THE JOURNAL OF INDUSTRIAL ECONOMICS 0022-1821
Volume LXV June 2017 No. 2

PRICES, LOCATIONS AND WELFARE WHEN AN ONLINE RETAILER COMPETES WITH HETEROGENEOUS BRICK-AND-MORTAR RETAILERS*

Wen-Chung Guo†
Fu-Chuan Lai‡

This study proposes a novel spatial model in which an online retailer competes with heterogeneous brick-and-mortar retailers. Consumers are assumed to be non-uniformly distributed along an urban-rural line, and online transactions provide savings in transportation costs at the expense of distaste costs. Among other results, we show that the surviving brick-and-mortar retailers eventually move toward densely populated (urban) areas after the entry of the online retailer. Consumer welfare, the policy of not taxing online business, and the socially optimal number of retailers are also analyzed.

I. INTRODUCTION

There are fundamental differences between physical and online retailing such that an online purchase is usually location-irrelevant, saving transportation costs for consumers, especially for rural residents, but is limited by a distaste cost. Competition from online businesses has had an impact on the structure of the retailing industry as shown in numerous empirical studies such as Goolsbee [2000, 2001], Brynjolfsson and Smith [2000], Brown and Goolsbee [2002] Chevalier and Goolsbee [2003], Prince [2007], Clay *et al.* [2002], Jin and Kato [2007], Goldmanis *et al.* [2010], and Ofek *et al.* [2011]. However, the previous studies seldom emphasize the influences of online business on the spatial configuration of physical retailers. In practice, it is observed that numerous physical retailers have moved into urban areas, close to highly dense populations, while online retailers often serve rural areas, which have fewer physical retailers. For instance, physical bookstores are nowadays narrowly distributed in areas of high traffic, central business districts, and university campuses.[1]

*We are deeply grateful to the Editor and two anonymous referees for their valuable comments. We also wish to express our gratitude to Benjamin Hermalin and Jan Brueckner for their helpful suggestions. All remaining errors are ours.

†Authors' affiliations: Department of Economics, National Taipei University and National Communications Commission, Taipei, Taiwan.
e-mail: guowc@ntu.edu.tw

‡Research Center for Humanities and Social Sciences, Academia Sinica and Department of Public Finance, National Chengchi University, Taipei, Taiwan.
e-mail: uiuclai@gate.sinica.edu.tw

[1] For instance, based on the state level data of the U.S. Census Bureau, the number of all retail stores per population decreased, on average, 17.77% from 1998 to 2010. However, this number

© 2017 The Editorial Board of The Journal of Industrial Economics and John Wiley & Sons Ltd

We analyze a simultaneous price and location game in which heterogeneous physical retailers compete with an online retailer, and face non-uniformly distributed consumers. Retailers are differentiated by their marginal costs, and more densely populated (urban) segments represent higher potential profits than less densely populated (rural) segments. Changes in prices, locations, and consumer welfare over the short, medium, and long run are considered. With respect to the tractability of the model, the assumption herein that prices and locations of retailers are chosen simultaneously is critical to avoid the existence problem in equilibrium, and differs from that made in the standard location-then-price game *à la* Hotelling [1929] in which no (pure strategy) subgame perfect equilibrium exists under linear transportation costs (d'Aspremont *et al.* [1979]).[2] To overcome this difficulty, Osborne and Pitchik [1987] allowed mixed strategies in prices to solve a unique symmetric pure location equilibrium by simulation, and Vogel [2008] proposed an auxiliary game in a circular market with uniformly distributed consumers and heterogeneous firms to obtain pure strategy equilibria along the equilibrium path. In fact, our game structure involving simultaneous choices of price and location was also employed by Lerner and Singer [1937], Anderson *et al.* [1992], and Gandhi *et al.* [2008].

Spatial models with non-uniform distributions of consumers have been analyzed by very few studies, which consider only cases with duopoly and quadratic transportation costs. For instance, pure-strategy location and price equilibria have been shown to exist when the density function is concave and not too concentrated (Neven [1986]), when the distribution is triangular, which yields asymmetric locations (Tabuchi and Thisse [1995]), and when the density function is neither too asymmetric nor too concave (Anderson *et al.* [1997]). In particular, the current model determines the price and location equilibrium in an environment with non-uniformly distributed consumers, heterogeneous physical retailers, and an online retailer.

Our analysis emphasizes the influences of the online retailer on spatial competition among physical retailers. Before the entry of the online retailer, spatial competition is shown to motivate the most competitive physical retailer to serve the most urbanized area, followed by the second-most competitive physical

is only 2.92% in New York State and 14.93% in Florida; meanwhile, the declines are above the national average in Oregon (16.51%) and Kansas (24.37%). We obtain a similar pattern for the number of bookstores (electronics stores), which decreased on average 34.55% (8.05%), decreased 22.36% (increased 13.84%) in New York and 33.62% (5.85%) in Florida, and decreased 75.83% (22.45%) in Oregon and 66.25% (22.14%) in Kansas, respectively. Grocery stores also showed the same pattern. In fact, rural bank closures have received considerable attention in the U.K. due to competition from Internet banking (Campbell and Sandhu [2015]).

[2] Anderson *et al.* [1992] suggested that the degree of flexibility of the price variable and the location variable can justify the setting of game structures: A simultaneous price and location game is plausible if relocation is costless, and a location-then-price game is suitable if relocation is prohibitively costly. The costs of relocation for physical retailers are in reality between these two extreme cases. We employ short-run, medium-run and long-run scenarios to to compensate for the restriction of our simultaneous game.

© 2017 The Editorial Board of The Journal of Industrial Economics and John Wiley & Sons Ltd

retailer, which serves the second-most urbanized area, and the order continues in that manner. The equilibrium locations can be characterized by a series of nonoverlapping and contiguous occupied segments. Our result of location settlement may be compared with those in recent studies of the sequential location game. Specifically, if the locations and prices are chosen in order by the online retailer, the firm with the lowest costs, the firm with the second lowest costs and so on, then the left-to-right iterative location settlement will be the same as in the case of monotonically decreasing density that was considered by Loertscher and Muehlheusser [2011]. Our contribution beyond their work is twofold: we allow endogenous pricing in contrast to their pure sequential location game, and we discuss the influences of online competition.

Consider the short-run scenario in which physical retailers are immutable and cannot shut down. We show that the entry of an online retailer reduces the prices of the physical retailers. The equilibrium online price can be either higher or lower than the offline prices. In the medium run, assume that the physical retailers may shut down (exit), but they cannot relocate. In the face of the online competition, some physical retailers with high marginal costs leave the market, so consumers who live close to those physical retailers are worse off, whereas individuals who live in a rural area where no retail service was previously available are better off.

In the long run, with the free exit and relocation of the physical retailers, the entry of an online retailer reduces both equilibrium prices and the number of physical retailers as compared to the equilibrium without the online retailer. In the face of this online competition, those physical retailers with high marginal costs eventually exit the market, and the remaining physical retailers move to more densely populated segments. At equilibrium, just as in the absence of the online retailer, physical retailers serve contiguous market segments, where each retailer's market is connected side by side. Meanwhile, the online retailer functions as a reservation price setter, and serves the segments not served by the physical retailers. Notably, if the entry of the online retailer is anticipated by physical retailers, then equilibrium prices and locations are identical to those in the long-run equilibrium. Intuitively, the anticipated entry of the online retailer will only replace the reservation price with the online purchasing cost. Therefore, the location settlement is consistent with that obtained by Loertscher and Muehlheusser [2011].

A welfare analysis of taxation and the optimal number of physical retailers is presented. The price margin of all retailers should be equal under optimal taxation to ensure efficiently located production, which cannot be achieved under a tax-free policy for online business. Additionally, the socially desirable number of physical retailers may be either greater or lower than that at equilibrium.

The urban (rural) area in our framework can be explained as the mainstream (niche) market, with reference to Bar-Isaac *et al.* [2012], who adopted the product design model to predict that a fall in search costs causes both the long-tail

© 2017 The Editorial Board of The Journal of Industrial Economics and John Wiley & Sons Ltd

effect and the superstar effect, and demonstrated that the more advantaged firms choose broad designs, while disadvantaged firms prefer niche ones. The results herein suggest an alternative that the online retailer in our framework appears to have the advantage of providing various products that fit specific interests in the niche market, while physical retailers may focus on the major interests in the mainstream market. Moreover, low-cost physical retailers will serve more mainstream markets than high-cost firms. Additionally, the introduction of an online retailer induces physical retailers to become more mainstream, while the online retailer serves the remaining niche interests.

An increasing number of studies are addressing competition between online retailers and physical retailers, and some of them involve the spatial characteristics, which are closely related to the current framework. Based on the circular spatial model of Salop [1979] in which locations of firms are exogenously given, Balasubramanian [1998] analyzed price competition between a direct channel (mail order) and conventional physical retailers, and found that consumers and competition among retailers may be affected by the market coverage controlled by the direct channel. Madden and Pezzino [2011] considered the social optimum in this competition between direct and conventional firms, and found that the number of conventional firms might be more or fewer than the social optimum. By using the linear spatial model of Hotelling [1929], Liu *et al*. [2006] employed a linear model to analyze how a physical incumbent can deter the entry of an online retailer by refraining from entering online commerce. By using a circular spatial framework, Loginova [2009] provided price and welfare analysis among physical retailers and perfectly competitive online retailers. In her framework, consumers may visit a physical retailer and learn their valuation for products (thus eliminating the uncertainty) and then return home and purchase from an online retailer. In the above studies, the locations of physical retailers are exogenous, and most of them only analyze symmetric equilibria. Instead, the location choices of physical retailers are endogenously solved in our model with heterogeneous costs for physical retailers.[3]

Our analysis suggests that neither zero taxation on the online retailer nor uniform taxation on all retailers is socially desirable. This finding may resolve a current contentious issue of the tax exemption on e-commerce, which has been viewed as a supportive policy for a fledgling Internet industry. Some policymakers argue that a giant retailer such as Amazon (with nearly 100 million customers) should be taxed to avoid unfair competition with physical retailers (see Stone [2011] for further details). Most transactions of online retailers in the United

[3] Recently, Foncel *et al*. [2011] developed a short-run (price) and long-run (price and location) model with a fully covered market in which one physical firm competes with one online firm. They showed the existence of two short-run equilibria and two specific long-run equilibria under some 'simple' mixed strategies on the price stage. Our model is more general than that of Foncel *et al*. [2011] in that we consider multiple physical retailers, free entry, urban/rural location, and the partial coverage of markets.

© 2017 The Editorial Board of The Journal of Industrial Economics and John Wiley & Sons Ltd

States are tax exempt, except where their facilities are located.[4] Several empirical studies of Goolsbee [2000], Ballard and Lee [2007], and Ahmed and Wirjanto [2008] supported the notion that sales tax induces online transactions due to tax avoidance. Recently, Ellison and Ellison [2009] found that online purchases will significantly decrease if the offline sales tax is eliminated; in addition, consumers still prefer purchasing from in-state retailers, even when a sales tax is considered. Einav *et al*. [2014] further used detailed eBay data to suggest that a state's sales tax reduces the behavior of online browsing as well as purchasing from the online sellers in the same state, while out-of-state online purchasing increases.

The rest of this paper is organized as follows. Section II introduces the model setting, and Section III analyzes the equilibrium without an online retailer. Section IV shows the short-run equilibrium when an online retailer enters the market unexpectedly, and all physical retailers are assumed to be immutable and cannot shut down. Then we present the medium-run equilibrium in which physical retailers can leave the market in Section V, and develop the long-run equilibrium in Section VI, where physical retailers are free to exit and locate wherever they wish. Section VII provides implications for the indirect taxation on e-commerce and the socially optimal number of physical retailers as well. Conclusions are finally drawn in Section VIII.

