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# How could the non-sustainable Easter Island have been sustained?☆

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ABSTRACT

The collapsing scenario of Easter Island has been analyzed by Brander and Taylor (1998) as a predator–prey model in a Malthusian world, in which the household is only concerned with its instantaneous utility. This paper develops an endogenous growth model with a renewable resource and analyzes the possibly non-sustainable growth as a steady state, in spite of the household being deeply concerned with all its future lifetime utility. Our analysis shows that the ignorance of future lifetimes in present decision-making is indeed crucial to economic non-sustainability. We then examine whether a deforestation tax set by the government could have reduced the resource exploration rate and thereby held back the economic collapse. We also demonstrate using phase-diagrams how such a tax can switch the economic dynamics from non-sustainability to sustainability.



## 1. Introduction

In history, many human societies have either collapsed or vanished; examples include the Maya cities in Central America, Angkor Wat in Asia and Easter Island in the Pacific Ocean, all of which have left behind monumental ruins. The scales of such ruins testify to the wealth and power of the dwellers in these societies. The story of Easter Island as described by Brander and Taylor (1998) and Diamond (2005) is perhaps the most vivid example, and we shall only briefly describe it as we introduce the motivation underlying our modeling.

### 1.1. The Brander & Taylor model of Easter Island

Easter Island is a small Pacific island covering an area of 66 square miles. The nearest land is 2300 miles away on the coast of Chile. The most visible evidence of a previous culture on Easter Island is its giant stone statues and the stone platforms on which they are placed. Since the carving, transportation, and erection of such statues would have required tremendous labor input and a lot of trees (to roll and move the statues), it is suggested that Easter Island would have been a complex populous society living in an environment rich enough so that meeting subsistence requirements would have been relatively easy, leaving ample time to devote

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to other activities. However, the population was estimated to be only about 3000 when encountered by European visitors in 1722, and the island had been found to be treeless. The question is: why would a once mighty society with an abundance of forests end up collapsing?

Brander and Taylor (1998) were the first to propose a formal model explaining the process of such a collapse. They set up a predator–prey model with man as the predator and the forest as the prey, and analyzed the population-resource dynamics under different parametric specifications. The Easter Island story is a typical example of non-sustainable growth, and the model of Brander and Taylor is a classic one with Malthusian population checks. The purpose of this paper is to propose a variant model that is more compatible with the sustainability problem we face today. We explain the detailed differences below.

### 1.2. Special features of our model

The first feature we embody in our model is the role of *future lifetimes*. As pointed out in the Brundtland Report (WCED, 1987), Ekin (1994) and Chichilnisky, Heal, and Vercelli (1998), the concern for the well-being of future lifetimes, particularly in so far as this is affected by their access to natural resources and environmental goods, is the key to the discussion of sustainability. In the typical Malthusian model à la Brander and Taylor (1998) and the literature that follows, agents are often assumed to have a temporal utility of their own consumption, and they do not care about their future lifetimes. This setting of short-sighted agents will arguably more easily lead to the possible collapse of the economy, because sustainability by definition refers to the perpetuation of economic activities for all infinite future time. As such, the more intriguing case, as in the discussion in most of the contemporary literature on economic sustainability, is to assume that agents are deeply concerned with all their future time.

The problem of non-sustainability in this short-sighted agent context is more interesting because one has to explain why the short-sighted decision may not be enough to prevent the disaster that will befall their future lifetimes. Put differently, as Martinez-Alier and O'Connor (1999) pointed out, in the discussion on sustainability, the ignorance of future lifetimes may be the cause of the economic collapse. We believe that meaningful economic policies can be proposed only when we explicitly embody the conflicts between a single time period and all lifetimes in our model.

The second feature we consider modifying in our model is the definition of sustainability. The definition of collapse in Brander and Taylor (1998) is related to the reduction in population size and the depletion of forest resources. The typical definition in the modern sustainability literature often refers to the constraint whereby the instantaneous utility should not compromise the utility of future lifetimes; see, e.g., Chichilnisky (1996), Pearce (1998), and Chichilnisky et al. (1998). If we are to consider the conflict between the single period and all future lifetimes, we should explicitly include the role of physical capital in our model since physical capital reflects a tradeoff of natural resources between preservation and utilization. However, physical capital accumulation is typically not included in a Malthusian model of population dynamics (Chu, 1998).

The third feature we would like to study in our model is related to several fundamental questions concerning the dynamics of an *unsustainable equilibrium*. Suppose agents have perfect foresight and are deeply concerned with all their future lifetimes. Then, at each time point $t$, we can calculate the discounted present value of their future utility. Let this present value be $v_t$. Following our discussion of the second feature above, a non-sustainable state suggests a decreasing pattern of $v_t$ with respect to $t$. Growth theory should provide answers to the following four questions: 1) Will this $v_t$ converge to a collapsing state when $t$ goes to infinity? 2) If it does converge to a collapsing state $v$, why would the agents *choose* an optimal consumption and saving path that converges to this non-sustainable state? 3) What rectification policy can the government adopt to prevent this from happening? 4) Along the lines of Laitner (1990), what is the phase-diagram if a policy changes a steady state from a non-sustainable state to a sustainable one?

As we suggested in the second and third features above, a path of economic growth may be unsustainable when the agents choose a very high resource depletion rate. We want to analyze when this is more likely to happen and whether this *laissez faire* state can be suppressed by a government policy. When we analyze this problem, what we have in mind is actually an *endogenous growth* model, which is typically different from the growth models that were used to study the Malthusian scenario for Easter Island. In a Malthusian model, the steady-state growth rate is zero and thus this structure can only deal with the *level* change in economic variables. Nevertheless, our endogenous growth model exhibits a non-zero steady-state growth rate in which agents' behavior is based on their optimal choice. In view of this, we can present the *rate* change of economic variables. We believe that this endogenous growth setup is a framework that is more consistent with the sustainability problem that we face today.

### 1.3. Previous literature

Several papers have tried to modify the Brander and Taylor (1998) paper along different directions. Reuveny and Decker (2000) incorporate the possibility of technological progress and population management into the Brander and Taylor model, and show that the fate of collapse might have been averted. However, this paper does not have physical capital accumulation, neither are there present agents' concerns of all their future lifetimes. Dalton, Coats, and Asrabadi (2005) assume that the agents' use of resources may be slower when they foresee resource depletion as a future trend. They show that when this institutional mechanism is added to the Brander–Taylor model, the feast-and-famine cycle in Easter Island may be dampened.

Erickson and Gowdy (2000) consider the accumulation of some kind of physical capital other than natural resources, and allow for their substitutability in production. However, their model still does not have endogenous growth. Finally, Pezzey and Anderies (2003) add a constraint to the minimum subsistence level of resource consumption and some institutional adaptations. They analyze the changes in equilibrium and overshoot in response to such adaptations. Again, they do not address the features we mentioned in the previous subsection.

In the context of growth theory, we should first ponder how the story of sustainability can be modeled in an endogenous growth framework. Conceptually, environmental quality can be regarded as a renewable resource, and the environmental pollution generated by economic activity plays the role of harvesting such resources (Aghion & Howitt, 1988). Indeed, when faced with environmental degradation, various policy instruments set by the government, such as green taxes, subsidies, and direct regulation, may be considered to be able to remedy it. The conventional wisdom argues that environmental policy hurts capital accumulation and economic growth through crowding out private expenditure (Huang & Cai, 1994 and Ligthart & van der Ploeg, 1994). The reason for this is that the public expends more resources to maintain environmental quality, leaving fewer resources to be used on productive activities, and hence deterring the balanced growth rate. Contrary to the preceding viewpoint, recent studies propose that environmental policies may have a favorable effect on economic growth via production channels.[1] For instance, an influential paper by Bovenberg and Smulders (1995) argues that the quality of the environment not only governs consumers' preferences, but also plays a role in production.[2] In their paper, an ambitious environmental policy may stimulate economic growth by assuming that environmental quality is beneficial to input productivity. A common feature of these studies is that they confine their models to a state of sustainable positive economic growth, thereby avoiding the issue of economic collapse.

