Chan and Maheu (2002), is a more advanced mixed GARCH-jump model, of which the conditional variance of asset returns is divided into two parts corresponding to moderate and large price movements resulting from normal and extreme news events, respectively. The dynamics of the two variances in a discrete-time setting are sketched by two conditionally independent autoregressive processes (Chan and Maheu 2002; Maheu and McCurdy 2004).

Instead of a jump-diffusion process in the mixed GARCH-jump model with a continuous sample path for the asset price dynamics, the VG-NGARCH model is a GARCH-type model that uses a variance-gamma (VG) process, a pure jump process having finite sum of absolute price movements during a defined time frame, to model the price dynamics to avoid the problem that the sum of absolute price movements during a finite time period is infinite. As pointed by Madan et al. (1998), the VG process is a purely jump Levy process of infinite activities characterizing a "high" arrival rate of jumps of different sizes and will adequately allow us to dispense with the need to consider the variant influences of news reports on the magnitude of price movements (Andersen 1996; Clark 1973; Ross 1989). With the VG process, the VG-NGARCH model captures the volatility clustering and the leverage effect by modeling the VG process's shape parameter in a nonlinear asymmetric autoregressive process. For this reason, the VG-NGARCH model is more informative and parsimonious compared to the GARJI model. The specification of a VG process is given in Appendix 1.

The goodness of fit of the VG-NGARCH and the GARJI model to the log of stock price returns of four major banks listed in the S&P 500 are given. Since latent random business times are introduced into the VG framework, to find parameter estimates, Monte Carlo expectation-maximization (MCEM) algorithm together with the Metropolis algorithm are implemented. The two estimation approaches are given in Appendix 2.

The structure of this article continues as follows. In Sect. 82.2, the two GARCH-type models, namely, the VG-NGARCH and the GARJI models, are introduced. In Sect. 82.3, results of the empirical study and the performance of the two types of GARCH models are presented and compared. Finally, a concluding remark is made on Sect. 82.4.

82.2 Model Specifications

This section gives introduction and specifications for the GARJI model and the VG-NGARCH model.

82.2.1 GARJI Model

Chan and Maheu (2002) proposed a GARCH-jump model with autoregressive conditional jump intensity, i.e., the GARJI model, in which the conditional variance of return innovations is divided into two distinct modules that define smooth and steep fluctuations in price driven by normal and extreme news events, respectively. The GARJI model employs two conditionally independent autoregressive processes for the two components in a discrete-time economy in which the trading period

$[0, T]$ is partitioned into T subintervals $(0, 1], (1, 2], \ldots, (T-1, T]$. The dynamics of the log-return $Y_t = \ln(S_t/S_{t-1})$ are as follows:

$$Y_t = \mu + \varepsilon_t, \quad t = 1, \ldots, T. \tag{82.1}$$

Here the return innovation ε_t is partitioned into two independent components $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ corresponding to normal and unusual price movements, respectively. Let $\mathcal{F}_{t-1}$ be the information set available at time $t-1$. Conditional on $\mathcal{F}_{t-1}$, the innovation from normal price movement

$$\varepsilon_{1,t}\big|\mathcal{F}_{t-1} \sim N\left(0, \sigma_t^2\right) \tag{82.2}$$

is normally distributed with the conditional variance, ${\sigma_t}^2$, being parameterized by a GARCH function of the previous return innovation ε_{t-1} as

$$\sigma_t^2 = a_0 + a_1(\varepsilon_{t-1} - c)^2 + a_2\sigma_{t-1}^2. \tag{82.3}$$

The parameters employed for the GARCH function are based on the work of Chan and Maheu (2002) and Maheu and McCurdy (2004), albeit in a more simplified range that accommodates the asymmetric feedback from positive through negative news while allowing for the ex post evaluation of the expected number of jumps through the interval $(t-2, t-1]$ as a result of the information set $\mathcal{F}_{t-1}$ at time $t-1$. The second component, $\varepsilon_{2,t}$, represents the jump innovation and is the discrepancy between the total jump size and the expected total jump size of the n_t jumps during $(t-1, t]$, i.e.,

$$\varepsilon_{2,t} = \sum_{j=1}^{n_t} U_{t,j} - \theta\lambda_t,$$

where $U_{t,j}$ is the jth jump size being normally distributed with mean θ and standard deviation δ and n_t denotes the number of jumps distributed according to Poisson with an autoregressive conditional jump intensity (ARJI)

$$\lambda_t = \lambda_0 + \rho\lambda_{t-1} + \gamma\xi_{t-1}. \tag{82.4}$$

The intensity residual, $\xi_{t-1} = \mathrm{E}(n_{t-1}|\mathcal{F}_{t-1}) - \lambda_{t-1}$, is defined as the difference between the filter expected number of jumps given $\mathcal{F}_{t-1}$, $\mathrm{E}(n_{t-1}|\mathcal{F}_{t-1})$ and the previous intensity λ_{t-1}. The probabilities of jumps to fluctuate periodically and cluster with a persistence parameter of $0 < \rho < 1$ are afforded by specifying the conditional intensity. The conditional density of the log-return Y_t given the information set $\mathcal{F}_{t-1}$ is

