The Journal of Risk and Insurance

 Vol. 84, No. S1, 299–317 (2017).
DOI: 10.1111/jori.12205

On the Failure (Success) of the Markets for Longevity Risk Transfer

Richard MacMinn
Patrick Brockett

Abstract

Longevity risk is the chance that people will live longer than expected. That potential increase in life expectancy exposes insurers and pension funds to the risk of not having sufficient funds to pay a longer stream of annuity benefits than promised. Longevity bonds and forwards provide insurers and pension funds with financial market instruments designed to hedge the longevity risk that these organization face. The European Investment Bank and World Bank have both discussed longevity bond issues, but those issues have failed due to insufficient demand. Forward contracts have also been created, but that market remains dormant. The extant literature suggests that these failures may be due to design or pricing problems. In this article the analysis shows that the market failure is instead due to a moral hazard problem.

Introduction

One of the largest sources of risk faced by pension funds and life insurance companies offering annuities is longevity risk. Longevity risk is the risk that members of a population live longer, on average, than expected when originally pricing the product. If the population is a pool of annuitants, then longevity risk is the risk that annuitants live longer on average than predicted in the life companies' mortality tables used to price the annuities. Longevity risk is an important societal problem because of the uncertainty concerning the longevity projections and because of the large exposure to longevity risk. The uncertainty of longevity projections is illustrated by the fact that life expectancy for men aged 60 was more than 5 years longer in 2005 than it was predicted to be in mortality projections made in the 1980s (Hardy, 2005). The significance of this uncertainty is further illustrated by noting that the amount at risk (exposure) in the U.S. defined benefit plans was estimated to be approximately $2.2 trillion in 2007 (Oppers et al., 2012) and covered 42 million participants. Swiss Re estimated the total global exposure to longevity risk at approximately $21 trillion(Burne, 2011).

Richard MacMinn is a Senior Research Fellow at National Chengchi University, Taipei, Taiwan and at The University of Texas, Austin, Texas. MacMinn can be contacted via e-mail: richard@macminn.org. Patrick Brockett is at the Department of Information, Risk, and Operations Management, The University of Texas at Austin McCombs School of Business, Austin, Texas.

Exposure to longevity risk is a serious issue, and yet, traditionally, life companies and pensions funds have had few means of managing it. Until recently, longevity risks were not securitized[1] and there were no longevity derivatives that institutions could use to hedge their longevity risk exposures. This state of affairs has changed, and the new life market for longevity linked financial instruments has begun to develop.[2] Most prominent among these financial instruments are longevity bonds, which are instruments in which at least one payment, and possibly more, depends on the realization of a survivor index.[3] Survivor or longevity bonds are designed to hedge the required annuity payments of an insurer or pension fund. Thus, the payoff of the longevity bond to the bond investor from the bond issuer decreases when survivorship increases (which is when the annuity provider or pension fund needs more money).

Two types of survivorship indices might be used. One is an index based on the actual survivorship of the annuitant pool insured (an indemnification payoff structure). A second possibility for the survivorship index is to base it on general population survivorship (an indexed payoff structure). In the index payoff bond structure there will generally be a difference between the payoffs required for the actual pool of survivors in the insurers annuity book of business versus the payoff for the pool of survivors in the population. The difference between the indemnity and index payoffs is known as basis risk.

There are other instruments designed to hedge longevity risk including longevity swaps and q-forward contracts.[4] The swap pays the difference between the indemnity payoff and the index payoff at each date while the q-forward exchanges the risky payoff "then" with a certain payoff.

Renewed interest in the notion of a survivor or longevity bond was initiated by Blake and Burrows (2001). Since then the literature on mortality and longevity risks and capital market instruments designed to hedge those risks has grown significantly. In 2003 Swiss Re successfully introduced a "mortality"-based security designed to hedge excessive mortality changes for its book of life insurance. The concern was mortality risk, that is, the risk of experiencing a higher death rate than was expected and priced. Since then mortality bonds have become common instruments for the transfer of mortality risk to the capital markets.

In 2004, the European Investment Bank (EIB) introduced a "longevity" bond designed to hedge longevity risk occurring with decreases in mortality (increased longevity). By utilizing such a financial instrument pension and annuity, providers could hedge the risk of adverse financial consequences arising from mortality improvements that were not anticipated and priced. The EIB longevity bond was ultimately not issued due to insufficient demand. In 2008 and 2009 the World Bank (WB) introduced longevity bonds

[1]The issues involved in the securitization of longevity risks are discussed further by Cowley and Cummins (2005) and Krutov (2006).

[2]See Blake et al. (2013) for a discussion of the new life market.

[3]As its name suggests, the survivor index is the proportion of some initial reference population that is still alive at some future time *t*.

[4]The q is common notation for mortality in actuarial science and so is used here to identify the forward contract.

in Chile, both of which also failed due to insufficient demand. In 2010, however, Swiss Re was successful in issuing a longevity bond through its special purpose vehicle Kortis Capital Ltd. In 2012, Additionally, Aegon N. V. successfully issued a longevity bond arranged by Deutsche Bank. Here we provide a financial market model that allows a theoretically sound explanation first, for the failure of some longevity risk transfer securities, and second, for a more struggling attempt at explaining the success of others.

