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Empirical analysis of stock indices under a regime-switching model with dependent jump size risks

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ABSTRACT

In this study, we propose a regime-switching model with dependent jump size risks to capture important characteristics of cyclical movements and abnormal shock events. We further demonstrate that the two-state model provides asymmetric and leptokurtic return features, and volatility clustering is observed empirically using 12 years of daily data for the S&P 500, Dow Jones Industrial Average (DJIA), and Nikkei 225 indices. In addition, our results indicate that the regime-switching model with dependent jump size risks is superior to the competing models.

1. Introduction

There has been an accelerating trend in recent decades to create passively managed mutual funds that are based on market indices, such as index funds or exchanged traded funds (ETFs). The stock market index provides reliable market information as well as a better understanding of market forces. It also creates a benchmark against which investors and money managers can measure. The stock market index is a useful tool used by investors and financial managers to describe the market and compare the returns on specific investments. According to the theory and numerous empirical evidence of the Efficient Market Hypothesis (EMH), it is impossible to consistently outperform the market without increasing the risk level. Additionally, a majority of mutual funds fails to outperform the market. Therefore, we can buy into the market through index-related funds with very low management fees. The so-called "index investing" is growing and prevailing not only because it aims to match market performance but also because it incurs very few expenses. There are many developed derivatives of stock indices such as stock index futures and stock index options. The derivatives of stock indices have become important tools with which to hedge risks. Therefore, it is vital to capture the dynamics of the stock market indices. Developing appropriate models to describe their dynamics and trends has drawn increasing attention from individual investors, fund managers, financial companies, researchers and the government.

In the early literatures, stock returns are assumed to follow a traditional geometric Brownian motion, including the Black–Scholes model (BSM), and this assumption is reasonable under relatively stable market conditions. However, the existence of cyclical price movements generates a series of regime-switching models on asset pricing. Hamilton (1989) first proposes the regime-switching model to capture the expansion–recession cycles for the growth rate of Gross National Product. The literature has shown that this model and its variants have been widely applied to analyze economic and financial time series (Bollen et al., 2000; Chang and Feigenbaum, 2008; Chun et al., 2014; Engel, 1994; Engel and Hamilton, 1990; Garcia and Perron, 1996; Goodwin, 1993; Hardy, 2001; Kim and Yoo, 1995; Schaller and van Norden, 1997; Schwert, 1989; Sola and Driffill, 2002).

In the past several decades, significant events including the dot-com bubble in 2000, the September 11 attacks in 2001, the end of the Iraq war in 2003, and the global financial crisis in 2008 occurred, leading to abnormal jumps in stock prices and returns (Lin et al., 2014; Su and Hung, 2011). Unfortunately, the regime-switching model cannot comprehensively describe dramatic changes in such a scenario, and in this paper we propose a regime-switching model with jump size risks to address the jump phenomenon in financial markets. Our model is not the first regime-switching model with jump risks. Elliott et al. (2007) proposed a Markov-modulated jump diffusion model to evaluate the European options. In the model, the market interest rate, jump frequency, mean, and volatility of the underlying asset price change over time according to the state of the economy, which is governed by a continuous Markov chain. In addition, Bo et al. (2010) investigated the same Markov model where the focus was on currency options. In a more recent paper, Chang et al. (2013) provided a closed-form solution for

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Table 1
Summary statistics of S&P 500 index return.

	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	Total
Panel A: trading days													
Number of days	251	252	248	252	252	252	252	251	251	253	252	252	3018
Max	0.0347	0.0465	0.0489	0.0557	0.0348	0.0162	0.0195	0.0213	0.0288	0.1096	0.0684	0.0430	0.1096
Min	−0.0285	−0.0600	−0.0505	−0.0424	−0.0359	−0.0165	−0.0169	−0.0185	−0.0353	−0.0947	−0.0543	−0.0398	−0.0947
Mean	0.0007	−0.0004	−0.0006	−0.0011	0.0009	0.0003	0.0001	0.0005	0.0001	−0.0019	0.0008	0.0005	7.88E-06
Std	0.0114	0.0140	0.0136	0.0164	0.0107	0.0070	0.0065	0.0063	0.0101	0.0258	0.0172	0.0114	0.0136
Skewness	0.0598	0.0007	0.0205	0.4251	0.0532	−0.1102	−0.0155	0.1028	−0.4941	−0.0337	−0.0605	−0.2110	−0.1088
Kurtosis	2.8535	4.3882	4.4478	3.6610	3.7589	2.8623	2.8493	4.1553	4.4481	6.6754	4.8510	4.9599	10.2871
Panel B: jump days													
In excess of 2%	14	18	12	23	10	0	0	2	6	31	27	12	155
Mean	0.0241	0.0287	0.0297	0.0314	0.0285	0.0000	0.0000	0.0279	0.0280	0.0409	0.0315	0.0276	0.0310
Std	0.0037	0.0067	0.0084	0.0103	0.0075	0.0000	0.0000	0.0076	0.0070	0.0215	0.0116	0.0072	0.0133
In excess of −2%	9	19	13	29	5	0	0	0	11	41	28	10	165
Mean	−0.0234	−0.0268	−0.0282	−0.0270	−0.0275	0.0000	0.0000	0.0000	−0.0265	−0.0411	−0.0353	−0.0340	−0.0313
Std	0.0026	0.0094	0.0077	0.0056	0.0058	0.0000	0.0000	0.0000	0.0051	0.0209	0.0160	0.0154	0.0135
In excess of ±2%	23	37	25	52	15	0	0	2	17	72	55	22	320
Mean	0.0055	0.0002	−1.88E-05	−0.0012	0.0024	0.0000	0.0000	0.0021	−0.0012	−0.0058	−0.0003	0.0019	−0.0011
Std	0.0239	0.0293	0.0304	0.0303	0.0302	0.0000	0.0000	0.0300	0.0298	0.0459	0.0369	0.0342	0.0340

their Markov-modulated jump diffusion model and empirically confirmed the existence of jump switching and clustering.

Table 1 reports summary statistics of the S&P 500 index returns from 1999 to 2010. In Panel A, the summary statistics are based on daily returns on trading days, and in Panel B, we show samples of large returns (jumps). From Panel A, we can see that the mean return is negative from 2000 to 2002 and in 2008. As is observed, the return volatility is larger in the same years than in other years. This may due to the Internet bubble in 2000 and the financial crisis in 2008. Generally, the dynamics of price and return of the S&P 500 can be classified into two states, expansion and recession. A state of recession is a period of low returns and high volatility, and a state of expansion is a period of high returns and low volatility.

Panel B in Table 1 presents the summary statistics of stock index returns on large return (jump) days where the return is in excess of ±2%.[1] Specifically, Panel B shows the number of jump days, the means, and the standard deviations of jump day returns. Except in 2004 and 2005, jumps appear every year while the jump frequency and the mean and standard deviation of jump day returns are state dependent. The mean frequency of the jumps in the entire period is 26.67, whereas the mean of jump frequencies in the recession state and expansion state is 46.5 and 16.75, respectively. Additionally, the means and standard deviations of jump day returns are higher in the recession state than those in the expansion state. When the information arrives, asset returns not only generate an abnormal jump but the mean and volatility of this jump size also vary under different states. Therefore, the mean and volatility of jump returns are dependent on different states of the economy.

In this paper, we propose a regime-switching model with dependent jump size risks, in which the jump size of the underlying asset changes over time according to the state of the economy for two main reasons. First, the past literature has documented strong empirical evidence of regime-switching behavior of stock market prices (Alizadeh and Nomikos, 2004; Hardy, 2001; Pan and Li, 2013; Rey et al., 2014; Schaller and van Norden, 1997; Schwert, 1989; Timmermann, 2000). Second, empirical observations also show that jump sizes in equity markets are not independent but seem to come together for a certain period. According to the previously observed features on large return days, we empirically find jump clustering, which means that jumps are more frequent in some periods than others, and different jump sizes under different states are also observed. Therefore, we incorporate both jump intensity and state-dependent jump sizes into the regime-switching model.

The regime-switching model with dependent jump size risks has the ability to capture cyclical movements as well as abnormal jump attributes of the underlying asset price. This paper extends the Markov-modulated diffusion model with independent jump risks (Chang et al., 2013; Lin et al., 2014) and empirically examines three stock indices, the S&P 500, DJIA and Nikkei 225 indices. The expectation maximization (EM) algorithm is applied to estimate the parameters of the model while also applying the Supplemented Expectation Maximization (SEM) algorithm to estimate the standard deviation of these parameters. From the empirically estimated parameters in the dynamic model and the derived stock prices, we show that the model is superior to the competing models in stock indices. The estimation results also indicate that our model may capture some critical empirically observed features of asset returns, including asymmetry, leptokurtosis, and volatility clustering. Moreover, the results suggest that jump frequencies and jump sizes are not independent, because high jump size risks are generally followed by continued high jump size risks for the period of the high arrival rate, and low jump size risks are generally followed by continued low jump size risks for the period of the low arrival rate. Therefore, the behaviors of jumps can address jump clustering or volatility clustering driven by jump frequencies and jump sizes.

In this paper, we propose a regime-switching model with dependent jump size risks, in which the jump size of the underlying asset changes over time according to the state of the economy. This paper contributes to the literature on asset pricing and risk management (Chang et al., 2013; Elliott et al., 2007; Elliott et al., 2010; Li et al., 2016; Lin et al., 2014, 2015; Merton, 1976; Su and Hung, 2011). First, we propose a more general jump size risk model, which advances the jump diffusion model to a regime-switching model with dependent jump size risks (RSMDJ) based on a reduced form of the regime-switching model. Second, we develop EM and SEM algorithms to estimate the parameters of the RSMDJ in the past estimation literature of the EM and SEM algorithm (Lange, 1995; Li et al., 2016; Lin et al, 2014; Lin et al., 2015; Mandelbrot and Benoit, 1963; Meng and Rubin, 1991). Finally, actual market data are used to examine the empirical fit performance. Past studies have provided strong empirical evidence of regime-switching behavior in the price in financial markets (Bollen et al., 2000; Chang et al., 2013; Chun et al., 2014; Elliott et al., 2007; Elliott et al., 2010; Garcia and Perron, 1996; Li et al., 2016; Lin et al., 2014; Lin et al., 2015; Rey et al., 2014). Compared to the competing models, our regime-switching model with dependent jump size risks can better explain the dynamics of the S&P 500, DJIA and Nikkei 225 indices. The empirical results are significant in capturing the asymmetry, leptokurtosis, and volatility clustering of stock returns.

The paper is organized as follows. Section 2 outlines the economic framework of the regime-switching model as well as the regime-

[1] We assume that price changes of less than 2% are noise.

switching models with independent and dependent jump size risks. Section 3 demonstrates the estimations and tests. Section 4 presents our empirical analysis and results. Section 5 presents the conclusions.

2. Modeling

This section outlines and compares the basic regime-switching model (RSM), the regime-switching model with independent jump risks (RSMIJ), and the regime-switching model with dependent size risks (RSMDJ). These models are developed for stock market indices returns to explain the economic implications of the models.

2.1. Regime-switching model

The parameters of the basic regime-switching model are functions of a Markov chain in which the states represent the hidden states of an economy or different stages of stock cycles. Suppose that there are two states; the state of low return and high volatility occurs (a recession state) when $q_t = 1$, while the state of high return and low volatility occurs (an expansion state) when $q_t = 2$. Here, q_t is the state variable that is unobservable at time t and is assumed to follow the first-order Markov process. If the probability of the state is dependent on the current regime, its mathematical expression regardless of prior states is expressed as

$$\Pr(q_t|q_{t-1}, \ldots, q_1) = \Pr(q_t|q_{t-1}). \tag{1}$$

Following the transition matrix, P is used to control the probability of regime-switching and is expressed as

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}, \tag{2}$$

where p_{ij} is the probability of transition from state i at time $t-1$ to state j at time t, which is given by

$$P(q_t = j|q_{t-1} = i) = p_{ij} \ , \ \ i, j = 1, 2. \tag{3}$$

Under state i at any time $t-1$, the probability of transition must meet the criterion given by

$$\sum_{j=1}^{2} p_{ij} = 1, \forall i = 1, 2. \tag{4}$$

Therefore, at a discrete time, the two states for the return on underlying assets can be expressed as

$$R_t = \begin{cases} \mu_1 + \sigma_1 Z_t & , q_t = 1 \\ \mu_2 + \sigma_2 Z_t & , q_t = 2 \end{cases}, \tag{5}$$

where R_t represents the asset returns at time t and Z_t denotes a standard normal distribution. When in recession state, the mean value is μ_1 and the volatility is σ_1. In an expansion state, the mean value is μ_2 and the volatility is σ_2. According to Engel and Hamilton (1990), the model could describe a variety of processes depending on the values given by the parameters. Thus, the model can better address the variation process of asset returns. However, this model cannot capture the jump feature when the financial market responds to unexpected events, such as the global financial crisis and the dot-com bubble. Therefore, a regime-switching model with independent jump risks is introduced in the following section.

