An Optimal MILP Formulation

for the Delay- and Delay Variation-Bounded Multicast Tree Problem*

Pi-Rong Sheu Hung-Yuan Tsai Shao-Chi Chen

Department of Electrical Engineering, National Yunlin University of Science & Technology

Touliu, Yunlin 640, Taiwan, R.O.C.

Email: sheupr@yuntech.edu.tw

Abstract

More and more multicast communications are becoming real-time. In real-time communications, messages must be transmitted to their destination nodes within a certain amount of time; otherwise the messages will be rendered futile. To support real-time multicast communications, computer networks have to guarantee an upper bound on the end-to-end delay from a source to each destination. This is known as *the multicast end-to-end delay problem*. On the other hand, if the same message fails to arrive at each destination at the same time, there will probably arise inconsistency or unfairness problem among users. This is related to *the multicast delay variation problem*. In the paper, we are concerned with minimizing multicast delay variation under multicast end-to-enddelay constraint. The problem has been proven to be NP-complete. After its NP-completeness, many heuristic algorithms have been developed for it. However, these heuristic algorithms can just generate suboptimal solutions. In this paper, we will propose a mixed integer linear programming formulation to yield optimal solutions for this difficult problem. Computer simulations testify that compared with existing heuristic algorithms, our new mixed integer linear programming formulation can yield solutions with smaller multicast delay variation under different multicast end-to-end delay constraint.

Keywords: Multicast Communication, Delay Variation, End-To-End Delay, Optimal Formulation, Linear Programming

摘 要

越來越多的多播通訊是屬於即時性，在即時性的通訊，資料必須在一定的時間期限內送達目的節點，否則這筆資料將變成無用。電腦網路要能夠支援即時性多播通訊，就必須要能夠保證起始節點到每一個目的節點的傳輸路徑的延遲是在一定的範圍之內，這就是所謂的多播通訊點對點延遲問題。另一方面，同一資料若無法在相同的時間傳送到多播通訊中的每一個目的節點，則可能會引起資料的不一致性或影響使用者之間的公平性，這就是所謂的多播通訊延遲差異問題。在本篇論文我們所要研究的主題是如何在多播通訊點對點延遲限制條件下，使得目的節點之間的傳輸延遲差異降到最低。這個問題已經被證明為 NP-完全性問題，許多啟發式演算法已被發展出來解決此一 NP-完全性問題。然而這些啟發式演算法只能產生次佳解。在本篇論文中，我們將提出一個「混合式整數線性規劃」以求此困難問題的最佳解。電腦模擬結果證明在不同的點對點延遲限制下，與目前的啟發式演算法做比較，我們的「混合式整數線性規劃」可以產生多播延遲差異較小的解。

關鍵詞：多播通訊, 延遲差異, 點對點延遲, 最佳化方程式, 線性規劃

1. Introduction

With the advancement in computer network technology, the applications of computer networks are becoming diverse, such as videoconferencing and on-line activities: shopping, game playing, and stock market screening and exchange. The new growth in network applications has boosted new requirements in communication modes, one of which is the provision of *multicast communications* [6][9]. In order to satisfy such a demand, several well-developed multicast routing protocols have been proposed to elevate the efficiency of network resources, including Distance-Vector Multicast Routing Protocol, Multicast Extensions to Open Shortest Path First, and Protocol-Independent Multicast [6].

Before transmitting multicast packets, most multicast routing protocols will establish a set of multicast paths called a multicast tree for efficiently transmitting the packets to each destination. The overall performance of a multicast routing protocol will largely depend on the efficiency of its multicast tree. Therefore, how to build an efficient multicast tree has become an important research topic in multicast communications. There has been plenty of literature dwelling on ways to establish competent multicast trees [6]. One of the most often considered multicast trees is referred to as the minimum cost multicast tree. (Every link in the multicast tree has a nonnegative

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cost. The cost of the multicast tree is defined as the sum of the costs of all the links in the entire multicast tree.) The minimum-cost multicast tree is also known as the Steiner tree [5], and finding such a tree is a famous NP-complete problem [5].

In real-time communications, packets must be transmitted to their destinations within a certain amount of time; otherwise the packets will be nullified. Videoconferencing and on-line game playing, for example, must receive fresh images and sounds. Even if the image or voice delays only several seconds, the user will become impatient. Another real-time demand is found in on-line stock market exchange, which tolerates absolutely no time delay in message transmissions or commercial transactions. To support real-time multicast communications, computer networks have to guarantee an upper bound on the end-to-end delay from a source to each destination. This is known as the multicast end-to-end delay problem [9].