II. MODEL SETTING

This study provides a framework of competition among multiple heterogeneous physical retailers and an online retailer.[5] Consider a simultaneous price and location game, in which only pure strategies of price and location are solved as follows. Assume that a mass of consumers are non-uniformly distributed in a linear market with a length L. Spatial population density follows $f(x) = a - bx$, $x \in [0, L]$, where $L = a/b$, $a > 0$, $b > 0$. This setting allows us to analyze an urban/rural configuration, where location 0 denotes the most urbanized area, and L represents the remotest rural area.[6] Each consumer purchases a product either from one of m heterogeneous physical retailers (denoted by $1, 2, \ldots, m$,

[4] A Supreme Court ruling from 1992 exempts retailers from sales taxes in states where they do not have a physical presence. Amazon therefore offers customers better prices without a sales tax in most areas, except for six states as of July, 2012. Recently, Amazon has adjusted its business model from that of a remote seller without any physical facilities in most states to a company with many distribution warehouses in order to get close to customers and reduce consumers' waiting costs. Amazon was forecast to collect sales tax in approximately half of all 50 states by 2014. Many small stores are concerned about a future where online competition leaves them without sufficient customers to survive (see Jopson [2012] for details).

[5] We allow only one online retailer, to avoid undercutting between online retailers. In practice, Amazon dominates the online bookstore market, with annual sales in 2011 at 6.87 times those of Barnes and Noble, the self-proclaimed world's largest physical book seller.

[6] The population density in our setting is triangular, going from a to 0 over the interval $[0, a/b]$. An alternative setting is to leave the geographical space $[0, L]$ fixed. Our results are generally valid under this alternative setting, but some additional assumptions on the space parameter L are required to ensure a partial coverage of markets in the benchmark case.

© 2017 The Editorial Board of The Journal of Industrial Economics and John Wiley & Sons Ltd

respectively) whose locations are determined endogenously at $x_i \in [0, L]$, or from a location-irrelevant online retailer (denoted by 0). The marginal costs for retailer i are denoted by constants c_i, and the marginal cost of the online retailer is a constant c_0. Assume that $c_1 < c_2 < c_3 \cdots < c_m$. Note that the assumption of heterogeneous costs is necessary for there to be an equilibrium of the kind analyzed with a non-uniform distribution. In order to make the algebraic expressions more transparent, we impose an additional restriction $c_i = i \cdot \Delta$, $i = 1, \ldots, m$, which means that the cost difference between any two most similar firms is simply a constant $\Delta > 0$. Their prices are denoted by p_i, $i = 0, 1, \ldots, m$, respectively. The number of physical retailers (m) is endogenously determined by a given considerable fixed cost F for each physical retailer. The utility of a customer located at x who purchases from a physical retailer i, $i = 1 \ldots m$, and online retailer 0, respectively, is

$$u_i(x) = v - p_i - k|x - x_i|, \quad i = 1, \ldots, m, \tag{1}$$

$$u_0 = v - p_0 - z, \tag{2}$$

where v denotes the reservation price, k represents the unit transportation cost, and z refers to consumer distaste for online purchases. This distaste cost includes the delay in receiving the product, the inability of consumers to inspect (i.e. touch, smell, hear, or directly see) the product beforehand, and uncertainty about the sellers' credit and shipping reliability. Note that the utility of not buying is zero. To ensure the existence of a pure strategy equilibrium, we assume b is sufficiently small and the cost difference Δ is large such that

$$b < \frac{2ka\Delta^2(i-j)^2}{\begin{array}{l}[v(2i-1) - \Delta i^2](v - i\Delta)^2 \\ -[v(2j-1) - \Delta j^2][(v - i\Delta)^2 - (i-j)^2\Delta^2]\end{array}},$$

$$\forall \quad i, j \text{ satisfying } 1 \leq j < i \leq m,$$

and

$$b < \frac{4Fk^2}{3(v - \Delta)}. \tag{3}$$

Notably, the right-hand side of the first inequality in (3) is always positive since $i > j$. These assumptions will be used later in the proof of Proposition 1 and will be explained after this proposition.[7] We also impose a technical assumption

[7] The assumption of linearly varying population density is crucial to deriving analytical solutions for prices and locations. In the general case of a monotonic decreasing density, an additional assumption that $f'(x)$ is small and satisfies a condition similar to equation (3) is required.

© 2017 The Editorial Board of The Journal of Industrial Economics and John Wiley & Sons Ltd

that no retailer can set prices higher than monopoly prices.[8] In the Benchmark case without the online retailer, the monopoly prices will be shown as $p_i^m = \frac{v+c_i}{2}$, $i = 1, 2, \ldots, m$. The above assumptions simplify our analysis to reach the conclusion that each physical retailer is a local monopolist. Precisely, a high-cost retailer i cannot earn more profit from moving to the location of a low-cost retailer j, which will be shown later in Proposition 1. Moreover, the market is assumed not to be entirely covered by the restrictions that v is not too high and F is not too low. Each consumer purchases zero or one product unit from the firm offering the highest utility level.

Four scenarios are analyzed to discuss the short-run, medium-run and long-run consequences of the entry of an online retailer. The first scenario describes the benchmark (B) case in which there are physical retailers and no online retailer. The second scenario is the short-run (SR) equilibrium where an online retailer makes an unexpected entry, yet the locations of physical retailers are immutable; in addition, they are unable to leave the market. The third scenario describes the medium-run (MR) equilibrium following this unexpected entry in which physical retailers can shut down, yet cannot relocate. The fourth scenario is the long-run (LR) equilibrium in which physical retailers are free to exit and locate wherever they desire.

III. EQUILIBRIUM WITHOUT AN ONLINE RETAILER

This section describes the construction of a benchmark model in which only physical retailers compete with each other in the market (the Benchmark Scenario B). Intuitively, each physical retailer prefers to locate in more densely populated (urbanized) areas. If the lowest-cost physical retailer serves the most urbanized area by setting a lowest monopoly price, then it can be shown that no other physical retailer can compete with the lowest-cost one. Consequently, the second lowest physical retailer occupies the next populated area with a second lowest monopoly price, and the order continues in that manner. Figure 1 describes the equilibrium configuration where no unoccupied market segment between physical retailers can be left. Assume that n_i^ℓ (n_i^r) is the left (right) consumer who is indifferent between buying from firm i and not buying. In equilibrium, $0 = n_1^\ell < x_1 < n_1^r = n_2^\ell < x_2 < n_2^r = n_3^\ell < \cdots < n_m^\ell < L$. Therefore, the utility function for consumers purchasing from retailer i is $u_i(x) = v - p_i - k|x - x_i|$. Define the market segment of retailer i as a set of consumers who prefer buying from retailer i, rather than either buying from other retailers or not buying. We call the market segment of retailer i non-overlapping (overlapping) with that of its right neighboring retailer $i+1$ if $n_i^r \leq n_{i+1}^\ell$ ($n_i^r > n_{i+1}^\ell$), which represents no direct competition between these two retailers. We also call the market segments

[8] Without this technical assumption, our proposed solution is still an equilibrium, while there exist other equilibria such that retailers set prices higher than the monopoly prices, and physical retailers still occupy non-overlapping and continuous segments.

© 2017 The Editorial Board of The Journal of Industrial Economics and John Wiley & Sons Ltd

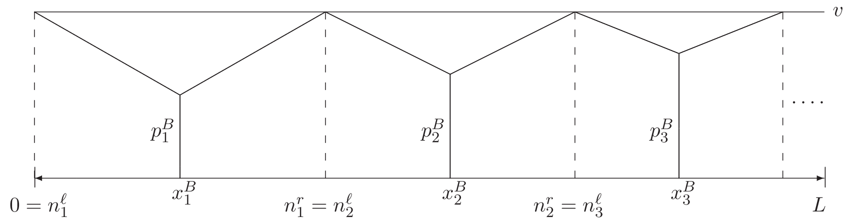

Figure 1
Total Purchasing Costs and Market Segments without an Online Retailer

of two neighboring retailers contiguous if $n_i^r \geq n_{i+1}^l$ which represents no gap between their market segments. Solving $u_i(x) = 0$ yields

$$n_i^r = x_i + \frac{v - p_i}{k}, \qquad n_i^\ell = x_i - \frac{v - p_i}{k}, \quad i = 1, 2, \ldots, m^B,$$

where superscript "*B*" refers to Scenario (B).[9] Therefore, the quantity supplied by physical retailer *i* is

$$(4) \qquad N_i = \int_{n_i^\ell}^{n_i^r} f(x)dx = \frac{2(v - p_i)(a - bx_i)}{k}, \quad i = 1, 2, \ldots, m^B,$$

and the profit functions are $\pi_i = (p_i - i \cdot \Delta) \cdot N_i - F$, $i = 1, 2, \ldots, m^B$. The equilibrium number of physical retailers m^B is determined by the fixed cost F with the condition $\pi_{m^B} = 0$.[10] We ignore the requirement that the number of physical retailers must be an integer for simplicity *à la* Salop [1979] and Madden and Pezzino [2011].[11] Notably, the quantities in equation (4) are decreasing in x, as is profit. The following proposition presents the equilibrium prices and locations.

Proposition 1. In the case without any online retailer, a unique pure strategy equilibrium exists. In this equilibrium, the physical retailers with low marginal costs occupy nonoverlapping, contiguous segments and the market segment for each retailer is connected with that of another retailer starting from the most urbanized area, and where the lowest cost retailer is the leftmost retailer,

[9] Hereinafter, the superscripts "*SR*", "*MR*", and "*LR*", refer to Scenarios in the short-run, medium-run, and long-run, respectively, in the later sections.

[10] In practice, fixed costs may be lower in rural areas than those in urban areas, because of lower real-estate costs. Our model is robust to accommodate fixed costs that decrease in the location variable x, provided the decrease is slow enough such that the demand advantage in urban areas is not altered.

[11] Under the requirement that the number of physical retailers is an integer, m^B is determined by $\pi_{m^B} > 0$ and $\pi_{m^B+1} < 0$.

© 2017 The Editorial Board of The Journal of Industrial Economics and John Wiley & Sons Ltd

followed by the second lowest cost retailer, and the order continues in that manner. The equilibrium prices are $p_i^B = \frac{v+i\cdot\Delta}{2}$, and the equilibrium locations are $x_i^B = \frac{v(2i-1)-i^2\cdot\Delta}{2k}$, $i = 1, 2, \ldots, m^B$. The equilibrium number of physical retailers m^B is increasing in a and v, and decreasing in b, F, and k.

The proof of Proposition 1 is involved. Our strategy is to construct a series of lemmas that are provided in the Appendix, which are intuitively described here. First, we show that no geographical point can accommodate more than one retailer, because low-cost retailers will undercut high-cost ones. Second, the rightmost firm m^B will set a local monopoly price $p_{m^B}^B = \frac{v+m^B\cdot\Delta}{2}$ and choose the location where its left market boundary is connected with that of the neighboring retailer, because the spatial population density is decreasing rightward. The second inequality in (3) prevents retailers from choosing overlapping market segments. Third, all low-cost retailers set lower prices and occupy denser market segments than those of high-cost retailers. Finally, there exists a unique pure strategy equilibrium in which any two neighboring retailers will be shown to set local monopoly prices $p_i^B = \frac{v+i\cdot\Delta}{2}$ and choose locations to occupy non-overlapping and contiguous segments, such that, the lowest-cost retailer has its left market boundary at $x=0$. The first inequality about b in (3) ensures that high-cost retailers will choose rural locations and cannot make additional profits by moving to the locations of low-cost retailers.