2.2. Regime-switching model with independent jump risks

The basic regime-switching model cannot capture the jump behavior of asset returns when extreme information appears in the market. In response, Lin et al. (2014) introduced the jump arrival and jump size into the regime-switching model, providing a comprehensive description of asset returns. The jump-diffusion model proposed by Merton (1976) also showed that the underlying assets are affected by significant information, and as such, the return is relevant to jump frequency and jump size. Further, the Markov-switching model of the Poisson jump process is also presented. It refers to a regime-switching model with independent jump risks wherein the jump terms are irrelevant to the state (Lin et al., 2014).

At discrete times, two states with independent jump risks can be expressed as

$$R_t = \begin{cases} \mu_1 + \sigma_1 Z_t + \sum_{n=1}^{N_t} \log Y_n & , q_t = 1 \\ \mu_2 + \sigma_2 Z_t + \sum_{n=1}^{N_t} \log Y_n & , q_t = 2 \end{cases}, \tag{6}$$

where the assumptions of Z_t, μ_i, σ_i, and q_t are the same as those of the regime-switching model. N_t is the number of jumps at time t, which is independent and follows a Poisson distribution of parameter λ. Y_n denotes the jump size that follows a log-normal distribution with mean μ_y and standard deviation σ_y. Both the mean and standard deviation also follow the first-order Markov chain switch between the states described in Section 2.1. However, we find that the mean and volatility of jump returns are dependent on different regimes. Therefore, we develop the RSMDJ, in which the jump size risks are dependent on different states.

2.3. Regime-switching model with dependent jump size risks

The RSMIJ assumes that the mean and volatility of sizes are consistent and unaffected by the state in which they are found. However, empirical observations show that jumps in the equity market are not independent and seem to have a different mean and volatility of jump sizes in different states. This can be observed from the dynamic process of the S&P 500 index returns. Suppose that the market is in a recession state; the jump arrival rate in the recession state should be higher (bad news comes frequently) than that in the expansion state. When significant information appears in the market, the jump size and volatility become larger when the information contains bad news. Therefore, the RSMDJ is explored in this paper to improve the characteristics of the terms $\{Y_n\}$, providing that the jump size is also affected by the state (expressed as $Y_{q_t,n}$).

At discrete times, two states with dependent jump size risks can be expressed as

$$R_t = \begin{cases} \mu_1 + \sigma_1 Z_t + \sum_{n=1}^{N_t} \log Y_{1,n} & , q_t = 1 \\ \mu_2 + \sigma_2 Z_t + \sum_{n=1}^{N_t} \log Y_{2,n} & , q_t = 2 \end{cases}, \tag{7}$$

where the assumptions of Z_t, μ_i, σ_i, and q_t are the same as those of the regime-switching model under no jump events. N_t denotes the number of jumps at time t, which is dependent and follows a Poisson distribution with parameter λ. $Y_{i,n}$ indicates the jump sizes that follow the log-normal distribution with a mean μ_{iy} and standard deviation σ_{iy} with state i. Both the mean and standard deviation are governed by a first-order Markov chain.

3. Estimation and tests

3.1. Estimation

It is assumed that return $\widetilde{R} = \{R_1, R_2, \ldots, R_T\}$ represents the observable data, whereas the number of jumps at each time interval $\tilde{N} = \{N_1, N_2, \ldots, N_T\}$ and regime $\widetilde{q} = \{q_1, q_2, \ldots, q_T\}$ are unobservable and consider missing data. These observed and missing data are collectively called complete data. The parameter space of the RSMDJ is expressed as

$$\Theta_{RSMDJ} = \left\{w_1, p_{11}, p_{22}, \mu_1, \mu_2, \mu_{1y}, \mu_{2y}, \sigma_1, \sigma_2, \sigma_{1y}, \sigma_{2y}, \lambda\right\} 0 \le \{w_1, p_{11}, p_{22}\} \le 1, \\ -\infty < \left\{\mu_1, \mu_2, \mu_{1y}, \mu_{2y}\right\} < \infty,\ 0 \le \{\sigma_1, \sigma_2, \sigma_{1y}, \sigma_{2y}\} \langle \infty, \lambda \rangle 0.$$

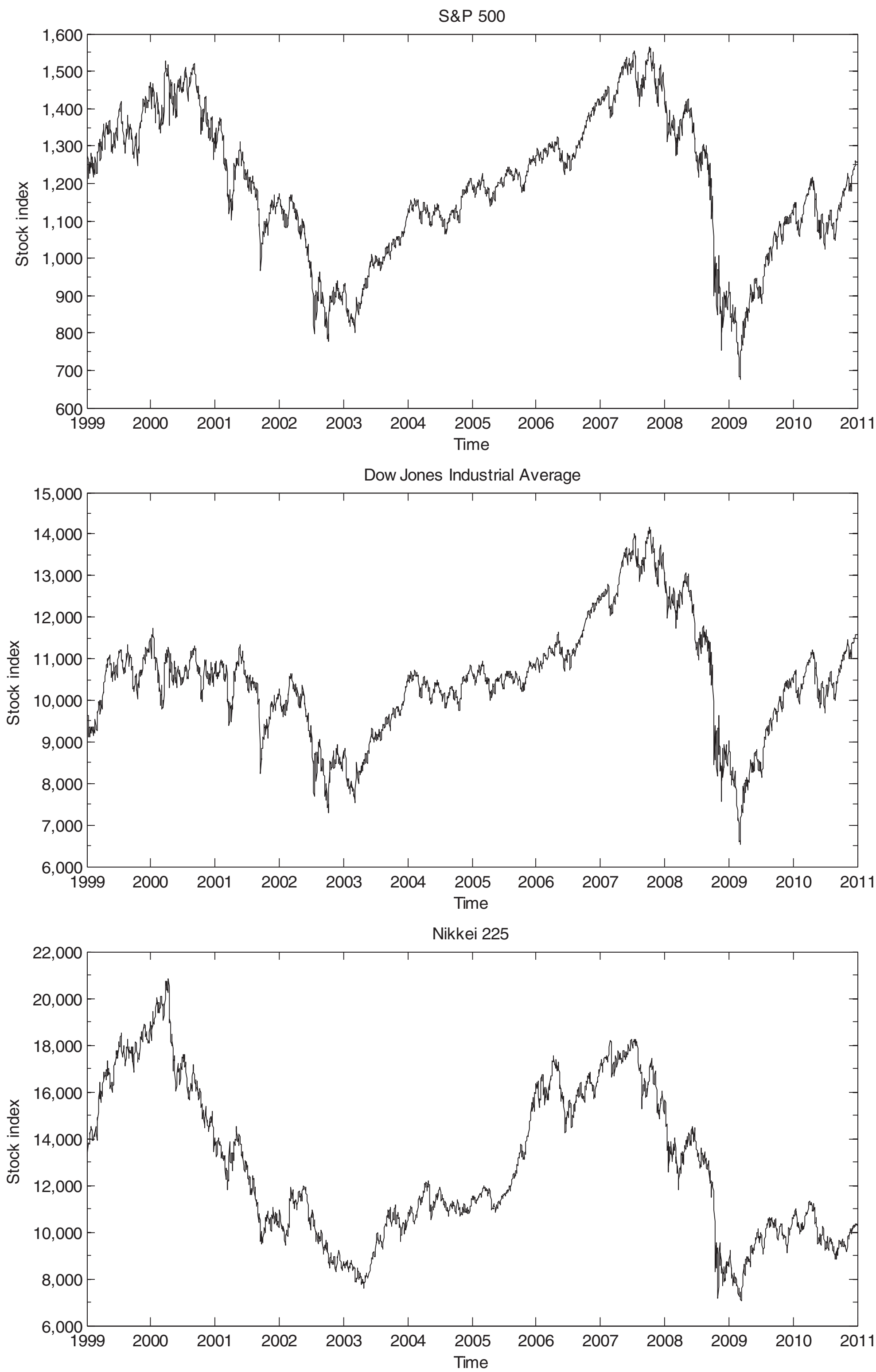

Fig. 1. Time series dynamic of S&P 500 index, Dow Jones Industrial Average index, and Nikkei 225 index.

Table 2
Summary statistics of Dow Jones Industrial Average Index Return.

	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	Total
Panel A: trading days													
Number	251	252	248	252	252	252	252	251	251	253	252	252	3018
Max	0.0280	0.0481	0.0437	0.0615	0.0353	0.0174	0.0204	0.0196	0.0252	0.1051	0.0661	0.0382	0.1051
Min	−0.0263	−0.0582	−0.0740	−0.0475	−0.0367	−0.0165	−0.0188	−0.0198	−0.0335	−0.0820	−0.0473	−0.0367	−0.0820
Mean	0.0009	−0.0003	−0.0003	−0.0007	0.0009	0.0001	0.0000	0.0006	0.0002	−0.0016	0.0007	0.0004	0.0001
Std	0.0102	0.0131	0.0135	0.0160	0.0104	0.0068	0.0065	0.0062	0.0092	0.0238	0.0152	0.0102	0.0127
Skewness	0.0431	−0.2812	−0.5680	0.4905	0.1117	0.0095	−0.0031	−0.1092	−0.6195	0.2220	0.0713	−0.1772	−0.0005
Kurtosis	2.8665	4.6802	6.9590	4.1861	4.0871	2.8642	3.0227	4.1903	4.5799	6.7269	5.1151	5.0991	10.3570
Panel B: jump days													
In excess of 2%	10	13	12	24	10	0	1	0	5	32	26	8	141
Mean	0.0226	0.0260	0.0278	0.0309	0.0285	0.0000	0.0281	0.0000	0.0274	0.0375	0.0284	0.0269	0.0297
Std	0.0024	0.0078	0.0084	0.0110	0.0077	0.0000	0.0079	0.0000	0.0074	0.0209	0.0103	0.0090	0.0133
In excess of −2%	6	18	12	24	5	0	0	0	9	40	19	10	143
Mean	−0.0231	−0.0273	−0.0300	−0.0271	−0.0275	0.0000	0.0000	0.0000	−0.0263	−0.0374	−0.0376	−0.0352	−0.0306
Std	0.0020	0.0094	0.0132	0.0069	0.0072	0.0000	0.0000	0.0000	0.0056	0.0177	0.0165	0.0171	0.0126
In excess of ±2%	16	31	24	48	15	0	1	0	14	72	45	18	284
Mean	0.0054	−0.0049	−0.0035	0.0019	0.0038	0.0000	0.0036	0.0000	−0.0008	−0.0041	0.0025	0.0024	−0.0007
Std	0.0229	0.0280	0.0312	0.0307	0.0291	0.0000	0.0290	0.0000	0.0288	0.0420	0.0371	0.0353	0.0328

The complete-data likelihood function under the RSMDJ is expressed as

$$L_C^{RSMDJ} = \left(\Theta_{RSMDJ}|\tilde{R},\tilde{q},\tilde{N}\right)$$
$$= \prod_{t=1}^{T}\Pr\left(R_t|q_t,N_t,\Theta_{RSMDJ}\right)\prod_{t=1}^{T}\Pr\left(N_t|\Theta_{RSMDJ}\right)\prod_{t=1}^{T}\Pr\left(q_t|q_{t-1},\Theta_{RSMDJ}\right)w_{q_1}.$$
$$= w_{q_1}\prod_{t=1}^{T}p_{q_{t-1}q_t}\prod_{t=1}^{T}\Pr\left(R_t|q_t,N_t,\Theta_{RSMDJ}\right)\prod_{t=1}^{T}\Pr\left(N_t|\Theta_{RSMDJ}\right)$$

