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The Challenge of Learning Physics Before Mathematics: A Case Study of Curriculum Change in Taiwan

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Abstract The aim of this study was to identify challenges in implementing a physics-before-mathematics curriculum. Obviously, students need to learn necessary mathematics skills in order to develop advanced physics knowledge. In the 2010 high school curriculum in Taiwan, however, grade 11 science students study two-dimensional motion in physics without prior learning experiences of trigonometry in mathematics. The perspectives of three curriculum developers, 22 mathematics and physics teachers, two principals, and 45 science students were obtained by interview. The results of qualitative data analysis revealed six challenges and suggested likely solutions. The national level includes political and social challenges, resolved by respecting teachers as professionals; the teacher level includes knowledge and teaching challenges, resolved by increasing teacher trans-literal capacities; and the student level includes learning and justice challenges, resolved by focusing on students' diverse developments in cross-domain learning.

Keywords Curriculum reform · Learning · Mathematics · Physics · Teaching

Background

Should we learn some things before others? Basic education tends to require a content-coherent curriculum with an appropriate learning sequence and content with few repetitions, such as learning addition (as a basic skill) before multiplication (as a more advanced skill) (Schmidt et al. 2005). A content-coherent curriculum across domains of knowledge, however, presents complications. The challenges of cross-domain content coherence may be more likely to occur in secondary education than early years, primary, and higher education. Secondary education is mostly subject-based, in which students have relatively little freedom to choose diverse courses in diverse sequences

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(Carr 2007). This challenge may become problematic in achieving content coherence between mathematics and physics. Physics teachers in both high school and higher education tend to view student mathematics competence as the basis for successful physics learning (Angell et al. 2004). The necessity of mathematics in physics is also acknowledged by scientists (Lützen 2011).

A physics-before-mathematics curriculum was formally launched in the 2010 Taiwan high school curriculum. Throughout the history of the national curriculum, relevant mathematics was taught before the physics; that is, related mathematics was taught in mathematics classrooms before its use in physics classrooms. In the new curriculum, grade 11 students choosing the science course package (i.e., students aiming to study sciences in higher education) study two-dimensional motion and dynamics in physics without any prior learning experiences of trigonometry or trigonometric functions in mathematics.

This study was conducted from 2011 to 2013, the first 2 years of the implementation of the physics-before-mathematics curriculum. It found that the curriculum is likely to create challenges at the national, teacher, and student levels, especially in a culture traditionally placing emphasis on content coherence (Cai et al. 2014). Documenting this historical event may elaborate the national, teacher, and student curricula, enhance the knowledge of cross-domain teaching and learning, give insights into how people respond to the curriculum, and serve as a valuable case study for future curricular designs. Education is not only a cognitive issue but also can be an affective, socio-cultural, and political one (Jablonka et al. 2013), and the results of this study are indicative of this fact.

National Curriculum: the Typical Curriculum Development Flow

The typical curriculum development flow is from nationally-intended to teacher-implemented to student-received curricula. In practice, most educational systems in the world follow this flow, as revealed in the Trends in International Mathematics and Science Study (International Association for the Evaluation of Educational Achievement 2005).

Curricular flow appears to base itself on the notion that national policies, as responses to social needs, precede academics; academic disciplines guide school subjects; and learning occurs through the teaching of subject matter. For example, in order to develop a coherent science and technology curriculum for the Netherlands, Geraedts et al. (2006) suggested a curricular decision-making framework going from the macro/state level (including the Ministry of Education, institutions, and publishers) to the meso/school level (including school and departments) to the micro/classroom level (including teachers and students). The flow also tends to distinguish between the roles of professors as experts in content knowledge and those of school teachers as experts in pedagogical content knowledge (Deng 2007).

The typical curricular flow inevitably creates gaps between the national and teacher curricula, which in turn may create challenges in student-received curricula. Burny et al.'s (2013) study showed that mathematics curriculum sequences may not be the same across countries, but some content can be learnt at earlier stages without being at the expense of learning outcomes. The empirical criteria for placing teaching content in appropriate sequences are still unresolved. Learning sequences as a political issue appear to be relatively rarely researched in science education but may have an important influence on students' learning in the sciences.

Teacher Curriculum: Subject-Based or Integrated Curriculum

Theoretically, curricula can be designed as subject-based or integrated. Geraedts et al. (2006) believe that a coherent science and technology curriculum may be achieved by considering the nature of the disciplines and the student experience of uninterrupted learning. Mathematics and science concepts, tools, and activities can be integrated to different degrees in mathematics and science teaching. As revealed by Lonning and DeFranco's (2010) theoretical model, which moves from independent mathematics, mathematics-focused, balanced mathematics and science, science-focused, to independent science. Mathematics can be viewed as conceptual, procedural, and professional skills for future careers (Wood et al. 2012). Mathematical objects and operations tend to be the basis for student understanding of mathematical functions in science concepts. For example, proportional knowledge, skills, and reasoning are the basis for full student understanding of pH values in the advanced high school chemistry curriculum (Park and Choi 2013). The interaction between mathematics and science appears to be necessary for successful science education.