Several rationales have been proposed for the failure of the EIB and WB issues. Blake, Cairns, and Dowd (2006) provide a number of possible reasons for the failure, including too short a time horizon on the bond to create an effective hedge, an excessive capital cost imposed on the hedgers, longevity risk that was transferred to a reinsurer rather than the capital markets, credit risk that would rest with the hedgers, and too much basis risk relative to the price. These are primarily comments about the design of the longevity bond. While also noting basis risk, Lin and Cox (2008) provide a different rationale for the failures; they price the 2003 mortality bond provided by Swiss Re through the special purpose vehicle Vita Capital I and then use the same model to price the EIB longevity bond issue. They find that the risk premium in the mortality bond is less than that in the longevity bond. More importantly, they find that the latter high risk premium was higher than the premium required by reinsurers. This advanced the claim that the high premium effectively caused the market failure. Chen and Cummins (2010) also build a pricing model that provides the same type of tail protection for hedgers as that found in the mortality bond, and from this model, they assert that previous failures were due to design flaws. Finally, Zelenko (2014), who was involved in the design and construction of the two attempted longevity bond issues in Chile, concludes in his piece that the reason for the market failures of the two attempted issues in Chile was the moral hazard problem. In referring to longevity risk his explanation of the problem is ". . . it is remote and of low probability, and while it could jeopardize the firm's solvency, the government would have no choice but to intervene and bail out insurers, since all firms would be hit at the same time. As a consequence, the optimal rational choice is to keep going or 'keep dancing.'"

The analysis here contributes to the literature by showing the source of the market failure is the moral hazard problem, that it exists even in the absence of any perceived government bailout, and that the optimal corporate choice, *ceteris paribus*, is to not hedge longevity risk. While the design flaws in the longevity bonds are not all explicitly modeled, the basis risk is. The analysis shows that the decision not to hedge is made both with and without basis risk. Hence, one of the most important components of the design flaw argument is itself flawed. The credit risk was not modeled here but it could be argued that the exchange-traded q-forward contracts would almost surely eliminate credit risk and the analysis shows that the moral hazard problem causes market failure for q-forwards as well as longevity bonds. Hence, the credit risk component of the design flaws argument is called into question. It has also been argued that too high a risk premium associated with the longevity instruments is the cause of the market failure. A greater risk premium would surely reduce the demand for the instrument and in the limit could cut off demand entirely. The analysis here, however, shows that even if the longevity instruments are perfectly priced, that is, embed the correct risk premium, the markets still fail. Hence, the overpricing argument must also be questioned. Finally, the *ceteris paribus* condition is relaxed and the firm is assumed to make an investment decision in a positive net present value (NPV) project in addition to the hedging decision; in this case, the analysis shows that if the corporation has a sufficiently good investment choice then that can counteract the moral hazard problem and lead to hedging.

For each hedging instrument introduced here we show, *ceteris paribus*, that the stock value of the hedged firm is less than that of the unhedged firm if the firm faces insolvency risk.[5] It is the insolvency risk that introduces the moral hazard problem for corporate management. Although hedging with the longevity-linked securities makes the annuity books of business more valuable and hence makes annuity holders better off, that same hedging reduces the value of the current shareholders' stake in the firm. Hence, corporate management acting in the interests of shareholders (and not the annuity holders) succumbs to the moral hazard problem and does not hedge. In the incentive effect section, we show two effects. First, if the insurer is faced with selecting between two books of annuity business, and one is riskier than the other, then the insurer has the incentive to take the riskier book.[6] Second, if the insurer can hedge in order to free reserves and uses these funds to invest in a positive NPV project, then the firm will make that investment if and only if its NPV exceeds the firm's put value. The last section provides concluding remarks.

This article is structured as follows: the financial market model for an insurer is constructed in the next section. The model is a complete financial market model like that first constructed by Arrow (1963), Debreu (1959), and subsequently by MacMinn (1987b) and others. Two financial instruments for hedging longevity risk, that is, longevity bond and q-forward contracts, are considered in subsequent sections. The longevity bond is considered because of the failures in attempted issues. For simplicity the longevity bond is first considered without basis risk and subsequently basis risk is added. The choice of a forward contract is made both because of the simplicity of the contract and also because of its design.[7]

[5]The results reported here are built upon corollaries of the 1958 Modigliani–Miller theorem and a Fisher separation theorem. A corollary of the 1958 Modigliani–Miller theorem shows that the value of the unhedged firm equals that of the hedged firm; for example, see MacMinn (1987a). Also, see MacMinn (1987b) for a similar corollary that shows the value of the insured firm to equal that of the uninsured firm. These corollaries show that no value is created by changing the composition of the contract set that finances the firm and imply that value may be reallocated from one group of stakeholders to another as the contract set is changed. The results here show that there is a reallocation of value and that it is from current shareholders to annuity holders. Then a Fisher separation theorem from MacMinn (2005) or an earlier lecture (MacMinn, 1984) shows that a CEO paid in salary and stock makes decisions on corporate account to maximize current shareholder value and that theorem is used to show that the CEO chooses not to hedge the longevity risk.

[6]This is a straightforward application of the risk-shifting analysis (e.g., see MacMinn, 1993) to the liability side.

[7]If the payoffs in the q-forward are the indemnity and index payoffs, then a portfolio of q-forwards would mimic a longevity swap. Hence, the swap is implicitly covered in the analysis.

A Financial Market Model With Longevity and Insolvency Risk[8]

Consider a corporation in a competitive economy operating between the dates $t = 0$ and 1. The dates $t = 0$ and 1 are subsequently referred to as "now" and "then," respectively. Decisions are made "now" and payoffs on those decisions are received "then." The economy is composed of corporations and risk-averse investors. Investors make portfolio decisions on personal accounts to maximize expected utility subject to a budget constraint.[9] The corporation is assumed to act on behalf of its shareholders.[10] The corporation of interest here is a life insurance company that is assumed to be in the annuity business.

The insurer's annuity contracts pay a specified benefit if the insured survives another year, but if the insured dies then the contract does not pay. The insurer forms a portfolio of contracts that we refer to as the book of annuity business. Let A denote the liability of the book of business and suppose that it depends the state of nature $\omega \in \Omega$ where Ω is the set of states of nature. Without loss of generality we assume that the liability increases in state, that is, $DA(\omega) > 0$.[11] Here we address the insurer's operating and hedging decisions.