### 1.4. Layout of this paper

With the story of Easter Island in mind, in this paper we construct an endogenous growth model with a renewable resource (a forest), and consider the equilibrium dynamics in response to government policies. We first demonstrate the existence of a steady state with decreasing utility value, which we define as a non-sustainable growth pattern, corresponding to the case of the collapse of Easter Island. This non-sustainable growth pattern is the optimal choice of agents, even though they care about the welfare of all their future lifetimes. We analyze when this will happen.

We then examine whether a deforestation tax set by a government can diminish deforestation and thereby hold back the economic collapse. A novel point is that we find that a restoration of sustainability embodies both a change in the steady state and a shift in the convergence–divergence zone in a phase diagram. We illustrate how the economy evolves in response to a change in the deforestation tax. We find that there is nothing the government can do if the degree of forest destruction is relatively high. However, the government could implement the deforestation tax to restore the economy's sustainability if the degree of forest destruction is not very high.

The remainder of this paper is organized as follows. Section 2 sets up the structure of the one-sector endogenous growth model and solves for the macroeconomic equilibrium. Section 3 examines the growth effect of adjusting the deforestation tax, and Section 4 deals with the economy's dynamic responses to an unanticipated rise in the deforestation tax. Finally, concluding remarks are provided in Section 5.

## 2. The model

Consider an economy consisting of a representative islander and a government agent. The islander produces a single composite commodity, which can be consumed or accumulated as physical capital (here, physical capital could be regarded as tools for fishing, hunting, felling, etc.). The government may collect a trust fund for forest preservation; the more that is collected, the higher the growth rate of the forest.

The representative islander is assumed to have an infinite planning horizon and perfect foresight. The islander derives utility from consumption $C$ and the forest stock $Z$[3]; her lifetime utility is specified as follows[4]:

$$\int_0^{\infty} [(1-a)\ \ln C + a\ \ln Z] e^{-\rho t} dt, \tag{1}$$

where $a(>0)$ measures the impact of the benefit of the forest stock on the islander's utility, and $\rho(>0)$ denotes the constant rate of time preference.[5]

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[1] Other channels through which an environmental policy may stimulate economic growth include an elastic labor supply (Chen, Lai, & Shieh, 2003), tax revenues recycled to subsidize intermediate goods' R&D (van Zon & Yetkiner, 2003), and the existence of an indeterminate equilibrium path (Itaya, 2008).

[2] Bovenberg and Smulders (1996, p.864) argue that "…environmental quality determines nature's capacity to grow, features an amenity value, and affects the living and working conditions in the economy."

[3] In line with the standard setting in the environmental economics literature (e.g., Chao, Laffargue, & Sgro, 2012), the environmental quality (the forest stock) is introduced into the islander's utility function.

[4] For notational convenience, in what follows the time subscript of all variables is omitted except in cases where it should be called to the reader's attention.

[5] In line with Bovenberg and Smulders (1995), in this paper the rate of time preference is specified as a constant. However, Yanase (2011) and Vella, Dioikitopoulos, and Kalyvitis (in press) alternatively specify that a better environment leads to more patience. The rationale for their specification can be explained intuitively. Lower pollution implies better health, and hence is association with a lower mortality rate. The household thus tends to be more patient and willing to postpone current consumption. As a result, it may be more plausible to specify that the islander's time preference rate is affected by the forest stock, and then examines the impact of a deforestation tax. For a more complete discussion on the endogenous time preference, see, e.g., Obstfled (1990) and Sarkar (2007).

In line with Bovenberg and Smulders (1995), output $Y$ is produced using physical capital $K$, deforestation flow $R$ and the forest stock $Z$. The production function is given by:

$$Y = K^{\alpha}R^{\beta}Z^{\eta};\ \alpha,\beta,\eta \in (0,\ 1),\ \alpha+\beta+\eta = 1. \tag{2}$$

At each instant of time, the representative islander is bounded by a flow constraint linking capital accumulation to any difference between her gross income and expenditure. The islander's budget constraint can then be written as follows:

$$\dot{K} = Y - C - \delta K - (\theta + p)R, \tag{3}$$

where $\delta$ is the physical capital depreciation rate, $\theta$ represents the basic expenditure on deforestation, e.g., the transportation cost, and $p$ denotes the deforestation tax, which is zero in a *laissez faire* state.

Following the approaches of Ligthart and van der Ploeg (1994) and Bovenberg and de Mooij (1997), the representative islander treats the forest stock as given since she believes that her activities are too insignificant to affect the forest stock. Based on the above assumption, the representative islander maximizes Eq. (1) subject to Eqs. (2) and (3) by choosing a sequence $\{C,K,R\}_{t=0}^{\infty}$. By letting $\lambda$ be the shadow value of the physical capital stock $K$, the current-value Hamiltonian can be formulated as follows:

$$H = [(1-a)\ \ln C + a\ \ln Z] + \lambda\left[K^{\alpha}R^{\beta}Z^{\eta} - C - \delta K - (\theta + p)R\right]. \tag{4}$$

The first-order conditions with respect to the indicated variables are:

$$C:\ (1-a)C^{-1} = \lambda, \tag{5a}$$

$$R: \beta K^{\alpha}R^{\beta-1}Z^{\eta} = \theta + p, \tag{5b}$$

$$K: \lambda\left(\alpha K^{\alpha-1}R^{\beta}Z^{\eta} - \delta\right) = -\dot{\lambda} + \lambda\rho. \tag{5c}$$

The optimal growth problem can be solved using expressions (5a)–(5c) together with Eq. (3), the transversality condition $\lim_{t\to\infty} \lambda K e^{-\rho t} = 0$, and the initially given physical capital $K_0$.

Eq. (5a) indicates that the islander equates the marginal utility of consumption to the marginal utility of physical capital. Expression (5b) reveals that the marginal benefit of deforestation is equal to the marginal cost of deforestation. Eq. (5c) indicates that the rate of return on physical capital equals the rate of return on consumption.

Now we specify the accumulation rule of the forest. The forest decreases due to deforestation $R$ and increases through the natural regeneration growth rate $g_0$. Moreover, the government may collect a trust fund $pR$ for forest preservation. The preservation fund per unit of forest determines the regeneration growth rate: $\phi \cdot (pR/Z)$. Thus, the evolution of the forest can be described by a formulation:

$$\dot{Z} = -R + [g_0 + \phi(pR/Z)]Z;\ 0 < \phi < 1. \tag{6}$$

The macroeconomic dynamics of the economy can then be described by (5a)–(5c), (3) and (6). These equations determine the endogenous variables $C$, $\lambda$, $K$, $R$, and $Z$.

We are now in a position to study the dynamic property of the balanced growth equilibrium. In particular, we want to know when the economy will converge to a state of collapse, just as in the case of Easter Island. From Eqs. (5a)–(5c), (3) and (6), the dynamic system can be expressed by:

$$\frac{\dot{C}}{C} = \Theta\alpha\left(\frac{R}{Z}\right)^{-\eta/\alpha} - \delta - \rho, \tag{7a}$$

$$\frac{\dot{K}}{K} = \Theta(1-\beta)\left(\frac{R}{Z}\right)^{-\eta/\alpha} - \frac{C}{K} - \delta, \tag{7b}$$

$$\frac{\dot{R}}{R} = \frac{\alpha}{1-\beta}\left(\frac{\dot{K}}{K}\right) + \frac{\eta}{1-\beta}\left(\frac{\dot{Z}}{Z}\right), \tag{7c}$$

$$\frac{\dot{Z}}{Z} = g_0 - (1-\phi p)\frac{R}{Z}, \tag{7d}$$

where $\Theta = [(\theta + p)/\beta]^{(\alpha-1)/\alpha} > 0$.

