科技部補助專題研究計畫成果報告
期末報告

高維度下具長域自我相斥隨機漫步，滲流與 Ising模型的兩點函數之臨界行為(第2年)

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處 理 方 式 ：
1.公開資訊：本計畫可公開查詢
2.「本研究」是否已有嚴重損及公共利益之發現：否
3.「本報告」是否建議提供政府單位施政參考：否

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中 文 摘 要 ： 在這此次計劃中， 我研究重心放在研究在高維度下具長域定向滲流，自我相斥隨機漫步與Ising 模型之兩點函數的臨界與漸進行為，此問題是與北海道大學數學系Akira Sakai教授合作的文章， 我們獲得兩點函數在臨界行為的收斂速度與漸進行為，並於2015年刊登在Ann. Prob. 期刊上，並且我們持續研究此類問題。 此外本人與成大物理系張書銓教授探討在二維三角晶格與蜂窩狀晶格上特殊的定向滲流之兩點函數的臨界行為與收斂速度也有兩篇文章分別發表于 J. Stat. Phys. 和 Physica A， 我們獲得在特殊的模型下兩點函數之收斂速度的上界估計與下界估計的結果，並且我們也持續研究此類的問題。

中 文 關 鍵 詞 ： 滲流，自我相斥隨機漫步，Ising 模型，兩點函數，臨界行為，Lace 展開，大分差，Berry－Esseen定理

英 文 摘 要 ： In this project, my main research is focused on the critical two-point functions for long-range percolation, self-avoiding walk and Ising model in high dimensions. This is joint work with professor Akira Sakai in the mathematics department at Hokkaido university. We obtained the rate of convergence and asymptotic behavior of two point functions and it has been published at Journal of ann. Probab. In 2015. We are doing this kind of problem now. In addition, I and professor Shu-Chiuan Chang in the physics department at national Cheng Kung university have some results for a version of directed percolation on the triangle lattice and honeycomb lattice. We obtained the upper and lower bounds of two point functions and our results bas been published J. Stat. Phys. and Physica A. I are doing the similar problems now.

英 文 關 鍵 詞 ： percolation, self-avoiding walk, Ising model, critical behavior, Lace expansion, Large derivation, Berry-Esseen theorem

Report on NCS program from from 2003 to 2015

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December 3, 2015

1 Work with professor Akira Sakai

Given a symmetric probability distribution

$$D(x) = O(L^{\alpha})\|\!|x|\!\|_L^{-d-\alpha}, \tag{1.1}$$

and there are $v_\alpha = O(L^{\alpha\wedge 2})$ and $\epsilon > 0$ such that

$$\hat{D}(k) \equiv \sum_{x\in\mathbb{Z}^d} e^{ik\cdot x} D(x) = 1 - v_\alpha |k|^{\alpha\wedge 2} \times \begin{cases} 1 + O((L|k|)^{\epsilon}) & [\alpha \neq 2], \\ \log \frac{1}{L|k|} + O(1) & [\alpha = 2]. \end{cases} \tag{1.2}$$

Note that if $\alpha > 2$, then $v_\alpha = \sigma^2/(2d)$ where $\sigma^2 = \sum_{x\in\mathbb{Z}^d} |x|^2 D(x)$. Moreover, if $L \geq 1$, there is a constant $\Delta \in (0, 1)$ such that

$$\|D^{*n}\|_\infty \leq O(L^{-d})\, n^{-\frac{d}{\alpha\wedge 2}} \quad [n \geq 1], \qquad 1 - \hat{D}(k) \begin{cases} < 2 - \Delta & [k \in [-\pi, \pi]^d], \\ > \Delta & [\|k\|_\infty \geq L^{-1}]. \end{cases} \tag{1.3}$$

The goal of this project is to overcome those difficulties and derive an asymptotic expression of the critical two-point function for the power-law decaying long-range models above the critical dimension, using the lace expansion. We have investigated crossover in the asymptotic expression when the power of the 1-step distribution of the underlying random walk changes.