$$f(Y_t|\mathcal{F}_{t-1}) = \sum_{n_t=0}^{\infty} f(Y_t|n_t, \mathcal{F}_{t-1}) \frac{e^{-\lambda_t}\lambda_t^{n_t}}{n_t!}, \tag{82.5}$$

where the conditional probability density $f(Y_t|n_t, \mathcal{F}_{t-1})$ is

$$\frac{1}{\sqrt{2\pi\left(\sigma_t^2 + n_t\delta^2\right)}}\exp\left(-\frac{(Y_t - \mu + \theta\lambda_t - \theta n_t)^2}{2\left(\sigma_t^2 + n_t\delta^2\right)}\right).$$

From Eqs. 82.1 to 82.5, it is clear that the main feature of GARJI model is the inclusion of both normal and extreme return innovations. Because these two types of innovations certainly affect future volatility differently. Nevertheless, it is impossible to identify the cutoff point between normal and extreme price movements by observing the log-returns. Consequently, only an ex post probability for the number of jumps, n_t, from the information set $\mathcal{F}_{t-1}$ at time t can be acquired, as n_t is non-observable. The ex post probability for n_t jumps given $\mathcal{F}_{t-1}$ is

$$P(n_t|\mathcal{F}_t) = \frac{f(Y_t|n_t, \mathcal{F}_{t-1})}{f(Y_t|\mathcal{F}_{t-1})} \times \frac{e^{-\lambda_t}\lambda_t^{n_t}}{n_t!}. \tag{82.6}$$

82.2.2 VG-NGARCH Model

To model stock price dynamics, Madan and Seneta (1990), Madan and Milne (1991), Madan et al. (1998), Carr et al. (2003), and Geman et al. (2001) considered the use of a VG process. In the following the specification of log-returns in terms of a VG process is given in a discrete-time setting. For $t = 1,\ldots, T$, the time-t log-return $Y_t = \ln(S_t/S_{t-1})$ can be formulated as

$$Y_t = m + \phi_t + \theta g_t + \varepsilon_t \tag{82.7}$$

where m denotes the mean of instantaneous return rate, g_t denotes a gamma-distributed random time change during the interval $(t-1, t]$, and ϕ_t denotes a time-varying parameter. The specification of a VG process is given in Appendix 1. Because of the characteristics of the VG process, the return innovation ε_t is conditionally Gaussian distributed as

$$\varepsilon_t\big|\mathcal{F}_{t-1} \sim N\left(0, \sigma^2 g_t\right). \tag{82.8}$$

It is worth noting that the conditional variance of the innovation ε_t depends on g_t during the interval $(t - 1, t]$. To accommodate the volatility clustering effect, the random time change g_t is considered to be gamma-distributed with a time-varying shape parameter v_t, specifically

$$g_t|\mathcal{F}_{t-1} \sim \text{gamma}(v_t, 1). \tag{82.9}$$

It is further assumed that the shape parameter v_t follows a nonlinear asymmetric NGARCH (1,1) process that depends on the previous return innovation ε_{t-1} and shape parameter v_{t-1}, respectively. The relation among them is as follows:

$$v_t = a_0 + a_1\left(\varepsilon_{t-1} - c\sqrt{v_{t-1}}\right)^2 + a_2 v_{t-1}, t \geq 1 \tag{82.10}$$

where $c>0$. The time-varying parameter ϕ_t in Eq. 82.7 is defined to be

$$\phi_t = v_t \ln\left(1 - \theta - \frac{1}{2}\sigma^2\right). \tag{82.11}$$

The skewness and kurtosis of log-return at time t are functions of the drift parameter θ, volatility σ, and the first four moments of the shape parameter v_t, which depend on the NGARCH parameters $\boldsymbol{\alpha} = (a_0, a_1, a_2, c)$. The skewness and kurtosis functions are given in Appendix 3. According to the skewness and kurtosis functions, the sign of the skewness relies on the sign of the drift parameter θ. Moreover, if $a_0 > 0$ and $a_1(\sigma^2 + c^2) + a_2 < 1$, then shape parameter becomes stationary, and

$$v_\infty = \lim_{t\to\infty} E(v_{t+1}) = a_0\left[1 - \left(\sigma^2 + c^2\right)a_1 - a_2\right]^{-1}. \tag{82.12}$$

From Eq. 82.18, the proposal transition density $\boldsymbol{f}$ of the target distribution, i.e., the posterior distribution $p(\boldsymbol{g}|\boldsymbol{Y};\boldsymbol{\Theta})$, is chosen to be the distribution of T independent gamma random variables with shape parameters $v_1 - 0.5, \ldots, v_T - 0.5$, respectively, and scale parameter $1/\kappa$. Specifically,

$$\boldsymbol{f} \propto \prod_{t=1}^{T} \exp(-\kappa g_t + (v_t - 1.5)\log(g_t)).$$

At the lth iteration of the independent Metropolis chain algorithm, a random sample of time changes $\boldsymbol{g} = (g_1, \ldots, g_T)'$ is drawn from the proposed transition density $\boldsymbol{f}$. The random sample $\boldsymbol{g}$ is accepted and $\boldsymbol{g}^{(l)} = \boldsymbol{g}$ with probability

$$\min\left\{\exp\left[-\sum_{t=1}^{T}\left(\frac{\delta_t}{g_t} - \frac{\delta_t}{g_t^{(l-1)}}\right)\right], 1\right\}; \tag{82.13}$$

otherwise, $\boldsymbol{g}^{(l)} = \boldsymbol{g}^{(l-1)}$.