Thus, $L_{iC}^{RSMDJ} = (\Theta_{RSMDJ}|\tilde{R})$ represents the incomplete-data likelihood function, which is expressed as

$$L_{iC}^{RSMDJ} = \left(\Theta_{RSMDJ}|\tilde{R}\right) = \sum_{q_1,q_2,\ldots,q_T=1}^{2}\ \sum_{N_1,N_2,\ldots,N_T=0}^{\infty} L_C^{RSMDJ} = \left(\Theta_{RSMDJ}|\tilde{R},\tilde{q},\tilde{N}\right).$$

However, if there are too many periods T, the possible combinations of $\{q_1, q_2, \ldots, q_T\}$ become too large, making the calculation of the incomplete-data likelihood function too difficult. Therefore, the EM algorithm proposed by Dempster et al. (1977) is used to determine the maximum likelihood estimations by the complete-data likelihood function. The EM algorithm has two steps, E and M. Step E takes the logarithm of the complete-data likelihood function and computes the conditional expectation given the observable return $\tilde{R}$ and previous stage parameters $\Theta^{(k-1)}{}_{RSMDJ}$. The expression for the log-complete-data likelihood function under the RSMIJ is given by

$$L_C^{RSMDJ} = \left(\Theta_{RSMDJ}\middle|\tilde{R},\tilde{q},\tilde{N}\right)$$
$$= \log w_{q1} + \sum_{t=2}^{T}\log p_{q_{t-1}q_t}$$
$$+\sum_{t=1}^{T}\left[-\lambda + n_t\log\lambda - \log n_t! - \frac{1}{2}\log\left[2\pi\left(\sigma_{q_t}^2 + n_t\sigma_{q_t y}^2\right)\right] - \frac{\left[R_t - \left(\mu_{qt} + n_t\mu_{q_t y}\right)\right]^2}{2\left(\sigma_{q_t}^2 + n_t\sigma_{q_t y}^2\right)}\right]$$

Assuming that the parameter of the iteration is $k-1$ and that $\Theta_{RSMDJ}^{(k-1)}$ has been obtained, then step E of iteration K can be expressed as

$$Q\left(\Theta_{\rm RSMDJ}\middle|\Theta_{\rm RSMDJ}^{k-1}\right) = E\left[\log L_C^{\rm RSMDJ}\left(\Theta_{\rm RSMDJ}\middle|\widetilde{R},\widetilde{q},\widetilde{N}\right)\middle|\widetilde{R},\Theta_{\rm RSMDJ}^{(k-1)}\right]$$
$$= \sum_{i=1}^{2}\log w_i\ \Pr\left(q_1 = i\middle|\widetilde{R},\Theta_{\rm RSMDJ}^{(k-1)}\right) + \sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{t=1}^{T}\log p_{ij}\ \Pr\left(q_1 = i\middle|\widetilde{R},\Theta_{\rm RSMDJ}^{(k-1)}\right)$$
$$+ \sum_{t=1}^{T}\sum_{n_t=0}^{\infty}(-\lambda + n_t\log\lambda - \log n_t!)\Pr\left(q_1 = i\middle|\widetilde{R},\Theta_{\rm RSMDJ}^{(k-1)}\right)$$
$$+ \sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{n_t=0}^{\infty}\log\Pr(R_t|q_t = i, N_t = n_t)\Pr\left(N_t = n_t\middle|\widetilde{R},\Theta_{\rm RSMDJ}^{(k-1)}\right)\Pr\left(q_t = i\middle|\widetilde{R},\Theta_{\rm RSMDJ}^{(k-1)}\right).$$

In step M, the parameter $\Theta_{RSMDJ} = \{w_1, p_{11}, p_{22}, \mu_1, \mu_2, \mu_{1y}, \mu_{2y}, \sigma_1, \sigma_2, \sigma_{1y}, \sigma_{2y}, \lambda\}$ should be determined and maximized. The function in the formula presented above can be divided into three parts,

Table 3
Summary statistics of Nikkei 225 index return.

	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	Total
Panel A: trading days													
Number	245	247	245	246	245	246	245	248	245	244	243	244	2943
Max	0.0489	0.0423	0.0722	0.0574	0.0333	0.0276	0.0246	0.0352	0.0360	0.1323	0.0503	0.0319	0.1323
Min	−0.0344	−0.0723	−0.0686	−0.0410	−0.0523	−0.0497	−0.0388	−0.0423	−0.0557	−0.1211	−0.0504	−0.0392	−0.1211
Mean	0.0014	−0.0013	−0.0009	−0.0009	0.0009	0.0003	0.0014	0.0002	−0.0007	−0.0020	0.0007	−0.0002	−0.0001
Std	0.0127	0.0144	0.0186	0.0162	0.0145	0.0113	0.0086	0.0125	0.0119	0.0293	0.0175	0.0132	0.0159
Skewness	0.1668	−0.4195	0.2025	0.2706	−0.5235	−0.3556	−0.2582	−0.1526	−0.5947	−0.2481	−0.0538	−0.2089	−0.2850
Kurtosis	4.0519	5.2450	4.2461	3.1708	3.5170	4.0103	4.7218	3.4131	5.0820	6.7541	3.5272	3.0392	9.0103
Panel B: jump days													
In excess of 2%	15	17	31	22	19	9	4	11	7	40	24	17	216
Mean	0.0286	0.0261	0.0321	0.0298	0.0270	0.0270	0.0271	0.0281	0.0277	0.0380	0.0381	0.0339	0.0300
Std	0.0082	0.0061	0.0104	0.0083	0.0052	0.0051	0.0050	0.0048	0.0048	0.0224	0.0158	0.0167	0.0125
In excess of −2%	12	19	30	31	23	9	2	13	17	46	29	18	249
Mean	−0.0267	−0.0296	−0.0304	−0.0260	−0.0283	−0.0289	−0.0295	−0.0289	−0.0277	−0.0431	−0.0422	−0.0399	−0.0310
Std	0.0051	0.0125	0.0109	0.0054	0.0074	0.0083	0.0083	0.0074	0.0077	0.0248	0.0246	0.0253	0.0142
In excess of ±2%	27	36	61	53	42	18	6	24	24	86	53	35	465
Mean	0.0040	−0.0033	0.0014	−0.0027	−0.0023	−0.0027	−0.0022	−0.0031	−0.0067	−0.0054	−0.0022	−0.0023	−0.0027
Std	0.0288	0.0299	0.0332	0.0285	0.0279	0.0283	0.0288	0.0286	0.0283	0.0470	0.0466	0.0442	0.0333

namely, the initial probability, the transition probability, and the remaining parameters. Thus, the parameter estimations of these three parts can be identified separately and their function of Q is maximized.

The first and second parts can be used to derive $\hat{w}_1$, $\hat{p}_{11}$, and $\hat{p}_{22}$ by constraints $\sum_{i=1}^{2} w_i = 1$ and $\sum_{i=1}^{2} p_{ij} = 1$ with a Lagrange multiplier. The parameter estimation of $\mu_1, \mu_2, \mu_{1y}, \mu_{2y}, \sigma_1, \sigma_2, \sigma_{1y}, \sigma_{2y}$, and λ can derive $d^1 Q(\Theta_{RSMDJ}|\Theta_{RSMDJ}^{(k-1)})$ and $d^2 Q(\Theta_{RSMDJ}|\Theta_{RSMDJ}^{(k-1)})$ through an EM gradient algorithm (Lange, 1995). The first- and second-order differentials are developed for each parameter of $Q(\Theta_{RSMDJ}|\Theta_{RSMDJ}^{(k-1)})$. The parameter of the next stage can then be identified using the formula given by

$$\Theta_{\mathrm{RSMDJ}}^{(k)} = \Theta_{RSMDJ}^{(k-1)} - a\left[d^2 Q\left(\Theta_{RSMDJ}|\Theta_{RSMDJ}^{(k-1)}\right)\right]^{-1} d^1 Q\left(\Theta_{RSMDJ}|\Theta_{RSMDJ}^{(k-1)}\right).$$

Thus,

$$\Theta_{\mathrm{RSMDJ}}^{(k)} = \arg\max_{\Theta} Q_{RSMDJ}\left(\Theta, \Theta^{(k-1)}\right),$$

where $a \in (0, 1)$ such that the parameter value does not exceed the parameter space.

Under the condition of the progressive increase of $Q(\Theta_{RSMDJ}|\Theta_{RSMDJ}^{(k-1)})$, steps E and M continuously iterate until constricted to a local maximum value. Finally, the SEM algorithm (Meng and Rubin, 1991) is used to approximate the variance of the estimation with the EM constringency rate.

3.2. Tests

This study uses the likelihood ratio (LR) as a testing model. The null hypothesis is $H_0 : \theta \in \Theta_0$ against the alternative hypothesis $H_1 : \theta \in \Theta_1 \backslash \Theta_0$, $\Theta_0 \subset \Theta_1$. The testing statistics are

$$\Lambda = 2(\ln L(R;\Theta_1) - \ln L(R;\Theta_0)),$$

where $\ln L(R;\Theta_i)$ is the log-maximum likelihood function under H_i. Under the null hypothesis and when the sample is large enough, the testing statistics Λ are distributed as $c^2(r)$, where r is the difference between the numbers of parameters in the two models. If $\Lambda > X^2_{r,1-\alpha}$, the null hypothesis is rejected.

In this study, we perform three LR tests as follows: test (a) is based on the BSM (normal distribution assumption for the return) against the RSM. When $\Lambda > X^2_{4,1-\alpha}$, the BSM is rejected and the RSM is proven to be better than the BSM. Test (b) is based on the RSM against the RSMIJ. When $\Lambda > X^2_{3,1-\alpha}$, the RSM is rejected and the RSMIJ is proven to be better than the RSM. Test (c) is based on the RSMIJ against the RSMDJ. When $\Lambda > X^2_{3,1-\alpha}$, the RSMIJ is rejected and the RSMDJ is proven to be better than the RSMIJ.

4. Empirical analysis

In this section, we first perform a data analysis for the three major stock indices in the world and then estimate the parameters in BSM, RSM, RSMIJ, and RSMDJ. The estimated results and LR tests will be presented as well.

4.1. Descriptions of the stock indices

Using 12 years of daily data from Jan. 1, 1999 to Dec. 31, 2010, we examined the three major stock market indices, including the S&P 500, DJIA, and Nikkei 225 indices. Fig. 1 shows the dynamics of these market indices over the past twelve years. All three indices have similar patterns and have at least two market cycles that experienced significant corrections in the 2000 and 2008 recessions. These dramatic market falls are associated with the dot-com bubble in 2000, the 9/11 terrorists attacks in 2001, the JPY Carry Trade and American subprime mortgage crisis in 2007 as well as the global financial crisis in 2008.

Tables 1, 2, and 3 show the descriptive statistics of the S&P 500, the Dow Jones Industrial Average, and the Nikkei 225 indices. Over the past twelve years, stock markets around the world have experienced two business cycles in which peaks occurred in 2000 and at the end of 2007, while bottoms occurred in 2003 and 2009. Accordingly, in the years from 2000 to 2002 and in the year 2008, there were two

Table 4
Parametric estimations and model test of stock index returns under BSM, RSM, RSMIJ and RSMDJ.