Self-avoiding walk (SAW) is a model for linear polymers. We define the two-point function for SAW on $\mathbb{Z}^d$ as

$$G_p^{\text{SAW}}(x) = \sum_{\omega:o\to x} p^{|\omega|} \prod_{j=1}^{|\omega|} D(\omega_j - \omega_{j-1}) \prod_{s<t} (1 - \delta_{\omega_s,\omega_t}), \tag{1.4}$$

where $p \geq 0$ is the fugacity, $|\omega|$ is the length of a path $\omega = (\omega_0, \omega_1, \ldots, \omega_{|\omega|})$ and $D : \mathbb{Z}^d \to [0, 1]$ is the $\mathbb{Z}^d$-symmetric non-degenerate (i.e., $D(o) \neq 1$) 1-step distribution for the underlying random walk (RW); the contribution from the 0-step walk is considered to be $\delta_{o,x}$ by convention. If the indicator function $\prod_{s<t}(1 - \delta_{\omega_s,\omega_t})$ is replaced by 1,

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then $G_p^{\text{SAW}}(x)$ turns into the RW Green's function $G_p^{\text{RW}}(x)$, whose radius of convergence p_c^{RW} is 1, as $\chi_p^{\text{RW}} \equiv \sum_{x\in\mathbb{Z}^d} G_p^{\text{RW}}(x) = (1-p)^{-1}$ for $p < 1$ and $\chi_p^{\text{RW}} = \infty$ for $p \geq 1$. Therefore, the radius of convergence p_c^{SAW} for $G_p^{\text{SAW}}(x)$ is not less than 1. It is known that $\chi_p^{\text{SAW}} \equiv \sum_{x\in\mathbb{Z}^d} G_p^{\text{SAW}}(x) < \infty$ if and only if $p < p_c^{\text{SAW}}$ and diverges as $p \uparrow p_c^{\text{SAW}}$. Here, and in the remainder of the paper, we often use "≡" for definition.

Percolation is a model for random media. Each bond $\{u, v\}$, which is a pair of vertices in $\mathbb{Z}^d$, is either occupied or vacant independently of the other bonds. The probability that $\{u, v\}$ is occupied is defined to be $pD(v - u)$, where $p \geq 0$ is the percolation parameter. Since D is a probability distribution, the expected number of occupied bonds per vertex equals $p\sum_{x\neq o} D(x) = p(1-D(o))$. The percolation two-point function $G_p^{\text{perc}}(x)$ is defined to be the probability that there is a self-avoiding path of occupied bonds from o to x; again by convention, $G_p^{\text{perc}}(o) = 1$.

The Ising model is a model for magnets. For $\Lambda \subset \mathbb{Z}^d$ and $\varphi = \{\varphi_v\}_{v\in\Lambda} \in \{\pm1\}^\Lambda$, we define the Hamiltonian (under the free-boundary condition) as

$$H_\Lambda(\varphi) = -\sum_{\{u,v\}\subset\Lambda} J_{u,v}\varphi_u\varphi_v, \tag{1.5}$$

where $J_{u,v} = J_{o,v-u} \geq 0$ is the ferromagnetic pair potential and inherits the properties of the given D, as explained below. The finite-volume two-point function at the inverse temperature $\beta \geq 0$ is defined as

$$\langle\varphi_o\varphi_x\rangle_{\beta,\Lambda} = \sum_{\varphi\in\{\pm1\}^\Lambda} \varphi_o\varphi_x\, e^{-\beta H_\Lambda(\varphi)} \Bigg/ \sum_{\varphi\in\{\pm1\}^\Lambda} e^{-\beta H_\Lambda(\varphi)}. \tag{1.6}$$

It is known that $\langle\varphi_o\varphi_x\rangle_{\beta,\Lambda}$ is increasing in $\Lambda \uparrow \mathbb{Z}^d$. Let $p = \sum_{x\in\mathbb{Z}^d} \tanh(\beta J_{o,x})$. The Ising two-point function $G_p^{\text{Ising}}(x)$ is defined to be the increasing-volume limit of $\langle\varphi_o\varphi_x\rangle_{\beta,\Lambda}$:

$$G_p^{\text{Ising}}(x) = \lim_{\Lambda\uparrow\mathbb{Z}^d} \langle\varphi_o\varphi_x\rangle_{\beta,\Lambda}. \tag{1.7}$$

Let $D(x) = p^{-1}\tanh(\beta J_{o,x})$.

For percolation and the Ising model, there is a model-dependent critical point $p_c \geq 1$ (from now on, we omit the superscript, unless it causes any confusion) such that

$$\chi_p \equiv \sum_{x\in\mathbb{Z}^d} G_p(x) \begin{cases} < \infty & [p < p_c], \\ = \infty & [p \geq p_c], \end{cases} \qquad \theta_p \equiv \sqrt{\lim_{|x|\to\infty} G_p(x)} \begin{cases} = 0 & [p < p_c], \\ > 0 & [p > p_c]. \end{cases} \tag{1.8}$$

The order parameter θ_p^{perc} is the probability that the occupied cluster of the origin is unbounded, while θ_p^{Ising} is the spontaneous magnetization, which is the infinite-volume limit of the finite-volume single-spin expectation $\langle\varphi_o\rangle_{\beta,\Lambda}^+$ under the plus-boundary condition. The continuity of θ_p at $p = p_c$ in a general setting is still a remaining issue.