82.3 Empirical Study

Due to the extended periods of market instability experienced by banks as a consequence of the extreme news reports associated with the 2008 financial crisis, this study selects four big commercial banks listed in the S&P 500, namely,

Table 82.1 Correlations of daily log-returns

Bank	Bank of America	J.P. Morgan	Citigroup	Wells Fargo
Bank of America	1	0.81168	0.80231	0.84943
J.P. Morgan Chase		1	0.71070	0.84059
Citigroup			1	0.71889
Wells Fargo				1

Table 82.2 Summary for daily log-returns

Bank	Bank of America	J.P. Morgan	Citigroup	Wells Fargo
Days	754	754	754	754
Mean	−0.00138	−0.00019	−0.00246	0.00003
St.dev	0.03769	0.03269	0.04294	0.03153
Min	−0.30408	−0.19694	−0.3056	−0.21034
Max	0.2409	0.19368	0.45729	0.28371
Range	0.54498	0.39062	0.76289	0.49406
Skewness	−0.47452	0.02702	0.67837	0.09241
Kurtosis	16.62389	9.24964	27.40201	15.55658

the Bank of America (BAC), J.P. Morgan Chase (JPM), Citigroup (Citi), and Wells Fargo (WF), to study their stock price dynamics. Daily price data including intraday highs and lows, and adjusted close prices, are collected from January 3, 2006 to December 31, 2009. Table 82.1 lists the correlation coefficients among them. Since all correlation coefficients exceed 0.7, their price movements are highly correlated. Table 82.2 lists the mean, standard deviation, skewness, and kurtosis for log-returns of the four banks.

82.3.1 Estimation and Validation

Parameter estimates and log-likelihood of the VG-NGARCH model for each bank are given in Table 82.3. Besides model fitting, two testing procedures were conducted and the results are also contained in Table 82.3.

The first one with the null hypothesis, H_0: $a_1 = a_2 = c = 0$, helps us to determine whether the VG-NGARCH model can be reduced to a simpler model, i.e., the VG model by Madan et al. (1998). Based on the likelihood ratio test, the autoregressive shape dynamics are strongly favored for all banks over the VG model with constant shape parameter. The second testing procedure is the Ljung-Box test, with H_0 describing the randomness of residuals. To compute Ljung-Box Q statistic, the lag is set to be 25, and the large p-values for all banks, as shown in Table 82.3, indicate that there is no significant serial correlation remaining among the residuals for all banks after their log-returns were fitted by the VG-NGARCH model.

Table 82.3 VG-NGARCH model estimates and tests

Parameter	Bank of America	J.P. Morgan	Citigroup	Wells Fargo
θ	−0.0058	−0.0060	−0.0076	−0.0039
σ	0.0218	0.0205	0.0239	0.0178
v_1	2.7839	2.3371	2.9322	3.5000
a_0	0.1682	0.1082	0.1651	0.1619
a_1	0.4038	0.3929	0.4629	0.4529
a_2	0.5347	0.5408	0.4808	0.5008
c	0.0081	0.0005	0.0003	0.0083
Log-likelihood	1,951.7	1,857.4	1,841.6	1,945.0
H_0: $a_1 = a_2 = c = 0$				
Log-likelihood	1,684.6	1,688.1	1,636.7	1,763.4
Likelihood ratio test	526.2	345.2	409.8	386.8
p-value	0.0000	0.0000	0.0000	0.0000
H_0: Residuals are random				
Ljung-Box Q (lag = 25)	8.9911	6.5697	11.5924	8.2869
p-value	0.9986	0.9999	0.9929	0.9991

Table 82.4 GARJI model estimates and tests

Parameter	Bank of America	J.P. Morgan	Citigroup	Wells Fargo
λ_0	0.1165	0.1072	0.1164	0.0754
ρ	0.3923	0.4799	0.3053	0.3626
γ	0.4570	0.3579	0.1034	0.5886
θ	0.0102	0.0154	0.0099	0.0080
δ	0.0218	0.0341	0.0306	0.0546
σ_0	0.0312	0.0279	0.0405	0.0283
a_0	0.0001	0.0001	0.0001	0.0001
a_1	0.3568	0.4351	0.545	0.3634
a_2	0.5872	0.4997	0.3856	0.5761
c	0.0356	0.0645	0.0218	0.0909
Log-likelihood	1,887.1	1,843.7	1,764.1	1,918.0
H_0: Residuals are random				
Ljung-Box Q (lag = 25)	43.7338	30.5120	31.4383	43.7169
p-value	0.012	0.2061	0.1761	0.012

