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An extension from network DEA to copula-based network SFA: Evidence from the U.S. commercial banks in 2009

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ABSTRACT

The main contribution of network DEA deals with the dual role of deposits in the bank production process. Deposits are first viewed as an intermediate output, produced by, e.g., fractions of labor and capital. This intermediate output is next used as an input in the second process, together with the remaining labor and capital, to produce output combinations. A problem occurs in that network DEA suffers from the difficulty of determining the fractions of labor and capital used in the first process. This research thus develops an economic model to characterize the underlying multi-stage technologies and proposes a copula-based econometric model to identify parameters of the structural equations, including the fractional parameters, by the maximum likelihood. Our model also estimates technical efficiencies of the stochastic production and cost frontiers. We collect data from U.S. banks in 2009 to illustrate the feasibility and usefulness of our modeling, and the results are promising.

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1. Introduction

There is long-standing controversy in the literature on how to define a bank's outputs and inputs. The asset (intermediation) approach characterizes banks as financial intermediaries between depositors and borrowers, whose liabilities such as deposits are considered to be inputs to the intermediation process, since banks can turn these funds into various loans. Bank assets such as loans and investments are taken to be outputs that generate various forms of revenues. On the other hand, the value-added (production) approach identifies the major categories of produced deposits (demand, time, and savings) and loans (real estate, commercial, and installment) as important outputs, defines purchased funds (federal funds purchased, large CDs, foreign deposits, other liabilities for borrowed money) as financial inputs, and treats government securities and other non-loan investments as unimportant outputs. See Berger and Humphrey (1992) for details.

Although each approach is capable of characterizing some aspects of the bank production processes and hence has its own advantages, such a classification often causes contradictory and incomparable results in, e.g., measures of technical efficiency (TE), as well as scale and scope economies across studies. Even worse, under the intermediation approach a bank with few deposits and more loans will be identified as technically efficient, but may be identified as inefficient on the basis of the production approach.

This dilemma of whether to consider deposits as an input or output is partially resolved by the previous literature under the framework of network data envelopment analysis (network DEA), which has been introduced in several forms.¹ Most traditional DEA

¹ One of the forms divides the entire production process into a series of inter-dependent production stages performed by different sectors of a firm, or subunits. The network model has been widely applied in the literature due to its capability in providing insight regarding the locations of inefficiency and process-specific direction to help managers promote the firm's TE. Eppen, Gould, Schmidt, Moore, and Weatherford (1998) and Färe, Grosskopf, and Whittaker (2007) introduce various network models. Pardalos, Hearn, and Hager (1997) and Cook et al. (2010) address some advances in this research field. There are many extensions and interesting applications of network DEA models in the literature. See, for example, Färe and

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models treat their reference technologies as black boxes, by assuming that inputs are converted in this box into outputs, as noted by Färe and Grosskopf (2000), Kao and Hwang (2008), and Cook, Liang, and Zhu (2010). The internal conversion process is commonly ignored, such that researchers are only required to specify what enters the box and what exits it. Intermediate products play no role as a result. Therefore, the traditional models are incapable of identifying the sources of inefficiency embedded in different sub-units of firms. Conversely, network DEA allows for the assessment of efficiency for each subunit of an organization and offers guidance to managers on where and how to improve each subunit's performance.

In a two-stage production process, the intermediate outputs produced by employing some input mix in the first stage are specifically utilized as inputs in the second stage to manufacture other outputs. The TE scores of both stages for a bank can then be respectively evaluated. Holod and Lewis (2011) propose using network DEA to treat deposits as intermediate outputs in the first production stage by employing some portions of labor and capital. In the second stage, deposits and the remaining fractions of labor and capital are defined as inputs to produce the outputs of loans, investments, and non-interest revenues. In this manner, deposits are considered neither an input nor an output and hence the aforementioned dilemma is avoided.

Although the idea of Holod and Lewis (2011) is novel, the fractions of labor and capital consumed in the first stage are usually unknown, unless disaggregate data for each subunit are available. These fractions are not estimable under the network DEA framework. Both DEA and the stochastic frontier approach (SFA) are popular methods in the past literature on efficiency analysis of firms, in which the former is classified as a non-parametric approach and the latter is categorized as a parametric approach.² Both have their own strengths and weaknesses, but to date, only DEA can formulate and gage the production efficiency that accounts for inter-sectoral linkages, while the role that SFA plays in this area is ignored. This paper develops an economic model that embodies the two-stage network production process for banks.

This dynamic modeling recognizes multiple bank production processes and is free from judging whether more or less deposits are consistent with higher bank efficiency. We then formulate an econometric model with simultaneous equations, on the basis of the economic model. A salient feature of our modeling is that the fractions of labor and capital, used in the first stage, can be directly estimated, along with the TE scores in the two production stages for each bank. More importantly, the estimation of the fractions requires only aggregate data, rather than disaggregate data,

Whittaker (1995), Seiford and Zhu (1999), Castelli, Pesenti, and Ukovich (2001), Sexton and Lewis (2003), Lewis and Sexton (2004), Prieto and Zofio (2007), Avkiran (2009), Bogetoff, Färe, Grosskopf, Hayes, and Taylor (2009), Chen (2009), Chen, Cook, Li, and Zhu (2009), Kao (2009, 2014), Tone and Tsutsui (2009, 2014), Kao and Hwang (2008, 2010), Vaz, Camanho, and Guimarães (2010), Holod and Lewis (2011), Yang and Liu (2012), Lewis, Mallikarjun, and Sexton (2013), and Matthews (2013). Seiford and Zhu (1999) are pioneers in applying two-stage network DEA to analyze the TE scores of U.S. banks, followed by Holod and Lewis (2011) who study TE of U.S. bank holding companies with the non-oriented network DEA method.

² Numerous studies have investigated efficiency-related issues ever since Farrell (1957). They are broadly classified into two major categories: SFA and DEA. The former dates back to Aigner, Lovell, and Schmidt (1977) and Meeusen and Van Den Broeck (1977), while the latter is pioneered by Charnes, Cooper, and Rhodes (1978) and Banker, Charnes, and Cooper (1984). A salient feature of non-parametric DEA is that it does not require assuming specific functional forms for a representative firm, but since DEA is deterministic in essence, it is unable to separate the inefficiency term from the noise component. Contrary to DEA, SFA is capable of distinguishing inefficiency from the noise component, but requires specifying a particular functional form and distributional assumptions on the composed errors. Readers can reference the excellent books written by Kumbhakar and Lovell (2000) and Coelli, Rao, O'Donnell, and Battese (2005) for SFA and DEA, respectively.

i.e., individual subsector data. This feature is particularly crucial, because almost all available databanks compile aggregate data of each firm, with few databanks collecting accounting items on the basis of the subsectors of firms. Naturally, our model is also suitable for other forms of firms characterized by a series of production processes.

The simultaneous equations model consists of the first-stage production frontier of deposits and a cost frontier in the second production stage, under the assumption that banks try to maximize the intermediate output quantity of deposits and pursue cost minimization in the production of their final outputs. This appears to be the first attempt in the literature to extend efficiency measurement accounting for either or both vertically- and horizontally-integrated production technologies from network DEA to network SFA. In the context of network SFA, the fractional parameters of the inputs explicitly enter both frontiers. It is noteworthy that cost share equations must be jointly estimated in order to help specifically identify the fractional parameters that are not estimable under a DEA setting, e.g., Holod and Lewis (2011). It is widely known that a firm's economic efficiency consists of technical efficiency and allocative efficiency, for which one notes that the estimation of allocative efficiency relies on the availability of input prices in both production stages, which are usually not supported by any databases. We thus ignore this issue.

Since the two-stage production processes are interrelated, the production and cost frontiers must be jointly determined. Their disturbance terms should also be correlated so as to embody the joint decision process. In this manner, it is difficult to construct a joint distribution for the two sets of error components without relying on the copula method. This method is based on Sklar's theorem that enables one to measure the dependence between random variables, and is suitable here for considering the inter-sectoral linkage between the two production stages.³ The copula methodology allows us to derive the joint distribution from the margins of a set of random variables on the basis of the copula family. Following Lai and Huang (2013), we choose the Gaussian copula to derive the copula-based likelihood function. Lai and Huang (2013) assert that omitting the dependency component will lead to biased technical efficiency measures and a loss of efficiency for parameter estimates.

The estimation of the fractional parameters is very important, because they are the core link between subunits adopting various sub-technologies, on the one hand, and show the distribution of some inputs among production stages, on the other hand. This enables us to deeply investigate the "black box" issue and describe a bank's entire production procedure in a more complete and correct manner. Without them, dynamic effects among multi-stage production processes disappear and inter-sectoral production processes are no longer interdependent. Consequently, the network model collapses to a more appropriate conventional DEA or SFA for illustrating a simple, one-stage production process.