We are interested in asymptotic behavior of $G_{p_c}(x)$ as $|x| \to \infty$. For the "uniformly spread-out" finite-range models, e.g., $D(x) = \mathbb{1}_{\{|x|=1\}}/(2d)$ or $D(x) = \mathbb{1}_{\{\|x\|_\infty\leq L\}}/(2L+1)^d$ for some $L \in [1, \infty)$, it has been proved [14, 17, 26] that, if $d > 4$ for SAW and the Ising model and $d > 6$ for percolation, and if d or L is sufficiently large (depending on the models), then there is a model-dependent constant A (= 1 for RW) such that

$$G_{p_c}(x) \underset{|x|\to\infty}{\sim} \frac{a_d/\sigma^2}{A|x|^{d-2}}, \tag{1.9}$$

where "$\sim$" means that the asymptotic ratio of the left-hand side to the right-hand side is 1, and

$$a_d = \frac{d\Gamma(\frac{d-2}{2})}{2\pi^{d/2}}, \qquad\qquad \sigma^2 \equiv \sum_{x\in\mathbb{Z}^d} |x|^2 D(x) = O(L^2). \tag{1.10}$$

This is a sufficient condition for the following mean-field behavior [1, 2, 3, 4, 23]:

$$\chi_p \underset{p\uparrow p_c}{\asymp} (p_c - p)^{-1}, \qquad\qquad \theta_p \underset{p\downarrow p_c}{\asymp} \begin{cases} \sqrt{p - p_c} & \text{[Ising]}, \\ p - p_c & \text{[percolation]}, \end{cases} \tag{1.11}$$

where "$\asymp$" means that the asymptotic ratio of the left-hand side to the right-hand side is bounded away from zero and infinity.

Let

$$d_c = \begin{cases} 2(\alpha \wedge 2) & \text{[Ising]}, \\ 3(\alpha \wedge 2) & \text{[percolation]}. \end{cases} \tag{1.12}$$

The proof of the above result is based on the lace expansion (e.g., [18, 23, 26]). The core concept of the lace expansion is to systematically isolate interaction among individuals (e.g., mutual avoidance between distinct vertices for SAW or between distinct occupied pivotal bonds for percolation) and derive macroscopic recursive structure that yields the random-walk like behavior (1.9). When $d > d_c$ and $d \vee L \gg 1$ (i.e., d or L sufficiently large depending on the models), there is enough room for those individuals to be away from each other, and the lace expansion converges [18, 23, 26]. The resultant recursion equation for G_p is the following:

$$G_p(x) = \begin{cases} \delta_{o,x} + \displaystyle\sum_{v\in\mathbb{Z}^d} pD(v)\, G_p(x-v) & \text{[RW]}, \\ \delta_{o,x} + \displaystyle\sum_{v\in\mathbb{Z}^d} \big(pD(v) + \pi_p(v)\big)\, G_p(x-v) & \text{[SAW]}, \\ \pi_p(x) + \displaystyle\sum_{\substack{u,v\in\mathbb{Z}^d \\ (u\neq v)}} \pi_p(u)\, pD(v-u)\, G_p(x-v) & \text{[Ising \& percolation]}, \end{cases} \tag{1.13}$$

where π_p is the lace-expansion coefficient. To treat all models simultaneously, we introduce the notation $f * g$ to denote the convolution of functions f and g in $\mathbb{Z}^d$:

$$(f * g)(x) = \sum_{v\in\mathbb{Z}^d} f(v)\, g(x-v). \tag{1.14}$$

Then the above identities can be simplified as (the spatial variables are omitted)

$$G_p = \begin{cases} \delta + pD * G_p & \text{[RW]}, \\ \delta + (pD + \pi_p) * G_p & \text{[SAW]}, \\ \pi_p + \pi_p * p\big(D - D(o)\delta\big) * G_p & \text{[Ising \& percolation]}. \end{cases} \tag{1.15}$$