Parameter estimates and log-likelihood of GARJI for each bank are given in Tables 82.4. After fitting GARJI model for each bank, residuals are diagnosed by the Ljung-Box test, with the lag being 25. From Table 82.4, the p-values of the Ljung-Box Q statistic for all banks under GARJI model are much smaller than those under the VG-NGARCH model. To be more specific, p-values for Bank of America and Wells Fargo are too small to provide strong evidence that there is significant serial correlation among the residuals for Bank of America and Wells Fargo after fitting the GARJI model to their log-returns.

Table 82.5 Information criteria

	Bank of America	J.P. Morgan	Citigroup	Wells Fargo
VG-NGARCH model				
AIC	−5.1610	−4.9083	−4.8662	−5.0278
SC	−5.1181	−4.8654	−4.8233	−5.0056
HQ	−5.1445	−4.8918	−4.8497	−5.0106
GARJI model				
AIC	−4.9579	−4.8426	−4.6474	−4.9465
SC	4.8904	−4.7752	−4.5799	−4.8801
HQ	−4.9319	−4.8167	−4.6214	−4.9209

82.3.2 Model Selection Based on Information Criteria

Since the two models are not nested, their goodness of fit is measured by the following three criteria based on log of the maximum likelihood and the number of parameters: Akaike information criterion (AIC), Schwarz criterion (SC), and Hannan-Quinn criterion (HQ). The formulas for the three criteria are as follows:

$$\begin{aligned}\text{AIC} &= -2(\mathcal{L}/T) + 2(k/T)\\ \text{SC} &= -2(\mathcal{L}/T) + k\log(T)/T\\ \text{HQ} &= -2(\mathcal{L}/T) + 2k\log(\log(T))/T\end{aligned}$$

where $\mathcal{L}$ is log of the maximum likelihood, k is the number of parameters, and T is the sample size. The model minimizing these information criteria is preferred.

Comparing log-likelihoods and the three information criteria for the VG-NGARCH and GARJI models listed on Tables 82.3, 82.4, and 82.5, the VG-NGARCH model not only has higher log-likelihood values but also has smaller values on AIC, SC, and HQ for all banks, suggesting that it provides not only better fitting but also more parsimonious model specification for these bank data.

82.3.3 Evaluation of Volatility Forecasts

This subsection evaluates the performance of each model on variance forecasts through comparing the out-of-sample volatility forecasts of the VG-NGARCH model with those of the benchmark GARJI. To assess out-of-sample forecasts, a range-based estimate of ex post volatility was calculated in compliance with the method of Parkinson (1980) and Maheu and McCurdy (2004) as follows:

$$\text{Range}_t = \sqrt{\eta}\,\log\left(P_{t,h}/P_{t,l}\right),$$

where $P_{t,h}$ and $P_{t,l}$ represent the intraday high prices and low prices, respectively. The parameter η in the above formula is the calibration parameter to make the range

Table 82.6 Out-of-sample variance forecasts

	Bank of America	J.P. Morgan	Citigroup	Wells Fargo
VG-NGARCH				
β_0	0.0000	0.0023	0.0049	0.0098
β_1	1.1626	1.1666	1.2582	1.0759
R^2	0.6575	0.6407	0.6482	0.6148
F	963.8306	895.3023	924.9720	801.2075
p-value	0	0	0	0
GARJI				
β_0	−0.0392	−0.0221	−0.0333	−0.0206
β_1	1.3681	1.3284	1.4201	1.2472
R^2	0.6500	0.6317	0.6470	0.5727
F	932.1904	861.0316	919.9395	672.7951
p-value	0	0	0	0

estimate of the unconditional variance equal the unconditional variance of daily returns. To conduct out-of-sample analyses, all models were reestimated using observations from January 2, 2008 to December 31, 2009. These out-of-sample estimates were kept to derive the out-of-sample forecast for the date-t conditional variance given $\mathcal{F}_{t-1}$, denoted as $\widetilde{\mathrm{V}}\mathrm{ar}(Y_t|\mathcal{F}_{t-1})$. Following the approach of Maheu and McCurdy (2004), the range-based ex post volatilities were regressed on the out-of-sample forecasts of the conditional variances as

$$\mathrm{Range}_t = \beta_0 + \beta_1\sqrt{\widetilde{\mathrm{V}}\mathrm{ar}(Y_t|\mathcal{F}_{t-}1) + \mathrm{error}_t}. \tag{82.14}$$

The coefficient of determination, R^2, of the regression tells us the proportion of the total variation for the range-based volatilities explained by the out-of-sample conditional variance forecasts. Hence, the model with higher R^2 is considered to be superior in forecasting volatilities. The R^2s of the regression models given in Eq. 82.14 are displayed on Table 82.6 for all banks under the VG-NGARCH and GARJI models, respectively. Since R^2s under the VG-NGARCH model are all larger than those under the GARJI model for all banks, the VG-NGARCH outperforms the GARJI model on out-of-sample volatility forecasts.