Suppose that the same packet fails to arrive at each destination at the same time. Inconsistency or unfairness problems will arise among users. For example, in a distributed database system, if the workstations receive the same packet asynchronously, their calculation results may be incongruous. Another possible dispute is found in on-line video gaming where several game participants connected with a game server do not receive the images and messages simultaneously. Third, in on-line shopping, if the packets are received among purchasers at a drastically asynchronous pace, the prospective buyers will lose faith and turn off. These are all related to the multicast delay variation problem [9]. The above examples reveal the command that the delay variation among all the paths from a source to each destination has to be kept within a definite limit.

Our research subject is concerned with the minimization of multicast delay variation under multicast end-to-end delay constraint. To be more specific, given a computer network, a source node, and a set of destination nodes, the goal is to find a multicast tree (namely a set of multicast paths) with the source as the "root" and the destinations as the "leaves" so that the multicast end-to-end delay between the source and each destination may fall within a tolerable range, and the multicast delay variation among the destinations may be the smallest. As shown in Figure 1, there are two paths between source vs and destination v1. Their end-to-end delays are 16 and 21, respectively. There are also two paths between source node vs and destination node v2. Their end-to-end delays are 19 and 10, respectively. If we merely consider multicast end-to-end delay constraint and set the permitted upper bound to be 19, then the selected path between vs to v1 should be that with delay 16, and the selected path between vs to v2 should be that with delay 10. Suppose only the minimization of multicast delay variation rather than multicast end-to-end delay constraint is considered. The selected path from vs to v1 then should be that with delay 21; the selected path from vs to v2 should be that with delay 19. The resulted multicast delay variation is 2, which is also the optimum value. Finally, if the multicast delay variation is to be minimized under multicast end-to-end delay constraint, and the permitted upper bound of the multicast end-to-end delay is set to be 19, the selected path from vs to v1 should be that with delay 16; the selected path from vs to v2 should be that with delay 19. The resulted multicast delay variation is 3, which is the optimum value under our multicast end-to-end delay constraint.

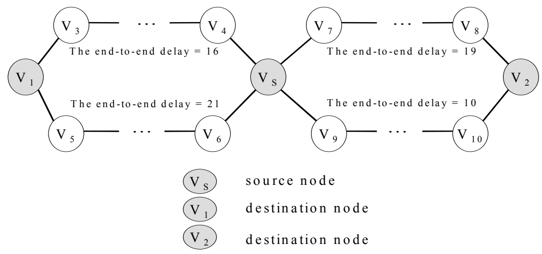

Figure 1 An illustration to DVBMT.

The issue first defined and discussed in [9] is that of minimizing multicast delay variation under multicast end-to-end delay constraint. The authors referred to this problem as *the delay- and delay variation-bounded multicast tree (DVBMT) problem* and have proved it to be NP-complete. After the NP-completeness of *DVBMT*, many heuristic algorithms have been developed for it. However, these heuristic algorithms can just obtain suboptimal solutions. In this paper, we will propose a mixed integer linear programming formulation which can generate optimal solutions for this difficult problem. Computer simulations show that compared with existing heuristic algorithms, our new mixed integer linear programming formulation can yield solutions with smaller multicast delay variation under different multicast end-to-end delay constraint.

The rest of the paper is organized as follows. In Section 2, the definition and complexity of DVBMT are given. In Section 3, an optimal mixed integer linear programming formulation for DVBMT is proposed. In Section 4, a comparison based on computer simulations will be made between the performances of our optimal formulation and an existing heuristic algorithm. Lastly, Section 5 concludes the whole research.

2. The Definition and Complexity of DVBMT

2.1 The Definition of DVBMT

We will use a weighted graph $G = (V, E)$ to denote a network, where the node set V represents the workstations or routers, and the edge set E represents the links between nodes. $n = |V|$ signifies the number of nodes in the network. We use ℓ_{ij} to denote the undirected link between nodes v_i and v_j. For each link ℓ_{ij} in E, we define *a link-delay function* $d : \ell_{ij} \rightarrow r^+$. It gives every link ℓ_{ij} a nonnegative value d_{ij}, which represents the necessary delay during the transmission of a packet in the link ℓ_{ij}.