As mentioned above, the classical models focus entirely on the banks' inputs, outputs and measure their relative efficiencies. This perspective often fulfills the purpose of analyzing the

³ Sklar's (1959, 1973) theorem offers a sound theoretical basis for copula methodology. The theorem has invoked many research fields of application, such as statistics, risk management, finance, economics, etc. Readers can refer to Nelsen (2006) and Trivedi and Zimmer (2007) for a detailed and comprehensive description of the copula method. A number of recent studies attempt to shed light on the interdependence issues into area of efficiency analysis, e.g. Bandyopadhyay and Das (2006), Smith (2008), Shi and Zhang (2011), Carta and Steel (2012), El Mehdi and Hafner (2013), Tsay et al. (2013), Lai and Huang (2013), Amsler, Prokhorov, and Schmidt (2014), and Repke (2014). It is notable that these articles aim to relax the independence assumption between the inefficiency term and disturbance term in the context of a single equation.

overall organizational inefficiencies. However, the classical models ignore internal operational structures of the banks and are not able to identify sources of inefficiency within the production process. Therefore, they fail to measure the corresponding subunit performance of the banks and hence provide limited information to bank managers on improving organizational efficiency.

The rest of this article is organized as follows. Section 2 develops a theoretical model that splits a bank's production into two inter-related processes. Two stochastic frontiers, a production frontier and a cost frontier, are established, exemplifying a relational network model. A simultaneous equations model is next derived and some estimation issues are discussed. Section 3 briefly describes our data. Section 4 performs the empirical study to demonstrate the usefulness of our model, while the last section concludes the paper.

2. Economic model

The first stage assumes a bank, without loss of generality, employs two inputs, i.e., labor and capital, denoted by a 2-vector x , in order to produce deposits (Z). Under this assumption of a single output, it seems to be a natural choice of employing a production function to characterize the technological relationship between inputs and the output. A cost function may alternatively be chosen as the cost function contains basically the same information that the production function contains, by relying on the principle of duality. The stochastic production frontier with conventional error components is expressed as:

$$Z = f(\alpha x) e^{\nu_1 - u_1}, \quad (1)$$

where $\alpha x = (\alpha_1 x_1, \alpha_2 x_2)$, (α_1, α_2) denotes the fractions of the bank's total employment of labor and capital that are specifically consumed for the first-stage production of Z , $\nu_1 \sim N(0, \sigma_{\nu_1}^2)$ is the random disturbance term uncontrollable by bank managers, and $u_1 \sim N^+(0, \sigma_{u_1}^2)$ is the output-oriented technical inefficiency and a non-negative random variable. Variables ν_1 and u_1 are conventionally assumed to be mutually independent. The remaining fractions, $1 - \alpha = (1 - \alpha_1, 1 - \alpha_2)$, of the inputs, together with the intermediate output Z , are used to manufacture final outputs, such as loans, investments, and off-balance sheet activities in the second stage. In this manner, α plays a pivotal role due to its ability to connect the two production processes.

It is noteworthy that vector α differs from the measure of input-oriented technical efficiency, say, b , as proposed by Atkinson and Cornwell (1993, 1994). The production function of an output y with input-oriented technical inefficiency can be formulated as:

$$y = f(bx),$$

where $0 < b \leq 1$ is a scalar and signifies the input-oriented TE measure. The higher the value of b is, the more technically efficient is the firm, and vice versa. Constant b represents the degree of TE, while α represents the distribution of resources between subunits. The estimation of α and b relies on the dual cost function, since the production function alone is unable to identify them. See below. For the case of multiple outputs, the above single-stage production function should be re-expressed as a production transformation function, i.e., $F(Y, bX) = 0$.

Since the second production stage constitutes the primary and conventional stage of a bank, we assume that the bank attempts to minimize its production cost $PC^*(\cdot)$ in this stage, by employing the remaining fractions of labor and capital, as well as the intermediate

output from the first stage, i.e., deposits, to produce an m -vector of final outputs Y :

$$\begin{aligned} PC^*\left(Y, \frac{W}{b}\right) &= \min_{b\tilde{\alpha}X} \left[\frac{W'}{b} (b\tilde{\alpha}X) | F(Y, b\tilde{\alpha}X) = 0 \right] \\ &= \frac{1}{b} \min_{b\tilde{\alpha}X} [W' (b\tilde{\alpha}X) | F(Y, b\tilde{\alpha}X) = 0] \\ &= \frac{1}{b} PC(Y, W), \end{aligned} \quad (2)$$

where $\tilde{\alpha}X = (\tilde{\alpha}_1 x_1, \tilde{\alpha}_2 x_2, \tilde{\alpha}_3 Z)'$ with $\tilde{\alpha}_1 = 1 - \alpha_1$, $\tilde{\alpha}_2 = 1 - \alpha_2$, and $\tilde{\alpha}_3 = 1$, W is the corresponding factor prices, $F(\cdot)$ is the implicit production transformation function, and b scales down the actual input mix arising from input-oriented managerial inability.

Eq. (2) is distinguishable from the previous literature in that it incorporates both the TE measure of b and fractional parameter α , in addition to the traditional technology parameters in a cost function. The assumption of cost minimization enables us not only to consider multiple outputs for a bank, but also to estimate all of the parameters of interest, including b , α , and the underlying technology parameters in the setting of simultaneous equations. The particular identification problem of α will be disentangled shortly.

A bank's demand function for the i th input can be derived by Shephard's Lemma (Shephard, 1953), i.e.:

$$\begin{aligned} \frac{\partial PC^*}{\partial (W_i/b)} &= b\tilde{\alpha}_i X_i \left(Y, \frac{W}{b} \right) \\ &= b\tilde{\alpha}_i X_i (Y, W) \end{aligned} \quad (3)$$

and

$$\frac{\partial PC^*}{\partial W_i} = \frac{\partial PC^*}{\partial (W_i/b)} \frac{\partial (W_i/b)}{\partial W_i} = b\tilde{\alpha}_i X_i \frac{1}{b} = \tilde{\alpha}_i X_i. \quad (4)$$

Taking a partial derivative of (2) with respect to W_i , we obtain:

$$\frac{\partial PC}{\partial W_i} = \frac{1}{b} \frac{\partial PC}{\partial W_i}.$$

Eq. (4) implies that:

$$X_i = \frac{1}{\tilde{\alpha}_i} \frac{\partial PC^*}{\partial W_i}.$$

The share equation of the i th input can be written as:

$$\begin{aligned} S_i(Y, W) &\equiv \frac{\partial \ln PC^*}{\partial \ln W_i} = \frac{\partial PC^*}{\partial W_i} \frac{W_i}{PC^*} = \frac{\tilde{\alpha}_i X_i W_i}{PC^*} \\ &= S_i \left(Y, \frac{W}{b} \right) \equiv \frac{\partial \ln PC^*}{\partial \ln (W_i/b)}. \end{aligned} \quad (5)$$

Eq. (5) indicates that the inclusion of technical inefficiency proportionally raises the expenditures of each input such that these input shares remain unchanged.

The actual expenditure and share equations are defined as:

$$\begin{aligned} E &= \sum_{i=1}^3 W_i X_i = \sum W_i \frac{PC^* S_i}{\tilde{\alpha}_i W_i} = PC^* \sum S_i \tilde{\alpha}_i^{-1} = PC^* G(Y, W) \\ \frac{W_i X_i}{E} &= \frac{W_i}{E} \frac{S_i PC^*}{\tilde{\alpha}_i W_i} = \frac{S_i \tilde{\alpha}_i^{-1}}{G}, \quad i = 1, 2, 3 \end{aligned} \quad (6)$$

where $G(Y, W) = \sum_{i=1}^3 S_i \tilde{\alpha}_i^{-1}$. Taking the natural logarithm with respect to E and adding error terms of ν_2 and ε_j , $j = 3-5$, to (6) to

account for random shocks out of the control of bank managers, we obtain:

$$\begin{aligned} \ln E &= \ln PC(Y, W) - \ln b + \ln G(Y, W) + \nu_2 \\ &= \ln PC(Y, W) + \ln G + \nu_2 + u_2, \end{aligned} \quad (7)$$

where $u_2 \equiv -\ln b$ is assumed to be a non-negative random variable independent of ν_2 , reflecting the additional cost incurred by technical inefficiency. Terms ν_2 and u_2 are distributed as $N(0, \sigma_{\nu_2}^2)$ and $N^+(0, \sigma_{u_2}^2)$, respectively. In addition, the actual share regression equation becomes:

$$\frac{W_i X_i}{E} = \frac{S_i \tilde{\alpha}_i^{-1}}{G} + \varepsilon_j, \quad j = 3, 4, 5. \quad (8)$$

As the three cost shares must sum up to unity, only two of them can be included to avoid the singularity problem of the covariance matrix of the disturbances. Herein, we arbitrarily choose to preclude the last input share.⁴

Recall that deposits are regarded as an intermediate product that is an output of the first production stage and is treated as an input to the second stage. The presence of this intersectoral linkage implies that the intermediate output should be considered as an endogenous (choice) variable, and has to be jointly determined within the two stages by managers. Therefore, the structural equations of (1), (7), and (8) must be simultaneously estimated to embody the treatment of deposits as neither an input nor an output and deal with the endogenous problem of the intermediate output by, e.g., the maximum likelihood. At the same time, the fractions of α_1 and α_2 can be estimated only under the framework of simultaneous equations, where (8) plays the key role of identifying them. The additional parameters of α_1 and α_2 cannot be estimated without (8). We are able to estimate α_1 and α_2 for each bank, provided panel data are available.⁵

It is noteworthy that the individual subsectors' input and output data of a firm are usually not available from accounting statements. In other words, the fractional parameters, e.g., of α_1 and α_2 , are not directly observable and have to be estimated. This frequently impedes empirical researchers from using network DEA to investigate the performance of different subunits in a firm, and thus some more or less ad hoc assumptions have to be made. The structural equations of (1), (7), and (8) are quite useful, since they provide a feasible way of jointly estimating all parameters of interest, including those fractional parameters, without the need for collecting subunits' data. Instead, the firm's aggregated data for relevant variables give enough information to identify the structural parameters and allow for the estimation of TE scores for each production stage or subsector.