82.3.4 Prediction of Large Price Movements or Jumps

In this subsection, the VG-NGARCH model is examined for its performance on predicting the probability of large price movements due to extreme events. Here, large price movement is defined to occur when the absolute log-return exceeds 0.05. From Eqs. 82.7, 82.8, and 82.9, the ex ante probability of large price movements is given by

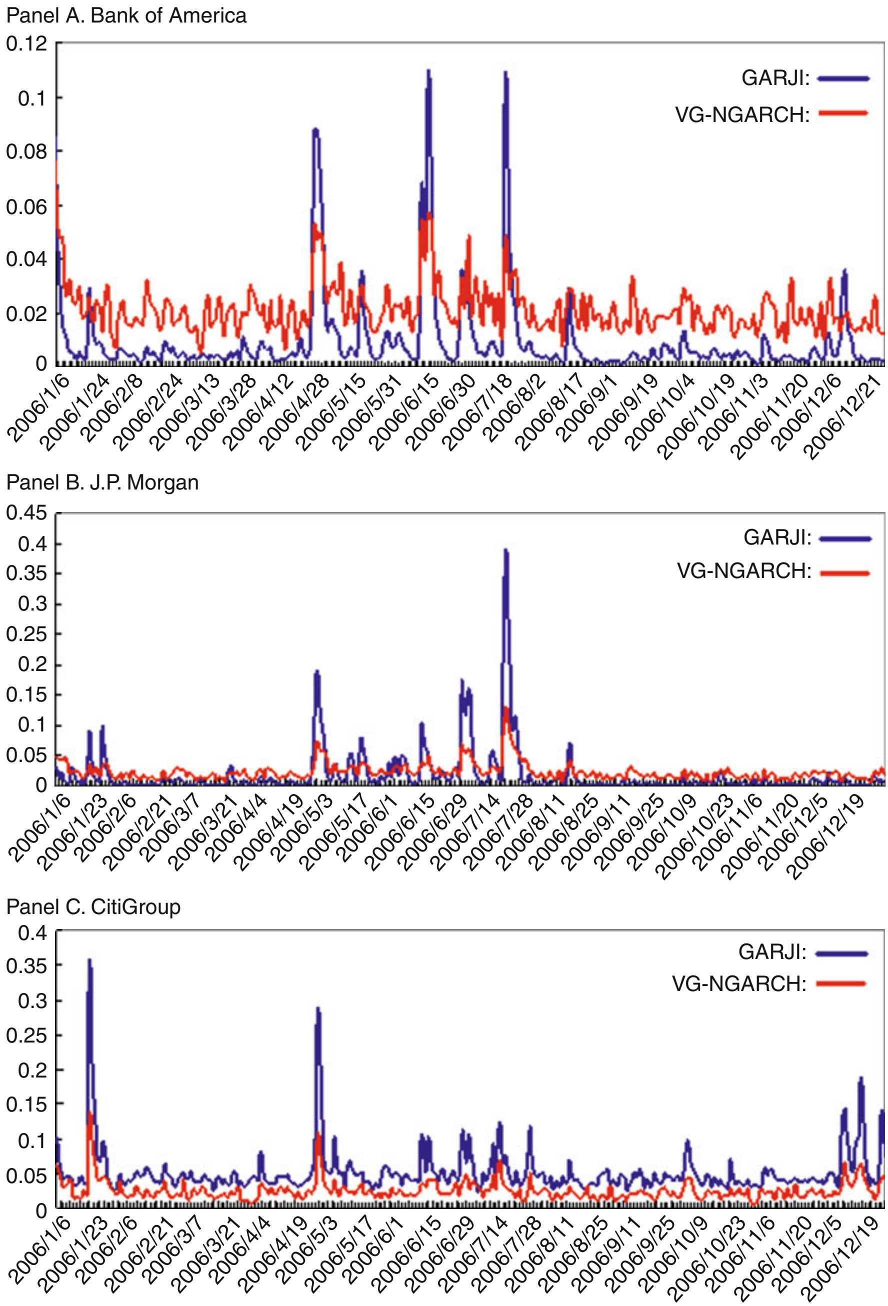

Fig. 82.1 (continued)