Index	Model	$\hat{p}_{11}$	$\hat{p}_{22}$	$\hat{\mu}_1$	$\hat{\mu}_2$	$\hat{\mu}_{1y}$	$\hat{\mu}_{2y}$	$\hat{\sigma}_1$	$\hat{\sigma}_2$	$\hat{\sigma}_{1y}$	$\hat{\sigma}_{2y}$	$\hat{\lambda}_1$	$-2\log\Lambda$
DJIA	BSM			−0.0000				0.0154					
				0.0002				0.0003					
	RSM	0.9824	0.9933	−0.0014	0.0005			0.0237	0.0102				841.33*
		(0.0064)	(0.0031)	(0.0007)	(0.0002)			(0.0003)	(0.0009)				
	RSMIJ	0.9828	0.9940	−0.0015	0.0007	−0.0001		0.0229	0.0081	0.0105		0.4110	13.25*
		(0.0048)	(0.0021)	(0.0104)	(0.0030)	(0.0582)		(0.0666)	(0.0004)	(0.0035)		(0.0172)	
	RSMDJ	0.9864	0.9935	−0.0012	0.0011	−0.0021	0.0002	0.0174	0.0079	0.0275	0.0100	0.2696	15.31*
		(0.0031)	(0.0021)	(0.0002)	(0.0004)	(0.0012)	(0.0043)	(0.0006)	(0.0008)	(0.0052)	(0.0009)	(0.0512)	
S&P 500	BSM			0.0001				0.0127					
				(0.0003)				(0.0005)					
	RSM	0.9798	0.9909	−0.0010	0.0004			0.0207	0.0082				822.23*
		(0.0053)	(0.0074)	(0.0005)	(0.0002)			(0.0011)	(0.0016)				
	RSMIJ	0.9815	0.9949	−0.0009	0.0005	−0.0002		0.0195	0.0069	0.0123		0.2319	72.56*
		(0.0039)	(0.0028)	(0.0005)	(0.0002)	(0.0198)		(0.0007)	(0.0003)	(0.0077)		(0.0079)	
	RSMDJ	0.9795	0.9791	−0.0006	0.0002	−0.0007	0.0044	0.0192	0.0068	0.0374	0.0025	0.0900	23.83*
		(0.0031)	(0.0021)	(0.0002)	(0.0004)	(0.0012)	(0.0043)	(0.0006)	(0.0008)	(0.0052)	(0.0009)	(0.0023)	
Nikkei 225	BSM			−0.0001				0.0159					
				(0.0003)				(0.0002)					
	RSM	0.9641	0.9903	−0.0016	0.0005			0.0257	0.0112				555.36*
		(0.0057)	(0.0024)	(0.0011)	(0.0003)			(0.0002)	(0.0003)				
	RSMIJ	0.9709	0.9932	−0.0011	0.0008	−0.0016		0.0250	0.0104	0.0113		0.3246	9.85*
		(0.0061)	(0.0010)	(0.0156)	(0.0907)	(0.0548)		(0.0089)	(0.0012)	(0.0088)		(0.0198)	
	RSMDJ	0.9754	0.9820	−0.0010	0.0002	−0.0035	0.0048	0.0176	0.0103	0.0339	0.0025	0.1096	21.61*
		(0.0069)	(0.0032)	(0.0052)	(0.0583)	(0.0402)	(0.0050)	(0.0074)	(0.0078)	(0.0055)	(0.0011)	(0.0018)	

Notation: BSM, RSM, RSMIJ, and RSMDJ are the estimation by using the MLE (maximum likelihood function) and EM algorithms, in normal distribution, the regime-switching model, the regime-switching model with independent jump risk, and the regime-switching model with dependent jump risks, respectively. The real numbers in the table are decimal presentations that round to four decimal places. “0” means the zero value and “0.0000” means it is smaller than 0.0001 and is zero rounded to four decimal places.

* The null hypothesis can be rejected at the 1% significance level.

recessions in which the stock market returns exhibited negative means with high volatility. On the contrary, an expansion of other years in which stock market returns exhibited positive means with low volatility was observed.

We define a jump day as a day in excess of $\pm 2\%$ stock returns; that is, there is a high probability of jump risks when the returns of the day are more than $+2\%$ or less than -2%. A further analysis of the returns and volatility in these jump days is presented in Panel B. Consistent with the results in Panel A, the means and standard deviations of jump day returns are dependent on different market conditions: a bear market or a bull market. That is, the absolute mean and standard deviation of returns in jump days are higher in recessions than in expansions.

From the above analysis, we can see that the mean and volatility of stock market returns and the jump risks are affected by market

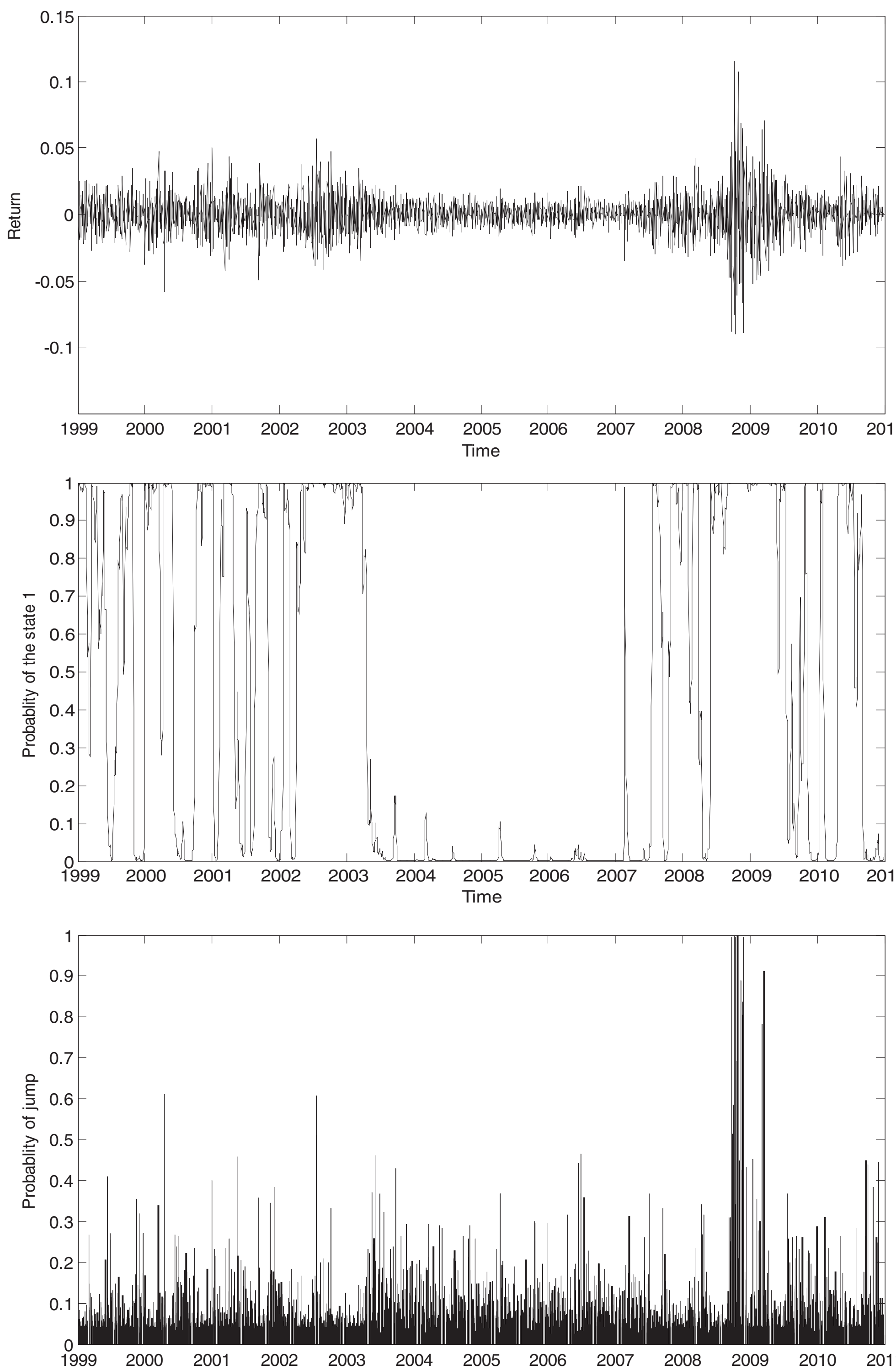

Fig. 2. The dynamics of return, the probabilistic dynamics of recession state, and the probabilistic dynamics of jumps for S&P 500 index return under RSMDJ.

conditions and undergo a regime switch phenomenon. During a bear market, the stock market experiences low returns and high volatility, which are accompanied by more numerous and volatile jumps. During a bull market, the stock market has high returns and low volatility accompanied by fewer and more steady jumps.

4.2. Estimations and tests

In this section, we estimate and test the BSM, RSM, RSMIJ, and RSMDJ models for the S&P 500, Dow Jones Industrial Average, and Nikkei 225 indices. The BSM uses a normal distribution to describe the

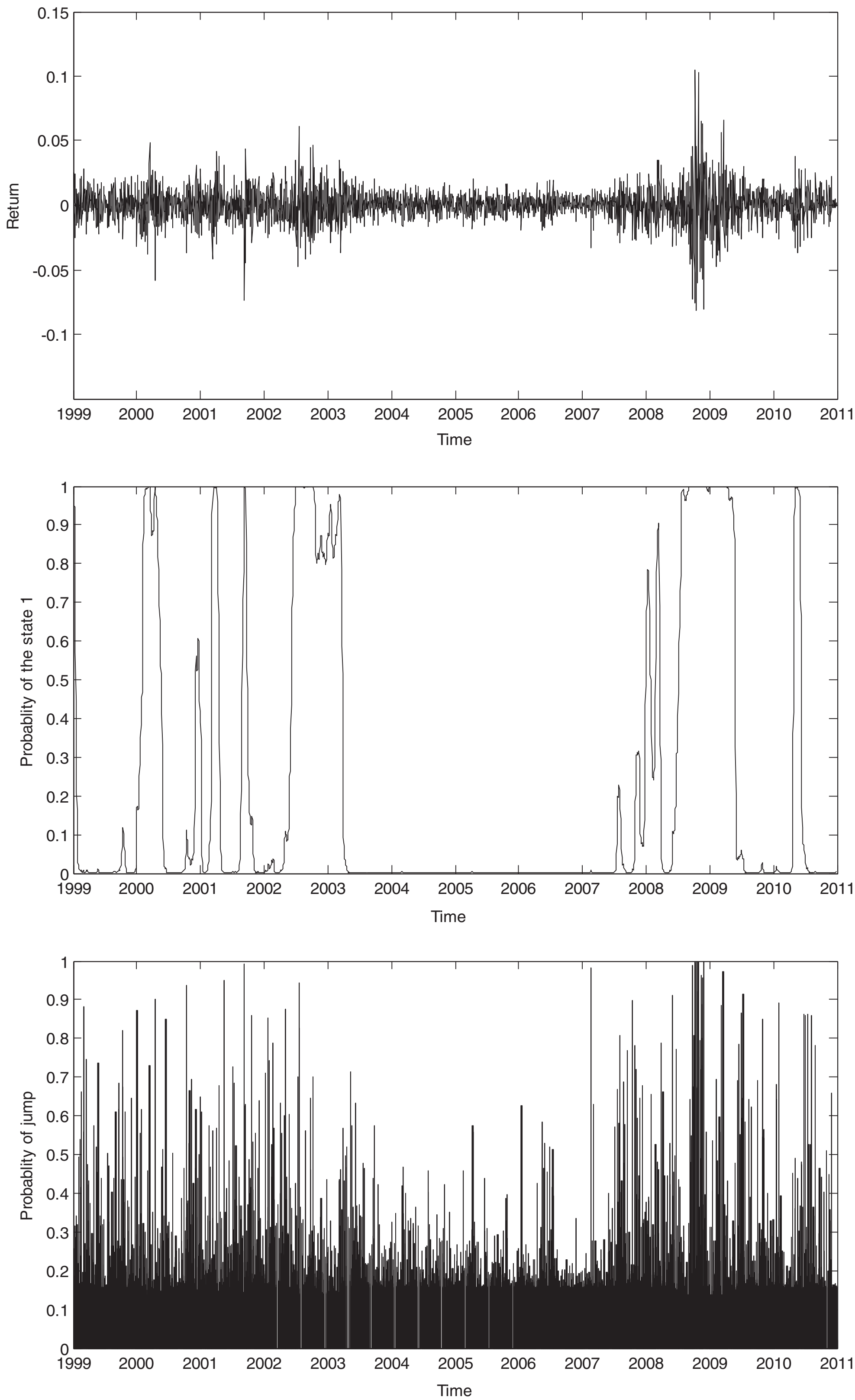

Fig. 3. The dynamics of return, the probabilistic dynamics of recession state, and the probabilistic dynamics of jumps for DJIA index return under RSMDJ.

return and estimate parameters with population means and standard deviation. The EM and SEM algorithms are applied to estimate the parameters of the RSMDJ.