In multicast communications, a multicast packet goes from a certain source node $v_s \in V$, passes some other nodes, and finally arrives at a set of destination nodes $M \subseteq V - \{ v_s \}$, where the set M is named as *the destination node set*. Its size is $m = |M|$ and every node in M is called *a destination node*. The packets to be transmitted will go along a multicast tree $T = (V_T, E_T)$ to each of the destination nodes in M. The multicast tree T is a spanning tree which joins the source node and all the destination nodes; that is, the source node and all the destination nodes constitute the leaves of T. Moreover, T may contain other nodes as relay ones which do not belong to M and source node v_s. We also define $P_T(v_s, v_w)$ as the path from source node v_s to destination node $v_w \in M$ in T. It is obvious that when a packet is transmitted from v_s to v_w through the path, the needed delay is $\sum_{\ell_{ij} \in P_T(v_s, v_w)} d_{ij}$.

In the following we will introduce two important qualities of service parameters in multicast communications: Δ and δ, which are defined in [9]. They represent the quality of service parameters required by the source and the destinations and are determined by users. (1) The multicast end-to-end delay constraint Δ: this parameter stands for an upper bound of all end-to-end delays associated with the paths from the source to each destination. The purpose of setting this parameter is to limit the time for packet transmissions in the network. If any end-to-end delay exceeds the upper bound, the packet will be counted useless. (2) The multicast delay variation δ: this parameter means that the difference of the maximum end-to-end delay and the minimum end-to-end delay among the paths from the source to all the destinations has to be kept within δ. The purpose of setting this parameter is to enable all the destinations to receive the same packets simultaneously as much as possible.

For the sake of convenience, we use Δ_T and δ_T to stand for the multicast end-to-end delay and the multicast delay variation in a multicast tree T. Based on these notations and definitions, we can now formally describe *the DVBMT problem* in our paper: Given a weighted digraph $G = (V, E)$, a source node $v_s \in V$, a destination node set $M \subseteq V - \{ v_s \}$, *a link-delay function* $d: \ell \rightarrow r^+$, $\ell \in E$, and a constant Δ, find an optimal multicast tree $T^* = (V_{T^*}, E_{T^*})$ which spans v_s and M such that $\Delta_{T^*} = \max_{v_w \in M} \sum_{\ell_{ij} \in P_{T^*}(v_s, v_w)} d_{ij} \le \Delta$, and

$$\delta_{T^*} = \min_{T} \left\{ \max_{v_u, v_w \in M} \left\{ \left| \sum_{\ell_{ij} \in P_T(v_s, v_u)} d_{ij} - \sum_{\ell_{ij} \in P_T(v_s, v_w)} d_{ij} \right| \right\} \right\}$$

where T denotes any multicast tree spanning v_s and M.

2.2 The Complexity of DVBMT

DVBMT has been proved to be NP-complete in [9]. After the NP-completeness of *DVBMT*, many heuristic algorithms have been developed for it [1][7][10][12]. Although these existing heuristic algorithms can produce solutions quickly, only suboptimal solutions can be obtained. On the other hand, an optimal formulation can generate optimal solutions as well as judge the quality of heuristic algorithms. In the next section, we will propose a mixed integer linear programming formulation which can generate optimal solutions for this difficult problem.

3. An Optimal Mixed Integer Linear Programming Formulation for DVBMT

In 1947, G. B. Dantzing proposed linear programming (LP) to solve complex optimization and scheduling problem [2]. Such a computing paradigm has been successfully employed in solving a variety of problems. Especially, LP and ILP (integer LP) have successfully been applied to a number of NP-complete problems, including many famous NP-complete problems in communication networks, such as the multicast routing problem. Recently, many researchers have attempted to adopt LP or ILP to solve various problems existing in wireless networks [3][4][8]. As a result, it is worthy to develop efficient LP or ILP formulations to yield the optimal solutions for DVBMT. In this section we will present a mixed integer linear programming formulation to optimally solve DVBMT. Our formulation is named MILP-for-DVBMT.