Practically, an integrated curriculum appears to create problems in content and timing across school subjects. Vahey et al. (2012) implemented an integrated curriculum for data literacy in a team-teaching-based secondary school. The curriculum carefully took account of the nature of the knowledge of diverse school subjects and sets up a clear sequence: social sciences as preparation, mathematics as formal learning, science as formal learning and application, and English language arts as concluding arguments. Despite the careful curriculum design and purposeful sample selection, the integrated curriculum still faced practical problems. Teachers expressed concern about whether to focus on their own subject or cross-subject teaching. They also felt controlled by the strict timing and content of the integrated curriculum. In addition, the timing of the content designed for this specific project may not be appropriate for the changing national curriculum.

Generally, the typical practice for most higher and secondary education is subject-based. Despite some advocates for general education in higher education (Laird and Garver 2010) and integrated curriculum in secondary education (Carr 2007), the major infrastructure of higher education is still domain- and vocation-based, and that of high schools is subject-based. The new trend in higher education tends to be decreasing support from the government, strong appeal for developing students procedural knowledge, and increasing need for funding from private industries (Williams 2007). This trend appears to further increase subject-based teaching and vocation-driven learning.

Student Curriculum: Transfer of Knowledge Between School Subjects

Traditionally, student academic achievements were seen as determined by general intelligence (Deary et al. 2007). As such, strong relationships are found between student achievements in different academic domains, such as mathematics and science (Chiu 2012) or mathematics and verbal skills (Marsh and Hau 2004). According to this paradigm, the transfer of knowledge between domains can be seen as a natural process for dealing with real problems in context and for achieving meaningful learning (Carr 2007).

Another line of research on intelligence argues that general intelligence may be not so general (Sternberg 2014). The distinctly different multiple intelligences imply that being good at one subject (e.g., mathematics) may not guarantee being good at another (e.g., physics) and vice versa. In other words, intelligences are content-specific combinations, which means that one may be good at only one intelligence or good at several specific intelligences at the same time (Davis et al. 2011). Transfer of knowledge appears to be significant for students with high intelligence or abilities (Meijer and Riemersma 2002). According to this paradigm, the transfer of knowledge between different school subjects, even for two similar subjects (e.g., mathematics and physics), may be problematic, with some students who are not good at either.

A top-down, student-centered curriculum may face challenges, especially in subject content. For example, when infusing higher-order thinking into all science classrooms in Israel, science educators encountered challenges in fitting the thinking goals to science content, designing reasonable teaching sequences based on learner thinking development, and developing teacher capacities to implement a thinking-based curriculum (Zohar 2013). Implementing a kindergarten curriculum focusing on discovery, play, and problem-solving in China faced the challenges of how to fit these focuses into mathematics concepts, properly manage teaching sequences, and increase teacher capacities in fulfilling both thinking and subject-content goals (Hu et al. 2014).

The Problem Context

In Taiwan, the national curriculum is mainly centralized and designed by the Ministry of Education (Huang 2012). As stated earlier, the new national curriculum for high school formally launched in 2010 and was developed following the typical curriculum development process, from national, to teacher, to student curricula. The majority of the content of the curriculum was designed by scholars of domain-specific academic disciplines, usually from higher education.

The curriculum allows Taiwanese grade 11 students who choose the science course package to study two-dimensional motion and dynamics in physics without any prior learning experiences of trigonometry. Table 1 shows the content of mathematics and physics topics that science students are taught in the three phases of the first semester of grade 11. Slightly later in the process of curriculum design, private publishers gradually began to design and publish textbooks and related teaching and learning materials based on the curriculum.

Public schools generally teach the topics and follow the schedules predetermined by the national curriculum although the general part of the curriculum provides some space for

Table 1 Content of mathematics and physics courses in the first semester of grade 11 science according to the 2010 curriculum in Taiwan

	Phase 1	Phase 2	Phase 3
Mathematics	Half knowledge of trigonometry[a]	Straight lines and circles	Vectors
Physics	Linear motion; Projectile motion	Static equilibrium; Newton's laws	Circular motion; Simple harmonic motion
(Mathematics for Physics)	(Full knowledge of trigonometry; Partial knowledge of vectors)	(More knowledge of vectors)	(Full knowledge of trigonometry and vectors)

[a] The other half knowledge of trigonometry is taught in grade 12

schools to fit the curriculum to their school context. Cram schools and private schools are less controlled by the curriculum and are likely to pre-teach students to supplement their mathematics knowledge. Cram schools form a popular private educational industry in Taiwan, aiming to enhance student achievement scores on school tests and university entrance examinations. Private and cram schools reflect Taiwanese parents' expectations of early and intensive preparation for academic success for their children (Tsai and Kuo 2008).

The design of the physics-before-mathematics curriculum, formally implemented from 2010 in Taiwan, follows the typical curricular flow. This study, therefore, sought to identify the challenges that the relevant people face at the national, teacher, and student curriculum levels, and suggestions for addressing these. The research questions are as follows:

1. What are the challenges in implementing a physics-before-mathematics curriculum in Taiwan?
2. What are the possibilities for addressing these challenges?