The insurer also faces the standard capital market risks such as interest rate and insolvency risks in addition to the longevity risk in its book of annuity business. The premium income is generated "now" and invested in an asset portfolio. The losses on the books of business occur "then" and depend on the operating and hedging decisions as well as the state of nature revealed. The premium income "now" is invested and the income on the investment is received "then." The investment payoff depends on the state of nature $\omega \in \Omega$. The following partially summarizes the notation used in the development of the model:

ω	state of nature.
$\Omega \equiv [0, \zeta]$	set of states of nature.
$\Phi(\omega)$	distribution function for states.
$p(\omega)$	basis stock price *now* in state ω· The basis stock are Arrow–Debreu securities that payoff one dollar then in a specific state and zero otherwise.[12]
$\Psi(\omega)$	sum of basis stock prices $\varepsilon \leq \omega$; $\Psi(\omega) = \int_0^\omega p(\varepsilon)d\varepsilon$.[13]

[8]This model is similar to that developed in MacMinn and Richter (2014); in the MacMinn and Richter manuscript, however, the authors compare the stock value of the firm that hedges mortality risk with index trigger on the mortality bond versus an indemnity trigger.

[9]The investor portfolio decisions yield the demands for all the stock and so determine the basis for the stock prices, which in turn form the means to value other financial instruments.

[10]The assumption is only for convenience. The corporate objective function can be derived; for example, see MacMinn (2005).

[11]$DA(\omega)$ represents the derivative of the liability with respect to the state variable.

[12]See MacMinn (2005) for a development of a complete financial market model with basis and corporate stock and for a derivation of the corporate executive officer's objective function.

[13]The sum of basis stock prices is not a distribution function; one can interpret the sum of all basis stock prices as the discount factor of a safe asset.

$\Gamma(\omega)$	premium income *then* on the book of business; $D\Gamma > 0$.
$\Delta(\omega)$	value *then* for assets held in reserve; $D\Delta(\omega) = 0$.
$A(\omega)$	liability on the annuity book of business; equivalently, the payoff *then* on the annuity book.
$\Pi(\omega)$	payoff on the business, that is, $\Pi(\omega) = \Gamma(\omega) + \Delta(\omega) - A(\omega); D\Pi = D\Gamma - DA > 0$.[14]
$B(\omega)$	payoff *then* on a longevity bond that uses the population mortality index.
a	forward price for mortality based security.
S^j	stock value *now* for $j = u, b, f, i$ where the superscripts represent unhedged, hedged with a longevity bond, hedged with a forward contract and hedged with a forward contract and invested in a positive NPV project, respectively.
L^j	liability value "now."
V^j	corporate value "now"where $V^j = S^j + L^j$.

Suppose the financial markets are competitive. In the absence of any insurance linked security, the stock market value of the insurer may be expressed as:[15]

$$S^u = \int_\Omega \max\{0, \Pi(\omega)\} d\Psi(\omega), \tag{1}$$

where $\max\{0, \Pi\}$ is the insurer's payoff in the absence of any hedging instrument. From the insurer's perspective the annuity book of business exposes the corporation to the risk that the insured live longer than expected and so we refer to it as longevity risk. The longevity risk may yield insolvency risk if the return on the premium income is not sufficient to cover the losses on the annuity book. This insolvency risk introduces the judgment-proof problem or equivalently a moral hazard problem (Shavell, 1986; MacMinn, 2002) with its associated incentive problems.

To note the insolvency risk, it is instructive to construct the liability value for the book of annuity business and then the put option value. The payout to annuity holders is $\min\{\Gamma + \Delta, A\}$ and the unhedged liability value is L^u where

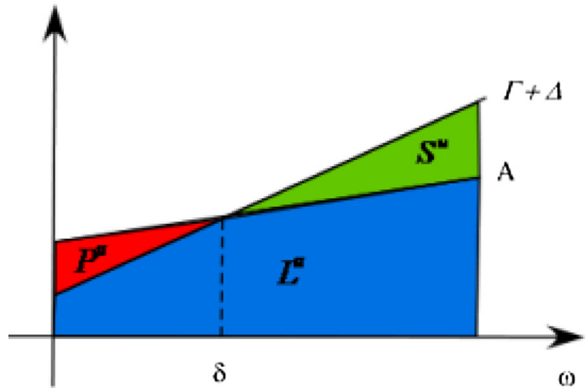
$$L^u = \int_\Omega \min\{\Gamma + \Delta, A\} d\Psi = \int_0^\delta (\Gamma + \Delta) d\Psi + \int_\delta^\zeta A d\Psi. \tag{2}$$

Note that the state δ is implicitly defined by the condition $\Gamma(\delta) + \Delta = A(\delta)$. The liability value is shown in Figure 1. Given insolvency risk, the firm puts the difference to the annuity holders. That put option value is P^u where

[14]This is a strong but useful simplifying assumption. The literature does support a positive correlation between assets and liabilities (e.g., Hsieh, Chen, and Ferris, 1994; Matsen and Thogersen, 2004; Novy-Marx and Rauh, 2011; Ang, Chen, and Sundaresan, 2013).

[15]An approximation of this value is shown in Figure 1. The values are actually the risk-adjusted present value of the areas shown in this and subsequent figures. The lines in the figures need not be straight.

Figure 1
Firm Values



$$P^u = \int_\Omega \max\{0, A - (\Gamma + \Delta)\} d\Psi = \int_0^\delta (A - (\Gamma + \Delta)) d\Psi. \tag{3}$$

If there is an implicit or explicit guarantee by the government, then the difference is put to the government rather than the annuity holders. The put value is shown in Figure 1.