In order to construct *MILP-for-DVBMT*, the variables are thus defined.

n : The number of nodes in the network

M: The destination set

m : The number of destination nodes

x_{ij}^{ks} : A binary variable. If ℓ_{ij} belongs to the path in R from source v_s to destination v_k, x_{ij}^{ks} =1; 0 otherwise.

y_{ij}^{s} : A binary variable. If ℓ_{ij} belongs to the multicast tree T_s^* rooted at source v_s, y_{ij}^{s}=1; 0 otherwise.

f_{ij}^{ks} : A flow variable associated with x_{ij}^{ks} to denote the flow in ℓ_{ij}-unit delay $\in$ R.

f_{ij}^{s} : A flow variable associated with y_{ij}^{s} to denote the flow in $\ell_{ij} \in T_s^*$.

d_{ij} : The necessary delay during the transmission of a packet in the link ℓ_{ij}.

Δ : The multicast end-to-end delay bound

δ: The multicast delay variation

***MILP-for-DVBMT*:**

Minimize:

$$\delta = \max_{v_u, v_w \in M} \left\{ \left| \sum_{\ell_{ij} \in P_T(v_s, v_u)} d_{ij} - \sum_{\ell_{ij} \in P_T(v_s, v_w)} d_{ij} \right| \right\} \quad (1.1)$$

Subject to:

Constraints from the path property:

$$\sum_{v_i \in V} x_{is}^{ks} = 0; \quad v_i \neq v_s \quad (2.1)$$

$$\sum_{v_i \in V} x_{ik}^{ks} = 1; \quad \forall v_k \in M, \quad v_k \neq v_i \quad (2.2)$$

$$\sum_{v_i \in V} x_{ij}^{ks} \leq 1; \quad \forall v_j \in V - M - v_s, \quad v_j \neq v_i \quad (2.3)$$

$$\sum_{v_i \in V} x_{ji}^{ks} - \sum_{v_i \in V} x_{ij}^{ks} = 0; \quad \forall v_j \in V - M - v_s \quad (2.4)$$

Constraints from the network flow property in a set R of paths:

$$\sum_{v_i \in V} f_{ij}^{ks} - \sum_{v_i \in V} f_{ji}^{ks} = \sum_{v_i \in V} x_{ij}^{ks}; \quad \forall v_j \in V - v_s, \quad v_j \neq v_i \quad (2.5)$$

$$x_{ij}^{ks} \leq f_{ij}^{ks} \leq (n-1) x_{ij}^{ks}; \quad \forall v_j \in V - v_s, \quad \forall v_i \in V, \quad v_j \neq v_i \quad (2.6)$$

Constraints from the multicast end-to-end delay bound:

$$\sum_{l_{ij} \in E} d_{ij} x_{ij}^{ks} \leq \Delta, \quad \forall v_k \in M \quad (2.7)$$

Constraints from the multicast delay variation bound:

$$\left| \sum_{l_{ij} \in E} d_{ij} x_{ij}^{k_1 s} - \sum_{l_{ij} \in E} d_{ij} x_{ij}^{k_2 s} \right| \leq \delta; \quad \forall v_{k_1}, v_{k_2} \in M, \quad v_{k_1} \neq v_{k_2} \quad (2.8)$$

Constraints from the domain definition:

$$x_{ij}^{ks} \in \{0,1\}; \quad \forall v_i, v_j \in V, \quad v_i \neq v_j \quad (2.9)$$

$$f_{ij}^{ks} \geq 0; \quad \forall v_i, v_j \in V, \quad v_i \neq v_j \quad (2.10)$$

Constraints for the transformation from a set of paths to a multicast tree:

$$m \times y_{ij}^{s} \geq \sum_{v_k \in M} x_{ij}^{ks} \geq y_{ij}^{s}; \quad \forall v_i, v_j \in V, \quad v_i \neq v_j \quad (3.1)$$

Constraints from the tree property:

$$\sum_{v_i \in V} y_{is}^{s} = 0; \quad v_i \neq v_s \quad (4.1)$$

$$\sum_{v_i \in V} y_{ik}^{s} = 1; \quad \forall v_k \in M, \quad v_k \neq v_i \quad (4.2)$$

$$\sum_{v_i \in V} y_{ij}^{s} \leq 1; \quad \forall v_j \in V - M - v_s, \quad v_j \neq v_i \quad (4.3)$$

$$\sum_{v_i \in V} y_{ji}^{s} \leq (n-1) \sum_{v_i \in V} y_{ij}^{s}; \quad \forall v_j \in V - M - v_s, \quad v_j \neq v_i \quad (4.4)$$