Table 4 shows the estimation and LR test results for the S&P 500 index of the four models: the BSM, RSM, RSMIJ, and RSMDJ. Under the BSM, the return follows the normal distribution with a mean of 0.0001 and a standard deviation of 0.0127. Under the RSM, we show that the transition probabilities $p_{11} = 0.9798$ and $p_{22} = 0.9909$ are close to 1, which means that the probability of remaining in regime one (two) in the flowing period is high. The market switches to another regime only after it remains in one regime for a long time, which denotes that state 1 remains for 99 days and transforms into state 2 on the 100th day when we suppose $p_{11} = 0.99$ and state 1 now. The estimated mean returns under Regimes 1 and 2 are −0.001 and 0.0004 and are

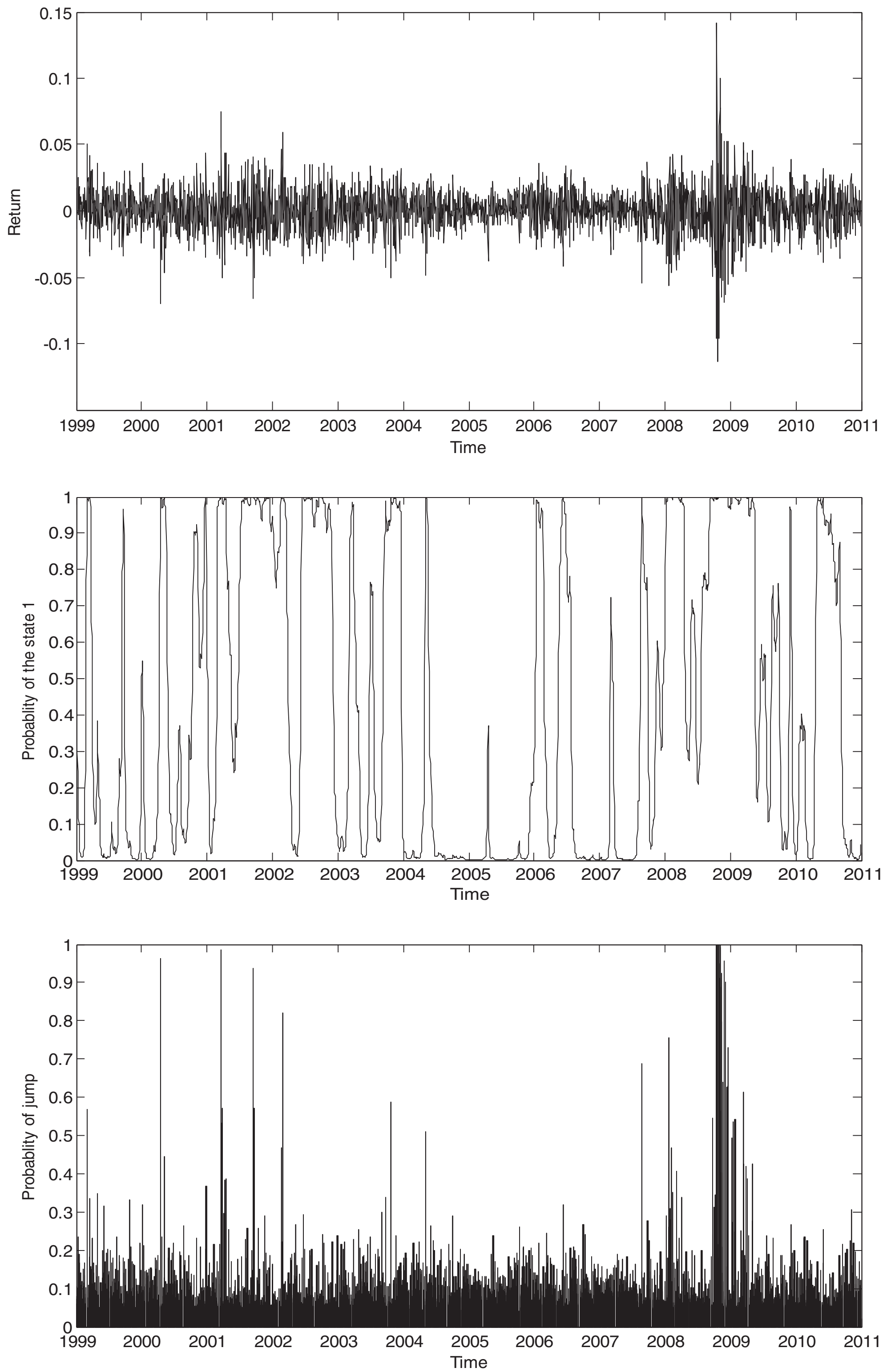

Fig. 4. The dynamics of return, the probabilistic dynamics of recession state, and the probabilistic dynamics of jumps for Nikkei 225 index return under RSMDJ.

Table 5
The moment formulas for regime-switching model with dependent jump risks.

Model
Regime-switching model with dependent jump risks
Mean
$\mu = \sum_{i=1}^{2} \pi_i (\mu_i + \lambda \mu_{iy})$
Variance
$\sigma^2 = \sum_{i=1}^{2} \pi_i \left[E\left(\sum_{n=1}^{N_t} , \log Y_{i,n} \right)^2 + 2E\left(\sum_{n=1}^{N_t} , \log Y_{i,n} \right)(\mu_i - \mu) + (\mu_i - \mu)^2 + \sigma_i^2 \right]$
Skewness
$\frac{1}{\sigma^3} \sum_{i=1}^{2} \pi_i \left[E\left(\sum_{n=1}^{N_t} , \log Y_{i,n} \right)^3 + 3E\left(\sum_{n=1}^{N_t} , \log Y_{i,n} \right)^2 (\mu_i - \mu) + 3E\left(\sum_{n=1}^{N_t} , \log Y_{i,n} \right)[(\mu_i - \mu)^2 + \sigma_i^2] + (\mu_i - \mu)^3 + 3(\mu_i - \mu)\sigma_i^2 \right]$
Kurtosis
$\frac{1}{\sigma^4} \sum_{i=1}^{2} \pi_i \left[E\left(\sum_{n=1}^{N_t} , \log Y_{i,n} \right)^4 + 4E\left(\sum_{n=1}^{N_t} , \log Y_{i,n} \right)^3 (\mu_i - \mu) + 6E\left(\sum_{n=1}^{N_t} , \log Y_{i,n} \right)^3 [(\mu_i - \mu)^2 + \sigma_i^2] \quad + 4E\left(\sum_{n=1}^{N_t} , \log Y_{i,n} \right)[(\mu_i - \mu)^3 + 3(\mu_i - \mu)\sigma_i^2] + (\mu_i - \mu)^4 + 6(\mu_i - \mu)^2 + 3\sigma_i^2 \right]$

Where

$$E\left(\sum_{n=1}^{N_t} \log Y_{i,n} \right) = \lambda \mu_{q_t y} \quad , \quad E\left(\sum_{n=1}^{N_t} \log Y_{i,n} \right)^2 = \mu_{q_t y}^2 (\lambda^2 + \lambda) + \sigma_{q_t y}^2 \lambda \quad , \quad E\left(\sum_{n=1}^{N_t} \log Y_{i,n} \right)^3 = \mu_{q_t y}^3 (\lambda^3 + 3\lambda^2 + \lambda) + 3\mu_{q_t y} \sigma_{q_t y}^2 (\lambda^2 + \lambda),$$

$$E\left(\sum_{n=1}^{N_t} \log Y_{i,n} \right)^4 = \mu_{q_t y}^4 (\lambda^4 + 6\lambda^3 + 7\lambda^2 + \lambda) + 6\mu_{q_{ty}}^2 \sigma_{q_t y}^2 (\lambda^3 + 3\lambda^2 + \lambda) + 3\sigma_{q_t y}^4 (\lambda^2 + \lambda) \quad \circ$$

associated with asymmetric estimated standard deviations 0.0207 and 0.0082, respectively. The results conform to the real process of market returns in which the regimes of a "bear market with high variance" and a "bull market with low variance" can be characterized.

Under the RSMIJ, we observe that the estimated results of the means, standard deviations and transition probabilities are similar to those under the RSM. The transition probability of regimes $p_{11} = 0.9815$ and $p_{22} = 0.9949$ remain close to 1, indicating that the probability of staying in regime one (two) in the flowing period is high. In the recession state, the mean return is positive and the volatility is high, while in the expansion state, the mean return is negative and the volatility is low. Moreover, compared to the estimated result of the RSM, the means are larger and volatilities are smaller, as explained by the jump term. The estimation of λ, the number of jumps between every time segment, is 0.2319. The mean and standard deviation of the jumps are -0.0002 and 0.0123, respectively, showing that the average return caused by the jump has a downward trend when significant events occur in the market within a 12-year sample period.

The last model is the RSMDJ developed by this study. The model considers the jump size in addition to the regime switch and jump terms. The estimated results of the means, standard deviations, and transition probabilities are similar to those in the RSM and RSMIJ. The transition probabilities of the regimes, $p_{11} = 0.9795$ and $p_{22} = 0.9791$, are also close to one. Compared to the jump terms of the RSMIJ, the estimated number of jumps λ is smaller than that under RSMIJ. Furthermore, under RSMDJ, the jump size and volatility are different across regimes. More specifically, the mean (volatility) of jump risks under RSMIJ is the same regardless of whether the state is in an expansion or a recession. However, under RSMDJ, the jump size and volatility are different across states and better correspond to reality. The mean and standard deviation of jumps under a recession regime are -0.0007 and 0.0374, respectively. In an expansion regime, the mean and standard deviation of jumps are 0.0044 and 0.0025. These results indicate that the mean of the jump events in the recession state is smaller and the volatility of the jump events is greater than those in an expansion state.

Jump risk models can better explain the behavior of the reaction of stock market indices to crises like the 2008–2009 financial crisis than the existing models. In this paper, the RSMDJ model incorporates dependent jump size risks into the regime-switching model to describe the reaction of stock market indices to crises such as the 2008–2009 financial crisis than existing models. With respect to the empirical evidence, we examine the three major stock market indices, including

Table 6
Moments estimations for Dow Jones Industrial Average, S&P 500 and Nikkei 225.

Index	Model	Mean	Variance	Skewness	Kurtosis
DJIA	Data	0.0001	0.0002	−0.0005	10.3570
	BSM	0.0001	0.0002	0.0000	3.0000
	RSM	0.0001	0.0001	−0.1198	4.5779
	RSMIJ	0.0003	0.0001	−0.1734	5.1760
	RSMDJ	0.0002	0.0002	−0.1421	7.1993
S&P 500	Data	0.0000	0.0002	−0.1088	10.2871
	BSM	0.0000	0.0002	0.0000	3.0000
	RSM	0.0000	0.0002	−0.1406	5.3884
	RSMIJ	0.0001	0.0002	−0.1478	5.6757
	RSMDJ	0.0001	0.0002	−0.1516	11.5411
Nikkei 225	Data	−0.0001	0.0003	−0.2850	9.0103
	BSM	−0.0001	0.0003	0.0000	3.0000
	RSM	−0.0001	0.0003	−0.1452	5.1911
	RSMIJ	0.0000	0.0003	−0.1179	5.2334
	RSMDJ	−0.0001	0.0002	−0.2973	7.6513

Notation: Data means the moments of the data for DJIA index log return, S&P 500 index log return, and Nikkei 225 return, BSM denotes using the normal distribution to compute the moments of the log return by the estimated result, RSM denotes using the regime switching model to compute the moments of the log return by the estimated result, RSMIJ presents using the regime-switching model with independent jump risk to evaluate the moments of the log return by the estimated result, and RSMDJ is using the regime-switching model with dependent jump risks to evaluate the moments of the log return by the estimated result. The real numbers in the table are decimal presentations that round to four decimal places. "0" means the zero value and "0.0000" means it is smaller than 0.0001 and is zero rounded to four decimal places.

the S&P 500, DJIA, and Nikkei 225 indices by using 12 years of daily data from January 1, 1999 to December 31, 2010 that includes the 2008–2009 financial crisis. The LR test results in the last column of Table 4 show that all three null hypotheses are concluded as being rejected. With a 0.05 significance level, (1) the RSM is better than the BSM; (2) the RSMIJ is better than the RSM; and (3) the RSMDJ is better than the RSMIJ. The results imply that the RSMDJ can better describe the dynamic process of stock index returns because it considers the possibility that jump sizes are different in terms of means and volatilities across regimes.