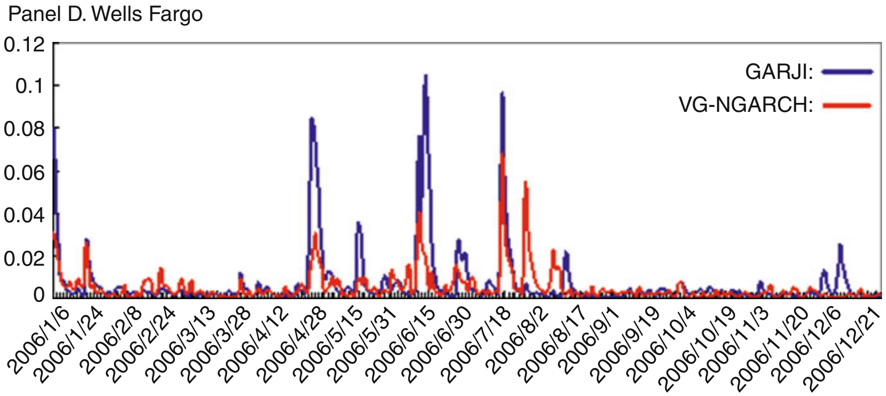

Fig. 82.1 Probabilities of large price movements or dumps during the year 2006

$$P(|Y_t|> 0.05|\mathcal{F}_{t-1}) = \int_0^\infty \left[1-\Phi\left(\frac{0.05-(m+\phi_t+\theta g_t)}{\sigma\sqrt{g_t}}\right)\right]h(g_t)dg_t + \int_0^\infty \Phi\left(\frac{-0.05-(m+\phi_t+\theta g_t)}{\sigma\sqrt{g_t}}\right)h(g_t)dg_t \tag{82.15}$$

where $h(g_t)$ denotes the probability density function of a gamma distribution with shape and scale parameters v_t and 1, respectively.

For the GARJI model, the performance of prediction on jumps is based on the ex post probability of at least one jump occurring, which is expressed as

$$P(n_t \geq 1|\mathcal{F}_t) = 1 - P(n_t = 0|\mathcal{F}_t), \tag{82.16}$$

where, following Eq. 82.6,

$$P(n_t = 0|\mathcal{F}_t) = e^{-\lambda_t}\frac{f(Y_t|0,\mathcal{F}_{t-1})}{f(Y_t|\mathcal{F}_{t-1}).}$$

After the parameters in Eqs. 82.15 and 82.16 are replaced by their estimates, $P(|Y_t|> 0.05|\mathcal{F}_{t-1})$ and $P(|n_t \geq 1|\mathcal{F}_{t-1})$ are estimated for the whole study period for each bank. In order to compare their performances, the probabilities belonging to Years 2006 and 2008 are plotted on Figs. 82.1 and 82.2, respectively. From Fig. 82.1, the ex ante probabilities for large price movements under the VG-NGARCH model are smoother than those resulting from the GARJI model over the same period, where no noteworthy extreme events occurred during 2006.

Thus, the ex post probabilities of jumps from the GARJI model tend to over predict the chance of jumps when price movements are moderate.

On the other hand, as Year 2008 has been well recognized by the occurrences of financial turbulence and crisis, Fig. 82.2 does demonstrate that much higher