The unhedged stock value may be expressed using the put option value as follows:

$$\begin{aligned} S^u &= \int_\delta^\zeta (\Gamma + \Delta - A) d\Psi \\ &= \int_0^\zeta (\Gamma + \Delta - A) d\Psi - \int_0^\delta (\Gamma + \Delta - A) d\Psi \\ &= \int_0^\zeta \Delta d\Psi + \int_0^\zeta (\Gamma - A) d\Psi + P^u \\ &= \int_0^\zeta \Delta d\Psi + \int_0^\delta (\Gamma + \Delta) d\Psi + \int_\delta^\zeta A d\Psi - \int_0^\zeta A d\Psi + P^u \\ &= \int_0^\zeta \Delta d\Psi + \int_0^\delta (\Gamma + \Delta) d\Psi - \int_0^\delta A d\Psi + P^u \\ &= \int_0^\zeta \Delta d\Psi. \end{aligned} \tag{4}$$

The fourth equality holds because the annuities are rationally valued, that is, $\int_\Omega \Gamma d\Psi = L^u$. Hence, in a competitive market and in the absence of hedging the insurer's stock value is the risk adjusted value of its reserves.

The corporate value for the insurer is the value of its stock plus its liability. Let V^u and denote the corporate value of the unhedged insurer. Then

$$\begin{aligned} V^u &= S^u + L^u \\ &= \int_\delta^\zeta (\Gamma + \Delta - A)d\Psi + \int_0^\delta (\Gamma + \Delta)d\Psi + \int_\delta^\zeta A(\omega)d\Psi \\ &= \int_\Omega (\Gamma + \Delta)d\Psi. \end{aligned} \tag{5}$$

A LONGEVITY BOND

Longevity bonds are designed to generate a cash flow similar to that of an annuity book of business. If the longevity bond issue is based on a population index of mortality then the cash flow will not match that of the insurer's book of business; that is, it would leave some basis risk for the insurer. It has been claimed that basis risk was one of the reasons for the failure of longevity bond issues.

No Basis Risk

We will first abstract from the existence of basis risk and consider the motivation of the insurer to invest in longevity bonds wherein there is no basis risk. No basis risk exists if $B(\omega) = A(\omega)$ for all ω. Hedging with a longevity bond requires that Γ, which represents the payoff from the invested premium income, be replaced in whole or in part by the payoff from a longevity bond instrument. Suppose here that the longevity bond is fairly priced at $\int_0^\zeta B d\Psi$ or equivalently the no arbitrage value. Then

$$\begin{aligned} \int_0^\zeta \Gamma d\Psi &= L^u \\ &< \int_0^\zeta B d\Psi \\ &= \int_0^\zeta A d\Psi \\ &= P^u + L^u. \end{aligned} \tag{6}$$

It follows that the firm must raise the difference, that is, P^u, in the financial markets. We will assume without loss of generality that the firm issues new shares to cover the investment in the longevity bond. Let S^{ob} and S^{nb} denote the value of the old and new shareholders, respectively. Since the firm must raise the put option value with new shares it follows that $S^{nb} = P^u$. With no longevity bond the annuity holders receive $\Gamma + \Delta < A$ for $\omega \leq \delta$ while they receive A for $\omega > \delta$. If the firm hedges with the longevity bond, then annuity holders receive $\min\{A, B + \Delta\}$. In this case of no basis risk, it follows that $B + \Delta > A$ for all states "then" and the hedged firm old shareholder value is

$$\begin{aligned} S^{ob} &= -S^{nb} + S^b \\ &= -P^u + \int_\Omega (B + \Delta - A) d\Psi \\ &= -P^u + \int_\Omega \Delta d\Psi - \int_\Omega (A - B) d\Psi \\ &= \int_\Omega \Delta d\Psi - P^u \\ &= S^u - P^u. \end{aligned} \tag{7}$$

Similarly, the shareholder value is S^b where

$$\begin{aligned} S^b &= S^{nb} + S^{ob} \\ &= P^u + \int_0^\zeta \Delta d\Psi - P^u \\ &= \int_0^\zeta \Delta d\Psi \\ &= S^u. \end{aligned} \tag{8}$$

Hence, the total shareholder value remains the same but the old shareholders face a reduction in value. The corporate executive officer has a fiduciary responsibility to act in the interests of current, equivalently old, shareholders, and hence, *ceteris paribus*, has a disincentive to hedge with the longevity bond since current shareholder value of the hedged firm is less than that of the unhedged firm.

The annuity value in this case is

$$\begin{aligned} L^b &= \int_0^\zeta B d\Psi \\ &= \int_0^\zeta A d\Psi \\ &= L^u + P^u, \end{aligned} \tag{9}$$

and so the corporate value net of the investment expenditure is

$$\begin{aligned} V^b - \mathrm{I} &= S^b + L^b - P^u \\ &= S^{nb} + S^{ob} + L^b \\ &= P^u + (S^u - P^u) + (L^u + P^u) - P^u \\ &= S^u + L^u \\ &= V^u. \end{aligned} \tag{10}$$

This is a corollary of the *1958* Modigliani–Miller theorem (e.g., see Modigliani and Miller, 1958; MacMinn and Martin, 1988).