Method

Participants

The research participants were 12 mathematics teachers, ten physics teachers, and 45 grade 11 science students (25 girls, 20 boys) in Taiwanese high schools. The teachers and students were selected by balancing genders, types of high schools, and the four areas (north, middle, south, and east) in Taiwan, a process designed to increase the representativeness of the sample (Lewis and Ritchie 2003). Then, by convenience sampling (Lincoln and Guba 1985), the interviewers recruited the participants who were easily available or referred to and who agreed to be interviewed.

The other participants were three curriculum developers and two high school principals. They were selected with the aim of clarifying issues raised in the teacher/student data collection and analysis process. The curriculum developers were professors with experience in national curriculum design. The principals were from one urban and one rural school. Tables 2 and 3 present the demographics of the participating teachers (including the mathematics teachers, physics teachers, principals, and professors) and students, respectively.

The students were the first cohort to formally experience the new 2010 high school curriculum since grade 10. This study was conducted in Grade 11 during the 2010 academic year (August 2010 to July 2011), when they had already formally chosen to study a multidisciplinary science package course, meaning that they were mainly aiming to study sciences (i.e., engineering, mathematics, medicine, national sciences, technology, etc.) in higher education.

In the present system, Taiwanese high school students can choose to study one package of courses from three choices: humanities and social sciences (package 1), physical sciences (package 2), and physical and biological sciences (package 3). Grade 11 students choosing package 2 or 3 courses (i.e., "science students" in this study) experience the physics-before-mathematics curriculum. They are taught advanced physics that needs use of some mathematics knowledge and skills to solve physics problems' (Table 1). The students choosing package 1 courses study basic physics, which emphasizes a qualitative understanding of physics knowledge and does not involve learning physics before learning related mathematics.

Table 2 Demographics of the participating teachers

ID	Gender	Job role	Affiliation	Age	Teaching year	Area in Taiwan
t01	male	mathematics teacher	high school	44	20	north
t02	male	mathematics teacher	high school	29	5	north
t03	male	mathematics teacher	high school	40	10	north
t04	female	mathematics teacher	high school	30	7	east
t05	female	mathematics teacher	high school	42	20	east
t06	male	mathematics teacher	high school	40	18	east
t07	female	mathematics teacher	high school	36	13	south
t08	female	mathematics teacher	high school	40	17	south
t09	female	mathematics teacher	high school	45	19	south
t10	female	mathematics teacher	high school	36	13	south
t11	female	mathematics teacher	high school	40	17	south
t12	male	mathematics teacher	high school	45	19	south
t13	male	physics teacher	high school	38	14	north
t14	male	physics teacher	high school	51	24	north
t15	male	physics teacher	high school	35	4	central
t16	male	physics teacher	high school	35	10	north
t17	female	physics teacher	high school	48	26	east
t18	male	physics teacher	high school	35	10	east
t19	male	physics teacher	high school	57	35	east
t20	male	physics teacher	high school	41	13	south
t21	male	physics teacher	high school	30	3	south
t22	male	physics teacher	high school	43	15	south
t23	female	principal	high school	NA[1]	NA	north
t24	male	principal	high school	NA	NA	central
t25	male	professor of mathematics	university	NA	NA	NA
t26	female	professor of mathematics and mathematics education	university	NA	NA	NA
t27	male	professor of vocational education	university	NA	NA	NA

The demographics, as indicated by "NA," are not presented in order to protect the participants' identities because there are relatively few principals and professors in Taiwan

Data Collection

The research participants were interviewed individually by one professor, seven high school teachers, and six research assistants, all of whom were trained to conduct semi-structured interviews. The interviewers asked the participants guiding questions in the interview. The participants' answers to each of the guiding questions were explored in depth by follow-up questions until the meaning of their answers was fully developed. The interviews lasted from 20 to 70 min and were audio recorded.

The participants were asked different guiding questions in the interview. The curriculum developers were interviewed using the following guiding questions:

1. What do you think about the relationship between physics and mathematics?

Table 3 Demographics of the participating students

ID	Gender	School sector	School type	Area in Taiwan
s01	female	private	high achieving	central
s02	female	private	high achieving	central
s03	female	private	high achieving	central
s04	female	public	community	north
s05	female	public	community	north
s06	female	public	high achieving	north
s07	female	public	high achieving	north
s08	female	public	community	north
s09	female	public	high achieving	north
s10	female	public	high achieving	north
s11	female	public	high achieving	east
s12	female	public	high achieving	east
s13	female	public	high achieving	east
s14	female	public	high achieving	east
s15	female	public	high achieving	east
s16	female	public	high achieving	east
s17	female	public	community	north
s18	female	public	community	north
s19	female	public	community	north
s20	female	public	community	south
s21	female	public	community	south
s22	female	public	community	south
s23	female	public	community	south
s24	female	public	community	south
s25	female	public	community	south
s26	male	public	high achieving	south
s27	male	public	community	north
s28	male	public	community	north
s29	male	public	community	north
s30	male	public	community	north
s31	male	public	community	north
s32	male	public	community	north
s33	male	public	community	north
s34	male	public	community	north
s35	male	public	community	north
s36	male	public	community	south
s37	male	public	community	south
s38	male	public	community	south
s39	male	public	community	south
s40	male	public	community	south
s41	male	public	high achieving	north
s42	male	public	high achieving	north
s43	male	public	high achieving	north
s44	male	public	high achieving	north
s45	male	public	high achieving	north