Repeated use of these identities yields[1]

$$G_p = \Pi_p + \Pi_p * pD * G_p, \tag{1.16}$$

where

$$\Pi_p(x) = \begin{cases} \delta_{o,x} & \text{[RW]}, \\ \displaystyle\sum_{n=0}^{\infty} \pi_p^{*n}(x) \equiv \sum_{n=0}^{\infty} (\underbrace{\pi_p * \cdots * \pi_p}_{n\text{-fold}})(x) & \text{[SAW]}, \\ \displaystyle\sum_{n=1}^{\infty} \big(-pD(o)\big)^{n-1} \pi_p^{*n}(x) & \text{[Ising \& percolation]}, \end{cases} \tag{1.17}$$

with the convention $f^{*0}(x) \equiv \delta_{o,x}$ for general f. When $d > d_{\rm c}$ and $d \vee L \gg 1$, there is a $\rho > 0$ such that $|\Pi_{p_{\rm c}}(x)|$ is summable and decays as $|x|^{-d-2-\rho}$ [14, 17, 26]. The multiplicative constant A in (1.9) and $p_{\rm c}$ can be represented in terms of $\Pi_{p_{\rm c}}(x)$ as

$$p_{\rm c} = \bigg(\sum_{x\in\mathbb{Z}^d} \Pi_{p_{\rm c}}(x)\bigg)^{-1}, \qquad A = p_{\rm c}\bigg(1 + \frac{p_{\rm c}}{\sigma^2} \sum_{x\in\mathbb{Z}^d} |x|^2\, \Pi_{p_{\rm c}}(x)\bigg). \tag{1.18}$$

In addition to the above properties (1.1)-(1.3), the n-step transition probability obeys the following bound:

$$D^{*n}(x) \le \frac{O(L^{\alpha\wedge 2})}{|\!|\!| x |\!|\!|_L^{d+\alpha\wedge 2}}\, n \times \begin{cases} 1 & [\alpha \ne 2], \\ \log |\!|\!| x |\!|\!|_L & [\alpha = 2]. \end{cases} \tag{1.19}$$

This is due to the following two facts: (i) the contribution from the walks that have at least one step which is longer than $c|\!|\!| x |\!|\!|_L$ for a given $c > 0$ is bounded by $O(L^\alpha)n/|\!|\!| x |\!|\!|_L^{d+\alpha}$; (ii) the contribution from the walks whose n steps are all shorter than $c|\!|\!| x |\!|\!|_L$ is bounded, due to the local CLT, by $O(\tilde{v}n)^{-d/2} e^{-|x|^2/O(\tilde{v}n)} \le O(\tilde{v}n)/|\!|\!| x |\!|\!|_L^{d+2}$ (times an exponentially

[1] For SAW, since $\|\pi_p\|_1 = o(1)$ as $d \vee L \to \infty$ and $\|G_p\|_\infty < \infty$ for every $p \le p_{\rm c}$ [14, 17],

$$\begin{aligned} G_p = \delta + pD * G_p + \pi_p * \underbrace{G_p}_{\text{replace}} &= \delta + pD * G_p + \pi_p * \big(\delta + pD * G_p + \pi_p * G_p\big) \\ &= (\delta + \pi_p) + (\delta + \pi_p) * pD * G_p + \pi_p^{*2} * \underbrace{G_p}_{\text{replace}} = \cdots \to (1.16). \end{aligned}$$

For percolation and the Ising model, since $D(o) = o(1)$ and $p\|\pi_p\|_1 = 1 + o(1)$ as $d \vee L \to \infty$ and $\|G_p\|_\infty \le 1$ for every $p \le p_{\rm c}$ [14, 17, 26],

$$\begin{aligned} G_p &= \pi_p + \pi_p * pD * G_p - pD(o)\pi_p * \underbrace{G_p}_{\text{replace}} \\ &= \pi_p + \pi_p * pD * G_p - pD(o)\pi_p * \big(\pi_p + \pi_p * pD * G_p - pD(o)\pi_p * G_p\big) \\ &= \big(\pi_p - pD(o)\pi_p^{*2}\big) + \big(\pi_p - pD(o)\pi_p^{*2}\big) * pD * G_p + \big(-pD(o)\big)^2 \pi_p^{*2} * \underbrace{G_p}_{\text{replace}} = \cdots \to (1.16). \end{aligned}$$

small normalization constant), where $\tilde{v}$ is the variance of the truncated 1-step distribution $\tilde{D}(y) \equiv D(y)\mathbb{1}_{\{|y|\leq c|x|\}}$ and equals

$$\tilde{v} = \sum_{y\in\mathbb{Z}^d} |y|^2 \tilde{D}(y) \leq O(L^{\alpha\wedge 2}) \times \begin{cases} \|\!|x|\!\|_L^{2-\alpha} & [\alpha < 2], \\ \log \|\!|x|\!\|_L & [\alpha = 2], \\ 1 & [\alpha > 2]. \end{cases} \tag{1.20}$$