In the context of system regression equations, the joint distribution of $\varepsilon_1 (= \nu_1 - u_1)$, $\varepsilon_2 (= \nu_2 + u_2)$, ε_3 , and ε_4 has to be constructed by the copula method. Based on Sklar's theorem, the joint cumulative distribution function (CDF) of a set of random variables, $F(\cdot)$, can be associated with the copula function, $C(\cdot)$, as:

$$F(\varepsilon_{1i}, \varepsilon_{2i}, \varepsilon_{3i}, \varepsilon_{4i}) = C(F_1(\varepsilon_{1i}), F_2(\varepsilon_{2i}), F_3(\varepsilon_{3i}), F_4(\varepsilon_{4i}); \rho), \quad (9)$$

where $F_j(\cdot)$, $j = 1-4$, denotes the one-dimensional marginal CDF, and ρ is a vector of dependence parameter that measures dependence

among the marginal CDFs. Therefore, the copula function allows us to model the correlated stochastic frontier regressions and is shown to be unique if all of the marginal CDFs are continuous. Since $0 < F_j(\cdot) \leq 1$, the copula function can also be viewed as a multivariate distribution of uniform $U[0,1]$ variables with the dependence parameter ρ .

The corresponding joint probability density function (PDF) can be obtained by taking derivatives of Eq. (9) with respect to $(\varepsilon_{1i}, \dots, \varepsilon_{4i})$, i.e.:

$$\begin{aligned} f(\varepsilon_{1i}, \varepsilon_{2i}, \varepsilon_{3i}, \varepsilon_{4i}) &= c(F_1(\varepsilon_{1i}), F_2(\varepsilon_{2i}), F_3(\varepsilon_{3i}), F_4(\varepsilon_{4i})) \\ &\times \prod_{j=1}^4 f_j(\varepsilon_{ji}). \end{aligned} \quad (10)$$

There are many forms of copula functions, e.g., the multivariate Student's t copula, Archimedean copula, Gumbel n-copula, Clayton n-copula, etc. Each of them imposes a different dependence structure. See Cherubini, Luciano, and Vecchiato (2004) for a complete review of the copula functions. Further extensions of our proposed approach to other copula functions should follow the same procedure with a similar calculation. Following Lai and Huang (2013), we select the Gaussian copula to derive the joint CDF of $\varepsilon_{1i}, \varepsilon_{2i}, \varepsilon_{3i}, \varepsilon_{4i}$, expressed as:

$$\begin{aligned} F(\varepsilon_{1i}, \varepsilon_{2i}, \varepsilon_{3i}, \varepsilon_{4i}) &= \Phi_4(\Phi^{-1}(F_1(\varepsilon_{1i})), \Phi^{-1}(F_2(\varepsilon_{2i})), \\ &\Phi^{-1}(F_3(\varepsilon_{3i})), \Phi^{-1}(F_4(\varepsilon_{4i})); \rho). \end{aligned} \quad (11)$$

Here, $\Phi^{-1}(\cdot)$ is the inverse CDF of the standard normal, and $\Phi_4(\cdot)$ is the CDF of a standard 4-variate normal distribution of the random variables with a mean vector of zeros and 4×4 correlation matrix $\rho = [\rho_{jk}]$, i.e.:

$$\rho = \begin{pmatrix} 1 & \rho_{12} & \rho_{13} & \rho_{14} \\ \rho_{12} & 1 & \rho_{23} & \rho_{24} \\ \rho_{13} & \rho_{23} & 1 & \rho_{34} \\ \rho_{14} & \rho_{24} & \rho_{34} & 1 \end{pmatrix}.$$

The corresponding Gaussian copula density can be written as:

$$\begin{aligned} f(\varepsilon_{1i}, \varepsilon_{2i}, \varepsilon_{3i}, \varepsilon_{4i}) &= c(F_1(\varepsilon_{1i}), F_2(\varepsilon_{2i}), F_3(\varepsilon_{3i}), F_4(\varepsilon_{4i}); \rho) \times \prod_{j=1}^4 f_j(\varepsilon_{ji}) \\ &= \frac{1}{|\rho|^{1/2}} \exp \left[\frac{-1}{2} \zeta'_i (\rho^{-1} - I_4) \zeta_i \right] \times \prod_{j=1}^4 f_j(\varepsilon_{ji}), \end{aligned} \quad (12)$$

where $\zeta_i = (\Phi^{-1}(F_1(\varepsilon_{1i})), \Phi^{-1}(F_2(\varepsilon_{2i})), \Phi^{-1}(F_3(\varepsilon_{3i})), \Phi^{-1}(F_4(\varepsilon_{4i})))'$, and I_4 is the 4×4 identity matrix.

After taking the natural logarithm, we write the log-likelihood function of the 4 multiple regression equations for a sample of n observations as:

$$\begin{aligned} \ln L(\theta) &= \sum_{i=1}^N \ln f(\varepsilon_{1i}, \varepsilon_{2i}, \varepsilon_{3i}, \varepsilon_{4i}) = \sum_{i=1}^N \ln c(F_1(\varepsilon_{1i}), F_2(\varepsilon_{2i}), \\ &F_3(\varepsilon_{3i}), F_4(\varepsilon_{4i}); \rho) + \sum_{j=1}^4 \sum_{i=1}^N \ln f_j(\varepsilon_{ji}) \\ &= \frac{-N}{2} \ln |\rho| - \frac{1}{2} \sum_{i=1}^N \zeta'_i (\rho^{-1} - I_4) \zeta_i + \sum_{j=1}^4 \sum_{i=1}^N \ln f_j(\varepsilon_{ji}), \end{aligned} \quad (13)$$

where θ is the unknown parameter vector of the system equations. Under the regularity conditions for the asymptotic maximum likelihood (ML) theory, the ML estimators can be shown to be consistent, asymptotically efficient, and asymptotically normal, provided that the copula function is correctly specified. The objective function of (13) can be reduced to the case that separately estimates the 4 equations by keeping only the last

⁴ In the conventional system estimation of (7) and (8), iterating on seemingly unrelated regressions until convergence generates maximum likelihood estimates (Kmenta & Gilbert, 1968), and Barten (1969) has shown that these estimates are invariant to which share equation is deleted. We are also estimate (7) and (8) by dropping the first (second) share equation and the estimation results are indeed robust.

⁵ Naturally, these estimators are subject to the problem of incidental parameters if the time period of each bank is finite. See, for example, Neyman and Scott (1948) and Hsiao (2003).

term, i.e., $L(\theta) = \sum_{j=1}^4 \sum_{i=1}^N \ln f_j(\varepsilon_{ji})$, since only the first term of

$\sum_{i=1}^N \ln c(F_1(\varepsilon_{1i}), F_2(\varepsilon_{2i}), F_3(\varepsilon_{3i}), F_4(\varepsilon_{4i}); \rho)$ captures the correlation

between equations. Lai and Huang (2013) claim that the separate regression may still give consistent estimates and valid standard errors under correctly specified marginal densities, but the standard errors are inefficient.

We present the PDFs of the composite errors of ε_1 and ε_2 as:

$$f_1(\varepsilon_{1i}) = \frac{2}{\sigma_1} \phi\left(\frac{\varepsilon_{1i}}{\sigma_1}\right) \Phi\left(\frac{-\varepsilon_{1i}\lambda_1}{\sigma_1}\right),$$

and

$$f_2(\varepsilon_{2i}) = \frac{2}{\sigma_2} \phi\left(\frac{\varepsilon_{2i}}{\sigma_2}\right) \Phi\left(\frac{\varepsilon_{2i}\lambda_2}{\sigma_2}\right),$$

where $\sigma_j = \sqrt{\sigma_{uj}^2 + \sigma_{vj}^2}$, $j = 1, 2$, $\lambda_j = \sigma_{uj}/\sigma_{vj}$, and $\phi(\cdot)$ is the standard normal PDF. Since those PDFs have no closed form, the computation of the corresponding CDFs is not an easy job. Some numerical integration or simulated ML procedures, e.g., Greene (2003, 2010), may be used to approximate the integration in computing $F_j(\varepsilon_{ji})$. Thanks to Tsay, Huang, Fu and Ho (2013), we employ the mathematical approximation functions to derive the closed form of $F_j(\varepsilon_{ji})$, which incurs an approximation error within 10^{-5} . We undertake the same procedure to derive the closed forms of $F_j(\varepsilon_{ji})$, $j = 1, 2$, which are presented in Appendix A.

One caveat is worth mentioning, i.e., the log-likelihood function of (13) is highly non-linear as it contains CDFs of ε_j and the inverse functions of those CDFs, along with the PDFs of ε_j being skewed normal distributions. The estimation of such a likelihood function is not that simple. Some simplified procedures may be required for the purpose of making the likelihood function converge easier in estimation and to at least have the corresponding parameter estimates be consistent. After getting all of the parameter estimates by maximizing Eq. (13), we evaluate the measure of technical efficiency (TE), proposed by Battese and Coelli (1988), for each sample bank.