2. What do you think about the relationship in the curriculum between physics and mathematics?

The mathematics and physics teachers were interviewed using the following guiding questions:

1. What are your perceptions, concerns, and teaching methods related to the past and present (2010) curricula you experience as a mathematics/physics teacher?
2. How related are mathematics and physics?
3. How related are mathematics and physics in teaching?
4. What are your reactions and your students' responses to the present curriculum, that students studying packages 2 and 3 (science-focused) courses will learn physics without some necessary mathematics knowledge or skills in grade 11?

The students were interviewed using following guiding questions:

1. Do you know that students in grade 11 studying packages 2 and 3 (science-focused) courses will learn physics without some necessary mathematics knowledge or skills? To what extent do you understand this? How do you know this? What are your opinions about this? How do you, your classmates, and your teachers face this situation?
2. How related are mathematics and physics? Please give your reasons for your answer. What mathematics knowledge do you need when you learn physics?

Data Analysis

The interviews were transcribed verbatim. Qualitative data analysis methods were used to analyze the transcriptions (Charmaz 2000; Corbin and Strauss 1990; Marton 1981; Miles and Huberman 1994; Strauss and Corbin 1990, 1998). The analysis focused on participants' responses to the national, teacher, and student curricula, respectively, and the themes were identified through the iterative process of open coding, constant comparison, and theme finding with partial support from the Atlas.ti Version 6.0.15 software (Atlas.ti GmbH, Berlin, Germany). The identified themes were interpreted and supported by representative quotes from the interview data of diverse participants.

In addition to the interview data, other sources were used to increase the trustworthiness, validity, and depth of data analysis (Clark et al. 2010; Ivankova et al. 2006; Lincoln and Guba 1985). The sources used to triangulate the interview data included (1) prolonged open data collections of relevant news (e.g., news about the upcoming high school curriculum separating trigonometry into two parts taught in different semesters (United Daily News, October 11, 2008)), government documents (e.g., curricula from the Ministry of Education, Taiwan), and school documents (e.g., content from a forum discussing the new high school curriculum, http://taiwaneducation.km.nccu.edu.tw/xms/content/show.php?id=368), and research reports (e.g., Li 2010); (2) materials (e.g., video records, presentations, and documents) from relevant teacher discussion groups and teaching groups face-to-face or via social media (e.g., a physics teacher's Facebook group for teaching physics) after the teachers' consent had been given; (3) field notes after the open data collections, participation in teacher groups, and interviews; (4) follow-up interviews with some participants in order to clarify the emerging themes or to add information; and (5) independent perspectives and collaborative discussions between three researchers (also the interviewers) about the emerging themes obtained in the data analysis process and the interview quotes used in this paper.

Results

Six challenges in implementing the physics-before-mathematics curriculum were identified based on the data analysis.

1. Political challenges: national curriculum emphasis on educational policy and university more than high school

At the level of the national curriculum, curriculum designers place greater emphasis on educational policies and university curricula than on high school student learning. The following quote shows the priority in the curriculum design by one of the mathematics curriculum developers.

> "[Three priorities are set in designing the mathematics curriculum.] First, the teaching content needs to prepare prior knowledge for the first-year mathematics-related courses in university, such as calculus, statistics, physical chemistry, introduction to computing, and economics .... Second, the national curriculum sets grade 10 as the last year of common courses for all students. The mathematics content needs to fit all students' needs (not just for science students) …. Then, third is cognitive development …. (Male, professor of mathematics in higher education, ID=t25 (Table 2))"

Based on communications with curriculum developers in each curriculum change, a mathematics teacher stated that:

> "When university teachers do not like to teach something, they let high school teachers teach it (Male, mathematics teacher, age 44, teaching year 20, north Taiwan, ID=t01)."

A curriculum developer further confirmed this top-down curriculum development system:

> "Professors determine the curricular framework, teaching content, and credit hours. Although there are forums for the public and school teachers to give their voice, basically, the curriculum has been pre-determined, and decisions have been made about how to implement the curriculum. So, the effect of the forum is not big. (Male, professor of vocational education in higher education, ID=t27)"

2. Social challenges: multiple interests intervening in education

In addition to the domain-specific section, the national curriculum has a "general" section, by which

"Schools can change the schedule of teaching content pre-determined by the national curriculum (Female, professor of mathematics and mathematics education in higher education, ID=t26)"

As such, professors appear to have a positive view of and assume easy solutions to the curriculum for students. That is, even if the curriculum is problematic, schools can resolve this themselves:

"If student prior knowledge of mathematics is not enough for learning physics, then a 'linking course' in the summer holiday may remedy the missing part in the curriculum (Male, professor of vocational education in higher education, ID=t27)."