For $\alpha \neq 2$, the inequality (1.19) is a discrete space-time version of the heat-kernel bound on the transition density $p_s(x)$ of an α-stable/Gaussian process:

$$p_s(x) \equiv \int_{\mathbb{R}^d} \frac{\mathrm{d}^d k}{(2\pi)^d}\, e^{-ik\cdot x - s|k|^{\alpha\wedge 2}} \leq \frac{O(s)}{|x|^{d+\alpha\wedge 2}}. \tag{1.21}$$

We also assume the following bound on the discrete derivative of the n-step transition probability:

$$\left| D^{*n}(x) - \frac{D^{*n}(x+y) + D^{*n}(x-y)}{2} \right| \leq \frac{O(L^{\alpha\wedge 2})\|\!|y|\!\|_L^2}{\|\!|x|\!\|_L^{d+\alpha\wedge 2+2}}\, n \qquad [|y| \leq \tfrac{1}{3}|x|]. \tag{1.22}$$

Here is the summary of the required properties of D.

Assumption 1.1. *The $\mathbb{Z}^d$-symmetric 1-step distribution D satisfies the properties (1.1), (1.2), (1.3), (1.19) and (1.22).*

Under the above assumption on D, we can prove the following theorem (c.f. [8]):

Theorem 1.2. *Let $\alpha > 0$, $\alpha \neq 2$ and*

$$\gamma_\alpha = \frac{\Gamma(\frac{d-\alpha\wedge 2}{2})}{2^{\alpha\wedge 2}\pi^{d/2}\Gamma(\frac{\alpha\wedge 2}{2})}, \tag{1.23}$$

and assume all properties of D in Assumption 1.1. Then, for RW with $d > \alpha \wedge 2$ and any $L \geq 1$, and for SAW, percolation and the Ising model with $d > d_\mathrm{c}$ and $L \gg 1$, there are $\mu \in (0, \alpha \wedge 2)$ and $A = A(\alpha, d, L) \in (0, \infty)$ ($A \equiv 1$ for random walk) such that, as $|x| \to \infty$,

$$G_{p_\mathrm{c}}(x) = \frac{\gamma_\alpha / v_\alpha}{A|x|^{d-\alpha\wedge 2}} + \frac{O(L^{-\alpha\wedge 2+\mu})}{|x|^{d-\alpha\wedge 2+\mu}}. \tag{1.24}$$

*As a result, by [**?**], χ_p and θ_p exhibit the mean-field behavior (1.11). Moreover, p_c and A can be expressed in term of Π_p in (1.17) as*

$$p_\mathrm{c} = \hat{\Pi}_{p_\mathrm{c}}(0)^{-1}, \qquad\qquad A = p_\mathrm{c} + \begin{cases} 0 & [\alpha < 2], \\ \dfrac{p_\mathrm{c}^2}{\sigma^2} \displaystyle\sum_x |x|^2 \Pi_{p_\mathrm{c}}(x) & [\alpha > 2]. \end{cases} \tag{1.25}$$

Remark 1.3. (a) The finite-range models are formally considered as the $\alpha = \infty$ model. Indeed, the leading term in (1.24) for $\alpha > 2$ is identical to (1.9).

(b) Following the argument in [14, 26], we can "almost" prove Theorem 1.2 for $\alpha > 2$ without assuming the bounds on $D^{*n}(x)$. The shortcoming is the restriction $d > 10$, not $d > 6$, for percolation. This is due to the peculiar diagrammatic estimate in [14], which we do not use in this paper.

(c) The asymptotic behavior of $G_{p_c}(x)$ in (1.9) or (1.24) is a key element for the so-called 1-arm exponent to take on its mean-field value [16, 19, 22, 25]. For finite-range critical percolation, for example, the probability that $o \in \mathbb{Z}^d$ is connected to the surface of the d-dimensional ball of radius r centered at o is bounded above and below by a multiple of r^{-2} in high dimensions [22]. The value of the exponent may change in a peculiar way depending on the value of α [19].