In order to deal with the dynamic equations of the economy, we follow Barro and Sala-i-Martin (1995) to define the consumption-capital ratio as $x = C/K$ and the deforestation rate as $h = R/Z$. From Eqs. (7a)–(7d), the dynamic system expressed in terms of the transformed variables $x$ and $h$ can be derived:

$$\frac{\dot{x}}{x} = -\Theta\eta h^{-\eta/\alpha} + x - \rho, \tag{8a}$$

$$\frac{\dot{h}}{h} = \frac{\alpha}{1-\beta}\left[\Theta(1-\beta)h^{-\eta/\alpha} - x - \delta - g_0 + (1-\phi p)h\right]. \tag{8b}$$

## 3. Balanced growth equilibrium

At the steady-state equilibrium, the economy is characterized by $\dot{x} = \dot{h} = 0$. Based on Eqs. (8a) and (8b), the corresponding steady-state values $\tilde{x}$ and $\tilde{h}$ are determined by:

$$-\Theta\eta\tilde{h}^{-\eta/\alpha} + \tilde{x} - \rho = 0, \tag{9a}$$

$$\Theta(1-\beta)\tilde{h}^{-\eta/\alpha} - \tilde{x} - \delta - g_0 + (1-\phi p)\tilde{h} = 0. \tag{9b}$$

Given that $x = C/K$ and that $\tilde{x}$ is constant along the balanced growth equilibrium, the consumption growth rate is identical to the capital growth rate. In the balanced-growth equilibrium, given Eqs. (2) and (5b), we know that the growth rates of output, capital, deforestation and the forest stock are all equal, i.e., each of $C$, $K$, $R$, $Z$ and $Y$ grows at a common rate $\tilde{\gamma}$. Furthermore, it follows from Eq. (7d) that, in the balanced-growth equilibrium, the common growth rate $\tilde{\gamma}$ can be expressed as follows:

$$\tilde{\gamma} = g_0 - (1-\phi p)\tilde{h}. \tag{10}$$

### 3.1. Case 1: the laissez faire scenario

It is quite clear from Eq. (10) that the common growth rate is $\tilde{\gamma} = g_0 - \tilde{h}$ when the government does not collect a trust fund for forest preservation, i.e., $p = 0$. This laissez faire growth rate may be negative either when the geographic condition is bad so that $g_0$ is small,[6] or when human exploitation rate is high ($\tilde{h}$ large). Along the balanced growth equilibrium, with a given initial forest stock $Z_0$, the time path of the forest stock can thus be expressed as:

$$Z_t = Z_0 e^{\tilde{\gamma}t} \tag{11}$$

when $\tilde{\gamma} < 0$, we know that $\dot{Z}_t < 0$ and $\ddot{Z}_t > 0$, implying that the forest stock $Z$ falls at a decreasing rate and eventually reaches zero. In our paper the situation where the balanced growth rate is non-positive is referred to as "the growth toward extinction." This case corresponds to what happened on Easter Island, perhaps due to their overexploiting the natural resources, leading to environmental degradation and cultural extinction.

At the first glance, (10) tells us that the imposition of a deforestation tax helps increase the forest growth rate. But in an endogenous growth model, the final outcome will depend on how individuals change their exploitation rate $\tilde{h}$ in response to this tax.

### 3.2. The impact of a deforestation tax

In this subsection we examine how the deforestation tax affects macroeconomic performance. From Eqs. (9a), (9b) and (10), the following steady-state relationship can be derived from straightforward comparative statics:

$$\frac{\partial\tilde{h}}{\partial p} = -\frac{\Theta(1-\alpha)(\theta+p)^{-1}\tilde{h}^{-\eta/\alpha} + \phi\tilde{h}}{\Theta\eta\tilde{h}^{-(\eta+\alpha)/\alpha} - (1-\phi p)}, \tag{12a}$$

$$\frac{\partial\tilde{x}}{\partial p} = \frac{\Theta\eta(\theta+p)^{-1}\tilde{h}^{-\eta/\alpha}[(1-\phi p)(1-\alpha) + \phi\eta(\theta+p)]}{\alpha\left[\Theta\eta\tilde{h}^{-(\eta+\alpha)/\alpha} - (1-\phi p)\right]}, \tag{12b}$$

[6] Diamond (2005 pp. 116–118) indicates that tree growth on Easter Island is much slower because of the higher latitude, lower rainfalls, and lower volcanic ash fall-out. Hence, natural regeneration growth cannot supplement the excess felling of trees on dry cold islands.

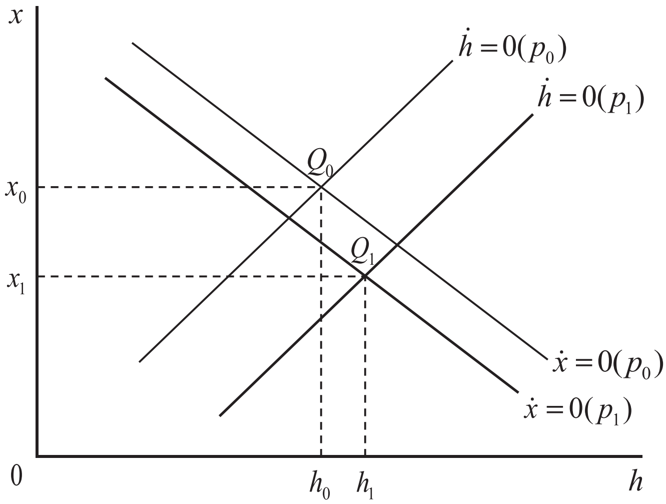


**Fig. 1.** The case when a higher $p$ increases the deforestation rate.

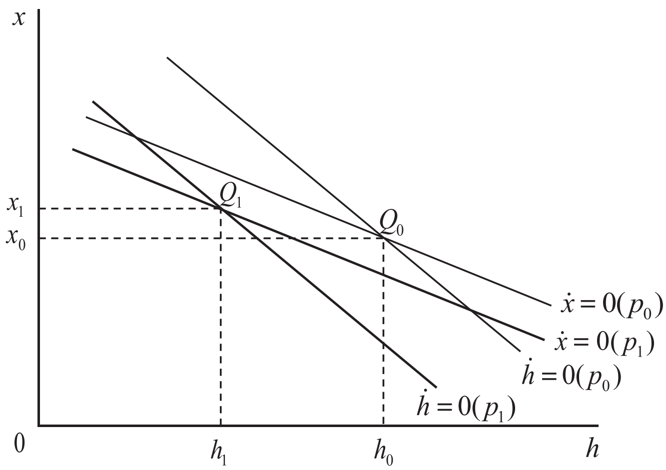


**Fig. 2.** The case when a higher $p$ decreases the deforestation rate.

$$\frac{\partial\tilde{\gamma}}{\partial p}=\frac{\Theta(\theta+p)^{-1}\tilde{h}^{-\eta/\alpha}[(1-\phi p)(1-\alpha)+\phi\eta(\theta+p)]}{\Theta\eta\tilde{h}^{-(\eta+\alpha)/\alpha}-(1-\phi p)}. \tag{12c}$$

It is clear from Eq. (12a)–(12c) that in the balanced growth equilibrium a rise in the deforestation tax may either boost or reduce the consumption–capital ratio, the deforestation rate and the common growth rate, depending upon the relative size between $\Theta\eta\tilde{h}^{-(\eta+\alpha)/\alpha}$ and $(1-\phi p)$.