The actual implementation of the curriculum in practice is that teachers' first choice is to follow the national curriculum in order to avoid any negative consequences:

- We discuss with our mathematics colleagues [whether it is possible to move the teaching of trigonometry and vectors to one-term earlier], but the conclusion is "no". The mathematics teachers worry that they will be sued … by parents and, in fact, cram schools. (Male, physics teacher, age 38, teaching year 14, north Taiwan, ID=t13)

Teachers feel the effects of the fixed curriculum, but professors do not. It is possible that this gap can be resolved by principals. As such, two high school principals, one from a rural school and the other from an urban school, were interviewed. The rural principal painted a desirable picture of flexibility in teaching schedules and good communication between teachers of different school subjects:

- We are a small country school. Most students have low socio-economic status, without money to go to cram school. Teachers of different subjects can communicate to change the teaching schedule and content. We also have summer camps for each subject, with one week for mathematics and one week for physics, to give students more teaching. (Female, principal of a country high school, north Taiwan, ID=t23)

The urban principal, however, was pessimistic about the possibility of a formal change in the schedule of teaching content in his school. It is also impossible to teach extra content during the summer holiday due to regulations set by the Ministry of Education. Multiple interests have become deeply involved in the curriculum design industry, including textbook publishers, teachers, and professors. The only way to address the problem is to have physics teachers teach mathematics based on their own personal or professional choice.

- The national curriculum can be changed, but the publishers have already published the textbooks, which are normally designed by professors and teachers. Change will increase the textbook publishers' costs, so they will not agree. Even if we have summer courses, teachers have to follow the schedule of the national curriculum and cannot teach new [next semester's] content …. Perhaps some physics teachers may teach some mathematics, but this is their personal choice. (Male, principal of a city high school, central Taiwan, ID=t24)

The national curriculum has opened space for flexibility in curriculum implementation. The actual situation, however, appears to be that the urban school has to strictly follow the national curriculum and educational policy because they face more powerful and conflicting interests than the rural school. For urban schools, the curriculum has become binding, and not changing it has become one of the major hidden curricula in school.

3. Knowledge challenges: increase in boundaries between different domains of knowledge and school subjects

Education as Preparation for Individual and National Competitiveness If cultivating professionals is one of the major aims of higher education and this is linked to national competitiveness, then high schools inevitably are viewed as preparatory institutions for universities. As a teacher commented:

"Engineers are the basis of our country. Physics is the basis of engineering. Mathematics is a tool, a preparation for other subjects, and cannot be changed without taking account of other subjects. (Male, physics teacher, age 38, teaching year 14, north Taiwan, ID=t13)."

In this vocational view, schools appear to become factories, aiming to create workers who need to make a living. Sciences appear to be the most important curricula and directly linked to economic development and the competiveness of a country. As such, high schools are accountable to universities, and in turn, universities are accountable to funders (including the government, parents, and private companies) who ask for domain-specific skilled workers.

Education as Change and Response to Change Human kind needs to create new, distinct, and specific knowledge and professionals who can drive responses to social and natural challenges, such as global warming (Chiu 2013). As such, the boundaries between different sciences appear to become gradually significant. Mathematics emphasizes abstraction, procedures, and theorems, while physics emphasizes scientific advances, concepts, and unified truth. The professors interviewed in this study revealed a conception of clear boundaries between academic fields:

- Purely mathematical reasoning is supposed to have no direct relationship with the real world … but it lets physicists see likely physical meanings…. Another example is earth science…. Earth science teaches and tests students on the "Coriolis force" to "fluid mechanics," which are not included in the physics curriculum because the two topics are a very recent development in physics history. (Male, professor of mathematics in higher education, ID=t25)
- Physicists see mathematics as a tool, but mathematics itself has its own thinking and beauty. (Female, professor of mathematics and mathematics education in higher education, ID=t26)

The notion of mathematics as a tool for physics appears to have been successfully transmitted from professors in higher education to their students (who would become high school teachers):

> "When students ask why they must learn such difficult mathematics, I say that physics uses mathematics (Female, mathematics teacher, age 36, teaching year 13, south Taiwan, ID=t07)."