(d) As described in (1.25), the constant A exhibits crossover between $\alpha < 2$ and $\alpha > 2$; in particular, $A = p_c$ for $\alpha < 2$. According to some rough computation, it seems that the asymptotic expression of $G_{p_c}(x)$ for $\alpha = 2$ is a mixture of those for $\alpha < 2$ and $\alpha > 2$, with a logarithmic correction:

$$G_{p_c}(x) \underset{|x|\to\infty}{\sim} \frac{\gamma_2/v_2}{p_c|x|^{d-2}\log|x|}. \tag{1.26}$$

One of the obstacles to prove this conjecture is a lack of good control on convolutions of the RW Green's function and the lace-expansion coefficients for $\alpha = 2$. As hinted in the above expression, we may have to deal with logarithmic factors more actively than ever. We are currently working in this direction.

2 Work with professor Shu-Chiuan Chang

Domany and Kinzel [10] defined a solvable version of compact directed percolation on the square lattice in 1981 as follows. For a fixed $p \in (0,1)$, each vertical bond is directed upward with occupation probability p (independently of the other bonds) and each horizontal bond is directed rightward with occupation probability 1. Furthermore, it is known that the boundary of the Domany-Kinzel model has the same distribution as the one-dimensional last passage percolation model [12]. Recently, one of the authors considered a version of directed percolation on the square lattice whose vertical edges occupied with a probability p_v and horizontal edges in the n-th row occupied with a probability 1 if n is even and p_h if n is odd [6]. Particularly for $p_h = 0$ or 1, that model reduces to the Domany-Kinzel model. In this article, we generalize further to consider a triangular lattice as follows. Instead of using regular triangles, it is easier to start from a square lattice with vertical probability y and horizontal probabilities 1 and x alternatively, then add diagonal edges from lower-left to upper-right or from lower-right to upper-left with probability d

Domany and Kinzel [10] defined a solvable version of directed percolation on the square lattice in 1981 as follows. For $p \in (0,1)$ fixed, each nearest neighbour vertical bond is directed upward with occupation probability p (independently of the other bonds) and each nearest neighbour horizontal bond is directed rightward with occupation probability one. It's known that the Domany-Kinzel model is a compact directed percolation and the boundary of the Domany-Kinzel model has the same distribution as the one dimensional last passage percolation model [12]. However the Domany-Kinzel model is essentially of a one-dimensional nature due to the restricted freedom in one spatial direction. To uncover the genuine nature of a two-dimensional directed percolation it is necessary to relax this unit-directional restriction. In this article, we consider a version of directed percolation on the square lattice whose vertical edges occupied with a probability p_v and horizontal edges in the n-th row occupied with a probability 1 if n is odd and p_h if n is even. Particularly, for $p_h = 0$ or 1 the model reduces to the Domany-Kinzel model. Besides, the model is not compact directed percolation for $p_h \in (0, 1)$ since the percolating configuration may has isolated vertices (as shown in Figure 1). However the volume of isolated vertices is finite almost surely, it is believed that the critical behavior of the model refer to the compact directed percolation universality class and not to the habitual directed percolation class.

The vertices (sites) of the triangular lattice are now located at a two-dimensional rectangular net $\{(m, n) \in \mathbb{Z} \times \mathbb{Z}_+ : -M \leq m \leq M \text{ and } 0 \leq n \leq N\}$. Consider the probabilities $x \in [0, 1]$, $y \in [0, 1)$ and $d \in [0, 1)$ but $(1-y)(1-d) \neq 1$, i.e., d and y should not be zero simultaneously, throughout this article, and the percolation always starts from the origin $(0, 0)$. We say that the vertex (m, n) is percolating if there is at least one connected-directed path of occupied edges from $(0, 0)$ to (m, n). Given any $\alpha \in \mathbb{R}$, let $N_\alpha = \lfloor \alpha N \rfloor = \sup\{m \in \mathbb{Z} : m \leq \alpha N\}$ with $N \in \mathbb{Z}_+$. It is clear that $\alpha \geq 0$ for the triangular lattice with diagonal edges from lower-left to upper-right and $\alpha \geq -1$ with diagonal edges from lower-right to upper-left. Let us define

$$\alpha_{\min} = \begin{cases} 0 & \text{if diagonal edges from lower-left to upper-right} , \\ -1 & \text{if diagonal edges from lower-right to upper-left} . \end{cases}$$