Proposition 1 asserts the existence and uniqueness of a pure strategy equilibrium at which the market segments of physical retailers are determined iteratively from the most urbanized area to the rural areas. The market is not fully covered because population density in the least densely populated (rural) area is too low to support a high-cost physical retailer, and the reservation price is assumed to be not high enough by assumption. This result of partial coverage is similar to that in Economides [1984] due to the setting of limited reservation price and departs from standard spatial models, such as those of Hotelling [1929], d'Aspremont *et al.* [1979], and Vogel [2008]. Typical Hotelling/Salop models are usually analyzed under the assumption that v is so large that prices are determined independently of v. However, for intermediate values of v, the equilibrium prices depend on v. Our result of local monopolists corresponds to the second case when the demand density is decreasing and v is limited such that the consumer between two neighboring retailers has a payoff zero. Since the spatial distribution in Proposition 1 runs from the retailer with the lowest cost to the retailer with the highest cost, physical retailers can individually function in a locally monopolistic manner and charge monopoly prices to their consumers.[12] This distribution of retailers in urban and rural segments provides further insight into spatial competition among retailers, and thus is in contrast to most theoretical location studies of firms with a uniform distribution of consumers.

[12] Note that the utility of the consumer at the edge of each market segment is zero by (1).

© 2017 The Editorial Board of The Journal of Industrial Economics and John Wiley & Sons Ltd

Proposition 1 also reveals that the equilibrium prices p_i^B, $i = 1, 2, \ldots, m^B$, which are local monopoly prices, are increasing in both v and marginal costs.[13] Therefore, a retailer that is closer to an urban area has a lower equilibrium price. Consequently, higher-cost retailers cannot benefit from moving to more urbanized segments and setting the same price as lower-cost retailers. Thus, lower-cost retailers locate closer to the more urbanized area in equilibrium. If a broad market is interpreted as an urban area with a high-density population, and a niche market is regarded as a rural area with a low-density population, then the result herein is consistent with that of Bar-Isaac *et al.* [2012], who found that low-cost firms try to attract a broad market, while high-cost firms target niches. Low-cost retailers in our framework enjoy high unit profits ($p_i^B - c_i$) and high mark-up ($\frac{p_i^B - c_i}{c_i}$). Additionally, since low-cost retailers have larger market segments, they are more isolated. These two properties about isolation and mark-up are similar to the findings of Vogel [2008], while multiple equilibria appear in his model.

The equilibrium profits π_i^B reveal that profits are decreasing from urban segments to rural segments, due to the decreasing density and increasing costs. This profit pattern is similar to the case of monotonically decreasing densities in Loertscher and Muehlheusser [2011], in which the settlement of locations occurring from the left (urban) to the right (rural) is implied in their sequential location game with homogenous production costs. In their model, high demand (left) segments attract more firms and result in more intense competition, but the advantage of locating in high demand areas is not fully offset by more entries. Therefore, earlier entries prefer to choose left segments. By contrast, in our heterogeneous cost framework, low-cost firms locate at urban (left) segments by choosing lower prices to prevent further entry from high-cost retailers, because a higher demand density is not fully offset by more entries.

The equilibrium number of physical retailers m^B is found to be positively related to the highest population density (a) and reservation price (v), because the profits of physical retailers increase in both parameters. Since population density declines in b, and the quantities of retailers decrease in k, the equilibrium m^B is negatively related to b and k. Specifically, the number of retailers increases as the density becomes more uniform (b decreases), because the market boundary $L = a/b$ increases and the rural segment allows more demand for high-cost retailers. This result is different from the finding of Loertscher and Muehlheusser (2011, pp. 652) that the number of entrants is minimized when the population distribution is uniform. Furthermore, m^B is decreasing in the parameter of cost differences Δ when b is close to zero. An increase in Δ has two

[13] Our result of monopoly prices crucially depends on the assumption of a limited reservation prices, which is different from the Hotelling-type models with a large reservation price. However, our setting is not rare. In fact, Economides [1984] showed that a lower reservation price induces local monopolists. Salop [1979] also considered the kinked equilibrium, which is similar to our contiguous and monopolistic market segment, where the consumer between two neighboring retailers has a payoff of zero.

© 2017 The Editorial Board of The Journal of Industrial Economics and John Wiley & Sons Ltd

opposing effects on m^B: the (direct) cost effect reduces the profit and the (indirect) location effect, which reduces each market segment for retailers and then pushes retailers to move toward the more urban segments and increases profits. When b is closer to zero, the difference of population densities of consumers between urban and rural areas is less. Therefore, the cost effect dominates the location effect, so the equilibrium m^B is lower.

IV. SHORT-RUN EQUILIBRIUM WITH MARKET ENTRY BY AN ONLINE RETAILER

This section describes the short-run equilibrium (Scenario SR), where all physical retailers are assumed to be immutable and cannot shut down operations, i.e., $x_i^{SR} = x_i^B$ and $m^{SR} = m^B$. Now an online retailer makes an unexpected entry into the market with a marginal cost c_0, where $c_0 + z < v$.[14] The entry of the online retailer results in competition with physical retailers, allowing consumers to purchase products either from the online retailer or from a nearby physical retailer. Therefore, any physical retailer cannot charge a price higher than $p_0 + z$. Thus, replacing v with $p_0 + z$ into Proposition 1 yields reaction functions $p_i^{SR} = \frac{p_0+z+c_i}{2}$, $i = 1, \ldots, m^{SR}$. Similarly, the profit of the online retailer is the price margin times the remaining market from the total market of all physical retailers minus the fixed cost of the online retailer (F_0):

$$\pi_0 = (p_0 - c_0)N_0 - F_0 = (p_0 - c_0)\left(\frac{a^2}{2b} - \sum_{i=1}^{m^{SR}} N_i\right) - F_0,$$

where $N_i = 2(p_0+z-p_i)(a-bx_i)/k$ from replacing v with p_0+z in equation (4). Assume that F_0 is not too large to ensure the market entry of the online retailer. Solving $\partial\pi_i/\partial p_i = 0$, $i = 1, \ldots, m^{SR}$, and $\partial\pi_0/\partial p_0 = 0$ simultaneously yields the equilibrium prices

$$p_0^{SR} = \frac{a^2 k}{6b\sum_{i=1}^{m^{SR}}\left[a - bx_i^{SR}\right]} + \frac{1}{3}\bar{c}^{m^{SR}} + \frac{2}{3}c_0 - \frac{z}{3}, \tag{5}$$

where $\bar{c}^{m^{SR}} = \frac{\sum_{i=1}^{m^{SR}}(a-bx_i^{SR})c_i}{\sum_{i=1}^{m^{SR}}(a-bx_i^{SR})}$ denotes the weighted average of marginal costs of physical retailers, and $p_i^{SR} = \frac{p_o^{SR}+z+c_i}{2}$, $i = 1, \ldots, m^{SR}$. In order to discuss the influence of marginal costs of physical retailers on the equilibrium prices, we keep the general c_i in the following proposition.

Proposition 2. When an online retailer enters the market unexpectedly and retailers are immutable and cannot shut down but prices are flexible, then equilibrium prices fall; that is, $p_i^{SR} < p_i^B$. Moreover, p_0^{SR} now increases in c_0 and k,

[14] If $c_0 + z \geq v$, then the online retailer has no cost advantage in entering the market.

© 2017 The Editorial Board of The Journal of Industrial Economics and John Wiley & Sons Ltd

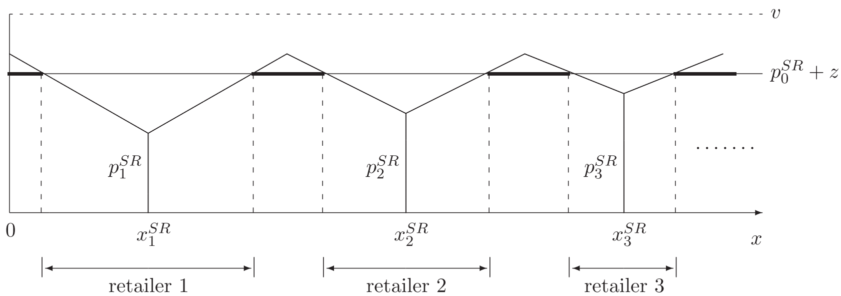

Figure 2
Market Segments in the Short-Run Equilibrium

and decreases in z, while $p_i^{SR}, i = 1, \ldots, m^{SR}$ increase in $c_0, k, c_j, j = 1, \ldots, m^{SR}$, and z, respectively.

Proposition 2 indicates that the entry of an online retailer to the market reduces the short-run equilibrium prices of physical retailers. Figure 2 displays the equilibrium configuration. It reveals that the entry of the online retailer causes the market to become fully covered, because online transactions are independent of location and, having entered the market, the online retailer serves all customers who do not purchase from any physical retailer. Additionally, all consumers benefit from price cuts of the physical retailers. Finally, the entry of the online retailer separates and shrinks the markets of all physical retailers, and the remaining markets (the segments within the bold line in Figure 2) are served by the online retailer.

In Proposition 2, increasing c_0, c_i or k increases the prices set by the online retailer and all physical retailers; meanwhile, increasing z reduces (increases) the equilibrium price(s) of the online retailer (physical retailers). Increasing k reduces spatial competition, allowing all retailers to charge higher prices. Since z only affects online purchases, increasing z reduces the competitive advantage of the online retailer (reduces p_0), allowing physical retailers to charge higher prices.

The influence of the marginal costs of each physical retailer or of the online retailer on the short-run equilibrium prices is global, whereas it is local in the benchmark case in Proposition 1, because the online retailer considers all costs of physical retailers to maximize profits. Vogel [2008] also observed that costs can have either a local or a global effect owing to the competition in a fully covered market. Notably, while Vogel [2008] required low-cost differences to ensure equilibrium, this work requires that the cost difference is not low. The above results imply that physical retailers with lower marginal costs are more isolated, because the distance between retailers i and $i+1$ is $\frac{v-(c_i+c_{i+1})/2}{k}$, which

© 2017 The Editorial Board of The Journal of Industrial Economics and John Wiley & Sons Ltd

decreases in marginal costs. The mark-up $\frac{p_i^{SR}-c_i}{c_i} = \frac{v-c_i}{2c_i}$ is higher for a retailer with lower costs. These two properties about isolation and mark-up are similar to the findings of Vogel [2008].

In the previous benchmark case with no online retailer, each physical retailer is a local monopolist, and its price is independent of the marginal costs of other retailers. However, the entry of the online retailer links all retailers. An increase in the marginal costs of any physical retailer or of the online retailer raises the prices of all retailers, and the influences are asymmetric. Since $\partial p_i^{SR}/\partial c_i = \frac{1}{2}\frac{\partial p_0^{SR}}{\partial c_i} + \frac{1}{2}$, $i = 1, 2, \ldots, m^{SR}$ and $\partial p_0^{SR}/\partial c_0 = 2/3$, the marginal costs of retailers variously affect the prices set by those retailers. Additionally, $\partial p_0^{SR}/\partial c_i = (1 - bx_i^{SR})/\sum_{i=1}^{m^{SR}} 3(a - bx_i^{SR})$, $i = 1, 2, \ldots, m^{SR}$, indicating that the price of the online retailer depends on the marginal costs of all physical retailers, with those of retailers whose marginal costs are low weighted more heavily. Intuitively, the pricing decision of the online retailer is more strongly based on those of physical retailers with cost advantages. Finally, the fact that $\partial p_i^{SR}/\partial c_0 > \partial p_j^{SR}/\partial c_0$, if $i<j$, suggests that the marginal cost of the online retailer more strongly affects those physical retailers with lower marginal costs.

The retail prices of physical retailers in Proposition 1 are independent of the transport cost, owing to the linear distribution of consumers. However, the transport cost affects the prices of physical retailers in Proposition 2, because the linkages among them are formed by the entry of the online retailer. Since the demand for the products of the online retailer is positively related to the transport cost, increasing the transport cost raises the market size of the online retailer, increasing the online price and allowing the physical retailers to charge higher retail prices. The distaste cost represents a disadvantage for the online retailer, explaining its negative effect on equilibrium online pricing; the prices of physical retailers are positively correlated with this distaste cost. According to Proposition 2, the comparison between p_0^{SR} and p_i^{SR}, $i = 1, 2, \ldots, m^{SR}$, is summarized as follows.