A graphical presentation will be helpful to our understanding of the comparative static results reported in Eq. (12a)–(12c). In both Fig. 1 and Fig. 2, the loci $\dot{x}=0$ and $\dot{h}=0$ trace all combinations of $x$ and $h$ that satisfy Eq. (9a) and (9b), respectively. Appendix A provides a detailed derivation of the slopes of the loci $\dot{x}=0$ and $\dot{h}=0$. Moreover, Figs. 1 and 2 present the phase diagrams corresponding to the cases where the degree of forest destruction is relatively high $\left((1-\phi p)>\Theta\eta\tilde{h}^{-(\eta+\alpha)/\alpha}\right)$ and the degree of forest destruction is relatively low $\left((1-\phi p)<\Theta\eta\tilde{h}^{-(\eta+\alpha)/\alpha}\right)$, respectively.[7] In Fig. 1, in response to a rise in the deforestation tax from $p_0$ to $p_1$, the $\dot{x}=0(p_0)$ schedule shifts leftward to $\dot{x}=0(p_1)$, and the $\dot{h}=0(p_0)$ schedule shifts rightward to $\dot{h}=0(p_1)$. The balanced growth

[7] From Eq. (7d), we know that $(1-\phi p)$ measures the impact of the degree of forest destruction on the dynamics of the forest.

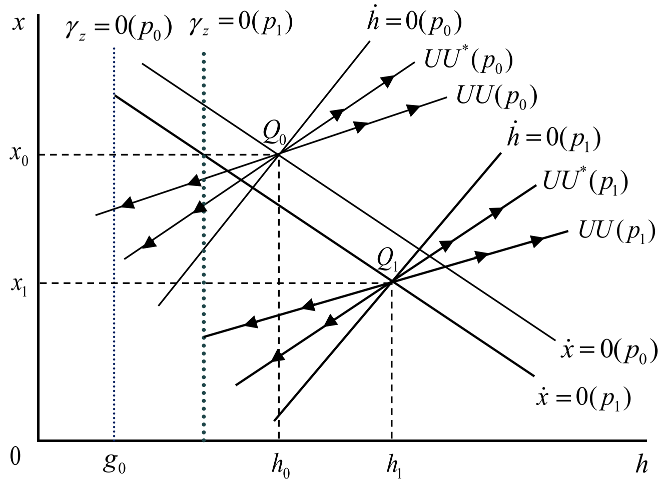


**Fig. 3.** Change in the equilibrium within the same converging zone.

equilibrium changes from $Q_0$ to $Q_1$. At the new stationary equilibrium, $\widetilde{h}$ rises from $h_0$ to $h_1$ but $\widetilde{x}$ falls from $x_0$ to $x_1$. Based on Eq. (10), the balanced growth rate is *lowered* from its initial level to a new level.

In Fig. 2, following a rise in the deforestation tax from $p_0$ to $p_1$, both $\dot{x} = 0(p_0)$ and $\dot{h} = 0(p_0)$ move leftward to $\dot{x} = 0(p_1)$ and $\dot{h} = 0(p_1)$, respectively. The balanced growth equilibrium changes from $Q_0$ to $Q_1$. At the new stationary equilibrium, $\widetilde{h}$ falls from $h_0$ to $h_1$ but $\widetilde{x}$ rises from $x_0$ to $x_1$. Based on Eq. (10), the balanced growth rate is *increased* from its initial level to a new level.[8]

The economic intuition for the above results is straightforward. On the one hand, a rise in the deforestation tax enhances the deforestation cost (referring to Eq. (5b)). In response to a rise in the deforestation cost, the islander is inclined to take action to lower the amount of deforestation. Due to the drop in deforestation, the future forest stock will increase, which in turn increases the marginal product of physical capital (henceforth *MPK*), thereby raising the balanced growth rate. On the other hand, a higher deforestation tax will reduce environmental degradation and thereby improve the quality of the environment, leading to a boost in the future forest stock. In the face of such a situation, the islander tends to sacrifice current consumption, thereby holding more physical capital in the future. Therefore, *MPK* is reduced in response, depressing the balanced growth rate. Given these two opposing forces, a rise in the deforestation tax has an ambiguous effect on the balanced growth rate.

As is well-known, sustainability is not merely a problem of comparative static analysis of steady states; even if an environmental policy changes the steady state from a bad one to a good one, our economy may not be able to move toward the new state. We will show below the key is the change of the stability zone.

## 4. Transitional adjustments of a shock in the deforestation tax

By using the graphical apparatus as in Figs. 1–2, we are now in a position to examine whether the government can set an appropriate deforestation tax to put a stop to the possibility of economic collapse. In addition, we would like to examine how the economy would evolve in response to a rise in the deforestation tax. We will highlight the analysis here; the complete dynamic analysis is more cumbersome and is presented in Appendix A.

### *4.1. The threshold zone for sustainable growth*

For the purpose of explaining the economic collapse, from Eq. (7d), we can construct the zero growth line of the forest in association with $p_0(=0)$, namely, $\gamma_Z = 0(p_0)$ line (see Fig. 3), which is the threshold level that leads the economy to move toward sustainability or collapse. Given $\partial\gamma_Z/\partial h < 0$ for $p \in [0, 1/\phi)$, we can infer that any point located to the right-hand side of the $\gamma_Z = 0(p_0)$ line is associated with $\gamma_Z < 0$, implying that the forest will eventually be depleted and that the economy will eventually collapse. By contrast, any point located to the left-hand side of the $\gamma_Z = 0(p_0)$ line is associated with $\gamma_Z > 0$, implying that the economy and forest will eventually exhibit sustainable growth. Moreover, given $(\partial h/\partial p)|_{\gamma_Z=0} > 0$ for $p \in (0, 1/\phi)$, we can deduce that, in response to a rise in the deforestation tax from $p_0$ to $p_1$, the $\gamma_Z = 0(p_0)$ schedule will shift rightward to $\gamma_Z = 0(p_1)$ in response.[9]

[8] Figs. 1 and 2 describe the situation $p < [(1-\alpha) + \phi\eta\theta]/\phi\beta$. Under the situation $p > [(1-\alpha) + \phi\eta\theta]/\phi\beta$, the figure is similar to Fig. 2, but $\widetilde{x}$ is lowered from its initial level to a new level. We skip the related discussion.

[9] By being reminded that $\gamma_Z = g_0 - (1-\phi p)h = 0$, we can then infer that the value of $h$ is negative as $p > 1/\phi$. To be specific, in response to a rise in the deforestation tax from $p_0$ to $p_1$, the $\gamma_Z = 0(p_0)$ schedule shifts leftward to $\gamma_Z = 0(p_1)$ which is located in the second quadrant in the $(x, h)$ -space. Moreover, given $\partial\gamma_Z/\partial h > 0$ for $p > 1/\phi$, we know that any point located to the right-hand side of the $\gamma_Z = 0(p_0)$ line is associated with $\gamma_Z > 0$.

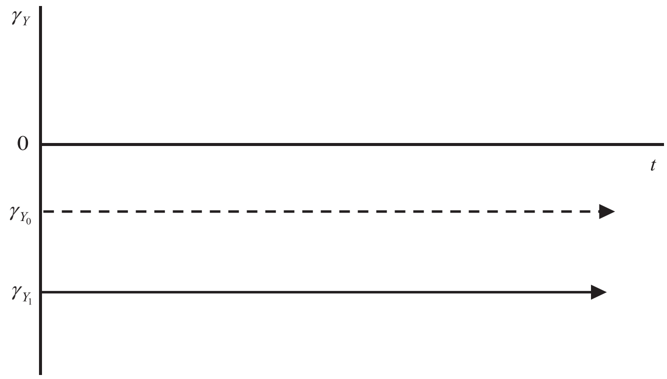


**Fig. 4.** Dynamics of $\gamma_Y$ in Fig. 3.

To characterize the impact of a change in the deforestation tax, based on Eqs. (7a) and (7d), we first define a threshold value of the deforestation tax, namely, $p^d$, that satisfies $\widetilde{\gamma} = 0$ at the balanced growth equilibrium. The threshold value is given by:

$$p^d = \frac{1}{\phi}\left[1 - g_0(\Theta\alpha/(\rho+\delta))^{-\alpha/\eta}\right].$$

If $p_1 \in (p^d, 1/\phi)$, the stationary equilibrium is located on the left-hand side of $\gamma_Z = 0(p_1)$, and the economy is characterized by a positive balanced equilibrium. This situation implies that the economy moves eventually to a configuration of sustainability. However, if $p_1 \in (0, p^d)$, the economy is featured by a negative balanced growth equilibrium and hence eventually moves to a configuration of non-sustainability.