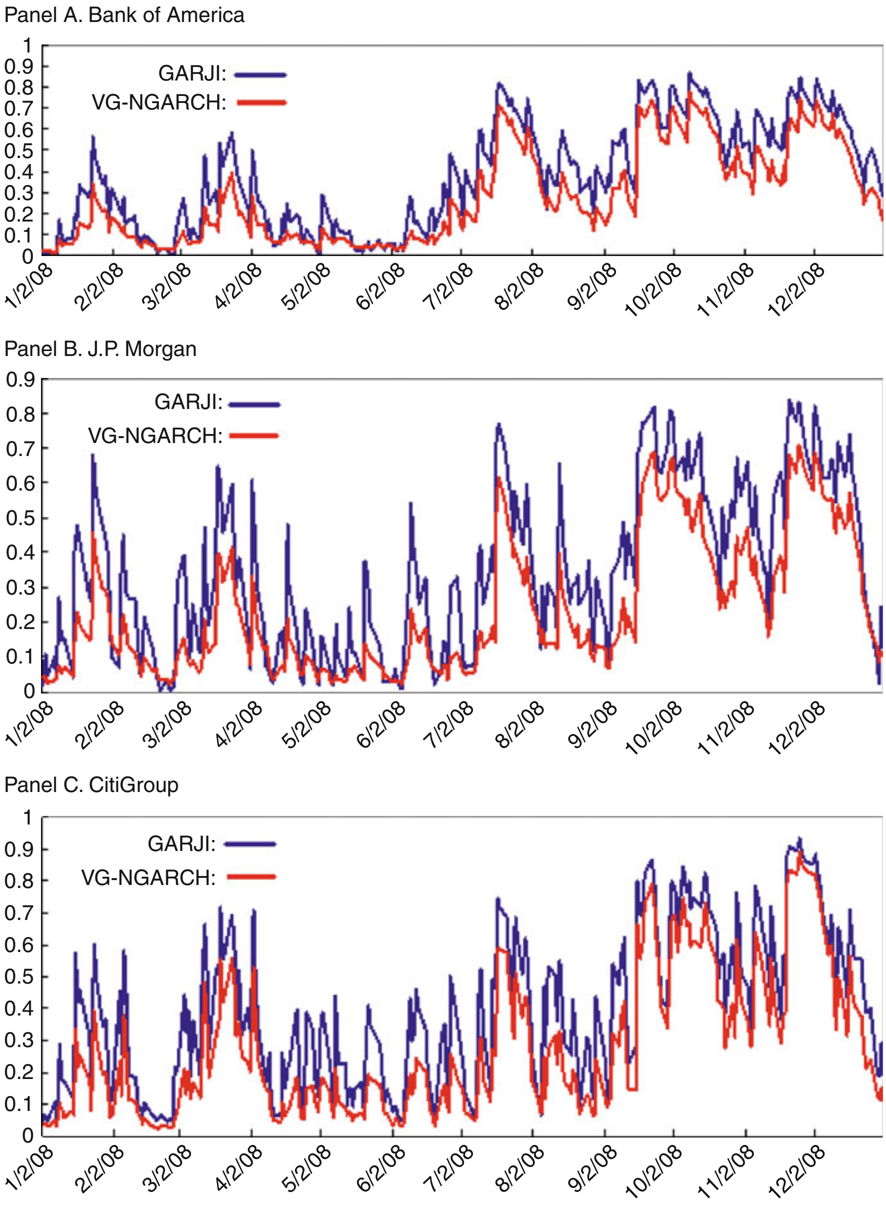

Fig. 82.2 (continued)

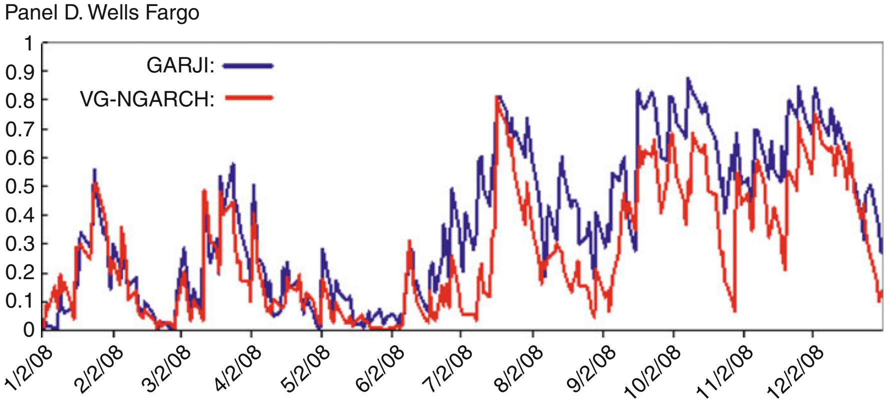

Fig. 82.2 Probabilities of large price movements or jumps during year 2008

probabilities of large price movements and jumps are predicted compared to those of Year 2006. Specifically, on September 16, 2008, the log-returns for Bank of America, J.P. Morgan, Citigroup, and Wells Fargo were −0.2398, −0.1066, −0.1642, and −0.1007, respectively. The corresponding ex ante probabilities of large price movements using the VG-NGARCH model were 0.7114, 0.4544, 0.6621, and 0.5368, respectively, while the ex post probabilities of jumps using the GARJI model were 0.8308, 0.6806, 0.7980, and 0.8348, respectively. Though both models show the ability to catch up large price movements or jumps, the VG-NGARCH model provides smoother and thus more reliable predictions than the GARJI model.

82.4 Conclusion

Differing from the GARJI model, for the VG-NGARCH model, based on a purely jump VG process, no cutoff point is required between normal and extreme price movements. In addition, instead of two independent autoregressive processes, a nonlinear asymmetric autoregressive process is used to model the shape parameter of the VG process. This makes the VG-NGARCH model more informative and parsimonious compared to the GARJI model. Furthermore, the empirical study demonstrates that through diagnosing the randomness of residuals, computing three information criteria (AIC, SC, and HQ), forecasting out-of-sample conditional volatility, and predicting the likelihood of large price movements or jumps, the VG-NGARCH model consistently outperforms the GARJI model. The superiority of the VG-NGARCH model relative to the benchmark GARJI model should improve the prediction ability of the occurrences of extreme events and hence is a better modeling approach to make financial management.

Appendix 1: Variance-Gamma Process

A VG process is a Brownian motion evaluated at a random business time modulated by a stochastic gamma process, to replace the role of Brownian motion. Specifically, at time s, a VG process X is given by

$$X(s) = \theta g(\mathrm{s}; 1, \gamma) + \sigma W(g(\mathrm{s}; 1, \gamma)), \tag{82.17}$$

where $g(\mathrm{s}; 1, \gamma)$ is the gamma process with unit mean rate and variance γ, W represents a standard Brownian motion, and θ and σ are the drift and volatility parameters, respectively. The extent of random time change $\Delta g = g(s;v,\gamma) - g(0;v,\gamma)$ is the increment of the gamma process during the interval $(0, s]$. Therefore, Δg follows gamma distribution with shape and scale parameters being vs and γ, respectively. Since the scale parameter γ can be transformed into one, it is set to one in our study.