Figs. 2 to 4 show the plots of returns, the dynamic probability of the recession state, and the dynamic probability of the jumps for the S&P

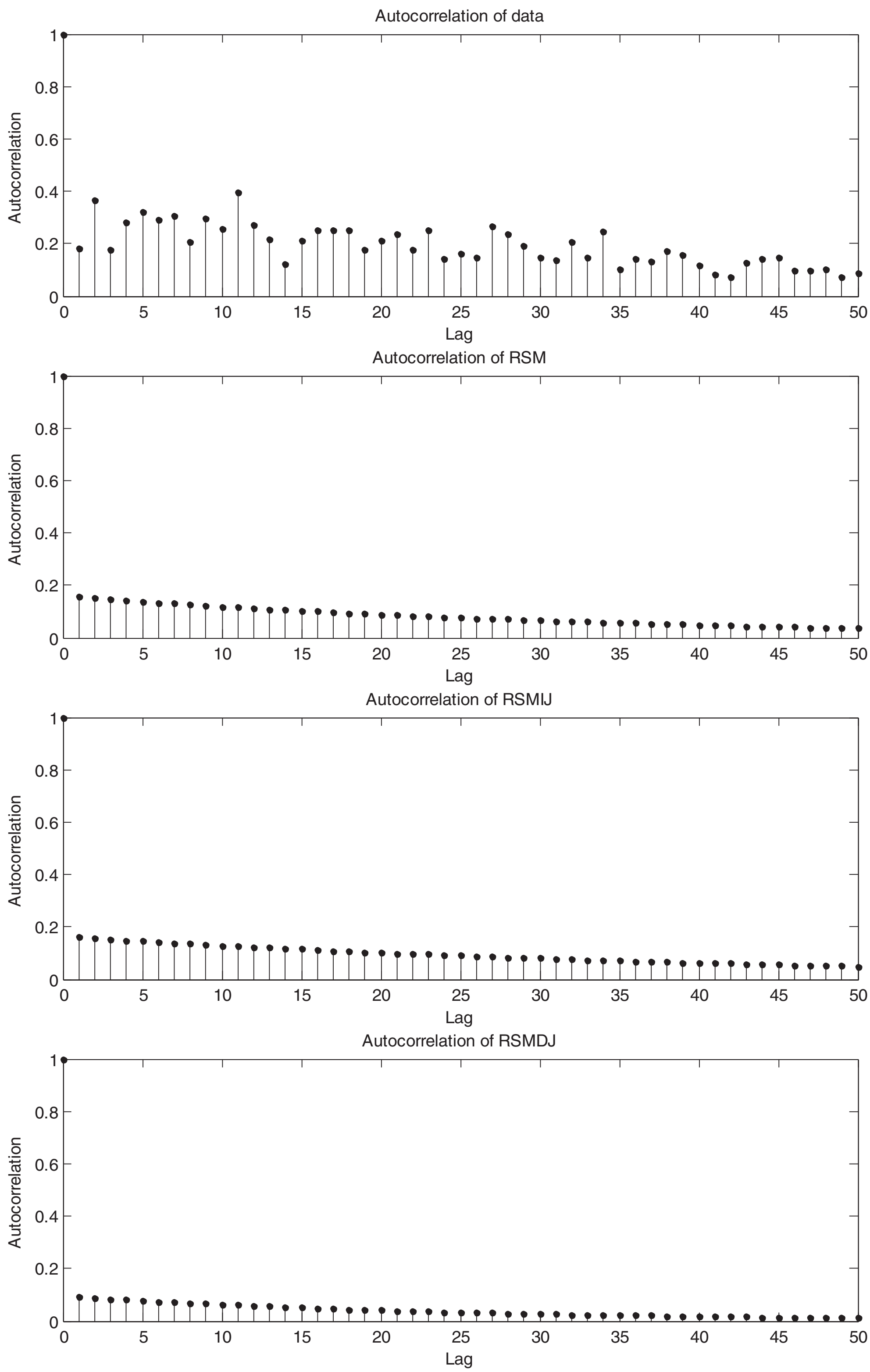

Fig. 5. Autocorrelations of squared returns in S&P 500 index under RSM, RSMIJ and RSMDJ.

Table 7
Autocorrelations of squared returns for RSM, RSMIJ and RSMDJ.

Model	Autocorrelation of squared return
RSM	$\frac{1}{\nu_{RSM}}[(\mu_1^2+\sigma_1^2)^2-(\mu_2^2+\sigma_2^2)^2]^2\frac{(-1+p_{11}+p_{22})^k(1-p_{11})(1-p_{22})}{(2-p_{11}-p_{22})^2}$
RSMIJ	$\frac{1}{\nu_{RSMIJ}}\left\{\left([(\mu_1^2+\sigma_1^2)^2-(\mu_1^2+\sigma_1^2)^2]^2+4\lambda\mu_y\sum_{i=1}^{2}\mu_i(\mu_i^2+\sigma_i^2)+4\lambda^2\mu_y^2(\mu_1-\mu_2)^2\right)\frac{(-1+p_{11}+p_{22})^k(1-p_{11})(1-p_{22})}{(2-p_{11}-p_{22})^2}+4\lambda\mu_y\frac{[1-(-1+p_{11}+p_{22})]^k}{2-p_{11}-p_{22}}\sum_{i=1}^{2}\mu_i(1-p_{2-i,2-i})(\mu_i^2+\sigma_i^2)\right\}$
RSMDJ	$\frac{1}{\nu_{RSMDJ}}\left\{\sum_{i=1}^{2}\pi_i(M_{ii}^k\text{-}\pi_i)[(\mu_i+\lambda\mu_{iy})^2+\sigma_i^2+\lambda(\mu_{iy}^2+\sigma_{iy}^2)^2]^2-2\prod_{i=1}^{2}\pi_i[(\mu_i+\lambda\mu_{iy})^2+\sigma_i^2+\lambda(\mu_{iy}^2+\sigma_{iy}^2)^2]^2\right\}$

Notation:(1) RSM denotes the regime switching model, RSMIJ presents the regime-switching model with independent jump risk, RSMDJ is the regime switching model with dependent jump risks.

(2) $\nu_{\mathrm{RSM}}=\pi_1(\mu_1^4+6\mu_1^2\sigma_1^2+3\sigma_1^4)+\pi_2(\mu_2^4+6\mu_2^2\sigma_2^2+3\sigma_2^4)-[\pi_1(\mu_1^2+\sigma_1^2)+\pi_2(\mu_2^2+\sigma_2^2)]^2$.。

$\nu_{RSMIJ}=\nu_{RSM}+C(Jump)$ 。

(3)
$$C(Jump)=E\left(\sum_{n=1}^{N_t}\log Y_n\right)^4-\left[E\left(\sum_{n=1}^{N_t}\log Y_n\right)^2\right]^2+4\left\{\sum_{i=1}^{2}\pi_i(\mu_i^2+\sigma_i^2)E\left(\sum_{n=1}^{N_t}\log Y_n\right)^2-\left[\sum_{i=1}^{2}\pi_i\mu_iE\left(\sum_{n=1}^{N_t}\log Y_n\right)\right]^2\right\}$$
$$+4\left[\sum_{i=1}^{2}\pi_i(\mu_i^3+3\mu_i\sigma_i^2)-\sum_{i=1}^{2}\pi_i\mu_i\sum_{i=1}^{2}\pi_i(\mu_i^2+\sigma_i^2)\right]E\left(\sum_{n=1}^{N_t}\log Y_n\right)+4\sum_{i=1}^{2}\pi_i\mu_i\left[E\left(\sum_{n=1}^{N_t}\log Y_n\right)^3-E\left(\sum_{n=1}^{N_t}\log Y_n\right)^2E\left(\sum_{n=1}^{N_t}\log Y_n\right)\right].$$

(4)
$$\nu_{RSMDJ}=\prod_{i=1}^{2}\pi_i[(\mu_i+\lambda\mu_{iy})^2+\sigma_i^2+\lambda(\mu_{iy}^2+\sigma_{iy}^2)^2]+\sum_{i=1}^{2}\pi_i\left[4\mu_i^2\sigma_i^2+3\sigma_i^4+4(\mu_i^2+\sigma_i^2)E\left(\sum_{n=1}^{N_t}\log Y_{i,n}\right)^2+E\left(\sum_{n=1}^{N_t}\log Y_{i,n}\right)^4\right]$$
$$-\sum_{i=1}^{2}\pi_i^2\left\{\sigma_i^4+4\mu_i^2\left[E\left(\sum_{n=1}^{N_t}\log Y_{i,n}\right)^2\right]^2-\left[E\left(\sum_{n=1}^{N_t}\log Y_{i,n}\right)^2\right]^2\right\}.$$

500 index, Dow Jones Industrial Average index, and Nikkei 225 index, respectively. Under the recession state, the probability of the S&P 500 index returns is higher from 2000 to 2002 and 2008 to 2009 than other years. Regime switches clearly occurred in 2003 and 2007, indicating that the markets at that time switched to the other regime. The probability of jumps shows that the return is relatively high and stable from 2003 to 2006, although this period experienced the dot-com bubble and the 9/11 terrorist attacks from 2000 to 2002. However, the jump probability of the returns significantly increased because of the subprime mortgage crisis and the global financial crisis from 2008 to 2009.

Furthermore, we observe the probabilistic dynamics of the recession state and the jump risks before, during and after the financial crisis in Fig. 2. We show that the RSMDJ can better correspond to reality. During the pre-crisis period (2004–2007), the probability of the recession state (refer to state 1 in the figure) being low means that the S&P 500 market is in an expansion. During the financial crisis period (2008–2009), the probability of the recession state being high means that the S&P 500 market is in a recession. During the post-crisis period (2010–2011), due to the European debt crisis, the probability of the recession state switching between high and low.

4.3. Asymmetry and leptokurtosis

We further analyze the derived formulas of the return skewness and kurtosis in this section. The skewness and kurtosis of each model are then compared to determine whether the characteristics of the empirical data can be identified under the four models, including the BSM, RSM, RSMIJ, and RSMDJ.

Timmermann (2000) derived the skewness and kurtosis equations of the RSM. Lin et al. (2014) derived the skewness and kurtosis equations of the RSMIJ. Appendix 1 shows the detailed derivation process of the skewness and kurtosis equations of the RSMDJ in Table 5. The equations of the RSMIJ are degraded to the skewness and kurtosis equations of the RSM if the jump terms (average jumps $\lambda=0$) are not considered. Meanwhile, the equations of the RSMDJ are degraded to the skewness and kurtosis equations of the RSMIJ if the regime does not associate the mean and variation of the jump terms, i.e., $\mu_{1y}=\mu_{2y}$ and $\sigma_{1y}=\sigma_{2y}$.

Table 6 reports the skewness and kurtosis as calculated by incorporating the parameter estimates in Table 4 into the formula for each model. The results of the S&P 500, Dow Jones Industrial Average, and Nikkei 225 indices show that the returns exhibit asymmetry and leptokurtosis, which is consistent with the results of Fama (1965), in which the asset returns show asymmetry, leptokurtosis, and fat tails.