### 4.2. Consequences of a shock in the deforestation tax

As mentioned in Section 3, a rise in the deforestation tax may either boost or depress the balanced growth rate, depending upon the relative degree of forest destruction. Thus the discussion can be broken down into the following cases.

#### 4.2.1. The scenario with an irreversible fate $\left((1-\phi p) > \Theta\eta\widetilde{h}^{-(\eta+\alpha)/\alpha}\right)$

In Fig. 3, suppose that the initial equilibrium $Q_0$ in association with $p_0 = 0$ is on the right-hand side of the $\gamma_Z = 0(p_0)$ line; the initial consumption–capital ratio and deforestation rate are $x_0$ and $h_0$, respectively. This implies that the forest is gradually vanishing by a non-positive steady-state growth rate (i.e., $\gamma_{Z_0} < 0$).

Suppose that at time $t = 0$ the government raises the deforestation tax from $p_0(=0)$ to $p_1(>0)$. In response to this tax change, both the $\dot{x} = 0(p_0)$ and the $\dot{h} = 0(p_0)$ schedules shift downward to $\dot{x} = 0(p_1)$ and $\dot{h} = 0(p_1)$, respectively. Meanwhile, the $\gamma_Z = 0(p_0)$ schedule shifts rightward to $\gamma_Z = 0(p_1)$. The steady-state equilibrium moves from $Q_0$ to $Q_1$; $\widetilde{h}$ rises from $h_0$ to $h_1$ but $\widetilde{x}$ falls from $x_0$ to $x_1$. It should be noted that, in association with a relatively higher degree of forest destruction, the new stationary equilibrium point $Q_1$ is still located on the right-hand side of $\gamma_Z = 0(p_1)$.

Two points should be noted here before we proceed to study the economy's dynamic adjustment. First, for expository convenience, let $0^-$ and $0^+$ denote the instant before and instant after the policy implementation, respectively. Second, as the deforestation tax increases from $p_0$ to $p_1$ at the moment $0^+$, the economy should exactly reach the point of the new stationary equilibrium $Q_1$ since the system is characterized by global instability. Based on these conditions, as depicted in Fig. 3, at time $0^+$ the economy will instantly jump from the initial point $Q_0$ to the new stationary point $Q_1$; and at the same time $h_0$ rises to $h_1$ and $x_0$ falls to $x_1$.

We can now discuss the focal point of this paper: how do the rate of output growth and the consumption–capital ratio adjust over time? Let $\gamma_Y$ be the rate of output growth. From Eqs. (2), (7c), (7d), and (8b), we have:

$$\gamma_Y = \frac{\dot{h}}{h} + \gamma_Z = \frac{1}{1-\beta}[\eta g_0 - \eta h(1-\phi p) - \alpha(x+\delta)] + \Theta\alpha h^{-\eta/\alpha}. \tag{13}$$

Following an increase in the deforestation tax, in Fig. 3, the economy will immediately jump from the initial point $Q_0$ to the new stationary point $Q_1$ at that instant of policy implementation. From $0^+$ onward, the economy stays put forever at point $Q_1$. In Fig. 4, the rate of output growth corresponding to the initial equilibrium $Q_0$ is $\gamma_{Y_0}\left(=\gamma_{Z_0}\right) < 0$.[10] Given $\partial\gamma_Z/\partial h < 0$ and Eq. (13), we can infer that, following a rise from $h_0$ to $h_1$ at the instant of policy implementation exhibited in Fig. 3, the rate of output growth will discretely fall

[10] It should be noted that both points of $Q_0$ and $Q_1$ are featured with $\dot{h} = \dot{x} = 0$.

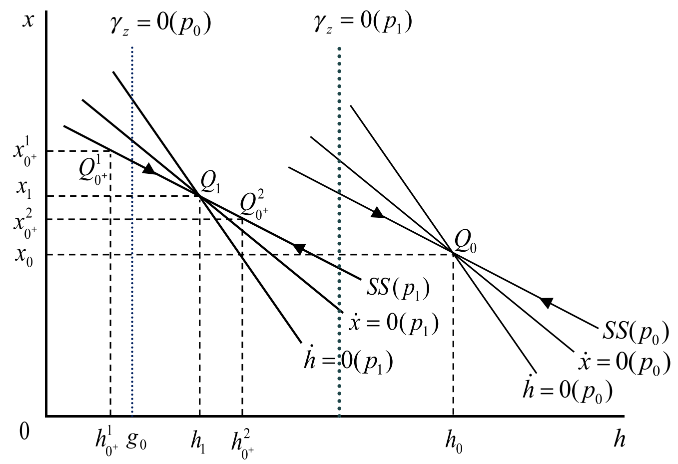


**Fig. 5.** Change in the equilibrium across different converging zones.

from $\gamma_{Y_0}$ to $\gamma_{Y_1}$, and then remain intact at the level $\gamma_{Y_1}$. As one can see, the economy in this case cannot escape from non-sustainability to sustainability even when the government raises the deforestation tax. The change in $\gamma_Y$ with respect to time is depicted in Fig. 4.

### 4.2.2. The scenario with a reversible fate $\left((1-\phi p)<\Theta\eta\tilde{h}^{-(\eta+\alpha)/\alpha}\right)$

In Fig. 5, suppose that the initial equilibrium $Q_0$ in association with $p_0(=0)$ is established on the right-hand side of the $\gamma_Z = 0(p_0)$ line; the initial consumption–capital ratio and deforestation rate are $x_0$ and $h_0$, respectively. Following a rise in the deforestation tax from $p_0(=0)$ to $p_1(>0)$ at time $t = 0$, both the $\dot{x} = 0(p_0)$ and $\dot{h} = 0(p_0)$ schedules shift leftward to $\dot{x} = 0(p_1)$ and $\dot{h} = 0(p_1)$, respectively. Meanwhile, the $\gamma_z = 0(p_0)$ schedule shifts rightward to $\gamma_z = 0(p_1)$. The new stationary equilibrium $Q_1$ may then be located on the right-hand side of $\gamma_Z = 0(p_1)$ or the left-hand side of it depending upon the size of the deforestation tax. In what follows we only deal with the case where the new stationary $Q_1$ is located on the left-hand side of $\gamma_Z = 0(p_1)$, i.e., $p_1 \in (p^d, 1/\phi)$.

Upon the shock of an increase in the deforestation tax, as depicted in Fig. 5, two adjustment paths are possibly present. If the degree of the islander's sensitivity to a boost in the deforestation tax is higher, then at the instant $0^+$, the economy will instantly jump from point $Q_0$ to a point like $Q^1_{0^+}$; at the same time $h_0$ falls to $h^1_{0^+}$ and $x_0$ rises to $x^1_{0^+}$. Thereafter, from $0^+$ onwards, $h$ continues to increase and $x$ continues to decrease as the economy moves along the $SS(p_1)$ curve toward its stationary point $Q_1$, where SS refers to the "stable branch" of the phase diagram. On the other hand, if the degree of the islander's sensitivity to a boost in the deforestation tax is lower, then at the instant $0^+$, the economy will instantly jump from point $Q_0$ to a point like $Q^2_{0^+}$; and $h_0$ falls to $h^2_{0^+}$ and $x_0$ rises to $x^2_{0^+}$. Thereafter, from $0^+$ onwards, $h$ continues to decrease and $x$ continues to increase as the economy moves along the $SS(p_1)$ curve toward its stationary point $Q_1$.