Corollary 1. The online retailer charges a higher price than those charged by physical retailers when z is relatively small and c_0 is relatively large. In particular, $p_0^{SR} > p_i^{SR}$ if and only if $z - \frac{c_0}{2} < \frac{a^2 k}{8b\sum_{i=1}^{m^{SR}}[a - bx_i^{SR}]} - \frac{1}{4}\bar{c}^{m^{SR}} - \frac{3}{4}c_i, i = 1, 2, \ldots, m^{SR}$.

Corollary 1 reveals that if the distaste cost is relatively small and the marginal cost of the online retailer is relatively large, then the online retailer charges a higher price than those of the physical retailers with high costs in order to reflect his convenience and cost level. This finding suggests that the price of the online retailer may be either lower or higher than that of the physical retailers. Empirically, whether the online retailers offer lower prices than physical retailers is ambiguous. For instance, Brynjolfsson and Smith [2000] find that prices on the Internet are 9-16% lower than physical prices, while Clay *et al.* [2002] find

© 2017 The Editorial Board of The Journal of Industrial Economics and John Wiley & Sons Ltd

that online retailers provide average prices similar to those of physical retailers. Moreover, online and physical retailers do not generally charge the same prices.

V. MEDIUM-RUN EQUILIBRIUM WHEN PHYSICAL RETAILERS MAY SHUT DOWN

This section describes the analysis of the medium-run equilibrium (Scenario MR) following the unexpected online entry in which physical retailers can shut down, yet cannot relocate, (i.e., $x_i^{MR} = x_i^{SR}$).[15] Denote $\pi_m(p_0, x) = \frac{(p_0+z-c_m)^2}{2k}(a - bx) - F$ as the monopolistic profit of retailer m located at x. Intuitively, those physical retailers with high marginal costs may shut down following the entry of the online retailer. Thus, the number of physical retailers falls in this situation. Precisely, the m^{SR}th retailer suffers a loss, since $p_0^{SR} + z < v$ implies $\pi_{m^{SR}}(p_0^{SR}, x_{m^{SR}}) < 0$. The equilibrium number of physical retailers m^{MR} is determined by $\pi_{m^{MR}}(p_0^{MR}, x_{m^{MR}}) = 0$, where the equilibrium price of the online retailer is

$$(6) \qquad p_0^{MR} = \frac{a^2 k}{6b\sum_{i=1}^{m^{MR}}(a - bx_i^{MR})} + \frac{1}{3}\bar{c}^{m^{MR}} + \frac{2}{3}c_0 - \frac{z}{3},$$

and where $\bar{c}^{m^{MR}} = \frac{\sum_{i=1}^{m^{MR}}(a-bx_i^{MR})c_i}{\sum_{i=1}^{m^{*MR}}(a-bx_i^{MR})}$ and the physical retailers' prices are $p_i^{MR} = \frac{p_0^{MR}+z+c_i}{2}$, $i = 1, 2, \ldots, m^{MR}$. Then, we have the following proposition.

Proposition 3. In the medium-run, the equilibrium number of physical retailers falls after the entry of the online retailer. Moreover, the prices are higher than the short-run prices when b is small: $p_0^{MR} > p_0^{SR}$ and $p_i^B > p_i^{MR} > p_i^{SR}$, $i = 1, 2, \ldots, m^{MR}$.

Proposition 3 states that those physical retailers with high costs will shut down upon the entry of the online retailer if $\pi_i(p_0^{SR}, x_i) < 0$. Those higher-cost retailers that suffer losses in Scenario SR will shut down in Scenario MR. These shutdowns have two opposing effects on the price of the online retailer. On the one hand, the price tends to rise in response to reduced competition among fewer retailers; on the other hand, the shutdown of high-cost retailers reduces the average cost of the remaining retailers, creating an incentive for the online retailer to lower its price. When b is small, the first effect dominates the second effect, and the equilibrium prices of all physical and online retailers increasingly exceed those in Scenario SR as the number of retailers falls. Notably, this condition on b cannot be implied by (3). With small b, $p_0^{MR} > p_0^{SR}$ is more likely. However, the retail prices of physical retailers are still lower than those in Scenario B.

[15] Relocation in practice takes a longer period than shutting down.

© 2017 The Editorial Board of The Journal of Industrial Economics and John Wiley & Sons Ltd

This finding can be applied to the aforementioned significant decrease in the number of physical bookstores following the entry of Amazon, and predicts that the medium-run prices will be higher than the short-run prices.

The above proposition is consistent with the significant impact of online business on traditional retail in recent years. For example, Amazon is a giant global retailer using the Internet medium to reach potential customers worldwide. The market entry of Amazon in 1995 led to a 34% decline in bookstores in the United States, from 13,403 at that point to 8,876 in 2012 (U.S. Census Bureau). Our theory is supported by empirical findings in Goldmanis *et al.* [2010] that diffusion of e-commerce affects the structure of retailing industry by lowering equilibrium price levels and reducing the number of producers, and forcing the exit of high-cost firms.

The following proposition states that the entry of the online retailer and the exit of a portion of the physical retailers (Scenario *MR*) are not to the benefit of all consumers.

Proposition 4. Following the entry of an online retailer, consumers may be either better off or worse off in the medium-run equilibrium than they were previously: (1) Consumers who are either served by an original physical retailer that does not shut down following the entry of the online retailer, or who were not previously served by an original physical retailer, are better off. (2) If consumers x are served by a physical retailer that shuts down following the entry of the online retailer, then they are worse off if (and only if) they are sufficiently close to their original retailer, such that $|x - x_i^B| < \frac{p_0^{MR}+z-p_i^B}{k}$ and $m^{MR} < i \leq m^B$.

The above result contrasts with the conventional wisdom that introducing a newly competitive entrant always benefits consumers. The first part of Proposition 4 states that some consumers benefit from the entry of the online retailer if they either are not served by any retailer, or are served by an original physical retailer that continues to operate. The former statement is obvious, while the latter arises from the price cut ($p_i^{MR} < p_i^B$) that follows the entry of the online retailer. The second part of Proposition 4 states that some consumers are made worse off by the shutdowns of their original nearby physical retailers after the entry of the online retailer, because they have no choice but to purchase from the online retailer or a remote physical retailer.[16]

VI. LONG-RUN EQUILIBRIUM

This section analyzes the long-run equilibrium in which physical retailers are free to exit and locate wherever they wish (Scenario LR). Similar calculations

[16] Recently, a once renowned physical bookstore, Borders, formally closed all its remaining stores on July 26, 2011. Many customers complained on blogs about the inconvenience of having to patronize other, more distant, bookstores.

© 2017 The Editorial Board of The Journal of Industrial Economics and John Wiley & Sons Ltd

to those for Section III demonstrate that the equilibrium locations in the long run are $x_i^{LR} = \frac{p_0^{LR}+z(2i-1)-\Delta i^2}{2k}$, $i = 1, 2, \ldots, m^{LR}$, and the equilibrium prices are $p_0^{LR} = \frac{a^2 k}{6b\sum_{i=1}^{m^{LR}}(a-bx_i^{LR})} + \frac{1}{3}\bar{c}^{m^{LR}} + \frac{2}{3}c_0 - \frac{z}{3}$, where $\bar{c}^{m^{LR}} = \frac{\sum_{i=1}^{m^{LR}}(a-bx_i^{LR})c_i}{\sum_{i=1}^{m^{LR}}(a-bx_i^{LR})}$ and $p_i^{LR} = \frac{p_0^{LR}+z+c_i}{2}$. The following proposition describes the configuration of the long-run equilibrium.

Proposition 5. In the long run, the physical retailers will contiguously serve the densely populated (urban) segments. The online retailer serves the remaining sparsely populated (rural) segments. Moreover, $p_i^{LR} < p_i^{MR}$ when b is small, $i = 1, 2, \ldots, m^{LR}$, and both p_i^{LR} and p_i^{MR} are less than p_i^B, and $x_i^{LR} < x_i^B$, $i = 1, \ldots, m^{LR}$, and $\partial p_0^{LR}/\partial c_0 < 0$, $\partial p_i^{LR}/\partial c_0 < 0$, $\partial x_i^{LR}/\partial c_0 > 0$. Therefore, physical retailers locate closer to each other and charge lower prices at long-run equilibrium than they would have charged in the absence of an online retailer.

Proposition 5 captures several assertions. First, all physical retailers move toward urban areas following the entry of an online retailer, with contiguous market segments; meanwhile, the online retailer serves the remaining rural segment.[17] This result is consistent with the evidence mentioned in footnote 1. Specifically, bookstores are nowadays mostly distributed in central urban areas and university campuses. Second, the long-run equilibrium prices are lower than the medium-run equilibrium prices if b is small. In fact, there are two opposite effects similar to those in Proposition 3. On the one hand, all physical retailers move toward urban areas, reducing the quantity of the online retailer. This decline arises from the fact that the online retailer no longer serves those high-demand areas. The price set by the online retailer falls, reducing the prices of all physical retailers. On the other hand, the equilibrium number of retailers can be either larger or smaller than that in Scenario MR. When b is small, the first effect dominates the second. Third, the inequality $x_i^{LR} < x_i^B$ implies that the quantities of the physical retailers will shrink owing to both location and price effects. The location effect refers to the fact that all physical retailers locate close to the most urbanized area. The price effect implies that the price of the online retailer falls and, correspondingly, the prices of the physical retailers decrease as well. The price of the online retailer falls because of a decline in demand, inducing more intense competition and ultimately reducing the profits of the physical retailers.

[17] This result is established by requiring that all consumers face the same unit transport cost, and this assumption is made in most location studies. However, if urban consumers face much higher transport costs (for example, owing to congestion, pollution, and high packing costs) than rural consumers, then some of the urban consumers may shop online, and some rural consumers shop offline. Formally, consider a different environment with two separated urban and rural segments, each of which is served by one physical retailer. A high transport cost in the urban segment results in partial coverage by the physical retailer, so some of the urban consumers who are far from any physical retailer will shop online. However, a low transport cost in the rural segment ensures full coverage by the physical retailer.

© 2017 The Editorial Board of The Journal of Industrial Economics and John Wiley & Sons Ltd

Fourth, unlike in Scenario B, the entry of the online retailer forces the physical retailers to move closer to each other and, in the long-run, toward the most urbanized area. Finally, the lower marginal cost of the online retailer causes the physical retailers to be less isolated and their markups to be lower.

If the urban (rural) area is interpreted as a mainstream (niche) market as in Bar-Isaac *et al.* [2012], the introduction of an online retailer induces physical retailers to become more mainstream, while the online retailer serves the niche interests. Notably, our discussion focuses on the competition effect of the Internet, following Bar-Isaac *et al.* [2012], who rely on the role of the Internet in reducing search costs.

Corollary 2. Consider consumers located in $(x^{LR}_{m^{LR}} + \frac{p_0^{LR}+z-c_{m^{LR}}}{2k}, x^{MR}_{m^{MR}})$ who were served by physical retailers that have moved to more densely populated areas without being replaced by any other physical retailer after the entry of an online retailer. These consumers are worse off if and only if they are sufficiently close to their original retailer, such that $|x - x_i^{MR}| < \frac{p_0^{LR}+z+p_i^{MR}}{k}$.

Corollary 2 states that some consumers are worse off after the entry of the online retailer. In particular, consumers who were located in $(x^{LR}_{m^{LR}} + \frac{p_0^{LR}+z-c_{m^{LR}}}{2k}, x^{MR}_{m^{MR}})$ and served by physical retailers in the long run suffer a loss because their formerly nearby physical retailers have moved to more densely populated areas, and no other physical retailer is available to fill the gap in the Scenario *LR*. These consumers have no choice but to purchase goods from the online retailer, and so are worse off. The following proposition compares the quantities of the online retailer and all physical retailers in the long-run.

Proposition 6. If the distaste cost for online purchases is low, the transportation cost is high, and the marginal cost of the online retailer is low, such that the market boundary of the physical retailers $m^{LR} \cdot \frac{p_0^{LR}+z+\frac{1}{2}(m^{LR}+1)\Delta}{k} < \frac{(2-\sqrt{2})a}{2b}$, then the quantity of the online retailer is larger than that of all physical retailers.