Appendix 2: Parameter Estimation: Monte Carlo EM and Metropolis Algorithm

Method of maximum likelihood is adopted for the VG-NGARCH model. However, since the random time changes $g_1, \ldots, g_T$ are unobservable, the parameters of the VG-NGARCH model are unidentifiable. To resolve this problem, the mean of instantaneous return rate m is set to the mean of the log-returns $Y_1, \ldots, Y_T$, namely, $\hat{m} = \overline{Y}$, and the Monte Carlo EM (MCEM) algorithm (Wei and Tanner 1990; McCulloch 1997) is employed to estimate the parameters $\mathbf{\Theta} = (\theta, \sigma, v_1, \boldsymbol{\alpha})'$, where $\boldsymbol{\alpha} = (a_0, a_1, a_2, c)$ is the NGARCH parameter.

To perform the MCEM algorithm, at each iteration a set of K samples of the unobservable random time changes, $\boldsymbol{g}^{(1)}, \ldots, \boldsymbol{g}^{(K)}$, where $\boldsymbol{g}^{(l)} = ({g_1}^{(l)}, \ldots, {g_T}^{(l)})$, $1 \leqq l \leqq K$, are drawn from the posterior distribution $p(\boldsymbol{g}|\boldsymbol{Y};\mathbf{\Theta})$, which is

$$p(\boldsymbol{g}|\,\boldsymbol{Y};\Theta) \propto \prod_{t=1}^{T} \exp\{-\kappa g_t - \delta_t/g_t + (v_t - 1.5)\log(g_t)\}, \tag{82.18}$$

where $v_t = a_0 + a_1\left(\varepsilon_{t-1} - c\sqrt{v_{t-1}}\right)^2 + a_2 v_{t-1}$ is the time-t shape parameter, and the coefficients κ and δ_t are

$$\kappa = \frac{\theta^2}{2\sigma^2} + 1 \quad \text{and} \quad \delta_t = \frac{\left(Y_t - \overline{Y} - \phi_t\right)^2}{2\sigma^2}.$$

Since Eq. 82.18 is not proportional to any density function of well-known distributions, it is not possible to directly sample the time changes, $\boldsymbol{g}^{(1)}, \ldots, \boldsymbol{g}^{(K)}$, from the posterior distribution $p(\boldsymbol{g}|\boldsymbol{Y};\mathbf{\Theta})$ at each iteration of the EM algorithm. Consequently, the Metropolis chain strategy is carried out here

(Metropolis et al. 1953; Hastings 1970). In the independent Metropolis chain algorithm, a random outcome is sampled from its target distribution π by generating a chain of size L as follows: at the nth step of the chain, if the chain is at a point $X_n = \boldsymbol{x}$, a candidate value $\boldsymbol{y}$ is sampled from a proposal transition density $f(\boldsymbol{y})$ for the next location X_{n+1}. The candidate $X_{n+1} = \boldsymbol{y}$ is accepted with probability

$$p(x, y) = \min\left\{\frac{\pi(\boldsymbol{y})f(\boldsymbol{x})}{\pi(\boldsymbol{x})f(\boldsymbol{y})}, 1\right\}.$$

An independent uniform random variate U is generated; if $U < p(\boldsymbol{x}, \boldsymbol{y})$, then $X_{n+1} = \boldsymbol{y}$; otherwise the step is rejected and the chain remains at $X_{n+1} = \boldsymbol{x}$. After L such steps, where L is sufficiently large, a realization is obtained from the target distribution π.

Appendix 3: Skewness and Kurtosis of Log-returns

The unconditional skewness and kurtosis of log-return at time t are expressed in terms of the model parameters and the first four moments of the shape parameter v_t, which are

$$\text{Skewness}(Y_t) = \frac{\left(2\theta^3 + 3\theta\sigma^2\right)E(v_t) + 3(\tau+\theta)\left(\theta^2+\sigma^2\right)V(v_t) + (\tau+\theta)^3 E^3(v_t - E(v_t))}{\left[\left(\theta^2+\sigma^2\right)E(v_t) + (\tau+\theta)^2 V(v_t)\right]^{3/2}}; \tag{82.19}$$

$$\text{Kurtosis}(Y_t) = \frac{3\left(\theta^2+\sigma^2\right)^2 E\left(v_t^2\right) + \left(3\sigma^4 + 6\theta^4 + 12\theta^2\sigma^2\right)E(v_t) + Q}{\left[\left(\theta^2+\sigma^2\right)E(v_t) + (\tau+\theta)^2 V(v_t)\right]^2}, \tag{82.20}$$

where $\tau = \ln(1 - \theta - \sigma^2/2)$; and

$$\begin{aligned} Q = {} & 6(\tau+\theta)^2\left(\theta^2+\sigma^2\right)\left[E\left(v_t^3\right) + E^3(v_t) - 2E(v_t)E\left(v_t^2\right)\right] \\ & + 4(\tau+\theta)\left(2\theta^4 + 3\theta^2\sigma^2\right)V(v_t) + (\tau+\theta)^4 E^4(v_t - E(v_t)). \end{aligned}$$

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