Fig. 6 depicts how $\gamma_Y$ will adjust over time in association with the dynamic paths in the $(x, h)$ plane in Fig. 5. Given that the initial equilibrium $Q_0$ is established on the right-hand side of the $\gamma_Z = 0(p_0)$ line, the rate of output growth corresponding to the initial equilibrium $Q_0$ is associated with $\gamma_{Y_0} < 0$.

As shown in Fig. 5, given $\partial\gamma_Z/\partial h < 0$, at time $0^+$ an immediate reduction in $h$ indicates that $\gamma_Y$ discretely rises on impact. If the economy jumps to point $Q^1_{0^+}$, then $h$ continues to decrease from $Q^1_{0^+}$ to $Q_1$, implying that $\gamma_Y$ falls at a decreasing rate.[11] The entire adjustment of $\gamma_Y$ will then be displayed at path (a) in Fig. 6. On the other hand, if the economy jumps to point $Q^2_{0^+}$, then $h$ continues to increase from $Q^2_{0^+}$ to $Q_1$, implying that $\gamma_Y$ rises at a decreasing rate. Therefore, the entire adjustment of $\gamma_Y$ is exhibited by path (b) in Fig. 6. The graphical analysis in Figs. 5 and 6 reveals that, in the face of the lower degree of forest destruction, the government could set an appropriate deforestation tax to move the economy from non-sustainability to sustainability.

### 4.3. Two changes in a phase diagram

In summarizing the above discussion, we see that there are two changes in the phase diagram associated with an increase in the forestation tax. The first is the change in the steady state equilibrium from $Q_0$ to $Q_1$, and the other is the shift in the $\gamma_Z = 0$ vertical line from $\gamma_Z = 0(p_0)$ to $\gamma_Z = 0(p_1)$ that separates the converging zones. In Fig. 3, the change in $Q$ caused by the tax increase remains in the same zone, so that the unsustainable state cannot be reversed. In Fig. 5, the new equilibrium $Q_1$ gets out of the originally unsustainable

[11] From Eqs. (9a), (9b) and (13) as well as the slope of the loci $\dot{x} = 0$, we can infer the following result: $\frac{\partial\dot{\gamma}_Y}{\partial h} = -\frac{1}{1-\beta}\left[\eta(1-\phi p) + \Theta\eta(1-\beta)\tilde{h}^{-(\eta+\alpha)/\alpha} + \alpha\frac{\dot{x}}{h}\right] < -\frac{\eta\tilde{h}^{-1}}{1-\beta}[\rho+\delta+g_0] < 0$.

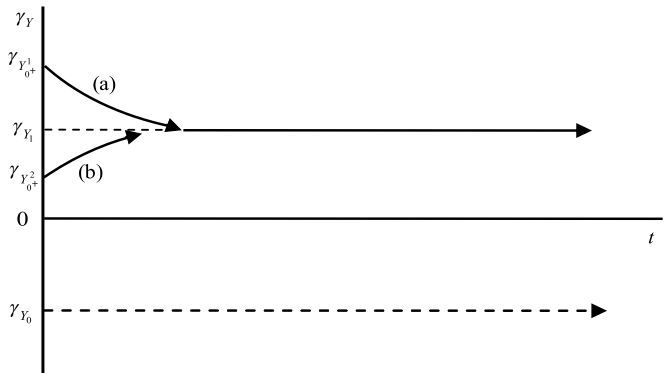


**Fig. 6.** Dynamics of $\gamma_Y$ in Fig. 5.

zone, and the starting point of a dynamic path jumps to a sustainable stable branch. Therefore, the key to a successful sustainable policy concerns not only the position of a steady state, but also the converging zone that covers the steady state.

Note that in an endogenous growth model, these two changes are caused by both direct shifts in parameters and by indirect shifts in agents' decisions in response to such parametric shifts. In particular, it is the reduced deforestation rate that alters the steady state control variables, which in turn change the steady state equilibrium and the converging zone. This comparative dynamic analysis seems to be more reasonable than that in the conventional exogenous growth setup for Easter Island.

## 5. Concluding remarks

This paper sets up an endogenous growth model with a renewable resource and applies it to the case of Easter Island which is a typical example of economic collapse. We then use the model to examine the possible consequence of adjusting the deforestation tax on the balanced growth rate and trace the dynamic responses of the output growth rate following an unanticipated rise in the deforestation tax.

Two main findings emerge from the analysis. First, a rise in the deforestation tax may either boost or deter the balanced economic growth rate, depending on the degree of forest destruction. Second, when the degree of forest destruction is relatively high, the economy cannot escape from non-sustainability to sustainability when the government raises the deforestation tax. However, when the degree of forest destruction is relatively low, the government can set an appropriate deforestation tax to move the economy from non-sustainability to sustainability. The restoration of sustainability involves two things: one is the change of steady states, and the other is the shift of converging zone. And it is the latter that saves the economic dynamics out of the collapsing trap.

Based on our analysis, it is clear that economic growth and the environment may be unsustainable, even if the agent is altruistic toward her offspring and optimizes on behalf of her offspring. This observation coincides with the general belief that sustainability is a problem mainly because there is an under-representation of the welfare of future lifetimes. However, when the government intervenes in the environmental price mechanism and sets an appropriate deforestation tax, it is possible that the agent may take action to lower the amount of deforestation, thereby making the environment sustainable. Presumably, this kind of remedy can be achieved only when the future lifetime's right to natural resources is protected by a very high level of law (such as the Constitution) that can over-ride the selfish concerns of the present single period.

## Appendix A

We shall concentrate on the shock of the deforestation tax, i.e., a change in $p$. Let $\tilde{x}$ and $\tilde{h}$ be the stationary values of $x$ and $h$, respectively. Then, linearizing the dynamic system (8a) and (8b) around the steady-state equilibrium yields:

$$\begin{bmatrix}\dot{x}\\ \dot{h}\end{bmatrix} = \begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}\begin{bmatrix}x-\tilde{x}\\ h-\tilde{h}\end{bmatrix} + \begin{bmatrix}a_{13}\\ a_{23}\end{bmatrix}dp, \tag{A1}$$

where

$$a_{11} = \tilde{x},\quad a_{12} = \frac{1}{\alpha}\left[\Theta\eta^2\tilde{x}\tilde{h}^{-(\eta+\alpha)/\alpha}\right],\quad a_{13} = \frac{1}{\alpha}\left[\Theta\eta(1-\alpha)(\theta+p)^{-1}\tilde{x}\tilde{h}^{-\eta/\alpha}\right],$$
$$a_{21} = -\frac{\alpha\tilde{h}}{(1-\beta)},\quad a_{22} = -\frac{\alpha\tilde{h}}{(1-\beta)}\left\{\frac{1}{\alpha}\left[\Theta\eta(1-\beta)\tilde{h}^{-(\eta+\alpha)/\alpha}\right]-(1-\phi p)\right\}$$
$$a_{23} = -\frac{\alpha\tilde{h}}{(1-\beta)}\left\{\frac{1}{\alpha}\left[\Theta(1-\alpha)(1-\beta)(\theta+p)^{-1}\tilde{h}^{-\eta/\alpha}\right]+\phi\tilde{h}\right\}.$$

The trace and determinant of the Jacobian are given by:

$$Tr = s_1 + s_2 = a_{11} + a_{22} = \tilde{x} - \frac{\alpha \tilde{h}}{(1-\beta)} \left\{ \frac{1}{\alpha} \left[ \Theta\eta(1-\beta)\tilde{h}^{-(\eta+\alpha)/\alpha} \right] - (1-\phi p) \right\}, \tag{A2}$$

$$Det = s_1 s_2 = a_{11}a_{22} - a_{12}a_{21} = -\frac{\alpha \tilde{x}\tilde{h}}{(1-\beta)} \left[ \Theta\eta\tilde{h}^{-(\eta+\alpha)/\alpha} - (1-\phi p) \right], \tag{A3}$$

where $s_1$ and $s_2$ are two characteristic roots of the dynamic system.