Intuitively, when z is small, k is high or c_0 is low, the online business has an advantage in competing with physical retailers, explaining why its quantity exceeds that of the physical retailers. In practice, the relative sizes of z, k, and c_0 vary among industries. Therefore, the quantities of online-related sales also vary across industries.

VII. INDIRECT TAXATION AND NUMBER OF RETAILERS

Until now, the role of indirect taxation has not been considered. The taxation of online commerce is still a contentious issue. In the early days of online commerce, the tax exemption for e-commerce was considered to support and nurture

© 2017 The Editorial Board of The Journal of Industrial Economics and John Wiley & Sons Ltd

a fledgling Internet industry. However, some policymakers assert that an equitable sales tax must be levied against such giant online retailers as Amazon. The current model provides a justification for the taxation of online businesses. For simplicity, let the per-unit sales tax be levied only on the producer side. Hence, the profit functions with taxes are $\pi_i' = (p_i - c_i - t_i)N_i$, $i = 0, 1, 2, \ldots, m$, where t_i represents the tax rate for retailer i. Presume that the policy maker who maximizes social welfare can choose a set of unit taxes t_i, but not the locations and prices. Under this new regime, prices and locations are still endogenously determined by retailers. The following proposition describes optimal taxation in the short run, medium run, and long run.

Proposition 7. Socially optimal taxation satisfies $t_i^J = p_0^J - z + i \cdot \Delta - 2c_0 + \frac{2}{3}t_0$, $J = SR, MR$, and LR. In other words, it requires that prices satisfy $p_i^J - i \cdot \Delta = p_0^J - c_0$, $i = 1, 2, \ldots, m^J$, $J = SR, MR$, and LR.

Proposition 7 reveals several implications of taxation in an economy that includes online businesses. First, a uniform tax fails to be socially optimal because physical retailers have heterogeneous costs. Second, optimal taxation should ensure that the price difference between any physical retailer and the online retailer equals their cost difference to maximize the efficiency of production, so it should also ensure that consumers are efficiently purchasing either from their nearby physical retailers or from the online retailer based on a social welfare aspect. Third, Proposition 7 provides a rationale for taxing online sales. A policy of tax-free online sales is generally not socially optimal, except when $t_i = p_0^{MR} - z + i \cdot \Delta - 2c_0$, $\forall\, i$. Restated, the socially optimal taxation may subsidize physical retailers with lower costs. In particular, when z and c_0 are low, the optimal tax levied on sales by the online retailer must be lower. Fourth, a higher tax rate should be imposed on a higher cost retailer to allocate production activities efficiently.[18,19] Notably, the condition in Proposition 7 is solved only for the differences between tax rates, because the purchase decisions of consumers depend only on the relative prices, as per equations (1) and (2). Therefore, this optimal taxation problem has an infinite number of solutions, as any tax pair that satisfies the condition in Proposition 7 is socially optimal.

[18] A sufficiently high tax on physical retailers may cause some retailers to shut down and the equilibrium number of physical retailers to decrease. However, the results in Proposition 7 are robust against changes in the number of physical retailers that are caused by taxation.

[19] We can derive the second-best solution of taxation when $b = 0$, in which the government imposes a unit tax on the online retailer and a uniform tax on all physical retailers. Analytical results suggest that the socially optimal tax on physical retailers must be higher than the tax on the online retailer when the distaste cost of online purchases is low, and the average cost of physical retailers is high. This is owing to the fact that, in this situation, the online retailer will set a higher price. Additionally, any price difference between the online and physical retailers will induce inefficient production. Therefore, the government should impose a higher tax on physical retailers to reduce the price difference in the market.

© 2017 The Editorial Board of The Journal of Industrial Economics and John Wiley & Sons Ltd

Consider a special case of unit taxation, in which the government cannot enact discriminatory taxation on physical retailers. Let $t_i = t_p$, $i = 1, 2, \cdots, m^{LR}$. Two cases with different levels of t_0 are compared: $t_0 = 0$ represents a policy of no tax on online sales, and $t_0 = t_p$ represents a policy of uniform taxation on all retail sales. Both tax policies have three effects on prices and locations. First, imposing a per unit tax on retailers is equivalent to increasing their marginal costs (cost effect). Second, the consequent decline in profits reduces the number of physical retailers (exit effect). Third, taxation may affect the locations of physical retailers (location effect).

If the exit effect is ignored, then the uniform tax policy will simply increase all equilibrium prices by the amount t_p without changing any locations, because p_0^{LR} is related to $\frac{1}{3}\bar{c}^{m^{LR}} + \frac{2}{3}c_0$ and $p_i^{LR} = \frac{p_0^{LR}+z+c_i}{2}$ before taxation, and the uniform taxation $t_i = t_p$, $i = 0, 1, \cdots, m^{LR}$ increases the term $\bar{c}^{m^{LR}}$ by an amount t_p and the term c_0 by an amount t_p, causing p_0^{LR} and p_i^{LR} to increase by an equal amount t_p so the locations of the physical retailers are unaffected. This result can be applied to the case of in-state online sellers, where sales tax is unavoidable for both online and physical purchases. Alternatively, a policy of zero tax on online sales will only increase p_0^{LR} by the amount $\frac{1}{3}t_p$ and p_i^{LR} by the amount $\frac{2}{3}t_p$, causing a cost effect and a location effect, as the physical retailers move toward the urban areas as a result of the tax advantage that is enjoyed by the online retailer. This finding suggests that sales tax induces out-of-state online purchases, consistent with Goolsbee [2000], Ballard and Lee [2007], and Einav *et al*. [2014], and also provides empirical implications on spatial configuration.

Given the equilibrium locations of retailers and prices, we further discuss the socially desirable number of firms. In the spirit of Salop [1979] and Madden and Pezzino [2011], the sum of transportation costs, production costs, and fixed entry costs is minimized. Considering the entry of the mth retailer, the additional total costs of all retailers are

$$\begin{aligned}\Phi(m) &= \int_{n_m^\ell}^{n_m^r} k|x - x_m| f(x)dx + (c_m - (c_0 + z)) \cdot \int_{n_m^\ell}^{n_m^r} f(x)dx - F \\ &= \frac{(a - bx_m)(c_m - c_0)(2z + c_0 - c_m)}{k} - F,\end{aligned}$$

where the first term is the transportation cost and the second term is the extra production cost when the customer switches from the online retailer to the mth physical retailer. Since $\Phi(m)$ is decreasing in m, the optimal number of physical retailers m° satisfies $\Phi(m^o) = 0$.

Intuitively, reducing the distaste cost for online purchase (z) or increasing the transportation cost (k) tends to reduce the socially desirable number of physical retailers. Comparing the long-run equilibrium with the socially optimum yields the following result.

© 2017 The Editorial Board of The Journal of Industrial Economics and John Wiley & Sons Ltd

Proposition 8. The socially desirable number of physical retailers is greater than that at the long-run equilibrium if $c_0 + \frac{2}{3}z > c_{m^{LR}}$.

The result in Proposition 8 can be compared with the literature of inefficient entry. Mankiw and Whinston [1986] found that excess entry appears under the case of a homogeneous good due to the fact that a new entrant steals business from incumbent firms, which may be reversed under the heterogeneous product case, because a new entrant provided additional variety to consumers. In the spatial case of a circular market *à la* Salop [1979], the business stealing effect dominates the variety effect and excess entry appears. In contrast, Madden and Pezzino [2011] studied a modified circular model with one firm located at the center of the circle and provided a reverse result of insufficient entry when the business stealing effect is not large.[20] In our framework, efficient entry is additionally affected by the cost-heterogeneity effect. If the marginal cost of the last entrant is low (high), relatively to that of the online retailer, the equilibrium number of physical retailers is insufficient (excessive), since some online purchases shift to the last entrant.

VIII. CONCLUSIONS

In the last two decades, physical retailers have encountered drastic competition from online business, which has revolutionized the retail industry. This work proposes a novel framework for analyzing the equilibrium prices, locations and number of firms, and the consumer welfare when an online retailer enters a linear market with areas of high demand (urban) and low demand (rural), and competes with physical retailers who have heterogeneous marginal costs. A unique pure strategy equilibrium is established in which physical retailers occupy non-overlapping and contiguous segments before the entry of the online retailer. This result contributes to the literature of pure sequential location games in providing a rationale with endogenous prices. Then, the chronology of the effects of an online retailer is demonstrated. In the short run, when all physical retailers are immutable and cannot shut down, the equilibrium prices fall and the online retailer serves some of the customers in market segments between those served by any two nearby physical retailers. In the medium run, when physical retailers may shut down but still cannot relocate, the reduction of competition among fewer retailers increases equilibrium prices. Some consumers are better off, while others are worse off. In the long run, when physical retailers are free to exit and can locate wherever they wish, all surviving physical retailers move

[20] Hsieh and Moretti [2003] derived a simple model and provide supported evidence to show that the low barriers to becoming a real estate agent result in wasteful entry in cities with high housing prices, due to the fact that real estate agents typically charge an equal percentage commission. Recently, Barwick and Pathak [2015] further employed agent heterogeneity and the nature of agent competition to estimate and find a significant social gain generated by cutting the commission rate in half.

© 2017 The Editorial Board of The Journal of Industrial Economics and John Wiley & Sons Ltd

toward urban areas, with the market segment of each connected to that of another, and the online retailer serves all remaining areas.

With respect to taxation, the policy of zero tax on online retailer sales is generally not socially optimal. The best taxation regime imposes a relatively low (high) sales tax on the online (physical) retailer(s) when the distaste cost and the marginal cost of the online retailer are relatively low. The socially desirable number of physical retailers exceeds that at the long-run equilibrium when the distaste cost and the marginal cost of the online retailer are sufficiently high.

APPENDIX

Before we proceed with the proof of Proposition 1, several lemmas are established as follows:

Lemma 1. Any geographical point cannot accommodate more than one physical retailer in an equilibrium.

Proof. Suppose there are two retailers j and j' located on the same point x, with $c_j < c_{j'}$, p_j must be equal to $p_{j'}$ since the retailer with a higher price will have zero quantity. However, both j and j' have incentives to set a slightly lower price to undercut the other until the prices are equal to the marginal cost $c_{j'}$. But this price cannot be sustained, since retailer j' has negative profits $(-F)$. Hence, any point cannot accommodate two retailers. A similar proof applies to the case with more than two retailers. ∎

Lemma 2. The rightmost retailer j_m sets a local monopoly price and chooses the location where its market segment is connected with that of the neighboring retailer when $b < \frac{4Fk^2}{3(v-c_1)^3}$. Moreover, this retailer will set the local monopoly price $p^*_{jm} = \frac{v+c_{jm}}{2}$.

Proof. Let j_m be the rightmost retailer, and its neighboring retailer is j_{m-1}. If the market segments of j_m and j_{m-1} are not connected, then retailer j_m has an incentive to move toward the left to occupy the unoccupied segment between them, because the left market segment is denser than the right one. Therefore, $n^l_{jm} \leq n^r_{j_{m-1}}$. Moreover, if $f(n^r_{jm}) > \frac{1}{2}f(n^l_{jm})$, firm j_m has no incentive to choose overlapping market segments with firm j_{m-1}. Therefore, $n^l_{jm} < n^r_{j_{m-1}}$ is excluded. Then, we have $n^l_{jm} = n^r_{j_{m-1}}$ implying this marginal consumer is indifferent among buying from retailer j_m, buying from retailer j_{m-1}, and not buying. For a given location x_{jm}, any profit cannot be higher than that under the monopoly price such that $\pi_{jm}(p_{jm}) \leq \pi_{jm}(p_{jm} = \frac{v+c_{jm}}{2})$ and from the condition of non-negative profits, we have $\pi_{jm}(p_m = \frac{v+c_{jm}}{2}) \geq 0$, which implies $x_{jm} \leq \frac{a(v-c_{jm})^2-2Fk}{b(v-c_{jm})^2}$. Substituting this upper bound of x_{jm} into the condition $f(n^r_{jm}) > \frac{1}{2}f(n^l_{jm})$ yields a sufficient condition $b < \frac{4Fk^2}{3(v-c_{jm})^3}$. Therefore, $b < \frac{4Fk^2}{3(v-c_1)^3}$ ensures the rightmost retailer chooses the location where its market segment is connected with that of the neighboring retailer.