3. The data

We compile the data for commercial banks of the U.S. in 2009 from the Report of Condition and Income (Call report). The main purpose of the empirical study is simply to illustrate the feasibility of the theoretical model developed in Section 2. We hope that the empirical study helps shed some light on the usefulness of our proposed model. A more complete study requires the use of data covering a longer period of time, such that the dynamic process of deposit gathering and loan expansion can be thoroughly examined, which may vary with the business cycles of the country.

The cross-sectional data contain 6182 observations after deleting some banks with either missing or extreme values. All nominal variables have been deflated by the consumer price index (CPI) with the base year of 1982. We identify three outputs, two inputs, and an intermediate output on the basis of the intermediation approach. The output variables include total loans (Y_1), investments (Y_2), and non-interest income (Y_3). The input variables are labor (X_1) and physical capital (X_2), while the intermediate output (Z) is purchased funds. The price of labor (W_1) is defined as the ratio of personnel expenses to the total number of full-time employees. The price of physical capital (W_2) is obtained by dividing the expenses on premises and fixed assets over the total dollar value of premises and fixed assets. The price of purchased funds (W_3) is given by the ratio of total interest expenses to total funds. Total expenditure (E)

Table 1
Sample statistics.

Variable name	Mean	Standard dev.	Minimum	Maximum
Total loans ^a (Y_1)	407042	572088	13388	6052256
Investments ^a (Y_2)	125305	186724	1481	1986616
Noninterest income ^a (Y_3)	4799	9601	52	120954
Labor (X_1)	67	83	5	751
Physical capital ^a (X_2)	11142	16015	89	147576
Funds ^a (X_3)	538085	723918	27030	7349168
Price of labor (W_1)	132	39	25	664
Price of physical capital (W_2)	0.3668	0.4866	0.0213	8.8389
Price of funds (W_3)	0.0177	0.0058	0.0008	0.0637

^a Measured in thousands of US dollars.

is equal to the sum of the above three items of expenditures on hiring the three inputs. The price of labor is arbitrarily chosen as the numeraire to impose the homogeneity constraint of the first degree in input prices.

Table 1 provides the descriptive statistics for the aforementioned variables. In 2009, an average U.S. bank granted US\$188 million in loans, engaged in investment up to US\$57 million, and earned non-interest income of US\$2 million. The average price of labor is about US\$60,000, the average price of physical capital is 0.3668, and the average price of funds is 0.0177. Most of the variables have quite large standard deviations relative to their sample means, meaning that the sample banks differ substantially in their output quantities and input prices.

4. Empirical results

We specify both production and cost functions of a bank as the translog form, because it is a flexible functional form providing a second-order approximation, commonly used by numerous practitioners.⁶ The cost function is required to satisfy the regularity conditions suggested by the fundamental microeconomic theory. The homogeneity and symmetry conditions can be directly imposed on Eqs. (7) and (8), while the monotonicity conditions in prices and outputs and concavity in prices can be verified after the parameters in Eqs. (7) and (8) are estimated.

We extend the copula-based maximum likelihood approach, first proposed by Lai and Huang (2013) under the framework of the SFA, to estimate a 4-equation system, consisting of Eqs. (1), (7), and (8). The highly non-linear nature of the log-likelihood functions, as mentioned above, challenges empirical researchers. Kumbhakar and Lovell (2000, p.165) suggest a two-step procedure to consistently estimate the system equations, which is a less efficient but computationally simpler method. We adopt their idea here and conduct simulations to confirm the appropriateness of the method.

In the first step, we view the 4 system equations as the seemingly unrelated regression models. The composed errors in (1) and (4) are assumed to be the standard disturbance terms for the time being, which bias the intercepts of the two equations as the means of the two composed errors are constants differing from zero, by assumption, which are absorbed by the intercepts. The application of the non-linear least squares potentially leads to consistent parameter estimates for the fractional parameters, as well as all of the slope parameters in (1) and (7). Monte Carlo simulations will be conducted and the results are suggestive of consistency for those estimates.

In the second step, these fractional and slope parameters are treated as given, and the share equations of (8) are overlooked

⁶ Other functional forms, such as the Cobb–Douglas, constant elasticity of substitution, and generalized Leontief forms, are also potential candidates. Moreover, the production and cost functions may take different forms.

since they merely function for identifying the fractional parameters. The constant terms of (1) and (7), along with the parameters that characterize the distributions of v and u and the dependence in the copula function, i.e., $\sigma_{u_1}, \sigma_{v_1}, \sigma_{u_2}, \sigma_{v_2}$, and ρ , are simultaneously estimated with respect to Eqs. (1) and (7) by the copula-based ML approach. Huang, Chen, Lin, and Chung (2014) examine the consistency property of the two-step procedure – similar to Kumbhakar and Lovell (2000) in essence – by Monte Carlo simulations under the framework of a semi-parametric regression model. Since the second-step procedure here is analogous to Huang et al. (2014), we shall not carry out simulations once more.

One problem in the second step worth mentioning arises from the treatment of the estimated fractional and slope parameters as given, so that the error terms contain estimation errors that are functions of explanatory variables shown in the first step and hence result in the variances of the error terms being heteroskedastic and the estimated standard errors of the parameter estimates being inconsistent. To correct for the inconsistency, caused by possible misspecification, we suggest using the procedure proposed by White (1982), which requires computing the sandwich-form of the covariance matrix of estimators in order to obtain correct standard errors.⁷

5. Simulation results

We now briefly describe our design of experiments, followed by evaluating the biasness and mean square error (MSE) of the proposed estimators using Monte Carlo simulations. This helps gain further insight into the performance of our estimators. Here, we specify the production function in the first production stage as the Cobb–Douglas form for simplicity and the cost function in the second stage as the translog form with a single output and three inputs as follows, where w_1 is arbitrarily chosen as the numeraire.

$$\ln y = a_0 + a_1 \ln(\alpha_1 x_1) + a_2 \ln(\alpha_2 x_2) + v_1 - u_1$$

$$\begin{aligned} \ln \left(\frac{E}{w_1} \right) = & c_0 + c_1 \ln y + d_2 \ln \left(\frac{w_2}{w_1} \right) + d_3 \ln \left(\frac{w_3}{w_1} \right) + c_{11} [\ln(y)]^2 + d_{22} \left[\ln \left(\frac{w_2}{w_1} \right) \right]^2 + d_{33} \left[\ln \left(\frac{w_3}{w_1} \right) \right]^2 \\ & + d_{23} \ln \left(\frac{w_2}{w_1} \right) \ln \left(\frac{w_3}{w_1} \right) + e_{21} \ln \left(\frac{w_2}{w_1} \right) \ln(y) + e_{31} \ln \left(\frac{w_3}{w_1} \right) \ln(y) + \ln(G) + u_2 + v_2 \end{aligned}$$

We set $\alpha_1 = 0.26$ and $\alpha_2 = 0.68$. The remaining true values of the coefficients are arbitrarily given as follows: $a_0 = 0.5$, $a_1 = 0.3$, $a_2 = 0.7$, $c_0 = 0.8$, $c_1 = 0.2$, $d_2 = 0.08$, $d_3 = 0.02$, $c_{11} = 0.1$, $d_{22} = -0.03$, $d_{33} = -0.05$, $d_{23} = 0.02$, $e_{21} = 0.2$, and $e_{31} = 0.7$. Input prices w_1 , w_2 , and w_3 are randomly drawn from uniform distributions $U(0, 1)$, $U(0.5, 2)$, and $U(0.5, 1.5)$, respectively. The quantities of inputs, x_1 , x_2 , and x_3 , and output, y , are independently drawn from normal distributions $N(5, 0.5)$, $N(3, 0.1)$, $N(5, 0.5)$, and $N(30, 1)$ respectively. The two-sided error of v is randomly drawn from $N(0, \sigma_v^2)$, and the

one-sided error of u is randomly drawn from $N^+(0, \sigma_u^2)$. Following Olson, Schmidt, and Waldman (1980), Fan, Li, and Weersink (1996), and Huang et al. (2014), we consider three sets of variance ratios and variances, i.e. $(\lambda_1, \sigma_2^2) = (1.24, 1.63)$, $(1.66, 1.88)$, and $(0.83, 1.35)$, where $\sigma_2^2 = \sigma_u^2 + \sigma_v^2$ and $\lambda = \sigma_u/\sigma_v$.

Table 2 presents the simulation results, for the cases of $(\lambda_1, \sigma_1^2) = (1.24, 1.63)$ and $(\lambda_2, \sigma_2^2) = (1.66, 1.88)$, in terms of biases and MSEs for the parameter estimates of the production function.⁸ We see that the estimators of α_1 and α_2 perform well even for the case of the smallest sample size, i.e., $N = 300$. Their biases and MSEs are quite small and decrease as the sample size increases. The estimator of $\ln(\alpha_1 X_1)$ has similar properties to those of α_1 and α_2 . Although the estimator of $\ln(\alpha_2 X_2)$ has a little larger bias, this bias is relatively small to its true value and diminishes with larger sample sizes, while its MSEs are relatively large, implying that this parameter cannot be accurately estimated. As expected, the estimator of the constant term exhibits quite large biases, irrespective of the sample sizes. Moreover, its MSEs are also large, which can be attributed to the fact it is confounded with the mean value of the inefficiency term.