As indicated in Eqs. (8a) and (8b), the dynamic system has two jump variables, $x$ and $h$. As a result, if $s_1 s_2 > 0$ holds, the steady-state equilibrium is locally determinate and there exists a unique growth path converging to it; and this model exhibits indeterminacy if $s_1 s_2 < 0$ holds.

It follows from Eqs. (8a) and (8b) that the general solution for $x$ and $h$ can be described by:

$$x(t) = \tilde{x}(p) + A_1 e^{s_1 t} + A_2 e^{s_2 t}, \tag{A4}$$

$$h(t) = \tilde{h}(p) + \frac{\alpha(s_1 - \tilde{x})}{\Theta\eta^2 \tilde{x}\tilde{h}^{-(\eta+\alpha)/\alpha}} A_1 e^{s_1 t} + \frac{\alpha(s_2 - \tilde{x})}{\Theta\eta^2 \tilde{x}\tilde{h}^{-(\eta+\alpha)/\alpha}} A_2 e^{s_2 t}, \tag{A5}$$

where $A_1$ and $A_2$ are as yet undetermined coefficients. Furthermore, Eq. (A3) reveals that this model exhibits two scenarios: local determinacy $\left((1-\phi p) > \Theta\eta\tilde{h}^{-(\eta+\alpha)/\alpha}\right)$ and local indeterminacy $\left((1-\phi p) < \Theta\eta\tilde{h}^{-(\eta+\alpha)/\alpha}\right)$.[12] From Eqs. (8a) and (8b), the slopes of the loci $\dot{x} = 0$ and $\dot{h} = 0$ are:

$$\left.\frac{\partial x}{\partial h}\right|_{\dot{x}=0} = -\frac{1}{\alpha} \left[ \Theta\eta^2 \tilde{x}\tilde{h}^{-(\eta+\alpha)/\alpha} \right], \tag{A6}$$

$$\left.\frac{\partial x}{\partial h}\right|_{\dot{h}=0} = (1-\phi p) - \frac{1}{\alpha} \left[ \Theta\eta(1-\beta)\tilde{h}^{-(\eta+\alpha)/\alpha} \right]. \tag{A7}$$

When provided with information concerning the direction of the arrows in Fig. A1 and A2, we can sketch all possible trajectories associated with the local determinacy case and the local indeterminacy case, respectively.[13] Under the local determinacy case, Fig. A1 shows that the $\dot{x} = 0$ schedule and the $\dot{h} = 0$ schedule intersect once at $Q_0$. In addition, the unstable branches $UU^*$ and $UU$ are associated with $A_2 = 0$ and $A_1 = 0$ in Eqs. (A4) and (A5), respectively. Moreover, all other unstable trajectories in Fig. A1 correspond to the values $A_2 \neq 0$ and $A_1 \neq 0$ in Eqs. (A4) and (A5). Under the local indeterminacy case, Fig. A2 shows that the $\dot{x} = 0$ schedule and the $\dot{h} = 0$ schedule intersect once at $Q_0$. In addition, the stable branch $SS$ and the unstable branch $UU$ are associated with $A_2 = 0$ and $A_1 = 0$ in Eqs. (A4) and (A5), respectively. Moreover, all other unstable trajectories in Figure A2 correspond to the values $A_2 \neq 0$ and $A_1 \neq 0$ in Eqs. (A4) and (A5). Appendix B and Appendix C provide a detailed derivation for all possible trajectories associated with the local determinacy case and the local indeterminacy case, respectively.

## Appendix B

This appendix derives the slopes of the loci $UU^*$ and $UU$ associated with $A_2 = 0$ and $A_1 = 0$, respectively, as well as the common feature of these divergent trajectories associated with $A_2 \neq 0$ and $A_1 \neq 0$, respectively. From Eqs. (A4) and (A5), we can infer the following results:

$$\left.\frac{\partial x}{\partial h}\right|_{UU^*} = \frac{a_{12}}{s_1 - a_{11}} \gtrless 0, \tag{B1}$$

$$\left.\frac{\partial x}{\partial h}\right|_{UU} = \frac{a_{12}}{s_2 - a_{11}} \gtrless 0. \tag{B2}$$

[12] For expository convenience, we assume that $s_2 > s_1 > 0$ if the steady-state equilibrium is locally determinate. If the model exhibits indeterminacy, we assume that $s_1 < 0 < s_2$.

[13] To save space, we only discuss two cases. First, under the situation where there is local determinacy, the $\dot{x} = 0$ locus is downward sloping and the $\dot{h} = 0$ locus is upward sloping. Second, under the situation where there is local indeterminacy, both the $\dot{x} = 0$ locus and the $\dot{h} = 0$ locus are downward sloping, and the $\dot{h} = 0$ locus is steeper than the $\dot{x} = 0$ locus.

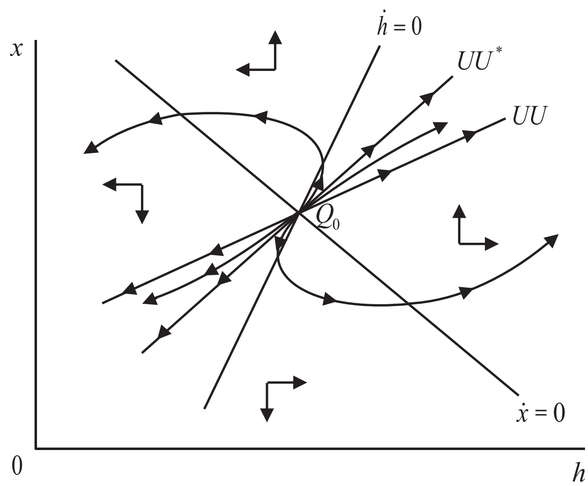


**Fig. A1 The local determinacy case.**

To save space, under $s_1 s_2 > 0$ we only mention that the $UU$ locus and the $UU^*$ locus are upward sloping. Moreover, from Eqs. (A2) and (A3), we can infer the following relationship:

$$\left(1-\frac{a_{22}}{s_1}\right)(s_1-a_{11}) = \frac{a_{21}a_{12}}{s_1}<0, \tag{B3}$$

$$-\frac{a_{22}}{a_{21}}\left(1-\frac{s_1}{a_{22}}\right) = \frac{a_{12}}{s_1-a_{11}}>0, \tag{B4}$$

where $(s_1 - a_{11}) > 0$ and $0 < (1 - s_1/a_{22}) < 1$. Substituting Eqs. (A7) and (B1) into Eq. (B4), we can observe that the $h = 0$ locus is steeper than the $UU^*$ locus.

Using Eqs. (A4) and (A5), we have the following feature of other divergent trajectories:

$$\lim_{t\to-\infty}\frac{\dot{x}}{\dot{h}} = \frac{a_{12}}{s_1-a_{11}}, \tag{B5}$$

$$\lim_{t\to\infty}\frac{\dot{x}}{\dot{h}} = \frac{a_{12}}{s_2-a_{11}}. \tag{B6}$$

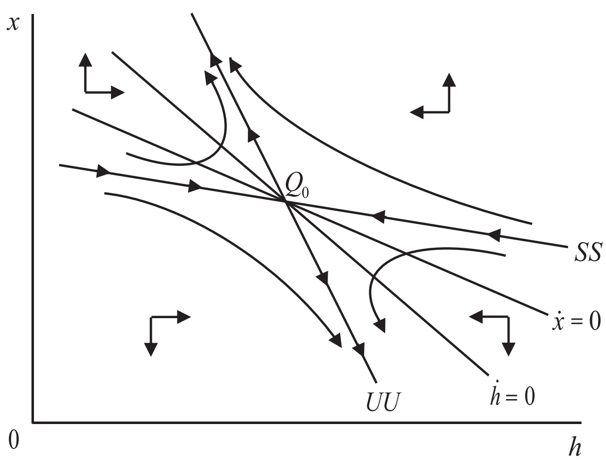


**Fig. A2 The local indeterminacy case.**

As a result, the common feature of these divergent paths is that they start from the unstable node $Q_0$ with the slope of the $UU^*$ locus, and their slope approaches asymptotically to that of the $UU$ locus as time goes by.