Consequently, the optimal price for firm j_m can be determined as follows. Consider the most profitable unilateral deviation $x'_{jm} = x_{jm} + \varepsilon$ and $p'_{jm} = p_{jm} - k\varepsilon$ so that the marginal consumer $\hat{x}_{j_{m-1}}$ is unchanged. It should be noted that ε is infinitesimally small so that we

© 2017 The Editorial Board of The Journal of Industrial Economics and John Wiley & Sons Ltd

do not need to worry about second-order effects of this deviation. The profit of retailer j_m along this deviation becomes $\pi_{jm}(\varepsilon) = \int_{\hat{x}_{jm}}^{x_{jm}+2\varepsilon+\frac{v-p_{jm}}{k}} f(x)dx \times (p_{jm} - k\varepsilon - c_{jm}) - F$. Differentiating $\pi_j(\varepsilon)$ with respect to ε at $\varepsilon = 0$ yields the first-order condition

$$\pi_j'(\varepsilon)|_{\varepsilon=0}$$

$$= \frac{-2b}{k}\left(p_{jm}^2 - \left(v + c_{jm} - 2k\left(\frac{a}{b} - x_{jm}\right)\right) \cdot p_{jm} + vc_{jm}\right.$$

$$\left. - k(v + c_{jm})\left(\frac{a}{b} - x_{jm}\right)\right) < 0,$$

$$\forall \quad p_{jm} < \frac{v + c_{jm}}{2}. \tag{A−1}$$

which yields the local monopoly price $p_{jm}^* = \frac{v+c_{jm}}{2}$. ■

Lemma 3. For any interior retailers, the left market segment is always contiguous to its left neighboring retailer, and the right market segment is never overlapping with that of its right neighboring retailer.

Proof. Obviously, those interior retailers have incentives to move toward the left to occupy denser market segments if the left market segment is not contiguous. Therefore, the left market segment of retailer i should be contiguous with that of its left neighboring retailer $i-1$. Moreover, if the right market segment is overlapping with that of its right neighboring retailer $i+1$ ($n_i^r > n_{i+1}^l$), then it is profitable for this interior retailer i to move toward the left, since the increased left-sided market is denser than the lost right-side market. ■

Lemma 4. Each retailer j_i, except the leftmost two retailers, sets an equilibrium price $p_{j_i}^* = \frac{v+c_{j_i}}{2}$.

Proof. From Lemmas 1 and 3, the market segments of these retailers are never overlapping with those of other retailers. If $p_{j_i} < \frac{v+c_{j_i}}{2}$, then it is clear that retailer j_i should increase its profit by raising p_{j_i} without relocation, since raising prices will not result in overlapping segments. Further from the assumptions $p_{j_i} \leq \frac{v+c_{j_i}}{2}$, we have proved that $p_{j_i}^* = \frac{v+c_{j_i}}{2}$ for all retailers except the leftmost two retailers. ■

Lemma 5. Each retailer j_i, except the leftmost two retailers, always sets lower prices than its right neighboring retailer j_{i+1}. That is, $p_{j_i} < p_{j_{i+1}}$, $i = 1, 2, \ldots, m-1$.

Proof. If $p_{j_i} \geq p_{j_{i+1}}$ for two neighboring retailers, then retailer j_{i+1} can undercut retailer j_i with a price $p'_{j_{i+1}} = p_{j_i} - \varepsilon$ or $p'_{j_{t+1}} = p_{j_{i+1}} - \varepsilon$, where ε is infinitesimally small, to take a denser market and raise its profit. ■

Lemma 6. All low-cost retailers, except the leftmost two retailers, occupy denser market segments rather than high-cost retailers. That is, $x_{j_i} < x_{j_{i+1}}$ iff $c_{j_i} < c_{j_{i+1}}$, $i = 3, 4, \ldots, m-1$.

© 2017 The Editorial Board of The Journal of Industrial Economics and John Wiley & Sons Ltd

Proof. If $c_{j_i} > c_{j_{i+1}}$ and j_{i+1} is the right neighboring retailer of j_i, then from Lemma 4, $p_{j_i} = \frac{v+c_{j_i}}{2} > p_{j_{i+1}} = \frac{v+c_{j_{i+1}}}{2}$, and it is profitable for retailer j_{i+1} to relocate to a position with denser population and undercut retailer j_i by keeping a local monopoly price $p'_{j_{i+1}} = p_{j_{i+1}}$. Therefore, low-cost retailers occupy denser market segments than those of high-cost retailers. ■

Lemma 7. The left boundary of the leftmost retailer j_1 must be $x=0$, and the equilibrium price $p^*_{j_1}$ and location $x^*_{j_1}$ of the leftmost retailer satisfies $p^*_{j_1} + kx^*_{j_1} = v$.

Proof. If the leftmost retailer locates at $x'_{j_1} = x^*_{j_1} - \varepsilon$ and sets a price $p'_{j_1} = v - k(x^*_{j_1} - \varepsilon)$, then it can raise its price to $p''_{j_1} = v - kx'_{j_1}$ and keep the same market segment. Therefore, its profit is increased. If the leftmost retailer locates at $x'_{j_1} = x^*_{j_1} + \varepsilon$ with a higher price such that $p'_{j_1} = p^*_{j_1} + k\varepsilon$, then there appears an unoccupied segment around $x=0$. Therefore, the leftmost retailer can increase its profit by moving toward the left to occupy this high-density market. ■

Lemma 8. If the market segments of the leftmost two retailers j_1 and j_2 are overlapping, then $f(\hat{x}_{j_1 j_2}) = \frac{kN_{j_1}}{p_{j_1}-c_{j_1}} = \frac{kN_{j_2}}{p_{j_2}-c_{j_2}}$, $c_{j_1} < c_{j_2}$, $p^*_{j_1} < p^*_{j_2}$, and $p^*_{j_1} - c_{j_1} > p^*_{j_2} - c_{j_2}$.

Proof. Plugging $x'_{j_2} = x_{j_2} + \varepsilon$, $p'_{j_2} = p_{j_2} + k\varepsilon$ into the profit function of retailer j_2 yields

$$\pi'_{j_2}(\varepsilon) = \int_{\frac{-p_{j_1}+(p_{j_2}+k\varepsilon)+k(x_{j_2}+\varepsilon+x_{j_1})}{2k}}^{x_{j_2}+\frac{v+p_{j_2}}{k}} f(x)dx \cdot (p_{j_2} + k\varepsilon - c_{j_2}) - F.$$

Differentiating $\pi'_{j_2}(\varepsilon)$ with respect to ε at $\varepsilon = 0$ yields

$$\begin{aligned}\left.\frac{d\pi'_{j_2}(\varepsilon)}{d\varepsilon}\right|_{\varepsilon=0} &= -f(\hat{x}_{j_1 j_2})(p_{j_2} - c_{j_2}) + \int_{\hat{x}_{j_1 j_2}}^{x_{j_2}+\frac{v+p_{j_2}}{k}} kf(x)dx \\ &= -f(\hat{x}_{j_1 j_2})(p_{j_2} - c_{j_2}) + kN_{j_2} = 0.\end{aligned}$$

Therefore, $p_{j_2} - c_{j_2} = \frac{kN_{j_2}}{f(\hat{x}_{j_1 j_2})}$. The profit function of retailer j_1 is

$$\pi_{j_1} = \int_0^{\frac{-p_{j_1}+p_{j_2}+k(x_{j_1}+x_{j_2})}{2k}} f(x)(p_{j_1} - c_{j_1})dx - F.$$

When retailer j_1 moves toward the left to $x'_{j_1} = x_{j_1} - \varepsilon$ and sets a higher price $p'_{j_1} = p_{j_1} + k\varepsilon$, its profit function is

$$\pi'_{j_1}(\varepsilon) = \int_0^{\frac{-p_{j_1}+p_{j_2}+k(x_{j_1}+x_{j_2})}{2k}-\varepsilon} f(x) \cdot (p_{j_1} + k\varepsilon - c_{j_1})dx - F.$$

© 2017 The Editorial Board of The Journal of Industrial Economics and John Wiley & Sons Ltd

Differentiating $\pi'_{j_1}(\varepsilon)$ with respect to ε at $\varepsilon = 0$ yields

$$\left.\frac{\partial \pi'_{j_1}(\varepsilon)}{\partial \varepsilon}\right|_{\varepsilon=0} = -f(\hat{x}_{j_1j_2})(p_{j_1} - c_{j_1}) + kN_{j_1} = 0.$$

Therefore, the indifferent consumer $\hat{x}_{j_1j_2}$ between retailer j_1 and retailer j_2 satisfies

$$\text{(A-2)} \qquad f(\hat{x}_{j_1j_2}) = \frac{kN_{j_1}}{p_{j_1} - c_{j_1}} = \frac{kN_{j_2}}{p_{j_2} - c_{j_2}}.$$

Now we prove $c_{j_1} < c_{j_2}$ by a contradiction. If $c_{j_1} > c_{j_2}$ and $p_{j_1} \geq p_{j_2}$, retailer j_2 will undercut retailer j_1 by relocation and keeping the same price p_{j_2} to obtain a greater market and profits. If $c_{j_1} > c_{j_2}$ and $p_{j_1} < p_{j_2}$, then $N_{j_1} > N_{j_2}$, because j_1 set a lower price in a denser segment. Hence, by (A-2) we have $p_{j_1} - c_{j_1} > p_{j_2} - c_{j_2}$, a contradiction. We can show that $N_{j_1} > N_{j_2}$ in equilibrium. If $N_{j_1} \leq N_{j_2}$, from (A-2) we have $p_{j_1} - c_{j_1} \leq p_{j_2} - c_{j_2}$, leading to $p_{j_1} < p_{j_2}$. Then, j_1 will undercut j_2 to obtain a greater market, a contradiction. Thus, we have $p_{j_1} - c_{j_1} > p_{j_2} - c_{j_2}$ in equilibrium by (A-2). ■

Lemma 9. The market segments of the leftmost two retailers j_1 and j_2 can never be overlapping.

Proof. From Lemma 8, we have $f(\hat{x}_{j_1j_2}) = \frac{kN_1}{p_{j_1}-c_{j_1}} = \frac{kN_2}{p_{j_2}-c_{j_2}}$. Rearranging this condition yields

$$f(\hat{x}_{j_1j_2}) \cdot (p_{j_1} - c_{j_1}) = k\int_0^{\hat{x}_{j_1j_2}} f(x)dx.$$

By intermediate value theorem, there exists a $\hat{\hat{x}}$, $0 \leq \hat{\hat{x}} < \hat{x}_{j_1j_2}$, such that $\int_0^{\hat{x}_{j_1j_2}} f(x)dx = f(\hat{\hat{x}}) \cdot \hat{x}$. Hence, we have $f(\hat{x}_{j_1j_2})(p_{j_1} - c_{j_1}) = f(\hat{\hat{x}})k\hat{x}_{j_1j_2}$, which can be rearranged as

$$\frac{f(\hat{x}_{j_1j_2})}{f(\hat{\hat{x}})}(p_{j_1} - c_{j_1}) = k\hat{x}_{j_1j_2}.$$

Since $f(\hat{x}_{j_1j_2})/f(\hat{\hat{x}}) < 1$, and $p_{j_1} \leq \frac{v_{j_1}+c_{j_1}}{2}$ by assumption, we then have $k\hat{x} < p_{j_1} - c_{j_1} \leq \frac{v+c_{j_1}}{2} - c_{j_1} = \frac{v-c_{j_1}}{2}$, which implies $\hat{x}_{j_1j_2} < \frac{v-c_{j_1}}{2k}$.