Basis Risk

Next, consider a case with basis risk. Suppose $A(\omega) > B(\omega)$ for $\omega < \alpha$ and $A(\omega) < B(\omega)$ for $\omega > \alpha$ as shown in Figure 2a. This might follow since those who buy annuities tend to live longer than the general population. This case creates basis risk if the payoff $B(\omega)$ is based on population mortality data rather than the mortality data from the firm's book of business. The basis risk is represented by the difference in payoffs in each state, that is, $A(\omega) - B(\omega)$. The value of the basis risk is

$$\int_0^{\zeta} (A - B)d\Psi > 0. \tag{11}$$

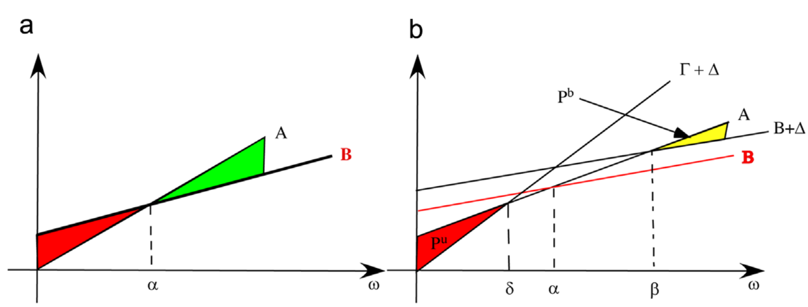
We suppose that the firm has raised the premium income and invested it. The value of the premium *now* is L^u as previously described. If the firm hedges with a longevity bond then it purchases the bond with the premium income and the additional amount necessary to cover the value of the longevity bond "now." The hedged firm has a liability value L^b where

$$L^b = \int_0^{\beta} A d\Psi + \int_{\beta}^{\zeta} (B + \Delta)d\Psi, \tag{12}$$

where β is implicitly defined by $B(\beta) + \Delta = A(\beta)$.

Suppose, without loss of generality, that enough new equity is issued so that the value of the new equity S^{nb} covers the difference in value.

FIGURE 2
Basis Risk



$$
\begin{aligned}
S^{nb} &= \int_0^{\zeta} B d\Psi - L^u \\
&= \int_0^{\zeta} B d\Psi - \int_{\delta}^{\zeta} A d\Psi - \int_0^{\delta} (\Gamma + \Delta) d\Psi \\
&= -\int_0^{\zeta} (A - B) d\Psi + P^u \\
&> 0.
\end{aligned}
\tag{13}
$$

In this case the current shareholder value is

$$
\begin{aligned}
S^{ob} &= S^b - S^{nb} \\
&= \int_0^{\beta} ((B + \Delta) - A) d\Psi + \int_0^{\zeta} (A - B) d\Psi - P^u \\
&= \int_0^{\beta} \Delta d\Psi + \int_{\beta}^{\zeta} (A - B) d\Psi - P^u \\
&= \int_0^{\zeta} \Delta d\Psi + \int_{\beta}^{\zeta} (A - (B + \Delta)) d\Psi - P^u \\
&= S^u - \left(P^u - P^b\right).
\end{aligned}
\tag{14}
$$

A hedge normally reduces the put option value so that $P^u - P^b > 0$ and so it follows that the stock value of the old shareholders of the hedged firm with longevity bonds facing basis risk and insolvency risk will be less than that of the unhedged firm value. Hence, the corporate executive officer acting in the interests of current shareholders does not have the incentive to hedge.

The corporate value may be expressed as

$$
\begin{aligned}
V^b &= S^b + L^b \\
&= \int_0^{\zeta} (\Gamma + \Delta) d\Psi \\
&= V^u.
\end{aligned}
\tag{15}
$$

This again demonstrates a corollary of the *MM58 theorem* holding, in this case, with basis and insolvency risk. This does not, however, imply indifference. The current shareholder value is reduced as shown in (14) and (16); hence, the corporate executive does not have the incentive to hedge longevity risk because it transfers value from shareholders to annuitants.

A FORWARD CONTRACT

Other hedging instruments are available. A q-forward contract was introduced in 2007 by JP Morgan (the q symbol being the common actuarial notation for mortality). The contract was designed to allow firms with longevity risk to replace that risk with a certain cash payment "then" as is the case in a forward contract. We introduce the simplest possible construct of a forward contract here to show both how such an instrument works as a hedge and what impact it has on corporate value. Suppose a forward contract is constructed for the book of annuity business. Let a be a constant that denotes the exercise value "then" and suppose that it is selected so that its risk-adjusted value "now" equals the risk-adjusted value of the annuity book "now", that is,

$$\int_{\Omega} a d\Psi = \int_{\Omega} A(\omega) d\Psi. \tag{16}$$

The payoff of the hedged firm "then" is $\Gamma + \Delta - A - (a - A)$.[16] It follows that the value of the hedged firm in this case is S^f where

$$\begin{aligned} S^f &= \int_{\Omega} (\Gamma + \Delta - A - (a - A)) d\Psi \\ &= \int_{\Omega} (\Gamma + \Delta) d\Psi - \int_{\Omega} a d\Psi \\ &= \int_{\Omega} (\Gamma + \Delta) d\Psi - \int_{\Omega} A d\Psi \\ &= \int_{\Omega} (\Gamma + \Delta - A) d\Psi \\ &= \int_{0}^{\delta} (\Gamma + \Delta - A) d\Psi + S^u \\ &= S^u - P^u. \end{aligned} \tag{17}$$

Hence, once again, *ceteris paribus*, the hedged stock value is less than the unhedged stock value; alternatively, the hedged stock value equals the unhedged stock value minus the value of the put option. There is no put option value in this case, that is, $P^f = 0$, since the full hedge eliminates the insolvency risk. There is also no basis risk. Also note that the capital cost of the longevity bond issue and the cost of the basis risk have been considered design flaws in the literature but neither is present in this case and the conclusion here is the same.

---

[16]It should be noted that the exercise value is less than the value of the premium income and other assets then; if that condition does not hold then the firm will not hedge with the futures contract because the corporate payoff would be negative in all states.