The notion of mathematics as a necessary tool for physics appears to create more challenges for physics teachers than for mathematics teachers. The physics curriculum appears to ask teachers to teach a substantial amount of (advanced) physics "content", which increases physics teachers' stress in covering all the content in time. Thus, physics teachers aticulated a desire that mathematics can "look after" physics:

> Physics and mathematics are almost the same and cannot be separated …. We do not have extra time for teaching physics-related mathematics [e.g., trigonometry] because the physics curriculum expects us to teach many new things, such as nanotechnology and astrophysics …. It should not be our [physics teachers'] job to teach mathematics. (Male, physics teacher, age 51, teaching year 24, north Taiwan, ID=t14)

On the other hand, mathematics can be independent from physics and focus on mathematical "thinking", rather than specific content. Mathematics teachers feel more relaxed and independent given the self-contained content:

- We teach 'mathematical thinking', not just content …. It is impossible to teach too much content. (Female, professor of mathematics and mathematics education in higher education, ID=t26)
- Perhaps physics teachers can change the order of teaching content by talking about things not so related to mathematics, such as sound waves and electric resistance. (Male, mathematics teacher, age 45, teaching year 19, south Taiwan, ID=t12)

In summary, the boundary between domains of knowledge in higher education appears to influence that between school mathematics and science curricula. The mathematics curriculum aims to increase thinking and decrease content. On the other hand, the physics curriculum has increased in content, in response to the fast and diverse development in the physical sciences, but has not decreased the need for mathematics as a necessary thinking tool. The heavy workload of the physics curriculum causes stress for physics teachers when mathematics does not support physics in the national curriculum and mathematics teachers therefore do not have a responsibility to support physics.

4. Learning challenges: missing knowledge of higher-order cross-domain learning

One question raised from in the previous section on "knowledge challenges" may be this: Between thinking and content, which should be the priority? Is it necessary that mathematics is a tool to be learned before physics? These questions may need to be answered by cognitive developmental researchers.

Ideally, education and curriculum focuses on student learning. Most of the cognitive development theories in psychology tend to focus on young children and general intelligence. Intelligent students generally have a larger knowledge base, higher processing speed, and better monitoring quality than less intelligent students (Steiner and Carr 2003). The gaps in understanding for cross-domain advanced science learning means that curriculum designers follow history and authorities in their academic fields:

- [There are] no child development theories for this situation …. We can see that vectors never independently existed in mathematics history. Physicists used space vectors first, and mathematicians supplemented plain vectors later …. So, it is better that physics teachers teach vectors first. A famous professor in Taiwan, who is good at both mathematics and physics, also believes that physics teachers should teach vectors first. (Male, professor of mathematics in higher education, ID=t25)

Focusing on thinking, not content, another curriculum designer expressed that learning the same content from different perspectives is of benefit to students:

- Physics teachers can teach mathematics, so there will be trigonometric functions based on both mathematics teachers' perspectives and physics teachers' perspectives. Such diverse perspectives will benefit our students. (Female, professor of mathematics and mathematics education in higher education, ID=t26)

Using multiple representations to learn content also appears to be an effective teaching approach in terms of elaboration. The problem may be that science students tend to see mathematics as critical for learning physics, which perhaps is a reflection of or response to their physics teachers' teaching. Learning physics before mathematics appears to confuse students and impact students in physics more than in mathematics.

- Mathematics affects physics. If you are not good at mathematics, then it [physics] will die a tragic death …. (Female, high-achieving school, north Taiwan, ID=s09)
- We learn physics vectors first and learn mathematics vectors later. When we return to do the previous physics using vectors, we feel that they cannot be linked together …. (Female, high-achieving school, east Taiwan, ID=s13)
- The curriculum is really bad because physics cannot be taught in detail, and I can only memorize it. Then, mathematics repeats in detail, but I forget how it [mathematics] is used in physics because when I learn physics, I learn by memorizing the related mathematics. (Male, community school, north Taiwan, ID=s30)
- My physics teacher only taught basic vectors …. The teacher was afraid to give us related problems …. This means that we actually did not learn the physics content …. (Male, community school, south Taiwan, ID=s36)

The above quotes show that science students are heavily reliant on mathematics as a basic ability if they are to learn advanced physics well, regardless of whether the students are high, middle, or low achievers. As such, if we ask the question, "Can science students learn advanced physics without mathematics?" the answer appears to be no, although individual differences may occur between different students in different learning contexts.

5. Justice challenges: Likely inequality in learning opportunities

The above analysis has shown that (1) students need related mathematics as a basic skill before they learn advanced physics, (2) the curriculum places physics before the necessary mathematics, and (3) teachers and principals of public schools, especially city schools, tend to strictly follow the national curriculum. These three factors indicate that students experience different realities even if there is only "one" national curriculum. The following three excerpts show that three kinds of science students are less vulnerable than others in the implementation of the physics-before-mathematics curriculum:

- Students with gifted education experiences and private schooling: I like physics, so I had learnt some related mathematics in junior high school. As such, I understood more. (Female, a private high-achieving school for Grades 7–12, central Taiwan, ID=s01)
- Students going to cram schools: I felt OK, … but I wouldn't have felt OK if I had not been to cram school [during the Grade 10 summer]. (Female, high-achieving school, north Taiwan, ID=s09)
- Students with high ability in sciences: My school always teaches very difficult sciences …. Sure, I could not understand. I'll understand when mathematics teaches it. (Male, high-achieving school, north Taiwan, ID=s42)

Educational inequality is likely to occur if some students are more privileged than others. Gifted education and private schooling (including cram schools) give students opportunities to learn extra, new, advanced content at earlier stages. High achievers in sciences appear to have confidence and the ability to integrate pieces of difficult knowledge, even if taught in chaotic sequences, into a full system. In other words, the physics-before-mathematics curriculum tends to negatively influence non-high-achieving science students taught only within the public school system.