Denote $\mathbb{P}$ as the probability distribution of the bond variables, and define the two point correlation function

$$\tau(N_\alpha, N) = \mathbb{P}((N_\alpha, N) \text{ is percolating}) .$$

It is appropriate to define some of the standard critical exponents and to sketch the phenomenological scaling theory of $\tau(N_\alpha, N)$. For $\alpha < \alpha_c$ and α close to α_c, the scaling theory of critical behavior asserts that the singular part of $\tau(N_\alpha, N)$ varies asymptotically as (c.f. [15])

$$\tau(N_\alpha, N) \approx \exp\Big(\frac{-BN}{(\alpha_c - \alpha)^{-\nu}}\Big) , \tag{2.1}$$

where the notation $f_{1,\alpha}(N) \approx f_{2,\alpha}(N)$ means that $\lim_{N\to\infty} \log f_{1,\alpha}(N)/\log f_{2,\alpha}(N) = 1$. The constants B and critical exponent $\nu \in (0, \infty)$ are universal constants and do not depend on α [11]. Note that there has been no general proof of the existence of the critical exponents. For $\alpha < \alpha_c$, the critical exponent of the correlation length $\nu = 2$ as shown below is the same as what was found in the Domany-Kinzel model [10, 27, 21, 20]. The consideration here generalizes and amends the corresponding results for the square lattice in Ref. [6]. The main purpose of this article is to find the critical value

$$\alpha_c = \frac{d - y - dy}{2(d + y - dy)} + \frac{1 - (1-d)^2(1-y)^2 x}{2(d + y - dy)^2} \tag{2.2}$$

for the triangular lattice with diagonal edges from lower-left to upper-right, and the critical value

$$\alpha_c = -\frac{3d + y - dy}{2(d + y - dy)} + \frac{1 - (1-d)^2(1-y)^2 x}{2(d + y - dy)^2} \tag{2.3}$$

for the triangular lattice with diagonal edges from lower-right to upper-left, such that

$$\lim_{N\to\infty} \tau(2N_\alpha, 2N) = \begin{cases} 1 & \text{if } \alpha > \alpha_c \ , \\ 0 & \text{if } \alpha < \alpha_c \ , \\ \frac{1}{2} & \text{if } \alpha = \alpha_c \ . \end{cases} \tag{2.4}$$

Notice that we use the same symbol α_c to denote the critical value for the triangular lattice with diagonal edges either from lower-left to upper-right or from lower-right to upper-left, because (2.4) and the following theorems apply to both cases. The meaning will be clear from context. We also obtain the values of ν and B for the triangle lattice. We use large derivation argument and the Berry-Esseen theorem to quantify the rate.

First we study the rate of convergence of $\tau(2N_\alpha, 2N)$ for a fixed α. For notation convenience, define

$$a = 1 + b - (d + y - dy)^2 \ , \quad b = (1-d)^2(1-y)^2 x \tag{2.5}$$

from now on, thus (2.2) and (2.3) can be written as

$$\alpha_c = \frac{1}{2} - \frac{y}{d+y-dy} + \frac{1-b}{2(1-a+b)}$$

for the triangular lattice with diagonal edges from lower-left to upper-right, and the critical value

$$\alpha_c = -\frac{1}{2} - \frac{d}{d+y-dy} + \frac{1-b}{2(1-a+b)}$$

for the triangular lattice with diagonal edges from lower-right to upper-left. Moreover, we define

$$\underline{\alpha} = \begin{cases} \alpha_{\min} & \text{if} \quad \alpha_c < -\frac{3}{4} + \frac{\sigma}{2} \ , \\ \frac{-3+\sqrt{(4\alpha_c+3)^2-4\sigma^2}}{4} & \text{if} \quad \alpha_c \geq -\frac{3}{4} + \frac{\sigma}{2} \ , \end{cases}$$

where the variance is given by

$$\sigma^2 = \frac{2(1-y)dy - 1 - b}{1-a+b} + \frac{(1-b)^2}{(1-a+b)^2} \tag{2.6}$$

for the triangular lattice with diagonal edges from lower-left to upper-right, and

$$\sigma^2 = \frac{2(1-d)dy - 1 - b}{1-a+b} + \frac{(1-b)^2}{(1-a+b)^2} \tag{2.7}$$

for the triangular lattice with diagonal edges from lower-right to upper-left. Here we again use the same symbol σ^2 to denote the variance for the two cases. Furthermore it is easy to see that $\underline{\alpha} < \alpha_c$, and for $\alpha \in (\underline{\alpha}, \alpha_c)$ we have

$$\left(\frac{4(\alpha+\alpha_c)+6}{\sigma^2}\right)(\alpha_c - \alpha) < 1 \ .$$