The value of the liability or equivalently, the annuity book business is

$$L^f = \int_0^{\zeta} A d\Psi, \tag{18}$$

and so corporate value is

$$\begin{aligned} V^f &= S^f + L^f \\ &= \int_{\Omega} (\Gamma + \Delta - A) d\Psi + \int_0^{\zeta} A d\Psi \\ &= \int_{\Omega} (\Gamma + \Delta) d\Psi \\ &= V^u. \end{aligned} \tag{19}$$

This again demonstrates the MM58 theorem holding. This does not, however, imply indifference. The current shareholder value is reduced as shown in (19) and so the corporate executive does not have the incentive to hedge longevity risk because it transfers value from shareholders to annuitants.

INCENTIVE EFFECTS

Next consider the incentive effects associated with forming books of annuity business. In particular, here we explore the incentive effects of a riskier book of business. Consider the firm with the choice of two books of business and suppose that one book is riskier than the other in the Rothschild–Stiglitz sense; that is, see Rothschild and Stiglitz (1970). We also consider instruments that allow the firm to hedge the risk of its chosen book of business and ask whether the firm has the incentive to hedge.

An Increase in Risk

Suppose the insurer can select one of two books of business. Let those books be Π_1 and Π_2 where Π_2 is the riskier book. Recall that $\Pi_j(\omega) = \Gamma_j(\omega) + \Delta - A_j(\omega)$ and suppose the increase in risk occurs in the annuity books. For simplicity suppose that book 2 is riskier than book 1 and that $A_2 = (1 + \theta)A_1 - \theta\mu$ where θ is a positive constant and μ is the common expected payoff of the books of business

$$\mu = \int_{\Omega} A_1 d\Phi = \int_{\Omega} A_2 d\Phi. \tag{20}$$

It follows that value of the less risky book of business is greater than the riskier book since

$$\int_{\Omega} A_1 d\Psi - \int_{\Omega} A_2 d\Psi = \theta \int_{\Omega} (\mu - A_1) d\Psi$$
$$= \theta\left(p\mu - \int_{\Omega} A_1 d\Psi\right) \tag{21}$$
$$> 0,$$

where p represents the sum of the basis stock prices or equivalently the discount factor for the safe asset and so $p\mu$ represents the value of a safe asset with the same expected payoff as the risk A_1 and so has a greater value.[17]

Suppose the corporate outsiders, for example, investors and annuitants, cannot observe the CEO's selection of a book of business; this selection introduces a hidden action, or equivalently a risk shifting problem for the corporation.[18]

Consider the stock and put option values given each book of business. Let P_j^u denote the put option value and S_j^u denote the stock value of the unhedged firm given book $j = 1, 2$. Letting δ_j be the boundary of the insolvency event, the stock and put option values are:

$$S_j^u = \int_{\delta_j}^{\zeta} (\Gamma + \Delta - A_j) d\Psi \tag{22}$$

$$P_j^u = \int_0^{\delta_j} \big(A_j - (\Gamma + \Delta)\big) d\Psi. \tag{23}$$

These values are shown in Figure 3a.

Also, note the difference in shareholder and put option values are:

$$S_1^u - S_2^u = \int_{\delta_2}^{\delta_1} (\Gamma + \Delta - A_2) \mathrm{d}\Psi + \int_{\delta_1}^{\zeta} (A_2 - A_1) \mathrm{d}\Psi > 0 \tag{24}$$

and

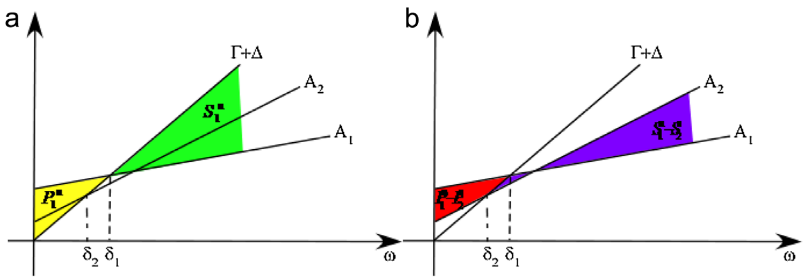
$$P_1^u - P_2^u = \int_0^{\delta_2} (A_1 - A_2) d\Psi - \int_{\delta_2}^{\delta_1} (A_1 - (\Gamma + \Delta)) d\Psi > 0\,. \tag{25}$$

From (26) it is clear that the corporate executive officer acting in the interests of shareholders has the incentive to select the riskier book of business and so shift risk from equity to liability asset holders. Since rational asset holders recognize this incentive to shift risk, the value of the annuity will be that for the second book of business and the rationally valued premium will have a payoff Γ_2 such that

[17]While this is intuitive, see MacMinn (1993) for a proof.

[18]For more on the risk-shifting or asset substitution problem, see MacMinn (1993) and Green (1984). These problems are also moral hazard problems.

FIGURE 3
(a) Asset Values; (b) Asset Substitution



$$\int_0^{\zeta} \Gamma_2 d\Psi = L_2^u, \tag{26}$$

and so from (24) it follows that the equity value may also be expressed as follows:

$$S_2^u = \int_0^{\zeta} \Delta d\Psi. \tag{27}$$

Since (28) would also have to hold if the manager selected A_1 we find that

$$S_1^u = \int_0^{\zeta} \Delta d\Psi - \int_0^{\zeta} (A_1 - A_2) d\Psi < S_2^u. \tag{28}$$

The inequality in (30) follows by (23). Hence, the hidden action problem, that is, the corporate outsiders not being able to observe the manager's choice of the annuity book, locks the manager into selecting the riskier annuity book. This analysis then shows that the same risk-shifting problem can occur on the liability side as on the asset side of the business and follow from the analysis in MacMinn (1993).