Table 3 shows the simulation results for the cost parameters. All estimators, except for the intercept, have quite small biases and MSEs. However, the estimator of $\ln Y_1$ has a little larger MSE. The above Monte Carlo simulations provide evidence supporting our proposed estimation procedure in the first estimation step, which relies on the NISUR. Note that the estimators of the constant terms perform poorly and therefore should be revised in the second estimation step. Further recall that we treat the estimates of the fractional and slope parameters as given in the second step.

Having presented that the parameter estimates have the desirable property of consistency, we proceed to perform an empirical study on the U.S. commercial banks. **Tables 4 and 5** report estimation results from the two-step procedures. Most of the parameter estimates of the production and cost functions achieve the 1% level of significance. Note that the standard errors of $\lambda_1, \lambda_2, \sigma_1, \sigma_2, \rho$,

and the two constant terms are corrected by the sandwich form, as shown in footnote 7. Using these estimates, we verify the regularity conditions required by the microeconomic theory and find that most of the observations have the correct signs, indicating that those parameter estimates can theoretically characterize the underlying production technologies used in the two production stages.

The fractional parameters of α_1 and α_2 are estimated to be equal to 0.26 and 0.68, respectively. This tells us that the sample banks allocate respectively 26% and 68% of their entire workforce and capital stock to lure deposits in the first production process. The results appear to be acceptable, since the first production stage mainly involves a fund collection process, where banks are committed to offer safe and convenient banking services to satisfy all customers' needs. Hence, a bank must accumulate a large amount of tangible assets like branches, ATMs, data processing and storage facilities, security technology equipment, etc. in this stage. The second production stage corresponds to revenue generation, where various funds collected in the first stage are transformed into mis-

⁷ Based on (13), the standard ML estimator has the inverse of the Fisher information matrix $I(\theta) = -E \left(\frac{\partial^2 \ln L(\theta)}{\partial \theta \partial \theta^T} \right)$ as the covariance matrix of the estimator $\hat{\theta}$. However, the covariance matrix of the quasi-maximum likelihood estimators has the so-called sandwich form: $I^{-1}(\theta) [S(\theta) S^T(\theta)] I^{-1}(\theta)$, where $S(\theta) = E \left(\frac{\partial \ln L(\theta)}{\partial \theta} \right)$ is the score function. Johnston and Dinardo (1997, pp. 428–430), provide a brief discussion of the quasi-maximum likelihood estimation of misspecified models and the derivation of the covariance matrix. There are two problems to be solved before deriving the partial derivative of $\partial \ln L / \partial \theta$. First, formula $I_{app}(A)$ and $I_{app}(B)$ contain the sign functions of $\text{sign}(A)$ and $\text{sign}(B)$, which causes the log-likelihood function to be discontinuous with respect to θ . We re-express $\ln L$ for different values of the sign function, i.e., $\text{sign}(\cdot) > 0$ and $\text{sign}(\cdot) < 0$. Second, it is difficult to calculate the partial derivative of $\partial \ln L / \partial \theta$, since $\ln L$ includes the function of $\Phi^{-1}(\cdot)$. We adopt the popular software of *Mathematica* to do the job.

⁸ We also conduct simulations assuming $(\lambda_1, \sigma_1^2) = (1.66, 1.88)$ and $(\lambda_2, \sigma_2^2) = (0.83, 1.35)$; $(\lambda_1, \sigma_1^2) = (0.83, 1.35)$ and $(\lambda_2, \sigma_2^2) = (1.24, 1.63)$. Since the simulation results are quite similar, we choose not to report them to save space.

Table 2

Biases and MSEs of the parameter estimates of the production function.

N	True value	300		500		1000	
		Bias	MSE	Bias	MSE	Bias	MSE
α_1	0.26	-0.0032	0.0027	-0.0020	0.0017	0.0003	0.0008
α_2	0.68	-0.0053	0.0080	-0.0030	0.0047	0.0007	0.0023
Constant	0.50	-0.7019	2.0471	-0.7228	1.4993	-0.7430	1.0446
$\ln(\alpha_1 X_1)$	0.30	0.0010	0.3355	0.0051	0.2020	-0.0019	0.0932
$\ln(\alpha_2 X_2)$	0.70	-0.0922	3.0351	-0.0808	1.8690	-0.0662	0.9417

Table 3

Biases and MSEs of the parameter estimates of the cost function.

N	True value	300		500		1000	
		Bias	MSE	Bias	MSE	Bias	MSE
Constant	0.80	0.9541	16.7176	0.8966	9.6189	0.8949	5.4052
$\ln Y_1$	0.20	-0.0053	5.7325	0.0323	3.1953	0.0231	1.6716
$\ln(W_2/W_1)$	0.08	0.0006	0.0155	-0.0012	0.0087	0.0020	0.0046
$\ln(W_3/W_1)$	0.02	0.0061	0.1098	0.0095	0.0638	0.0133	0.0303
$0.5 * (\ln Y_1)^2$	0.10	9.16E-06	0.5165	-0.0122	0.2879	-0.0068	0.1508
$0.5 * \ln(W_2/W_1)^2$	-0.03	-0.0003	0.0009	-0.0002	0.0005	0.0007	0.0003
$0.5 * \ln(W_3/W_1)^2$	-0.05	-0.0001	0.0016	-0.0009	0.0009	-3.74E-05	0.0004
$\ln(W_2/W_1) * \ln(W_3/W_1)$	0.02	0.0002	0.0008	-4.39E-05	0.0005	-0.0007	0.0003
$\ln(W_2/W_1) * \ln Y_1$	0.20	-0.0003	0.0035	0.0004	0.0021	-0.0017	0.0010
$\ln(W_3/W_1) * \ln Y_1$	0.70	-0.0021	0.0105	-0.0024	0.0060	-0.0026	0.0030

Table 4

Parameter estimates of the production function.

Variable	Parameter estimates	Standard errors
α_1	0.2618***	0.0067
α_2	0.6761***	0.0091
Constant	9.4628***	0.0018
$\ln(\alpha_1 X_1)$	0.9812***	0.0081
$\ln(\alpha_2 X_2)$	0.0593***	0.0067
$0.5 * \ln(\alpha_1 X_1) * \ln(\alpha_1 X_1)$	-0.0047*	0.0027
$0.5 * \ln(\alpha_2 X_2) * \ln(\alpha_2 X_2)$	0.0050***	0.0013
$\ln(\alpha_1 X_1) \ln(\alpha_2 X_2)$	-0.0005	0.0016
λ_1	1.5041***	2.23E-05
σ_1	0.5581***	2.28E-05

* Denotes significance at the 10% level.

** Denotes significance at the 1% level.

cellaneous loans and investments in corporate and government securities, and used to engage in some off-balance sheet activities. In this stage, banks earn their traditional interest revenues and capital gains from loans and investments, as well as non-traditional income, fees, and commissions, from providing financial services to, e.g., credit card holders, wealth management clients, letters of credit issuers in international trade, and new financial derivative products. These services appear to require more labor input. Here, the estimated values of α_1 and α_2 lead us to infer that a representative bank allocates 74% and 32% of its entire workers and capital, respectively, to fulfill the second production process. Huang, Lin, and Chen (2017) apply similar network SFA to estimate the efficiency scores for Chinese commercial banks spanning 2002–2015. Their estimates of α_1 and α_2 are equal to 0.59 and 0.61, respectively.

We use the foregoing parameter estimates to evaluate the individual TE scores in the two production stages, denoted by TE_1 and TE_2 , respectively. The average value of TE_1 from the production frontier is equal to 0.6717 with a standard deviation of 0.1127, reflecting that an average bank could be technically efficient if it can produce around 33% more intermediate output. The average TE_2 score implied by the cost frontier is equal to 0.7758 with a standard deviation of 0.0919. This implies that an average bank is fully cost efficient, should it cut roughly 28.9% ($=1/0.7758 - 1$) of its current input mix. Both TE_1 and TE_2 are accurately estimated due to their small standard deviations relative to the individual

Table 5

Parameter estimates of the cost function.