6. Teaching challenges: Increase in physics teachers' burden to teach mathematics and mathematics teachers' confusion when teaching repeated content

The national curriculum sets grade 11 as the time when students choose either social science or natural science course packages. Grade 11 physics for science students needs mathematics to understand advanced physics knowledge. Like their science students, physics teachers see mathematics as a necessity for physics:

"Physics cannot survive without mathematics. Grade 11 is the most difficult time, as students have not yet learned much mathematics (Male, physics teacher, age 57, teaching year 35, east Taiwan, ID=t19)."

As such, all the physics teachers interviewed in this study use mathematics to teach physics. However, they manage their mathematics teaching for physics in different ways.

- No mathematics teaching and limited physics teaching: Many problems cannot be used for teaching like before … because trigonometry is not taught …. I cannot and am not suited to teaching trigonometry to help students. (Male, physics teacher, age 30, teaching year 3, south Taiwan, ID=t21)
- Some mathematics teaching for specific physics content: In the present curriculum, if I need some mathematics, I'll have to teach mathematics first. For instance, today if I use $\cos\Theta$, then I tell students what $\cos\Theta$ means. (Male, physics teacher, age 35, teaching year 4, central Taiwan, ID=t15)
- Extensive mathematics teaching and using mathematics in physics: If I need to use mathematics, I will teach students repeatedly …. Given the new [physics-before-mathematics] curriculum, I have to spend much more time in mathematics …. For example, when we teach the projection of light, a physics problem has given you $\sin\Theta$, and $\sin\Theta$ is $1/n$. Therefore, you have to know $\cos\Theta$ is the root of $(1-1/n^2)$. (Female, physics teacher, age 48, teaching year 26, east Taiwan, ID=t17)

Mathematics teachers, on the other hand, feel confused by the physics-before-mathematics curriculum largely because of their students' responses. They do not know how to teach the mathematics that has been taught by their physics colleagues.

> When physics needs mathematics that has not been taught yet, physics will teach and use all of the mathematics .… [Mathematics teachers] later teach from the very beginning .… Students feel the sequence is very strange … (Male, mathematics teacher, age 40, teaching year 10, north Taiwan, ID=t03)
> If physics has taught mathematics, mathematics teachers will think whether I should teach this in detail or quickly, as students should have already learned this. (Male, mathematics teacher, age 29, teaching year 5, north Taiwan, ID=t02)

The physics-before-mathematics curriculum appears to increase physics teachers' burden by requiring them to teach mathematics or risk negatively impacting students' physics abilities. Mathematics teachers are puzzled by the curriculum because of their students' negative responses to repeated teaching of mathematics content.

Discussion

Six challenges are identified in the implementation of the physics-before-mathematics curriculum. The identified challenges have been elaborated at the three levels of typical curriculum development flow. The national curriculum appears to include political and social challenges. The teacher curriculum includes knowledge and teaching challenges. The student curriculum includes learning and justice challenges. A particular concern is that professors (i.e., curriculum developers) and teachers are traditionally viewed as professionals in the curriculum development and implementation system; in fact, they are at the middle level that reflects concerns from society and students. The following is a discussion of the results and possible approaches to address the challenges, answering research question 2.

National Curriculum: Acknowledging Political and Social Challenges with Respect To Teachers as Professionals

Educators tend to accept unequal political power in education and follow national policy, even when they do not completely agree with the policy and are not completely sure how the policy is justified (Hart 2001). Taiwanese educators, as revealed in this study, generally obey the order of the Ministry of Education and pay limited attention to voices from society.

The societal barriers against a desirable student curriculum, however, are of concern to educators. The barriers need to be acknowledged, confronted, and overcome through a flexible curriculum that recognizes teachers as professionals. Cram schools, private educational organizations (including private schools), textbook publishers, and parents have gradually played an increasing role in education since the 1993 curriculum reform in Taiwan (Chiu and Whitebread 2011). The private educational sector appears to force public schools toward a fixed, powerless, ineffective system. Can public school teachers work together to fight for their students and their educational ideals? Collaboration between professors and teachers may be a solution because they are the professionals in the curriculum development and implementation.

Teacher Curriculum: Actively Confronting Knowledge and Teaching Challenges by Increasing Trans-literal Capacities

A curriculum formally divided into several school subjects inevitably faces issues of boundaries and coherence across knowledge domains. A professor-developed curriculum may increase the boundaries between school subjects because research-based knowledge is often incoherent, scattered, and sometimes equivocal (Niemi 2008). The content differences and generation gaps between older and younger sciences create occasional situations that a specific science tends to be the basis for another, as in mathematics for physics, chemistry for biology, and physics for earth science.