A hedging instrument could only be useful in this setting of the hidden action problem if the instrument provided some credible assurance to asset holders that the manger would select the safer project. The longevity bond does not provide credible assurance if it increases the put option value. The forward contract has the potential to provide the necessary assurance to asset holders. If it eliminates insolvency risk, then the annuity assets will be appropriately valued.

A Positive NPV Project

The positive NPV investment introduced is introduced here. Given the insolvency risk, it creates an underinvestment problem similar to that found in the literature; for example, see Myers (1977) and subsequent contributions by Mayers and Smith (1987), MacMinn (1987b), and Garven and MacMinn (1993). In one version of the underinvestment problem a sufficiently levered firm underinvests because the shareholders cannot capture enough of the project NPV. One mechanism that has been shown to solve this problem is to issue a bond that includes an insurance covenant designed to ensure repayment of the face value; that is, see Garven and MacMinn (1993). The story here is altered somewhat because the analysis shows that the corporate executive can add value for current shareholders by hedging and so freeing reserves to invest in a sufficiently positive NPV project.

To see this, consider investment opportunities of the firm. Recall that Δ represents assets that must be held in reserve for the current book of annuity business. Now suppose that such a reserve is no longer necessary if the firm hedges. This would allow the firm to pay dividends or make investments. Consider a generic positive NPV project. Suppose the firm may invest Δ in a project that yields $E > \Delta$. The risk adjusted NPV of the investment is

$$\int_{\Omega}(-\Delta + E)d\Psi > 0. \tag{29}$$

Suppose the firm hedges using a forward contract and that that action removes any constraint limiting the use of Δ. This frees the firm to make the positive NPV investment. As we have noted, before the investment the hedged corporate payoff is $\Gamma + \Delta - a$. After the investment the payoff becomes $\Gamma + E - a$. Let S^i denote the shareholder value given the hedge and investment. We want to know if $S^i > S^u$, where the unhedged value is

$$S^u = \int_{\delta}^{\zeta} (\Gamma + \Delta - A)d\Psi, \tag{30}$$

where δ is implicitly defined by the condition $\Gamma + \Delta = A(\delta)$ and where the value given the hedge and investment is

$$S^i = \int_{0}^{\zeta} (\Gamma + E - a)d\Psi. \tag{31}$$

Hence,

$$\begin{aligned} S^i - S^u &= \int_0^{\zeta} (\Gamma + E - a)d\Psi - \int_{\delta}^{\zeta} (\Gamma + \Delta - A)d\Psi \\ &= \int_0^{\zeta} (\Gamma + E - a)d\Psi - \int_0^{\zeta} (\Gamma + \Delta - A)\mathrm{d}\Psi + \int_0^{\delta} (\Gamma + \Delta - A)\mathrm{d}\Psi \\ &= \int_0^{\zeta} (-\Delta + E)d\Psi + \int_0^{\zeta} (A - a)\mathrm{d}\Psi + \int_0^{\delta} (\Gamma + \Delta - A)\mathrm{d}\Psi \\ &= npv - \int_0^{\delta} (A - (\Gamma + \Delta))d\Psi \\ &= npv - P^u. \end{aligned} \tag{32}$$

There is nothing that says that the project NPV must be greater than the original put value; however, if it is, then the firm has a clear incentive to hedge and use the reserve to invest in a sufficiently positive NPV project.

One argument concerning the failure of the longevity bond and other longevity links financial instruments is that insurers lack the incentive to hedge due to a dearth of positive NPV investment projects. To lend empirical support for such an argument consider Tobin's q in the insurance industry versus other industries.[19] Tobin's q ratio is a representation of monopoly rents and Tobin argued that a ratio greater than one would imply more investment since the market value of the new capital investment would exceed its replacement cost; for example, see Lindenberg and Ross (1981) and Tobin (1978). Erickson and Rothberg (2015) calculate the q ratios by industry and report a q ratio of 1.16 and 1.12 in the 1990s and 2000s for life insurance companies, respectively (i.e., see Table 4 in Erickson and Rothberg, 2015); the q ratios for other industries were generally higher.[20] As mentioned previously, there have been two successful longevity risk bonds, one by Swiss Re and the other by Aegon N.V. Swiss Re and Aegon N.V. both have the SIC code 6411, which is for Insurance Agents, Brokers, and Service. Cummins, Weiss, and Xie (2006) study mergers and acquisitions and find that the Tobin q of acquirers in the Insurance Agents, Brokers, and Service designation was 2.2. The potentially larger Tobin q for Swiss Re and Aegon N. V. may also indicate availability of larger positive NPV projects that, in accordance with the results of this article, would motivate hedging their longevity risk.

CONCLUDING REMARKS

This is a cautionary tale for the construction of markets for longevity risk transfer. If insurers in the annuity market view the hedging decision in isolation or have no sufficiently positive NPV projects, then the analysis shows that the publicly held and traded insurer has nothing to gain and indeed some shareholder value to lose by hedging. The failure of the longevity bond issue attempts may be as simple as this. What is more, if the books of annuity business can be rearranged into riskier books then that may increase shareholder value while also increasing the value being put to annuity holders or the government and that represents an additional moral hazard problem. If, however, the insurer can free reserves to invest in a positive NPV project by hedging then that has the potential to not only make shareholders better off but also the annuity holders.

REFERENCES

Ang, A., B. Chen, and S. Sundaresan, 2013, Liability Driven Investment With Downside Risk, *Journal of Portfolio Management*, 40(1): 71-87.

[19]The q in Tobin's q is not to be confused with that in q-forward. The first refers to the ratio of the market value of the firm divided by its replacement cost while the latter refers to the mortality rate.

[20]This idea for empirical support using Tobin's q ratio came from a reviewer, whom we thank.