Variable	Parameter estimates	Standard errors
Constant	6.4443***	0.0101
$\ln Y_1$	0.0779***	0.0288
$\ln Y_2$	0.3176***	0.0182
$\ln Y_3$	0.3161***	0.0184
$0.5 * \ln Y_1 * \ln Y_1$	0.1858***	0.0037
$0.5 * \ln Y_2 * \ln Y_2$	0.1223***	0.0019
$0.5 * \ln Y_3 * \ln Y_3$	0.0181***	0.0018
$\ln Y_1 * \ln Y_2$	-0.1148***	0.0021
$\ln Y_1 * \ln Y_3$	-0.0354***	0.0021
$\ln Y_2 * \ln Y_3$	-0.0008	0.0015
$\ln W_3$	2.1701***	0.0052
$0.5 * \ln W_3 * \ln W_3$	0.2302***	0.0004
$\ln W_2 * \ln W_3$	-0.0200***	0.0004
$\ln W_3 * \ln Y_1$	0.0105***	0.0003
$\ln W_3 * \ln Y_2$	0.0028***	0.0002
$\ln W_3 * \ln Y_3$	-0.0001	0.0002
$\ln W_2$	-0.1072***	0.0034
$0.5 * \ln W_2 * \ln W_2$	0.0057***	0.0003
$\ln W_2 * \ln Y_1$	0.0026***	0.0003
$\ln W_2 * \ln Y_2$	-0.0002	0.0002
$\ln W_2 * \ln Y_3$	-0.0012***	0.0002
λ_2	1.8053***	0.0842
σ_2	0.3393***	0.0082
ρ	-0.7840***	0.0129

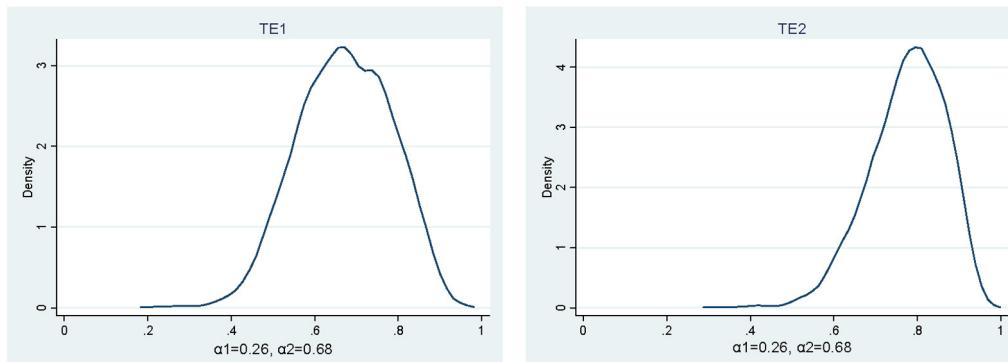
*** Denotes significance at the 1% level.

means. Our model allows for identifying the fractional parameters and estimating the technical efficiency scores in the two production processes, enabling bank managers to adopt valid strategies to improve operational performance in both stages. Moreover, the value of the correlation coefficient between TE_1 and TE_2 is as high as 0.8861, implying that the more efficient the production of deposits is, the more efficient is the second production stage for final products. Bank managers are recommended to increase their managerial ability at the first stage, optimizing the allocation of inputs, since this may foster cost efficiency in the second stage.

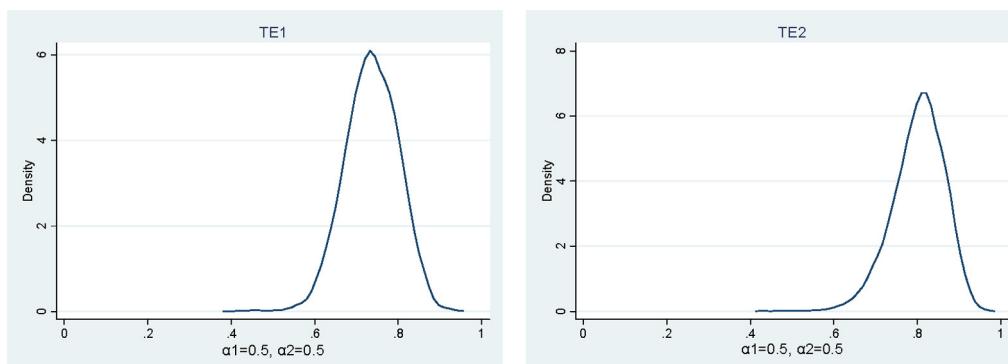
We can calculate the measure of output elasticity by summing up the partial derivatives of (log)output with respect to each

$$(log)input, i.e., \epsilon = \sum_{i=1}^2 \partial \ln f / \partial \ln x_i. If \epsilon is greater than, equal to,$$