Constraints from the network flow property in a multicast tree:

$$\sum_{v_i \in V} f_{ij}^{s} - \sum_{v_i \in V} f_{ji}^{s} = \sum_{v_i \in V} y_{ij}^{s}; \quad \forall v_j \in V - v_s, \quad v_j \neq v_i \quad (4.5)$$

$$y_{ij}^{s} \leq f_{ij}^{s} \leq (n-1) y_{ij}^{s}; \quad \forall v_j \in V - v_s, \quad \forall v_i \in V, \quad v_j \neq v_i \quad (4.6)$$

Constraints from the domain definition:

$$y_{ij}^{s} \in \{0,1\}; \quad \forall v_i, v_j \in V, \quad v_i \neq v_j \quad (4.7)$$

$$f_{ij}^{s} \geq 0; \quad \forall v_i, v_j \in V, \quad v_i \neq v_j \quad (4.8)$$

Given a weighted graph $G = (V, E)$ with $|V| = n$, a source node $v_s \in V$, a destination node set $M \subseteq V - \{v_s\}$, *a link-delay function* $d: \ell \rightarrow r^+$, $\ell \in E$, and a constant Δ, formulation *MILP-for-DVBMT* is able to generate an optimal multicast tree $T^* = (V_{T^*}, E_{T^*})$ which spans v_s and M.

Let us explain *MILP-for-DVBMT* in detail. Inequalities (2.1) to (2.6) will form a set of paths with minimum delay variation (see Inequality (2.8)) from the source v_s to each destination v_k under the end-to-end delay constraint (see Inequality (2.7)).

Inequality (2.1) states that no link is incident to the source v_s since v_s is the starting node of each path. Inequality (2.2) states that exactly one link is incident to the ending node of each path (i.e., each destination node v_k). Inequality (2.3) states that with a path, at most one link is incident to each relay node v_j . Inequality (2.4) states that the number of links incident to a relay node v_j must be equal to the number of links out off v_j .

Inequalities (2.6) and (2.7) are derived from the usual conservation of flow constraints. From a flow viewpoint, multicasting can be imaged as a virtual packet allocation where the source v_s generates $m \le x \le n-1$ virtual packets and routes them along a set of paths such that each destination and each relay node can receive one virtual packet (see Inequalities (2.2) and (2.3)). During the routing of virtual packets, a relay node v_j will take one virtual packet for itself (see Inequality (2.5)) and re-route it to other nodes. Thus, the maximum flow (number of virtual packets) f_{ij} along with any link ℓ_{ij} with $x_{ij}^{ks} \neq 0$ is at most $n-1$ (see Inequality (2.6)). Inequalities (2.9) and (2.10) express the integrality of the variables x_{ij}^{ks} and non-negativity of the variables f_{ij}^{ks} , respectively.

Inequality (3.1) is used for transforming variables x_{ij}^{ks} associated with a set of paths to variables y_{ij}^{s} associated with a tree. It forces y_{ij}^{s} to be 1 if $\sum_{v_k \in M} x_{ij}^{ks}$ =1, 0 otherwise. Inequalities (4.1) to (4.6) will transform the set R of paths obtained from Inequalities (2.1) to (2.10) to a multicast tree T_s . The meanings of Inequalities (4.1) to (4.6) are similar to Inequalities (2.1) to (2.7) except Inequalities (4.4) and (2.4). Inequalities (4.4) says that the out-degree of a node v_j in T_s is at most $n-1$ if its in-degree is 1. If its in-degree is 0, then its out-degree is also 0.

Inequalities (4.7) and (4.8) express the integrality of the variables y_{ij}^{s} and non-negativity of the variables f_{ij}^{s} , respectively. Finally, Function (1.1) is used to minimize the multicast delay variation associated with the final broadcast tree T_s^* .

4. Computer Simulations

In the section we will examine the efficiency of MILP-for-DVBMT through computer simulations. Our comparisons will be made between our MILP-for-DVBMT and a well-known multicast tree heuristic algorithm for DVBMT named DDVCA [10]. We will evaluate their multicast delay variations under different multicast end-to-end delay bounds.