Arrow, K. J., 1963, The Role of Securities in the Optimal Allocation of Risk-Bearing, *Review of Economic Studies*, 31: 91-96.

Blake, D., A. Cairns, G. Coughlan, K. Dowd, and R. MacMinn, 2013, The New Life Market, *Journal of Risk and Insurance*, 80(3): 501-558.

Blake, D., A. J. G. Cairns, and K. Dowd, 2006, Living with Mortality: Longevity Bonds and Other Mortality-linked Securities, *British Actuarial Journal*, 12(1): 153-197.

Burne, K., 2011, Swiss Re Longevity-Risk Deal Opens Door to More, *Wall Street Journal*, January 7. Available at http://www.wsj.com/articles/SB10001424052748703730704576065871937194158 (accessed April 14, 2012). New York.

Chen, H., and J. D. Cummins, 2010, Longevity Bond Premiums: The Extreme Value Approach and Risk Cubic Pricing, *Insurance: Mathematics and Economics*, 46(1): 150-161.

Cummins, J. D., M. A. Weiss, and X. Xie, 2006, Market Concentration, Vertical Integration and Bancassurance: Consolidation and the "Insurance Middleman," *ARIA Annual Meeting, Washington, DC*.

Debreu, G., 1959, *Theory of Value: An Axiomatic Analysis of Economic Equilibrium* (New Haven, CT: Yale University Press).

Erickson, G. S., and H. N. Rothberg, 2015, Knowledge Assets in Services Across Industries and Across Time, *Investigaciones Europeas de Direccion y Economia de la Empresa*, 21: 58-64.

Garven, J. R., and R. D. MacMinn, 1993, The Underinvestment Problem, Bond Covenants and Insurance, *Journal of Risk and Insurance*, 60(4): 635-646.

Green, R., 1984, Investment Incentives, Debt, and Warrants, *Journal of Financial Economics*, 13: 115-136.

Hardy, M., 2005, A Matter of Life and Death, *Financial Engineering News*, 17: 20.

Hsieh, S.-J., A. H. Chen, and K. R. Ferris, 1994, The Valuation of PBGC Insurance Premiums Using an Option Pricing Model, *Journal of Financial and Quantitative Analysis*, 29(1): 89-99.

Krutov, A., 2006. Insurance-linked securities: An emerging class of financial instruments. *Financial Engineering News*: No. 48, March-April 2006, p. 2007, 2016.

Lin, Y., and S. H. Cox, 2008, Securitization of Catastrophe Mortality Risks, *Insurance Mathematics & Economics*, 42(2): 628-637.

Lindenberg, E. B., and S. Ross, 1981, Tobin's q Ratio and Industrial Organization, *Journal of Business*, 54(1): 1-32.

MacMinn, R. D., 1984, *Financial Markets and the Fisher Model* (Austin, TX: University of Texas).

MacMinn, R. D., 1987a, Forward Markets, Stock Markets, and the Theory of the Firm, *Journal of Finance*, 42(5): 1167-1185.

MacMinn, R. D., 1987b, Insurance and Corporate Risk Management, *Journal of Risk and Insurance*, 54(4): 658-677.

MacMinn, R., 1993, On the Risk-Shifting Problem and Convertible Bonds, in: C.-F. Lee. ed., *Advances in Quantitative Analysis of Finance and Accounting* (Oxford, UK: Elsevier Science & Technology), pp. 181–200.

MacMinn, R. D., 2002, On the Judgment Proof Problem, *Geneva Papers on Risk and Insurance Theory*, 27: 143-152.

MacMinn, R. D., 2005, *The Fisher Model and Financial Markets* (Singapore: World Scientific Publishing).

MacMinn, R. D., and J. D. Martin, 1988, Uncertainty, the Fisher Model, and Corporate Financial Theory, in: *Research in Finance* (Greenwich, CT: JAI Press), pp. 227-264.

MacMinn, R., and A. Richter, 2014, *The Choice of Trigger in an Insurance Linked Security: The Mortality Risk Case* (Normal, IL: Illinois State University).

Matsen, E., and O. Thogersen, 2004, Designing Social Security—A Portfolio Choice Approach, *European Economic Review*, 48(4): 883-904.

Mayers, D., and C. W. Smith Jr., 1987, Corporate Insurance and the Underinvestment Problem, *Journal of Risk and Insurance*, 54(1): 45-54.

Modigliani, F., and M. H. Miller, 1958, The Cost of Capital, Corporation Finance and the Theory of Investment, *American Economic Review*, 48(3): 261-297.

Myers, S. C., 1977, The Determinants of Corporate Borrowing, *Journal of Financial Economics*, 5: 147-175.

Novy-Marx, R., and J. Rauh, 2011, Public Pension Promises: How Big Are They and What Are They Worth? *Journal of Finance*, 66(4): 1211-1249.

Oppers, S. E., K. Chikada, F. Eich, P. Imam, J. Kiff, M. Kisser, M. Soto, and T. Sun, 2012, The Financial Impact of Longevity Risk, in: *Global Financial Stability Report: The Quest for Lasting Stability* (Washington, DC: International Monetary Fund).

Rothschild, M., and J. E. Stiglitz, 1970, Increasing Risk: I. A Definition, *Journal of Economic Theory*, 2: 225-243.

Shavell, S., 1986, The Judgement Proof Problem, *International Review of Law and Economics*, 6: 45-58.

Tobin, J., 1978, Monetary Policies and the Economy: The Transmission Mechanism, *Southern Economic Journal*, 37: 421-431.

Zelenko, I., 2014, Longevity Risk and the Stability of Retirement Systems: The Chilean Longevity Bond Case, *Journal of Alternative Investments*, 17(1): 35-54.