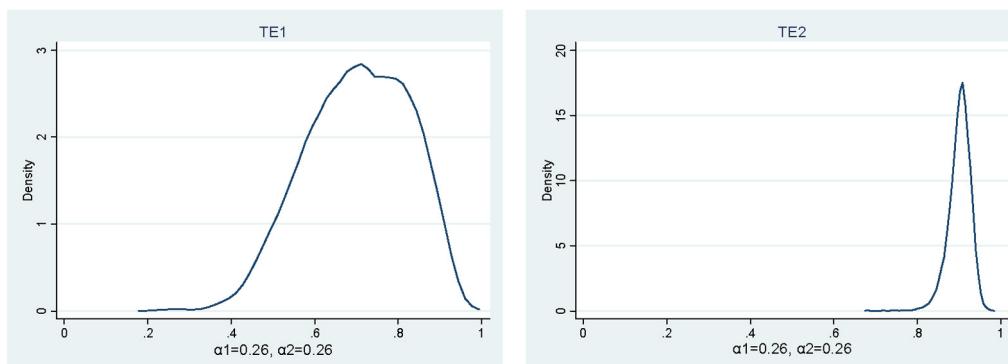
Panel A



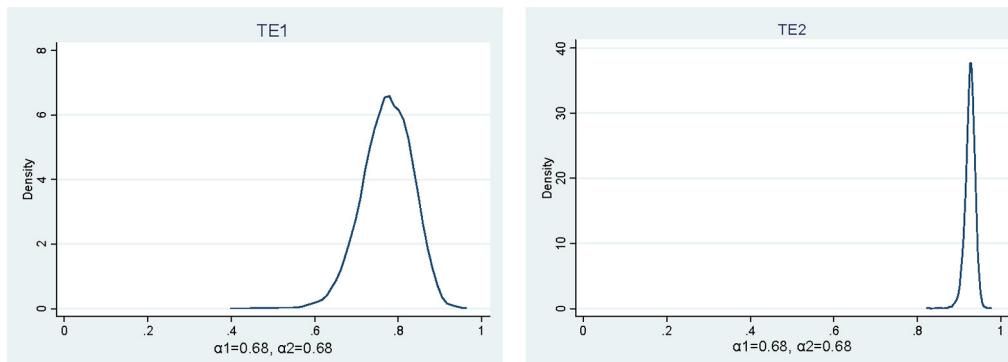
Panel B



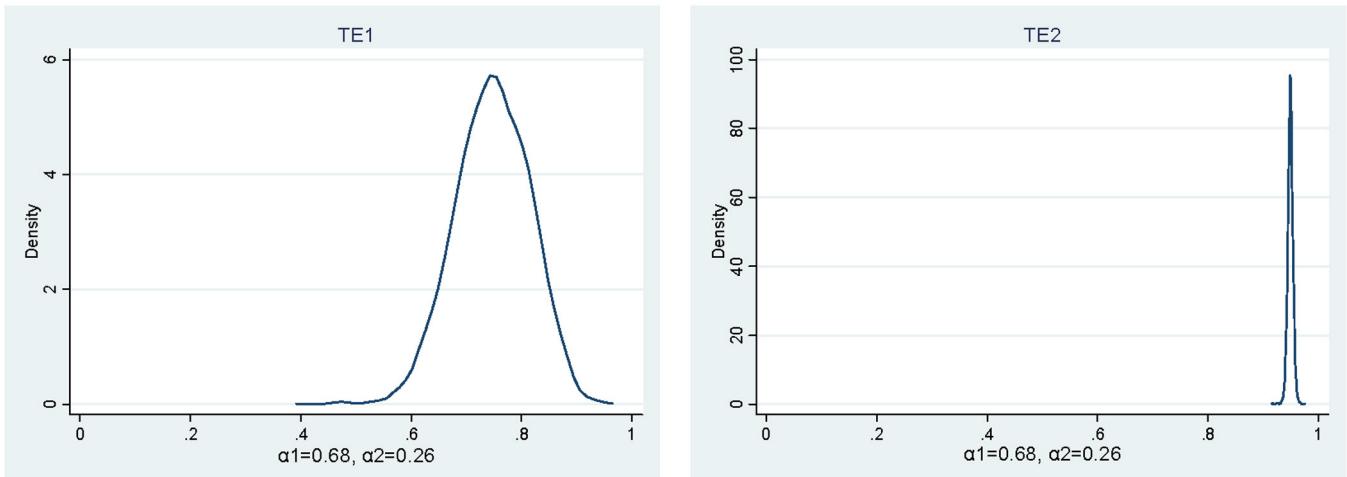
Panel C



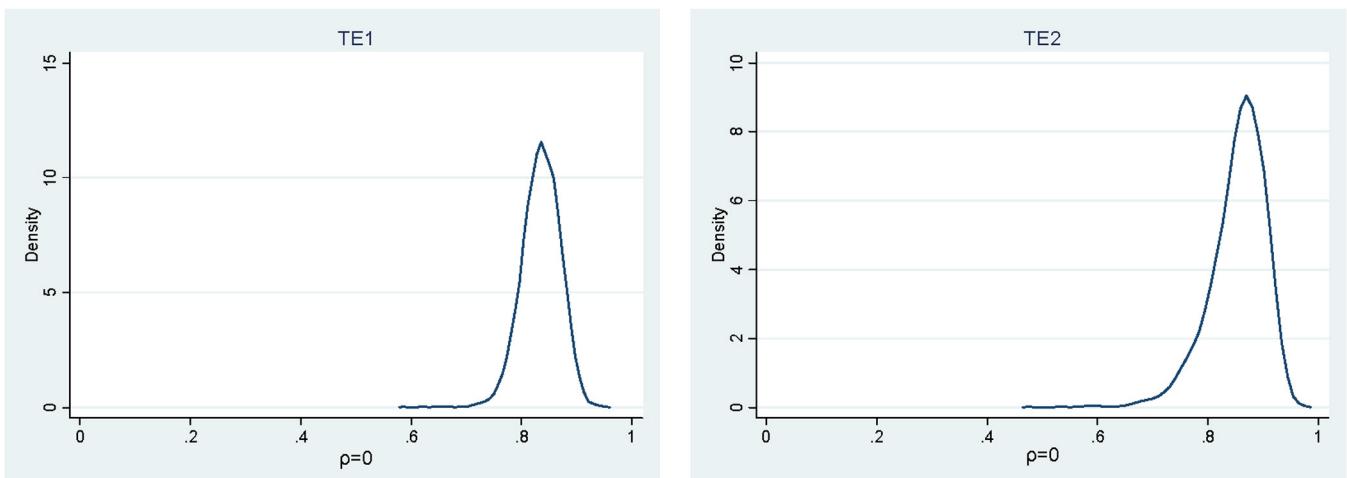
Panel D

**Fig. 1.** Kernel densities of estimated TE scores.

Panel E



Panel F



Panel G

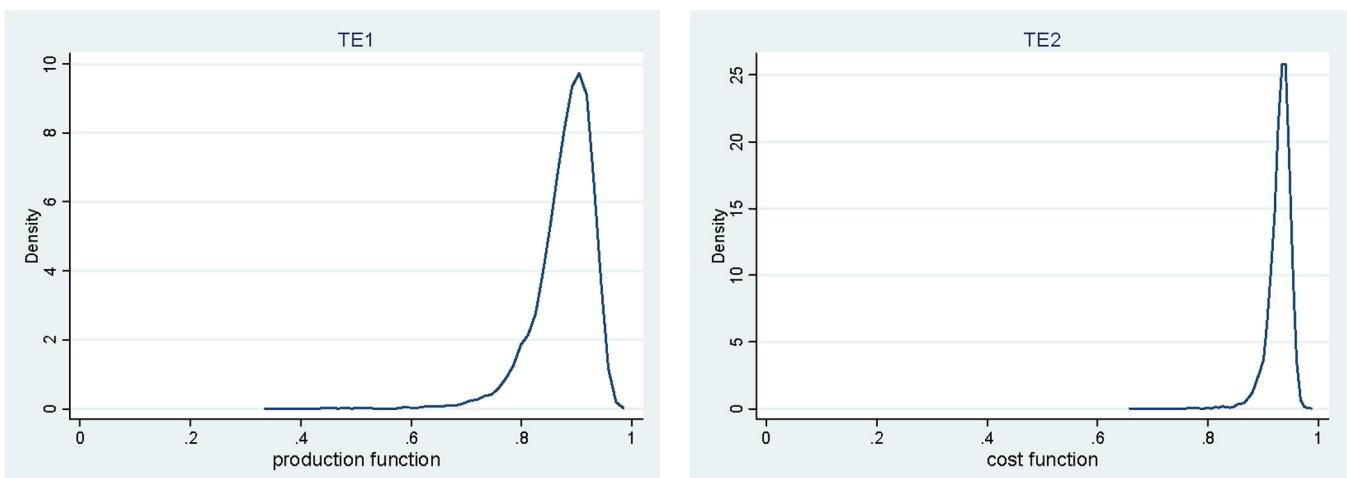


Fig. 1. (Continued)

or less than unity, then the technology exhibits increasing, constant, or decreasing returns to scale (IRS, CRS, or DRS). Our empirical result regarding the mean value of scale economies is equal to 1.93, reflecting that an average U.S. bank exhibits IRS in the first stage. We may conclude that the sample banks should keep expanding their production scale in order to enjoy the advantage of economies of scale, since doubling all their inputs would raise outputs by more than double.

The coefficient estimates of the cost frontier in the second stage permit us to evaluate the measures of scale economies (SE) and scope economies (SC).^{9,10} The average SE measure is equal to 1.05 with a standard deviation of 0.0408. The U.S. banks are, on average, operating under increasing returns to scale technology, which is consistent with the findings of [Whealock and Wilson \(2012\)](#), [Hughes and Mester \(2013\)](#), and [Restrepo, Kumbhakar, and Sun \(2013\)](#), to mention a few. The sample banks benefit from economies of size and therefore are suggested to expand their production scale in order to reduce their long-run average cost. The average SC measure is equal to 0.07, indicating the presence of scope economies. It is preferable for those U.S. banks producing multiple outputs to concentrate on a single output or a few outputs. This finding tends to support the formation of financial conglomerates that are capable of providing an array of financial products within an organization in such a way as to share various resources, like computer equipment and clients' information.

The application of the network DEA model requires knowing the distribution of some inputs among alternative production stages—that is, the fractional parameters must be known, *a priori*. This requirement does not generally hold and cannot be estimated in the context of DEA. [Holod and Lewis \(2011\)](#) thus propose a modified model that avoids this requirement and can assess the efficiency of each bank at the expense of failing to evaluate the efficiency of each stage. [Kao and Hwang \(2010\)](#) impose restrictions on the values of fractional parameters in the range of [0.6, 0.9]. These restrictions may not be consistent with the true condition and hence give rise to undesirable estimation results. To validate this assertion, we re-estimate our model under the assumption that the values of α_1 and α_2 are arbitrarily given for several combinations of them. [Table 7](#) shows the results of average TE scores in the two stages, which are denoted by TE_1^* and TE_2^* , respectively, for different sets of α_1 and α_2 . The column “Diff” represents the difference in the average TE scores between (TE_1, TE_2) and (TE_1^*, TE_2^*) . For the case of $\alpha_1 = 0.5$ and $\alpha_2 = 0.5$, i.e., half the labor and physical capital are consumed in the first stage, the average values of TE_1^* and TE_2^* are roughly 74% and 80%, respectively, which are significantly different from the corresponding average values of TE_1 and TE_2 . The results reveal that the fractional parameters play important roles in determining efficiency scores in different stages. We also consider other cases, such as $(\alpha_1 = 0.26, \alpha_2 = 0.26)$, $(\alpha_1 = 0.68, \alpha_2 = 0.68)$, and $(\alpha_1 = 0.68, \alpha_2 = 0.26)$, and the results are presented in the second to fourth rows of [Table 7](#). All of the paired differences between (TE_1, TE_2) and (TE_1^*, TE_2^*) attain statistical significance at the 1% level.

We conduct another experiment, where the intersectoral production processes are assumed to be mutually independent, i.e.,

⁹ The formula of scale economies is written as $SE = C(Y, W) / \sum_{j=1}^3 Y_j C_j(Y, W)$,

where $C_j(Y, W)$ is the partial derivative of $C(Y, W)$ with respect to the j th output. The measure of returns to scale is increasing, constant, or decreasing, when the SE is greater than, equal to, or less than unity.

¹⁰ Following [Kim \(1986\)](#), we formulate scope economies as: $SC = [C(Y_1 - 2\xi_1, \xi_2, \xi_3) + C(\xi_1, Y_2 - 2\xi_2, \xi_3) + C(\xi_1, \xi_2, Y_3 - 2\xi_3) - C(Y_1, Y_2, Y_3)] / C(Y_1, Y_2, Y_3)$, where $\xi_j (j=1-3)$ denotes 10% of the mean value of the j th output. If SC is greater than (less than) zero, then the economies (diseconomies) of the product mix prevail.

setting $\rho = 0$. The conclusion of the statistical tests is still the same as above, as shown in the fifth row of [Table 7](#). We finally consider the traditional treatment, i.e., banks employ a bundle of inputs to produce an array of outputs in a single stage. Efficiency scores can be evaluated either by a production function that contains two inputs, labor and capital, and an output, deposits, or by a cost function that includes three inputs, labor, capital, and funds, and three outputs, loans, investments, and non-interest income. Average efficiency measures are reported in the bottom two rows of [Table 7](#). These average values are significantly different from those of TE_1 and TE_2 .

[Fig. 1](#) plots the kernel densities of the estimated TE scores for all cases considered, where Panel A corresponds to our empirical results with estimated $\hat{\alpha}_1 = 0.26$ and $\hat{\alpha}_2 = 0.68$. The remaining panels of B to G draw the kernel density for each of the cases considered in [Table 7](#), in which the fractional parameters are not estimated, but given. Obviously, shapes of the kernel densities of the remaining panels deviate substantially away from those in Panel A, implying that the distributions of those TE scores are indeed dissimilar.

In sum, [Table 7](#) and [Fig. 1](#) present that the mean TE is sensitive to different values of α_1 and α_2 , depends on the presence of ρ , and varies with the assumption of a single stage or multiple production stages. If α_1 and α_2 are given incorrect values, then the resulting TE score tends to be misleading. This suggests that the fractional parameters should be either given directly on the grounds of disaggregated data or estimated by an appropriate econometric model like the one proposed by this article. In addition, employing the copula method is necessary since it is able to account for the dependence between the production frontier and the cost frontier, characterizing the two production processes of banks.