4.1 Network Model

The network model G (V, E) employed by our computer simulations comes from the one proposed in [11]. In the model, n nodes are randomly scattered on a square checkered map where each node is located on an integer coordinate. The relative distance between the nodes is measured by their distance in the map. As far as any two nodes v_u and v_w on the network are concerned, the probability of forming a link between them is $P(\{v_u, v_w\}) = a_1 \exp \frac{-dist(v_u, v_w)}{La_2}$, where $dist(v_u, v_w)$ denotes the distance between nodes v_u and v_w. L represents the maximum distance between any two nodes, i.e., L = $\underset{v_u, v_w \in V}{Max}\{dist(v_u, v_w)\}$. The values of a_2 and a_1 fall between (0, 1]. If a_1 becomes larger, there will be more links on the network; that is to say, the links will become denser. If a_2 becomes larger, there will be longer links on the network; that is, there will be fewer shorter links. In our computer simulations, both parameters are set to be a_1=0.20 and a_2=0.25, respectively. Finally, the transmission delay between a pair of nodes v_u and v_w is set to $dist(v_u, v_w) \times \theta + 1$, where θ will be selected from (0, 1] randomly for each link.

4.2 Simulation Results

In our experiments, three different cases will be considered. In case 1, the number of destination nodes in the multicast group is equal to 3. The multicast end-to-end delay bound is fixed at 50. Figure 2 shows the multicast delay variations generated by MILP-for-DVBMT and DDVCA when the network sizes under consideration begin with 10 nodes, 15 nodes, 20 nodes, 25 nodes, and up to 30 nodes. It is apparent that the multicast delay variations of our MILP-for-DVBMT are less than those of DDVCA.

In case 2, the number of destination nodes in the multicast group is equal to 3 and the number of nodes in the network is 15. Figure 3 presents the multicast delay variations generated by MILP-for-DVBMT and DDVCA when the multicast end-to-end delay bound varies from 50 to 250 at a step of 50. It can be observed that the multicast delay variations of our MILP-for-DVBMT are smaller than those of DDVCA.

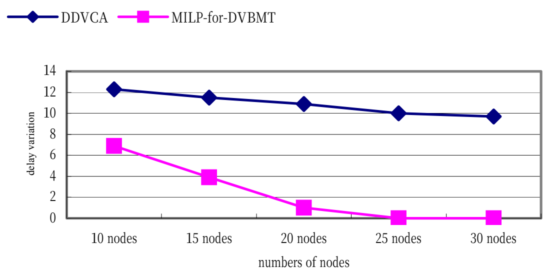

Figure 2 Simulation results for case 1: number of destination nodes=15 and multicast end-to-end delay bound=50.

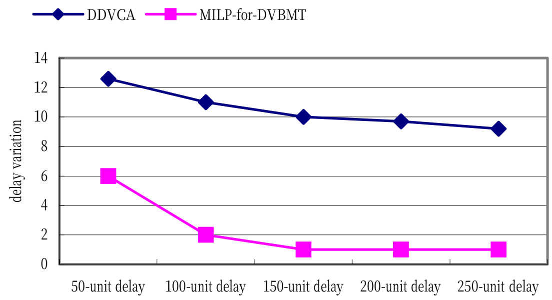

Figure 3 Simulation results for case 2: number of nodes=15 and number of destination nodes=50.

In case 3, the number of nodes in the network is equal to 15 and the multicast end-to-end delay bound is set to 50. Figure 4 states the multicast delay variations generated by MILP-for-DVBMT and DDVCA when the number of destination nodes begin with 3 nodes, 6 nodes, 9 nodes, and up to 12 nodes. We discover that the multicast delay variations of our MILP-for-DVBMT are shorter than those of DDVCA.

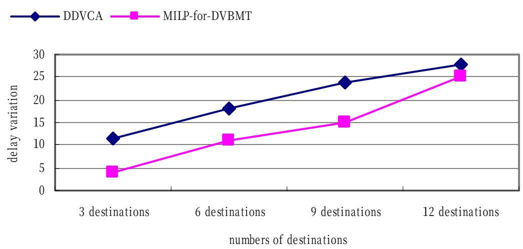

Figure 4 Simulation results for case 3: number of nodes=15 and multicast end-to-end delay bound=50.

5. Conclusions

In this paper, we have discussed the DVBMT problem, which has been proved to be NP-complete. Up to now, many heuristic algorithms have been developed for it. In spite of the short execution time of these existing heuristic algorithms, the solutions generated by them cannot be guaranteed to be optimal. Based on linear programming technique, in this paper we have designed a mixed integer linear programming formulation, MILP-for-DVBMT, to yield optimal solution for DVBMT. Computer simulations show that compared with DDVCA, our MILP-for-DVBMT can obtain multicast trees with smaller multicast delay variations under different multicast end-to-end delay constraints.

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