## Appendix C

This appendix derives the slopes of the loci $SS$ and $UU$ associated with $A_2 = 0$ and $A_1 = 0$, respectively, as well as the common feature of these divergent trajectories associated with $A_2 \neq 0$ and $A_1 \neq 0$, respectively. Using (A4) and (A5), the slopes of the loci $SS$ and $UU$ can be obtained:

$$\left.\frac{\partial x}{\partial h}\right|_{SS} = \frac{a_{12}}{s_1 - a_{11}} < 0, \tag{C1}$$

$$\left.\frac{\partial x}{\partial h}\right|_{UU} = \frac{a_{12}}{s_2 - a_{11}} \gtrless 0. \tag{C2}$$

Moreover, from Eqs. (A2) and (A3), we can infer the following relationship:

$$\left(1 - \frac{a_{22}}{s_2}\right)(s_2 - a_{11}) = \frac{a_{21}a_{12}}{s_2} < 0, \tag{C3}$$

where $(1 - a_{22}/s_2) > 0$ and $(s_2 - a_{11}) < 0$. Based on Eq. (C3), it is clear that the $UU$ locus is downward sloping, $(\partial x/\partial h)|_{UU} = a_{12}/(s_2 - a_{11}) < 0$. Then, from Eqs. (A6), (C1), and (C3), we have:

$$\left.\frac{\partial x}{\partial h}\right|_{SS} - \left.\frac{\partial x}{\partial h}\right|_{\dot{x}=0} = \frac{s_1 a_{12}}{(s_1 - a_{11})a_{11}} > 0, \tag{C4}$$

$$-\frac{a_{22}}{a_{21}}\left(1 - \frac{s_2}{a_{22}}\right) = \frac{a_{12}}{s_2 - a_{11}}. \tag{C5}$$

Eqs. (C4) and (C5) indicate that the $UU$ locus is steeper than the $\dot{h} = 0$ locus and the $\dot{x} = 0$ locus is steeper than the $SS$ locus.

Using Eqs. (A4) and (A5), we have the following feature of other divergent trajectories:

$$\lim_{t \to -\infty} \frac{\dot{x}}{\dot{h}} = \frac{a_{12}}{s_1 - a_{11}}, \tag{C6}$$

$$\lim_{t \to \infty} \frac{\dot{x}}{\dot{h}} = \frac{a_{12}}{s_2 - a_{11}}. \tag{C7}$$

As a result, the common feature of these divergent paths is that they start from $\infty$ $(-\infty)$ with the slope of the $SS$ locus, and their slope asymptotically approaches that of the $UU$ locus as time goes by.

## References

Aghion, P., & Howitt, P. (1988). *Endogenous growth theory*. Cambridge, MA: MIT Press.
Barro, R. J., & Sala-i-Martin, X. (1995). *Economic growth*. New York: McGraw-Hill.
Bovenberg, A. L., & de Mooij, R. A. (1997). Environmental tax reform and endogenous growth. *Journal of Public Economics*, *63*, 207–237.
Bovenberg, A. L., & Smulders, S. (1995). Environmental quality and pollution-augmenting technological change in a two-sector endogenous growth model. *Journal of Public Economics*, *57*, 369–391.
Bovenberg, A. L., & Smulders, S. (1996). Transitional impacts of environmental policy in an endogenous growth model. *International Economic Review*, *37*, 861–893.
Brander, J. A., & Taylor, M. S. (1998). The simple economics of Easter Island: A Ricardo-Malthus model of renewable resource use. *American Economic Review*, *88*, 119–138.
Chao, C. C., Laffargue, J. P., & Sgro, P.M. (2012). Environmental control, wage inequality and national welfare in a tourism economy. *International Review of Economics and Finance*, *22*, 201–207.
Chen, J. H., Lai, C. C., & Shieh, J. H. (2003). Anticipated environmental policy and transitional dynamics in an endogenous growth model. *Environmental and Resource Economics*, *25*, 233–254.
Chichilnisky, G. (1996). An axiomatic approach to sustainable development. *Social Choice and Welfare*, *13*, 231–257.
Chichilnisky, G., Heal, G. M., & Vercelli, A. (1998). *Sustainability: Dynamics and uncertainty*. MA: Kluwer Academic Publishers.
Chu, C. Y. C. (1998). *Population dynamics: A new economic approach*. New York: Oxford University Press.
Dalton, T. R., Coats, R. M., & Asrabadi, B. R. (2005). Renewable resources, property-rights regimes and endogenous growth. *Ecological Economics*, *52*, 31–41.
Diamond, J. (2005). *Collapse: How societies choose to fail or succeed*. New York: Viking Books.
Ekin, P. (1994). The environmental sustainability of economic progresses: A framework for analysis. In Jeroen C. J. M. van der Bergh, & J. van der Straaten (Eds.), *Toward sustainable development: Concepts, methods and policy*. Washington D.C.: International Society for Ecological Economics.
Erickson, J.D., & Gowdy, J. M. (2000). Resource use, institutions, and sustainability: A tale of two Pacific island cultures. *Land Economics*, *76*, 345–354.
Huang, C. H., & Cai, D. (1994). Constant returns endogenous with pollution control. *Environmental and Resource Economics*, *4*, 383–400.

Itaya, J. I. (2008). Can environmental taxation stimulate growth? The role of indeterminacy in endogenous growth models with environmental externalities. *Journal of Economic Dynamics and Control*, *32*, 1156–1180.
Laitner, J. (1990). Tax changes and phase diagrams for an overlapping generations model. *Journal of Political Economy*, *98*, 193–220.
Ligthart, J. E., & van der Ploeg, F. (1994). Pollution, the cost of public funds and endogenous growth. *Economics Letters*, *46*, 339–349.
Martinez-Alier, J., & O'Connor, M. (1999). Distributional issue: An overview. In Jeroen C. J. M. van der Bergh (Ed.), *Handbook of Environmental and Resource Economics*. Cheltenham, UK: Elward Elgar.
Obstfled, M. (1990). Intertemporal dependence, impatience, and dynamics. *Journal of Monetary Economics*, *26*, 45–75.
Pearce, D. (1998). *Economics and environment: Essays on ecological economics and sustainable development*. MA: Edward Elgar Publishing, Inc.
Pezzey, J. C. V., & Anderies, J. M. (2003). The effect of subsistence on collapse and institutional adaptation in population-resource societies. *Journal of Development Economics*, *72*, 299–320.
Reuveny, R., & Decker, C. S. (2000). Easter Island: Historical anecdote or warning for the future? *Ecological Economics*, *35*, 271–287.
Sarkar, J. (2007). Growth dynamics in a model of endogenous time preference. *International Review of Economics and Finance*, *16*, 528–542.
van Zon, A., & Yetkiner, I. H. (2003). An endogenous growth model with embodied energy-saving technical change. *Resource and Energy Economics*, *25*, 81–103.
Vella, E., Dioikitopoulos, E. V., & Kalyvitis, S. (2014). Green spending reforms, growth and welfare with endogenous subjective discounting. *Macroeconomic Dynamics* (in press).
World Commission on Environment and Development (1987). *Our common future*. The Brundtland Report. Oxford: Oxford University Press.
Yanase, A. (2011). Impatience, pollution, and indeterminacy. *Journal of Economic Dynamics and Control*, *35*, 1789–1799.