6. Conclusion

This paper extends the network DEA model to a copula-based network SFA model that embodies multi-stage production processes for banks under the framework of simultaneous equations. These equations are derived from economic models under the assumption of output maximization in the first stage and cost minimization in the second stage. Our model allows for correlated composite errors, which arise from two subunits of a bank that are connected in series by intermediate outputs and reflect the joint decision making for the two subunits made by bank managers. In this manner, the entire production process is no longer treated as a “black box”, as previous research studies did, and the efficiency measures of different subsectors can be respectively estimated. More importantly, the fractional parameters are not required to be known, *a priori*, but rather can be estimated using aggregated, instead of disaggregated, data of firms—a salient feature of our model. The theoretical model can be transformed into an econometric model, consisting of four simultaneous equations, for the purpose of identifying the additional fractional parameters.

In the empirical study, deposits are treated as an intermediate output produced in the first production stage and described by the translog production frontier that is a function of some portions of labor and capital, rather than the entire employment of both inputs. This intermediate output is next viewed as an input in the second production stage, together with the remaining portions of labor and capital, to produce loans, investments, and non-interest incomes, in the context of the translog cost frontier. Under the copula-based network SFA framework, the fractional parameters of the two inputs involved in the first stage can be estimated by relying on additional information from the cost share equations. The resulting simultaneous equations, formed by a stochastic production frontier, a stochastic cost frontier, and two cost share equations, can be consistently estimated by the ML whose likelihood function is derived by the copula methods of [Lai and Huang \(2013\)](#). This allows for estimating the mutual dependence among equations.

Table 6

Summary statistics of estimated TE scores in both production stages.

	Mean	Standard deviation	Minimum	Maximum	Correlation
TE ₁	0.6717	0.1127	0.2009	0.9703	0.8832
TE ₂	0.7758	0.0919	0.3027	0.9843	

Table 7The mean TE scores of various sets of α_1 and α_2 .

	TE ₁ *	Diff.	TE ₂ *	Diff.
$\alpha_1 = 0.50, \alpha_2 = 0.50$	0.74	-0.0649***	0.80	-0.0267***
$\alpha_1 = 0.26, \alpha_2 = 0.26$	0.70	-0.0315***	0.90	-0.1257***
$\alpha_1 = 0.68, \alpha_2 = 0.68$	0.77	-0.1014***	0.93	-0.1511***
$\alpha_1 = 0.68, \alpha_2 = 0.26$	0.75	-0.0757***	0.95	-0.1741***
$\rho = 0$	0.84	-0.1636***	0.85	-0.0784***
Production function	0.88	-0.2059***	-	-
Cost function	-	-	0.93	-0.1546***

*** Denotes significance at the 1% level.

Adopting the idea of a two-step procedure, which is in essence similar to Kumbhakar and Lovell (2000), we successfully solve the convergence problem specifically for a highly non-linear simultaneous equations model. To justify this procedure, we conduct Monte Carlo simulations to show the consistency of the parameter estimates. The efficiency score of each subsector and the fractional parameters can then be assessed and identified. The knowledge of the technical efficiencies in and the resources' allocation among different stages provides detailed information to regulatory authorities, bank managers, and industry consultants, concerning the effects of mergers and acquisitions, capital regulations, deregulation of deposit rates, removal of geographic restrictions on branching and holding company acquisitions, etc. on bank performance and probability of failure.

Using data of U.S. commercial banks in 2009, we demonstrate that the fractions of a bank's labor and capital consumed in the first-stage production are estimable and equal to 0.26 and 0.68, respectively (see Table 4). This implies that a representative bank is apt to allocate relatively less of its workforce and more of its physical capital in the first production stage to satisfy customers' financial needs. This intermediate output, along with the remaining inputs of labor and capital, is next used to produce three final outputs in the second production stage, which is primarily a revenue generation process. The average TE score in the first stage is equal to around 0.6717 (see Table 6), meaning that an average bank is manufacturing around 67.17% of the best-practice bank's output for a given input mix. The average TE measure in the second stage is equal to 0.7758 (see Table 6), implying that an average bank should cut 28.90% of its current expenditures to reach the cost frontier. Evidence shows that the major source of inefficiency comes from the first stage, rather than the second stage. Banks are suggested to enhance their managerial ability, particularly in the first production process, in order to effectively raise output quantities for a given input mix.

Appendix A.

This Appendix derives the formula of the approximated CDF of the composed error (ε), proposed by Tsay et al. (2013). It is known that $\varepsilon_1 = v_1 - u_1$ has the PDF as follows:

$$f(\varepsilon_1) = \frac{2}{\sigma_1} \phi\left(\frac{\varepsilon_1}{\sigma_1}\right) \Phi\left(-\frac{\lambda_1 \varepsilon_1}{\sigma_1}\right)$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ represent the standard normal PDF and CDF, respectively. The corresponding CDF of ε_1 is expressed as:

$$F(A) = \int_{-\infty}^A f(\varepsilon_1) d\varepsilon_1 = \int_{-\infty}^A \frac{2}{\lambda_1 \sigma_1} \phi\left(\frac{\varepsilon_1}{\sigma_1}\right) \Phi\left(-\frac{\lambda_1 \varepsilon_1}{\sigma_1}\right) d\varepsilon_1 \\ = \frac{2}{\sigma_1} \int_{-\infty}^A \left(\int_{-\infty}^{-\frac{\lambda_1 \varepsilon_1}{\sigma_1}} \phi(\xi) d\xi \right) \phi\left(\frac{\varepsilon_1}{\sigma_1}\right) d\varepsilon_1 = \frac{2}{\sigma_1} I(A)$$

Function $I(A)$ has no closed form and cannot be used to derive the required likelihood function. However, its approximation, $I_{app}(A)$, is shown to be equal to:

$$I_{app}(A) = \frac{1}{2b} \left[1 + \text{erf}\left(\frac{bA}{\sqrt{2}}\right) \left(\frac{1 - \text{sign}(A)}{2} \right) \right] - \frac{1}{4\sqrt{b^2 - a^2} c_2} \\ \exp\left(\frac{a^2 c_1^2}{4(b^2 - a^2) c_2}\right) \left\{ 1 + \text{erf}\left[\frac{-ac_1 - \sqrt{2}A(b^2 - a^2)c_2 \text{sign}(A)}{2\sqrt{b^2 - a^2} c_2}\right] \right\}$$

where $a = -\lambda_1/\sigma_1 < 0$, $b = 1/\sigma_1$, $c_1 = -1.09500814703333$, $c_2 = -0.75651138383854$, $\text{sign}(A) = 1, 0, -1$ depending on $A >, =, < 0$, and the error function $\text{erf}(z)$ is defined as:

$$\text{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt = 2 \int_0^{\sqrt{2}z} \phi(t) dt \\ = 2\Phi(\sqrt{2}z) - 1 \\ \approx 1 - \exp(c_1 z + c_2 z^2) = g(z)$$

Tsay et al. (2013) show that the choice of constants c_1 and c_2 is to ensure that the error function $\text{erf}(z)$ can be well approximated by $g(z)$, for $z \geq 0$. The introduction of the error function and $g(z)$ into the above equation is important, because the integration of $I(A)$ can then be analytically approximated by $I_{app}(A)$. Replacing $I(A)$ with $I_{app}(A)$, we obtain the approximated CDF of $F(A)$, i.e.:

$$F_{app}(A) = \frac{2}{\sigma_1} I_{app}(A)$$

We now write the PDF of $\varepsilon_2 = v_2 + u_2$ as:

$$f(\varepsilon_2) = \frac{2}{\sigma_2} \phi\left(\frac{\varepsilon_2}{\sigma_2}\right) \Phi\left(\frac{\lambda_2 \varepsilon_2}{\sigma_2}\right)$$

The corresponding CDF of ε_2 is similarly expressed as:

$$F(B) = \int_{-\infty}^B f(\varepsilon_2) d\varepsilon_2 = \int_{-\infty}^B \frac{2}{\lambda_2 \sigma_2} \phi\left(\frac{\varepsilon_2}{\sigma_2}\right) \Phi\left(\frac{\lambda_2 \varepsilon_2}{\sigma_2}\right) d\varepsilon_2 \\ = \frac{2}{\sigma_2} \int_{-\infty}^B \left(\int_{-\infty}^{-\frac{\lambda_2 \varepsilon_2}{\sigma_2}} \phi(\xi) d\xi \right) \phi\left(\frac{\varepsilon_2}{\sigma_2}\right) d\varepsilon_2 = \frac{2}{\sigma_2} I(B)$$

Since function $I(B)$ has no closed form, we must derive its approximation, i.e., $I_{app}(B)$, which is shown to be:

$$I_{app}(B) = \frac{1}{4\sqrt{b^2 - a^2} c_2} \exp\left(\frac{a^2 c_1^2}{4(b^2 - a^2) c_2}\right) \\ \left\{ 1 - \text{erf}\left[\frac{-ac_1 + \sqrt{2}B(b^2 - a^2)c_2 \text{sign}(B)}{2\sqrt{b^2 - a^2} c_2}\right] \right\} \\ + \frac{\text{erf}\left(\frac{bB}{\sqrt{2}}\right)}{2b} \frac{1 + \text{sign}(B)}{2}$$

where $a = \lambda_2/\sigma_2 > 0$, $b = 1/\sigma_2$, $c_1 = -1.09500814703333$, $c_2 = -0.75651138383854$ and $\text{sign}(B)$ is similarly defined to $\text{sign}(A)$. Replacing $I(B)$ with $I_{app}(B)$, we get the approximated CDF of $F(B)$, i.e.:

$$F_{app}(B) = \frac{2}{\sigma_2} I_{app}(B).$$

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