Moreover, from Lemma 5 $p^*_{j_1} + kx^*_{j_1} = v$, we have $x_{j_1} = \frac{v-p_{j_1}}{k}$, which is not less than $\frac{v-(\frac{v+c_{j_1}}{2})}{k}$, because of the assumption $p_{j_1} \leq \frac{v+c_{j_1}}{2}$. Therefore, we have $x_{j_1} \geq \frac{v-c_{j_1}}{2k}$, which contradicts $\hat{x}_{j_1j_2} < \frac{v-c_{j_1}}{2k}$, since $x_{j_1} < \hat{x}_{j_1j_2}$. Therefore, the leftmost two firms can never be overlapping. ■

Lemma 10. The leftmost two retailers j_1 and j_2 will set local monopoly prices $p_{j_1} = \frac{v+c_{j_1}}{2}$ and $p_{j_2} = \frac{v+c_{j_2}}{2}$. Furthermore, they have a cost advantage over other retailers. That is, $c_{j_1} < c_{j_2} < c_{j_3}$, where j_3 is the third retailer from the left.

© 2017 The Editorial Board of The Journal of Industrial Economics and John Wiley & Sons Ltd

Proof. From Lemma 9, these two retailers are never overlapping with other retailers. If $p_{j_1} < \frac{v+c_{j_1}}{2}$, $i=1$ or 2, then the retailers can profitably raise their prices without relocations. Furthermore, $c_{j_1} < c_{j_2} < c_{j_3}$ by the same argument in Lemma 6. ■

Proof of Proposition 1. For a given location, the profit for any physical retailer can never be higher than the profit of being a local monopolist. Lemmas 1-10 ensure that each physical retailer is a local monopolist, which maximizes its profit by setting a price such that $p_i^* = \frac{v+i\cdot\Delta}{2}$. Then, $\pi_i = \frac{(v-c_i)^2}{2k}(a-bx_i)-F$, which is decreasing in x. Therefore, each physical retailer is motivated to move toward the left (i.e., the highly urbanized area). Lemma 5 and Lemma 6 ensure that the lowest-cost retailer is the leftmost retailer, followed by the second lowest-cost retailer, and the order continues in that manner. Starting from retailer 1, the leftmost location is determined by $n_1^\ell = 0$, implying that $x_1^* = \frac{v-c_1}{2k}$. Consequently, retailer 2 can only move to $n_1^r = n_2^\ell$ such that $x_2^* = \frac{v-c_1}{k} + \frac{v-c_2}{2k}$ to enjoy a local monopoly position. By induction, the rightmost retailer m is located at $x_m^* = \left(\sum_{i=1}^m \frac{v-c_i}{k}\right) + \frac{v-c_m}{2k} = \frac{v(2i-1)-\Delta i^2}{2k}$. To avoid undercutting, the assumption of (3) ensures that b is sufficiently small to guarantee the equilibrium described in Proposition 1. The condition that retailer i cannot benefit from moving to the location of retailer j and set the same price as retailer j, $j<i$ is $(p_i - c_i)N_i \geq (p_j - c_i)N_j$, implying that

$$\text{(A}-3) \qquad b < \frac{a(c_i - c_j)^2}{x_i(v-c_i)^2 - x_j\left((v-c_i)^2 - (c_i-c_j)^2\right)}, \qquad \text{for all} \quad j < i,$$

which is exactly the assumption (3). Since the RHS of (A-3) is positive, a sufficiently small b guarantees the equilibrium. Since m^B satisfies $\pi_m = \frac{(v-m\Delta)^2}{2k}(a - b\frac{v(2m-1)-\Delta m^2}{2k}) - F = 0$, comparative statics are implied by the differentiation on the implicit function $\pi_m = 0$ and $\partial\pi_m/\partial m < 0$, $\partial\pi_m/\partial b < 0$, $\partial\pi_m/\partial a > 0$, $\partial\pi_m/\partial F < 0$, and $\partial\pi_m/\partial k < 0$. ■

Proof of Proposition 2. The profit of the online retailer is

$$\begin{aligned}
\pi_0 &= (p_0 - c_0)\cdot N_0 - F_0 \\
&= (p_0 - c_0)\left\{\frac{a^2}{2b} - \frac{2mp_0a}{k} - \frac{2mza}{k}\right. \\
&\quad \left. - \left[\sum_{i=1}^m \left(\frac{2p_ibx_i}{k} - \frac{2p_0bx_i}{k} - \frac{2zbx_i}{k} - \frac{2p_ia}{k}\right)\right]\right\} - F_0.
\end{aligned}$$

Solving $\partial\pi_0/\partial p_0 = 0$ and using $p_i = \frac{p_0+z+c_i}{2}$, $i = 1, 2, \ldots, m$ yields the equilibrium prices in Proposition 2. From (2), we have $u_0 > 0$ and so $p_0^{SR} + z < v$. Thus, $p_i^{SR} < p_i^B$, $\forall i = 1, 2, \ldots, m^B$. Moreover, the second-order condition is satisfied, since $\frac{\partial^2\pi_0}{\partial p_0^2} = \frac{-4}{k}\left(ma - \sum_{i=1}^m bx_i\right) < 0$. The comparative statics are easily shown. According to (5), $\partial p_0^{SR}/\partial c_0 > 0$, $\partial p_0^{SR}/\partial k > 0$, and $\partial p_0^{SR}/\partial z < 0$, $\frac{\partial p_0^{SR}}{\partial c_i} = \frac{\left(a-bx_i^{SR}\right)}{3\left[m^{SR}a - b\sum_{i=1}^{m^{SR}} x_i^{SR}\right]} > 0$, $\partial p_i^{SR}/\partial c_0 > 0$, $\partial p_i^{SR}/\partial k > 0$, $\frac{\partial p_i^{SR}}{\partial c_i} = \frac{1}{2}\frac{\partial p_0^{SR}}{\partial c_0} + \frac{1}{2} > 0$, and $\frac{\partial p_i^{SR}}{\partial z} = \frac{1}{2}\frac{\partial p_0^{SR}}{\partial z} + \frac{1}{2} = -\frac{1}{3} + \frac{1}{2} > 0$. ■

© 2017 The Editorial Board of The Journal of Industrial Economics and John Wiley & Sons Ltd

Proof of Corollary 1. Since

$$p_0^{SR} - p_i^{SR} = \frac{p_0^{SR} - z - c_i}{2} > 0 \quad \text{iff} \quad p_0^{SR} > z + c_i,$$

we have the following condition:

$$z - \frac{c_0}{2} < \frac{a^2 k}{8b\left[\sum_{i=1}^{m^B}(a - bx_i^{SR})\right]} - \frac{1}{4}\bar{c}^m - \frac{3}{4}c_i.$$

■

Proof of Proposition 3. Since $\pi_i^{SR} < \pi_j^{SR}$, while $i>j$, $i,j = 1,2,\ldots,m^{SR}$, at least the physical retailer with the highest marginal cost shuts down because $\pi_{m^{SR}} < 0$. Then we can find an equilibrium number of physical retailers m^{MR} such that $\pi_{m^{MR}}^{MR} = 0$ and $m^{MR} < m^{SR}$. The equilibrium prices p_0^{MR} and p_i^{MR}, $i = 1,2,\ldots,m^{MR}$ can be derived by a proof similar to the one for Proposition 2. Moreover, $p_0^{MR} - p_0^{SR} = \frac{a^2 k}{6b}\left(\frac{1}{\sum_{i=1}^{m^{MR}}(a-bx_i^{MR})} - \frac{1}{\sum_{i=1}^{m^{SR}}(a-bx_i^{SR})}\right) + \frac{1}{3}\left(\bar{c}^{m^{MR}} - \bar{c}^{m^{SR}}\right) > 0$ when b is small. Since $m^{MR} < m^{SR}$, $\bar{c}^{m^{MR}} - \bar{c}^{m^{SR}} < 0$, because the weighted average of marginal cost is smaller than that before the entry of the online retailer, due to the shutdown of the highest cost physical retailer, subsequently leading to a decline in the new average marginal costs. Finally, $p_0^B > p_0^{MR}$, since $v > p_0^{MR} + z$. ■

Proof of Proposition 4.

(1) Those consumers are better off, because $p_i^B < p_i^{SR} < p_i^{MR}$ from Corollary 1 and Proposition 3, implying that $p_i^B + k|x - x_i^B| > \min\left\{p_i^{MR} + k|x - x_i^{MR}|, p_0^{MR} + z\right\}$, $i = 1,2,\ldots,m^{MR}$.
(2) Those consumers x who are originally served by a physical retailer i ($i > m^{MR}$), which shuts down after the entry of the online retailer ($m^B \geq i > m^{MR}$), are worse off if and only if $p_i^B + k|x - x_i^B| < p_0^{MR} + z$, implying that $|x - x_i^{MR}| < \frac{p_0^{MR}+z-p_i^B}{k}$, which is positive since $p_i^B < p_i^{SR} < p_0^{SR} + z < p_0^{MR} + z$. ■

Proof of Proposition 5. When physical retailers are free to move to another location, each one has an incentive to move closer to highly populated areas. This is owing to the fact that in the medium-run equilibrium, their markets are connected by the market of the online retailer. Thus, the equilibrium locations and prices are derived similarly to Proposition 1, with $x_i^{LR} < x_i^B$. Replacing v with $p_0^{LR} + z$ yields

$$x_i^{LR} = \sum_{j=2}^{i}\left(\frac{p_0^{LR} + z - c_j}{k}\right) + \frac{p_0^{LR} + z - c_1}{2k} = \frac{p_0^{LR} + z(2i-1) - \Delta i^2}{2k},$$
$$i = 1,2,\ldots,m^{LR},$$

and $p_i^{LR} = \frac{p_0^{LR}+z-c_i}{2}$, $i = 1,\ldots,m^{LR}$. When the online retailer raises its price, the profit of the online retailer is

$$\pi_0^{+} = (p_0 - c_0)N_0 - F_0$$

© 2017 The Editorial Board of The Journal of Industrial Economics and John Wiley & Sons Ltd

$$= (p_0 - c_0) \int_{x_{m^{LR}} + \frac{p_0 + z - p_i}{k}}^{\frac{a}{b}} (a - bx)dx - F.$$

Next, when the online retailer considers a lower p_0, the profit function is

$$\pi_0^- = (p_0 - c_0)\left\{ \frac{a^2}{2b} - \frac{2mp_0 a}{k} - \frac{2mza}{k} \right.$$
$$\left. - \left[\sum_{i=1}^{m} \left(\frac{2p_i bx_i}{k} - \frac{2p_0 bx_i}{k} - \frac{2zbx_i}{k} - \frac{2p_i a}{k} \right) \right] \right\} - F_0.$$

Obviously, $\partial\pi_0^+/\partial p_0 < \partial\pi_0^-/\partial p_0$, because the online retailer can reduce its price to take a proportion of the market segments from each physical retailer. Using $\partial\pi_0^-/\partial p_0 = 0$ and $p_i = \frac{p_0+z-c_i}{2}$, $i = 1, 2, \ldots, m^{LR}$ yields the equilibrium prices in Proposition 5. Finally, $p_i^{LR} < p_i^{MR}$ is shown by contradiction. If $p_i^{LR} > p_i^{MR}$, then $\pi_i^{LR} > \pi_i^{MR}$ and so $m_i^{LR} \geq m_i^{MR}$. This subsequently leads to

$$p_0^{LR} - p_0^{MR}$$
$$= \frac{a^2 k}{6b} \left(\frac{1}{\sum_{i=1}^{m^{LR}} (a - bx_i^{LR})} - \frac{1}{\sum_{i=1}^{m^{MR}} (a - bx_i^{MR})} \right)$$
$$- \frac{1}{3} \left(\bar{c}^{m^{LR}} - \bar{c}^{m^{MR}} \right) > 0, \text{ if } b \text{ is small}.$$