The BSM assumes a normal distribution return, a skewness of zero and the kurtosis at 3, so it cannot capture the asymmetry and leptokurtosis features. However, the other three regime-switching models can successfully capture the leptokurtosis. Take the DJIA, for example; the kurtosis estimates are 4.5779 for RSM, 5.1760 for RSMIJ, and 7.1993 for RSMDJ. The kurtosis for the sample data is 10.3570, indicating that the RSMDJ provides the best empirical fit among these regime-switching models. When comparing the kurtosis difference (in standard RSM) between jump risk regime-switching models, the RSMIJ is only marginally better than the RSM; however, the RSMDJ captures the dynamic of the data much better.

In the fifth column in Table 6, except for the BSM, the skewness estimates are all negative, which accords with the empirical data. The skewness estimation of the RSMDJ is similar to that of the RSMIJ when the absolute skewness of the return is smaller than 0.2, such as the DJIA and S&P 500 index. However, for the Nikkei 225 with an absolute skewness larger than 0.2, the skewness estimation of the RSMDJ is much closer to the actual value (the skewness of the data) than that of other competing models. The results suggest that the model with dependent jump size risks, low jump size risks in good times and high jump size risks in bad times capture both the asymmetry and leptokurtosis features and comprehensively describe the nature of the "fat tails" in the return data.

4.4. Volatility clustering

The time series of financial asset returns often exhibit the volatility-clustering property where large changes in prices tend to cluster together and small changes tend to cluster together, which was first observed and proposed by Mandelbrot (1963). Cont (2005) further defined volatility clustering as a positive, significant, and slowly decaying autocorrelation function in squared or absolute returns. Figs. 2 to 5 show the time series of index returns. We can observe that the volatility-clustering behavior is obvious in the S&P 500, Dow Jones Industrial, and the Nikkei 225 indices, respectively.

Table 7 shows the inference equations[2] of the autocorrelation function for the squared returns under the three regime-switching models. To compare the empirical autocorrelations, the model estimations are

[2] The autocorrelation function equation under the RSM and RSMIJ was provided by Lin et al. (2014). Appendix B shows the detailed derivation process of the autocorrelation function equation of the squared returns under the RSMDJ.

derived from the autocorrelation function equations in Table 7 with estimated parameter values from Table 6. Figs. 5, 6, and 7 are the time series of the return autocorrelations under each model for the S&P 500, Dow Jones Industrial Average, and Nikkei 225 indices, respectively. In the figures, the autocorrelations reveal obvious slowly decaying patterns. Moreover, the autocorrelations for squared returns under the RSM, RSMIJ, and RSMDJ models are also very similar to those of actual data. This evidence indicates that regime-switching models can accurately describe the clustering volatility characteristics of the index returns.

5. Conclusions

This paper develops a regime-switching model with dependent jump size risks, in which asset return and jump size risks switch over

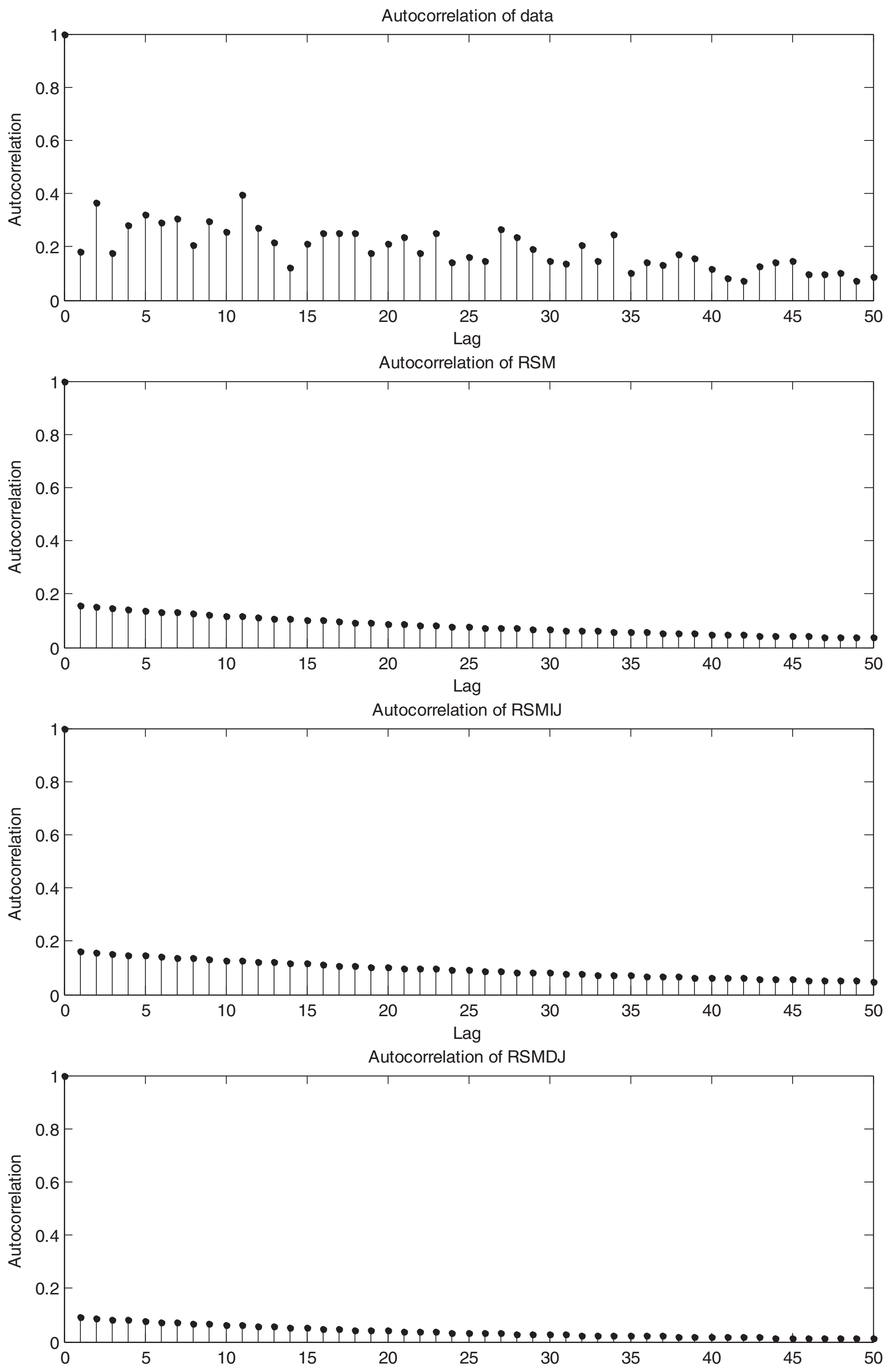

Fig. 6. Autocorrelations of the squared returns in DJIA index under RSM, RSMIJ and RSMDJ.

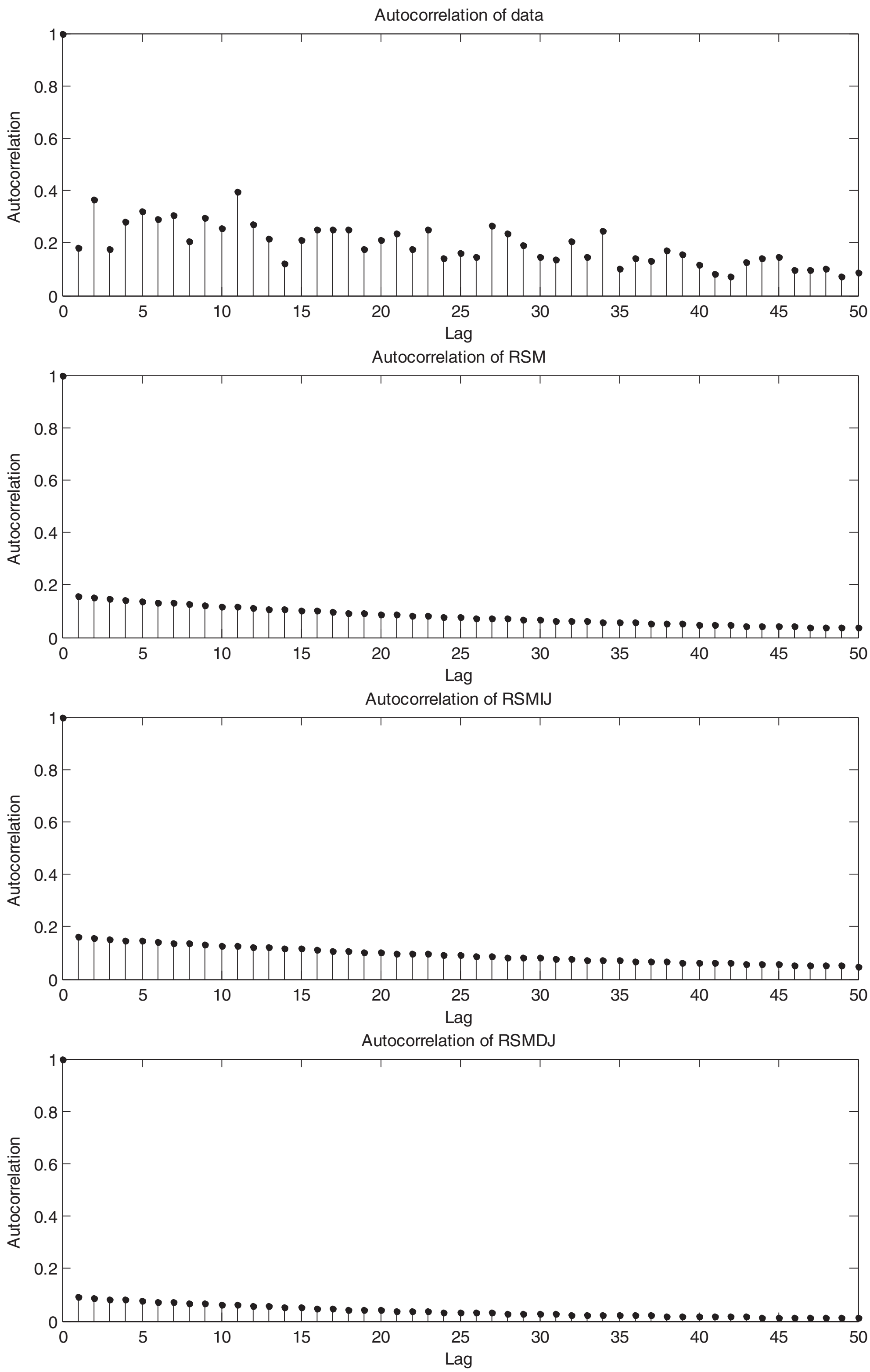

Fig. 7. Autocorrelations of the squared returns in Nikkei 225 index under RSM, RSMIJ and RSMDJ.

time according to the state of the economy. We then begin to estimate the parameters of the model with EM and SEM algorithms and demonstrate empirically observed features using 12 years of data for the S&P 500, Dow Jones Industrial Average, and Nikkei 225 indices. The empirical results show that the model captures the asymmetric, leptokurtic, volatility clustering, and jump clustering features of the market indices in the stock market. The LR tests also show that the RSMDJ is better than the competing models, such as the BSM, RSM, and RSMIJ, for capturing the dynamic process of financial asset returns, suggesting that the regime-switching model with dependent jump size risks has a superior empirical fit over other regime-switching models.

This paper has some implications and possible extensions. First, our RSMDJ model can help investors determine a superior investment strategy when a jump event occurs. During a recession state, investors may benefit if they underweight their investments in the stock index or sell the stock index futures. Similarity, under the expansion state, an investor's superior investment strategy is to overweight their investments in the stock index in order to benefit from the upward market trend. Because the size of the jump risks is larger in the recession state than that in the expansion state, in the recession state with high jump risks, investors would be better off if they adopt a conservative strategy in which they underweight their investments in the stock index or sell the stock index futures to hedge their portfolios. Investors should be more aware of the jump risks for the jump size during a recession. In addition, Elliott et al. (2010) study a mean-variance portfolio selection problem in a hidden Markovian regime-switching model. Accordingly, we can propose our regime-switching model with jump risks to describe the mean-variance portfolio selection problem.