The results of this study reveal that the mathematics curriculum aims to increase depth and reduce breadth in order to teach mathematical thinking. The physics curriculum aims to increase the breadth of new physics developments and keep the original depth without increasing the time required to teach it (cf., Murdock 2008; Schwartz et al. 2009). The trend of larger gaps between different academic disciplines in higher education inevitably increases the possibility of debates over the content and sequences between difference school sciences in the national curriculum.

Professors are professionals in their respective knowledge domains. Secondary school teachers tend to be professionals in teaching the domains (Beswick 2007). Professors (scientists) and school teachers need to acknowledge their partial knowledge and understanding of each other's roles. Their understanding of other domains is also weak. In addition, they do not have full knowledge of student development and learning. The acknowledgement of these weaknesses may promote collaboration between professors and teachers from different domains for the sake of improving students' education. Deliberate work in evidence- and practice-based educational research needs to be undertaken to identify the missing knowledge of the barriers between professors, between teachers, and between professors and teachers within and across domains.

Crossing boundaries between knowledge domains appears to be a necessary practice in dealing with problems in real contexts. Trans-literal and technological capacities may need to be part of teacher training programs in order to increase teachers' freedom and ability to teach students to face domain-related problems in their current and future lives.

Trans-literal Capacity The mathematics curriculum calls for external connections with life (Askew et al. 2012; Szendrei 2007) and is closely linked to sciences, especially physics (Fiss 2012). Conversely, physics teachers can also teach mathematics (e.g., trigonometry and vectors) in order to help their students to learn physics. A similar situation occurs with earth science. Earth science teachers need to teach about the "Coriolis force" and "fluid mechanics" if this content is included in the national curriculum and university entrance examinations. Similar situations also occur in higher education. For example, social science departments normally teach statistics related to their academic disciplines, such as educational and psychological statistics, structural equation modeling, and item response theory being taught in educational psychology departments, without reliable support from statistics or mathematics departments.

Technological Capacity Teacher autonomy with institutional collaboration in creating open educational resources using modern technology may help increase trans-literacy and reduce educational inequality. Some successful cases include MOOCs, Khan Academy, and the teacher education in sub-Saharan Africa program (Murphy and Wolfenden 2012). A physics teacher, with partial support from a mathematics teacher, was invited by this present study to create a set of teaching programs on "mathematics for physics". The teaching lectures and presentations have been shared on YouTube (https://www.youtube.com/watch?v=XiPPGhRRhTE) and SlideShare (http://www.slideshare.net/MeiShiuChiu/01-16379816). This teaching program may supplement the limited time allotted for teaching trigonometry and vectors in the first semester of the grade 11 physics curriculum in Taiwan.

Student Curriculum: Overcoming Learning and Justice Challenges by Focusing on Students' Diverse Cognitive, Affective, and Social Developments in Cross-Domain Learning

At the level of the student-experienced curriculum, the challenges are related to diverse (cognitive) development and educational inequality. The general practice is that students tend to learn physics using a superficial approach, feeling frustrated when solving physics problems using mathematics skills learned through rote memorization. In public schools, some physics teachers aim to completely teach related mathematics, but most physics teachers appear to teach a small amount of mathematics and limit their physics teaching. High-achieving and wealthy students have the advantage of support from private schooling and experience relatively fewer negative impacts as a result of the physics-before-mathematics curriculum than non-high-achieving and poor students.

Who wins and loses in the physics-before-mathematics curriculum? The mathematics curriculum and related pedagogical changes are likely to influence students' science learning from both cognitive and affective perspectives (Lin et al. 2013). This study shows that science students tend to have negative affective responses to the curriculum due to having insufficient mathematics skills to solve physics problems. The most significant losers appear to be non-high-achieving and low-income science students who cannot afford private and cram schooling. This will be a tragedy for science education in terms of educational equality.

Perhaps one option is to design the science curriculum by focusing on students' diverse cognitive, affective, and socio-economic needs. There are old and young sciences in terms of science history. The science content, however, is sequenced in the national high school curriculum based mainly on the history of each academic discipline. Curriculum developers may need to notice the fact that there are diverse subjects co-existing in school and that students learn all these subjects at the same time. Mirroring science history development to student cognitive development appears to be a logical approach for scientists but may be problematic for learners. For instance, we need to make the decision: Between thinking and content, which should we focus on? Between coherence between different school subjects and a self-contained curriculum for each subject, which is more important? Both answers need to be obtained through empirical research with learners.

Research has shown that multiple representations may deepen student knowledge and cultivate student capacity for flexible thinking (Triantafillou et al. 2013). This notion was also discussed by the mathematics educator interviewed in this study. The acknowledgement of the benefit to students of learning mathematics via different routes suggests that physics teachers need to assume responsibility for teaching relevant mathematics skills and concepts in physics classrooms. Physics textbook designs and teacher training courses may need to incorporate mathematics for physics to increase physics teachers' confidence and capacity to teach related mathematics. Future research may need to focus on creating a detailed way to manage teaching methods, and examining the effectiveness of teaching approaches that cross domains.

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