Since the first term is negative, while the second term is not far from zero, $p_0^{LR} < p_0^{MR}$ when b is small, a contradiction. ■

Proof of Corollary 2. Consumers located on $x \in (x_{m^{LR}}^{LR} + \frac{p_0^{LR}+z-c_{m^{LR}}}{2k}, x_{m^{MR}}^{MR})$ are originally served by some physical retailers, and then are served by the online retailer. They are worse off if and only if $p_i^{MR} + k|x - x_i^{MR}| < p_0^{LR} + z$, leading to conclusion of the corollary. ■

Proof of Proposition 6. According to Proposition 5, the market boundary of physical retailers is $\bar{x} = \sum_{i=1}^{m^{LR}} (\frac{p_0^{LR}+z-c}{k}) = m^{LR} \frac{p_0^{LR}+z+\frac{1}{2}(m^{LR}+1)\Delta}{k}$. The total quantities of physical retailers are

$$\sum_{i=1}^{m^{LR}} N_i = \frac{1}{2}(a + a - b\bar{x})\,\bar{x},$$

while the quantity of the online retailer is

$$N_0 = \frac{1}{2}aL - \sum_{i=1}^{m^{LR}} N_i = \frac{a^2}{2b} - \frac{(2a - b\bar{x})\bar{x}}{2}.$$

© 2017 The Editorial Board of The Journal of Industrial Economics and John Wiley & Sons Ltd

Therefore,

$$N_0 - \sum_{i=1}^{m^{LR}} N_i = \frac{a^2 - 4ab\bar{x} + 2b^2\bar{x}^2}{2b} > 0, \quad \text{iff} \quad \bar{x} < \frac{(2-\sqrt{2})a}{2b},$$

which is satisfied when z is low, k is high, and c_0 is low. ■

Proof of Proposition 7. With unit taxation, the equilibrium prices are

$$p_0^J = \frac{a^2 k}{6b\sum_{i=1}^{m^J}\left(a - bx_i^j\right)} - \frac{1}{3}\bar{c}^{m^j} + \frac{2}{3}(c_0 + t_0) - \frac{z}{3} = p_0^J + \frac{2}{3}t_0,$$
$$j = SR, MR, LR,$$

and $p_i^J = \frac{p_0^J + z + i\cdot\Delta + t_i}{2}$. The Pareto optimality requires prices satisfying

$$p_i^J + k|x - x_i^J| < p_0 + z \quad \text{iff} \quad i\cdot\Delta + k|x - x_i| < c_0 + z, \quad \forall x \in [0, L],$$

which implies $z - k|x - x_i^J| < p_i^J - p_0^J$ iff $z - k|x - x_i^J| < i\cdot\Delta - c_0$ for all x. Therefore, if $p_i - p_0 < c_i - c_0$, then there exists an x such that $p_i - p_0 < z - k|x - x_i| < c_i - c_0$ violating this inequality. Similarly, $p_i^J - p_0^J > i\cdot\Delta - c_0$ also leads to a contradiction. Hence, $p_i^J - p_0 = i\cdot\Delta - c_0$ is a necessary condition for Pareto optimum. Plugging p_0 and p_i into this condition yields

$$\frac{p_0^J + \frac{2}{3}t_0 + z + i\cdot\Delta + t_i^J}{2} - p_0^J - \frac{2}{3}t_0 = i\cdot\Delta_j - c_0,$$

and so $t_i^J - \frac{2}{3}t_0 = p_0^J - z + i\cdot\Delta - 2c_0$. ■

Proof of Proposition 8. In the long-run equilibrium, if the profit of the m^{LR}th retailer $\pi_M = \frac{(p_0^{LR} + z - m^{LR}\Delta)^2 (a - bx_{m^{LR}}^{LR})}{2k} - F = 0$, which implies $F > \frac{(p_0 + z - m^{LR}\Delta)^2 (a - bx_{m^{LR}}^{LR})}{2k}$ and therefore, $\phi(m^{LR}) < \frac{a - bx_{m^{LR}}^{LR}}{k}[(m^{LR}\Delta - c_0)(2z + c_0 - m^{LR}\Delta) - \frac{(p_0 + z - m^{LR}\Delta)^2}{2}] < \frac{-(3c_0 + 2z - 3m^{LR}\Delta)(2z + c_0 - m^{LR}\Delta)}{2} < 0$ under the condition $c_0 + \frac{2}{3}z > c_{m^{LR}}$, since $p_0 > c_0 + z$. Hence, the socially desirable number of physical retailers is greater than that in the long-run equilibrium. ■

REFERENCES

Ahmed, S. and Wirjanto, T.S., 2008, 'The Impact of Sales Taxation on Internet Commerce – An Empirical Analysis,' *Economic Letters*, 99, pp. 557–560.

Anderson, S.P.; de Palma, A. and Hong, G. S., 1992, 'Firm Mobility and Location Equilibrium,' *Canadian Journal Economics*, 25, pp. 76–88.

Anderson, S.P.; Goeree, J.K. and Ramer, R., 1997, 'Location, Location, Location,' *Journal of Economic Theory*, 77, pp. 102–127.

d'Aspremont, C.; Gabszewicz, J.J. and Thisse, J.F., 1979, 'On Hotelling's "Stability in Competition",' *Econometrica*, 47, pp. 1145–1150.

© 2017 The Editorial Board of The Journal of Industrial Economics and John Wiley & Sons Ltd

Balasubramanian, S., 1998, ‘Mail versus Mall: A Strategic Analysis of Competition Between Direct Marketers and Conventional Retailers,’ *Marketing Science*, 17, pp. 181–195.

Ballard, C.L. and Lee, J., 2007, ‘Internet Purchases, Cross-Border Shopping, and Sales Taxes,’ *National Tax Journal*, 60, pp. 711–725.

Bar-Isaac, H.; Caruana, G. and Cuñat, V., 2012, ‘Search, Design and Market Structure,’ *The American Economic Review*, 102, pp. 1140–1160.

Barwick, P.J. and Pathak, P.A., 2015, ‘The Costs of Free Entry: An Empirical Study of Real Estate Agents in Greater Boston,’ *The RAND Journal of Economics*, 46, pp. 103–145.

Brown, J.R. and Goolsbee, E.A., 2002, ‘Does the Internet Make Markets More Competitive? Evidence from the Life Insurance Industry,’ *Journal Political Economy*, 110, pp. 481–507.

Brynjolfsson, E. and Smith, M.D., 2000, ‘Frictionless Commerce? A Comparison of Internet and Conventional Retailers,’ *Management Science*, 46, pp. 563–585.

Campbell, J. and Sandhu, S., 2015, ‘Campaigners Condemn Rural Bank Closures’ (*The Independent*, Jan. 3rd, London, England).

Chevalier, J. and Goolsbee, A., 2003, ‘Measuring Prices and Price Competition Online: Amazon.com and BarnesandNoble.com,’ *Quantitative Marketing and Economics*, 1, pp. 203–222.

Clay, K.; Krishnan, R.; Wolff, E. and Fernandes, D., 2002, ‘Retail Strategies on the Web: Price and Non-price Competition in the Online Book Industry,’ *The Journal of Industrial Economics*, 50, pp. 351–367.

Economides, N., 1984, ‘The Principle of Minimum Differentiation Revisited,’ *European Economic Review*, 24, pp. 345–368.

Ellison, G. and Ellison, S.F., 2009, ‘Search, Obfuscation, and Price Elasticities on the Internet,’ *Econometrica*, 77, pp. 427–452.

Einav, L.; Knoepfle, D.; Levin, J. and Sundaresan, N., 2014, ‘Sales Taxes and Internet Commerce,’ *American Economic Review*, 104, pp. 1–26.

Foncel, J.; Guyot, M. and Jouneau-Sion, F., 2011, ‘The Shop around the Corner in the Internet Age,’ *Recherches écon de Louvain*, 77, pp. 47–85.

Gandhi, A.; Froeb, L.; Tschantz, S. and Werden, G.J., 2008, ‘Post-Merger Product Repositioning,’ *The Journal of Industrial Economics*, 56, pp. 49–67.

Goldmanis, M.; Hortaçsu, A.; Syverson, C. and Emre, Ö., 2010, ‘E-Commerce and the Market Structure of Retail Industries,’ *The Economic Journal*, 120, pp. 651–682.

Goolsbee, A., 2000, ‘In a World without Borders: The Impact of Taxes on Internet Commerce,’ *The Quarterly Journal of Economics*, 115, pp. 561–576.

Goolsbee, A., 2001, ‘Competition in the Computer Industry: Online Versus Retail,’ *The Journal of Industrial Economics*, 49, pp. 487–499.

Hotelling, H., 1929, ‘Stability in Competition,’ *Economic Journal*, 39, pp. 41–57.

Hsieh, C.T. and Moretti, E., 2003, ‘Can Free Entry Be Inefficient? Fixed Commissions and Social Waste in the Real Estate Industry,’ *Journal of Political Economy*, 111, pp. 1076–1122.

Jin, G.Z. and Kato, A., 2007, ‘Dividing Online and Offline: A Case Study,’ *The Review of Economic Studies*, 74, pp. 981–1004.

Jopson, B., 2012, ‘Amazon Finds Upside to Sales Tax Payment,’ *Financial Times*, July 10.

Lerner, A.P. and Singer, H.W., 1937, ‘Some Notes on Duopoly and Spatial Competition,’ *The Journal of Political Economy*, 45, pp. 145–186.

Liu, Y.; Gupta, S. and Zhang, Z.J., 2006, ‘Note on Self-Restraint as an Online Entry-Deterrence Strategy,’ *Management Science*, 52, pp. 1799–1809.

Loertscher, S. and Muehlheusser, G., 2011, ‘Sequential Location Games,’ *The RAND Journal of Economics*, 42, pp. 639–663.

Loginova, O., 2009, ‘Real and Virtual Competition,’ *The Journal of Industrial Economics*, 57, pp. 319–342.

Madden, P. and Pezzino, M., 2011, ‘Oligopoly on a Salop Circle with Centre,’ *The B.E. Journal of Economic Analysis & Policy*, 11, Article 2.

© 2017 The Editorial Board of The Journal of Industrial Economics and John Wiley & Sons Ltd

Mankiw, N.G. and Whinston, M.D., 1986, 'Free Entry and Social Inefficiency,' *The RAND Journal of Economics*, 17, pp. 48–58.

Neven, D., 1986, 'On Hotelling's Competition with Non-Uniform Customer Distributions,' *Economics Letters*, 21, pp. 121–126.

Osborne, M.J. and Pitchik, C., 1987, 'Equilibrium in Hotelling's Model of Spatial Competition,' *Econometrica*, 55, pp. 911–922.

Ofek, E.; Katona, Z. and Sarvary, M., 2011, "'Bricks and Clicks": The Impact of Product Returns on the Strategies of Multichannel Retailers,' *Marketing Science*, 30, pp. 42–60.

Prince, J.T., 2007, 'The Beginning of Online/Retail Competition and Its Origins: An Application to Personal Computers,' *International Journal of Industrial Organization*, 25, pp. 139–156.

Salop, S.C., 1979, 'Monopolistic Competition with Outside Goods,' *Bell Journal of Economics*, 10, pp. 141–156.

Stone, B., 2011, 'Amazon May Soon Need to Collect Sales Tax,' *BloonbergBusiness week* (https://www.bloomberg.com/news/articles/2011-06-02/amazon-may-soon-need-to-collect-sales-tax)

Tabuchi, T. and Thisse, J.F., 1995, 'Asymmetric Equilibria in Spatial Competition,' *International Journal of Industrial Organization*, 13, pp. 213–227.

Vogel, J., 2008, 'Spatial Competition with Heterogeneous Firms,' *Journal of Political Economy*, 116, pp. 423–466.

© 2017 The Editorial Board of The Journal of Industrial Economics and John Wiley & Sons Ltd