Second, the model developed herein can be applied to pricing derivatives. Many recent studies have proposed regime-switching models to price derivatives. Elliott et al. (2007) price options under a generalized Markov-modulated jump diffusion model. Bo et al. (2010) focus on currency options. Chang et al. (2013) provide close-form solutions for option prices under a Markov-modulated jump diffusion model. Lin et al. (2015) price a foreign exchange option in the currency cycle with jump risks. Lin et al. (2014) propose a recursive formula for a participating contract that embeds a surrender option in a regime-switching model with jump risks. Li et al. (2016) use a regime-switching jump diffusion model to price derivatives while modeling a CO2 emission allowance.

Third, due to the discrete nature of a jump risk, the market is incomplete, and conventional riskless hedging is difficult to obtain. Therefore, the issue of hedging with jump risk remains an important challenge in the field of risk management. Zhou (2001) develops a theory to incorporate jump risk into the default process to explain the term structure of credit spreads. Su and Hung (2011) analyze the influences of jump dynamics, heavy-tails and skewness on a value-at-risk estimation. Egami and Yamazaki (2013) use the jump-diffusion model to describe defaults in a bank's loan/credit business portfolios for credit risk management.

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Appendix A. The moment formulas under RSMDJ

Suppose the return follows RSMDJ, then

$$R_t \,|\, q_t, N_t \sim Normal\left(\mu_{q_t} + N_t\mu_{q_t y}, \sigma^2_{q_t} + N_t\sigma^2_{q_t y}\right).$$

Define $\pi = (\pi_1, \pi_2)$ as stationary probability $P = \begin{bmatrix} p_{11} & 1-p_{11} \\ 1-p_{22} & p_{22} \end{bmatrix}$ of Markov chain. The average return becomes

$$E(R_t) = E[E(R_t\,|\,q_t, N_t)] = E\left(\mu_{q_t} + N_t\mu_{q_t y}\right) = \pi_1\left(\mu_1 + \lambda\mu_{1y}\right) + \pi_2\left(\mu_2 + \lambda\mu_{2y}\right).$$

And the k - th central moment of return is

$$\begin{aligned} E(R_t-\mu)^k &= E\left\{E\left[(R_t-\mu)^k|q_t\right]\right\} \\ &= \sum_{i=1}^{2} \Pr(q_t = i) E\left(\mu_1 + \sigma_t Z_t + \sum_{n=1}^{N_t} \log Y_{i,n} - \mu\right)^k \\ &= \sum_{i=1}^{2} \Pr(q_t = i)\left[\sum_{j=0}^{k} C_j^k \sum_{s=0}^{k-j} C_j^{k-j} (\mu_i-\mu)^j \sigma_i^s E(Z_t)^s E\left(\sum_{n=1}^{N_t} \log Y_{i,n}\right)^{k-j-s}\right]. \end{aligned}$$

When $k = 2$, the variance is

$$\begin{aligned} E(R_t-\mu)^2 = &\sum_{i=1}^{2} \Pr(q_t = i) \\ &\times \left[\sum_{j=0}^{2} C_j^2 \sum_{s=0}^{2-j} C_j^{2-j} (\mu_i-\mu)^j \sigma_i^s E(Z_t)^s E\left(\sum_{n=1}^{N_t} \log\ \ Y_{i,n}\right)^{2-j-s}\right] \end{aligned}$$

$E(Z_t)^s = 0$ as $s = 1$ and $E(Z_t)^s = 1$ as $s = 0, 2$. Therefore we should only consider $s = 0, 2$,

$$\mathrm{E}(R_t-\mu)^2 = \sum_{i=1}^{2}\pi_i \left[E\left(\sum_{n=1}^{N_t} \log Y_{i,n}\right)^2 + 2(\mu_i-\mu)E\left(\sum_{n=1}^{N_t} \log Y_{i,n}\right) + (\mu_i-\mu)^2 + \sigma_i^2\right].$$

The 3rd central moment of return is

$$\begin{aligned} E(R_t-\mu)^3 = &\sum_{i=1}^{2} \Pr(q_t = i) \\ &\times \left[\sum_{j=0}^{3} C_j^3 \sum_{s=0}^{3-j} C_j^{3-j} (\mu_i-\mu)^j \sigma_i^s E(Z_t)^s E\left(\sum_{n=1}^{N-t} \log Y_{i,n}\right)^{3-j-s}\right] \end{aligned}$$

where $E(Z_t)^s = 0$ as, $s = 1, 3$, and $E(Z_t)^s = 1$ as $s = 0, 2$. Thus we should only consider $s = 0, 2$

$$\begin{aligned} E(R_t-\mu)^3 = \sum_{i=1}^{2}\pi_i \Bigg[&E\left(\sum_{n=1}^{N_t} \log Y_{i,n}\right)^3 + 3(\mu_i-\mu)E\left(\sum_{n=1}^{N_i} \log Y_{i,n}\right)^2 \\ &+3\left[(\mu_i-\mu)^2 + \sigma_i^2\right]E\left(\sum_{n=1}^{N_t} \log Y_{i,n}\right)^1 + (\mu_i-\mu)^3 + 3(\mu_i-\mu)\sigma_i^2\Bigg]. \end{aligned}$$

Therefore the formula of skewness is $E(R_t - \mu)^3/Var(R_t)^{1.5}$ and the 4th central moment becomes

$$E(R_t-\mu)^4 = \sum_{i=1}^{2} \Pr(q_t = i)\left[\sum_{j=0}^{4} C_j^4 \sum_{s=0}^{4-j} C_j^{4-j} (\mu_i-\mu)^j \sigma_i^s E(Z_t)^s E\left(\sum_{n=1}^{N_t} \log Y_{i,n}\right)^{4-j-s}\right]$$

$E(Z_t)^s = 0$ as $s = 1, 3$; $E(Z_t)^s = 1$ as $s = 0, 2$ and $E(Z_t)^s = 3$ as $s = 4$. Thus we should only consider $s = 0, 2, 4$.

$$\begin{aligned} E(R_t-\mu)^4 = \sum_{i=1}^{2}\pi_i \Bigg[&E\left(\sum_{n=1}^{N_t} \log Y_{i,n}\right)^4 + 4(\mu_i-\mu)E\left(\sum_{n=1}^{N_t} \log Y_{i,n}\right)^3 \\ &+6\left[(\mu_i-\mu)^2 + \sigma_i^2\right]E\left(\sum_{n=1}^{N_t} \log Y_{i,n}\right)^2 \\ &+4\left[(\mu_i-\mu)^3 + 3(\mu_i-\mu)\sigma_i^2\right]E\left(\sum_{n=1}^{N_t} \log Y_{i,n}\right)^1 + (\mu_i-\mu)^4 \\ &+6(\mu_i-\mu)^2\sigma_i^2 + 3\sigma_i^2\Bigg] \end{aligned}$$

Therefore the formula of kurtosis becomes $E(R_t - \mu)^4/Var(R_t)^2$ where

$$\mu = \pi_1\left(\mu_1 + \lambda\mu_{1y}\right) + (1-\pi_1)\left(\mu_2 + \lambda\mu_{2y}\right)$$

$$E\left(\sum_{n=1}^{N_t} \log Y_{q_t,n}\right)^1 = \lambda\mu_{q_t y}\ , \quad E\left(\sum_{n=1}^{N_t} \log Y_{q_t,n}\right)^2 = \mu^2_{q_t y}\left(\lambda^2 + \lambda\right) + \sigma^2_{q_t y}\lambda\ ,$$

$$E\left(\sum_{n=1}^{N_t} \log Y_{q_t,n}\right)^3 = \mu^3_{q_t y}\left(\lambda^3 + 3\lambda^2 + \lambda\right) + 3\mu_{q_t y}\sigma^2_{q_t y}\left(\lambda^2 + \lambda\right)\ ,$$

$$\begin{aligned} E\left(\sum_{n=1}^{N_t} log\ Y_{q_t,n}\right)^4 &= \mu^4_{q_t y}\left(\lambda^4 + 6\lambda^3 + 7\lambda^2 + \lambda\right) \\ &\quad +6\mu^2_{q_t y}\sigma^2_{q_t y}\left(\lambda^3 + 3\lambda^2 + \lambda\right) + 3\sigma^4_{q_t y}\left(\lambda^2 + \lambda\right). \end{aligned}$$

Appendix B. The autocorrelations function of squared return under RSMDJ

Assume the return follows RSMDJ, the autocorrelation function between time t and t-k is

$$\rho_k = \frac{Cov\left(R_t^2, R_{t-k}^2\right)}{\sqrt{Var\left(R_t^2\right)\cdot Var\left(R_{t-k}^2\right)}} \tag{B.1}$$

where

$$R_t^2 = \left[\left(\mu_1 + \sigma_1 Z_t + \sum_{n=1}^{N_t}\log Y_{1,n}\right) I_{\{q_t=1\}} + \left(\mu_2 + \sigma_2 Z_t + \sum_{n=1}^{N_t}\log Y_{2,n}\right) I_{\{q_t=2\}}\right]^2$$

k denotes the number of lags, and variance refer to the Appendix A. Define $\pi = (\pi_1, \pi_2)$ as the stationary probability $P = \begin{bmatrix} p_{11} & 1-p_{11} \\ 1-p_{22} & p_{22}\end{bmatrix}$ of Markov Chin. M_{ij}^k denotes the transition probability from state i to state j. And the covariance between R_t^2 and R_{t-k}^2 becomes

$$\begin{aligned} Cov\left(R_t^2, R_{t-k}^2\right) &= \pi_1\left(M_{11}^k - \pi_1\right)\left[\left(\mu_1 + \lambda\mu_{1y}\right)^2 + \sigma_1^2 + \lambda\left(\mu_{1y}^2 + \sigma_{1y}^2\right)^2\right]^2 \\ &+ \pi_2\left(M_{22}^k - \pi_2\right)\left[\left(\mu_2 + \lambda\mu_{2y}\right)^2 + \sigma_2^2 + \lambda\left(\mu_{2y}^2 + \sigma_{2y}^2\right)^2\right]^2 \\ &- 2\pi_1\pi_2\left[\left(\mu_1 + \lambda\mu_{1y}\right)^2 + \sigma_1^2 + \lambda\left(\mu_{1y}^2 + \sigma_{1y}^2\right)^2\right]^2\left[\left(\mu_2 + \lambda\mu_{2y}\right)^2 \right. \\ &\left. + \sigma_2^2 + \lambda\left(\mu_{2y}^2 + \sigma_{2y}^2\right)\right]^2. \end{aligned}$$

And the denominator of Eq. (B.1) becomes

$$\begin{aligned} Var\left(R_t^2\right) = \pi_1\pi_2&\left[\left(\mu_1 + \lambda\mu_{1y}\right)^2 + \sigma_1^2 + \lambda\left(\mu_{1y}^2 + \sigma_{1y}^2\right)^2\right] \\ &\left[\left(\mu_2 + \lambda\mu_{2y}\right)^2 + \sigma_2^2 + \lambda\left(\mu_{2y}^2 + \sigma_{2y}^2\right)^2\right] \\ &+ \sum_{i=1}^{2}\pi_i\left[4\mu_i^2\sigma_i^2 + 3\sigma_i^4 + 4(\mu_i^2 + \sigma_i^2)E\left(\sum_{n=1}^{N_t}\log Y_{i,n}\right)^2 + E\left(\sum_{n=1}^{N_t}\log Y_{i,n}\right)^4\right] \\ &- \sum_{i=1}^{2}\pi_i^2\left[\sigma_i^4 + 4\mu_i^2\left[E\left(\sum_{n=1}^{N_t}\log Y_{i,n}\right)\right]^2 - \left[E\left(\sum_{n=1}^{N_t}\log Y_{i,n}\right)^2\right]^2\right] \end{aligned}$$

where

$$\begin{aligned} \mu &= \pi_1\left(\mu_1 + \lambda\mu_{1y}\right) + (1-\pi_1)\left(\mu_2 + \lambda\mu_{2y}\right)E\left(\sum_{n=1}^{N_t}\log Y_{q_t,n}\right) \\ &= \lambda\mu_{q_t y}\ , E\left(\sum_{n=1}^{N_t}\log Y_{q_t,n}\right)^2 = \mu_{q_t y}^2\left(\lambda^2 + \lambda\right) + \sigma_{q_t y}^2\lambda. \end{aligned}$$

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