Theorem 2.1. *Given $x \in [0,1]$, $y \in [0,1)$, $d \in [0,1)$ with $(1-y)(1-d) \neq 1$ and the critical aspect ratio α_c in (2.2) or (2.3), the asymptotic behavior of the two point correlation function is*

$$\begin{cases} \tau(2N_\alpha, 2N) & \approx \exp\big(-2NI(\alpha)\big) \quad \text{for} \quad \alpha < \alpha_c \ , \\ \tau(2N_\alpha, 2N) & = \frac{1}{2} + O(\frac{1}{\sqrt{N}}) \quad \text{for} \quad \alpha = \alpha_c \ , \\ 1-\tau(2N_\alpha, 2N) & \approx \exp\big(-2NI(\alpha)\big) \quad \text{for} \quad \alpha > \alpha_c \ , \end{cases} \tag{2.8}$$

where

$$\frac{1}{\sigma^2}(\alpha_c - \alpha)^2 \leq I(\alpha) \leq -\ln y \quad \text{for} \quad \alpha \in (\alpha_{\min}, \alpha_c) \tag{2.9}$$

$$\frac{1}{\sigma^2}(\alpha_c - \alpha)^2 \leq I(\alpha) \leq \frac{\frac{1}{\sigma^2}(\alpha_c - \alpha)^2}{1 - \big(\frac{4(\alpha+\alpha_c)+6}{\sigma^2}\big)(\alpha_c - \alpha)} \quad \text{for} \quad \alpha \in (\underline{\alpha}, \alpha_c) \tag{2.10}$$

$$\frac{\frac{1}{\sigma^2}(\alpha_c - \alpha)^2}{1 + \big(\frac{4(\alpha+\alpha_c)+6}{\sigma^2}\big)(\alpha - \alpha_c)} \leq I(\alpha) \leq \frac{1}{\sigma^2}(\alpha_c - \alpha)^2 \quad \text{for} \quad \alpha > \alpha_c \ . \tag{2.11}$$

Furthermore,

$$\begin{cases} \tau(2N_\alpha, 2N) \leq \exp\big(\frac{-2N}{\sigma^2}(\alpha_c - \alpha)^2\big) & \text{for} \quad \alpha \in (\underline{\alpha}, \alpha_c) \ , \\ 1-\tau(2N_\alpha, 2N) \leq \exp\big(\frac{2(\alpha-\alpha_c)}{\sigma^2}\big) \exp\Big(\frac{\frac{-2N}{\sigma^2}(\alpha_c-\alpha)^2}{1+\big(\frac{4(\alpha+\alpha_c)+6}{\sigma^2}\big)(\alpha-\alpha_c)}\Big) & \text{for} \quad \alpha > \alpha_c \ . \end{cases} \tag{2.12}$$

By Theorem 2.1, we have the following theorem:

Theorem 2.2. *Given $x \in [0,1]$, $y \in [0,1)$, $d \in [0,1)$ with $(1-y)(1-d) \neq 1$ and the critical aspect ratio α_c in (2.2) or (2.3), inequalities*

$$\frac{\tau(2N_\alpha, 2N+1)}{\tau(2N_\alpha, 2N)} \leq 1 \ , \qquad \frac{\tau(2N_\alpha, 2N+2)}{\tau(2N_\alpha, 2N)} \leq 1$$

hold and the asymptotic behavior of the two point correlation function is

$$\begin{cases} \tau(N_\alpha, N) & \approx \exp\big(-NI(\alpha)\big) \quad \text{for} \quad \alpha < \alpha_c \ , \\ \tau(N_\alpha, N) & = \frac{1}{2} + O(\frac{1}{\sqrt{N}}) \quad \text{for} \quad \alpha = \alpha_c \ , \\ 1-\tau(N_\alpha, N) & \approx \exp\big(-NI(\alpha)\big) \quad \text{for} \quad \alpha > \alpha_c \ . \end{cases}$$

Remark 2.3. *Theorem 2.1 and Theorem 2.2 lead to the following information:*

1. *The function $I(\alpha)$ for $\alpha \neq \alpha_c$ does not have a simple expression. However, for the original Domany-Kinzel model on the square lattice (i.e., $d = 0$, $x = 1$), it is given by*

$$I(\alpha) = \alpha \ln\Big(\frac{\alpha}{(1-y)(1+\alpha)}\Big) - \ln\big(y(1+\alpha)\big) \ . \tag{2.13}$$

2. *For $d = 0$, the expressions of α_c in (2.2), (2.3) and the expressions of σ^2 in (2.6), (2.7) reduce to those for the square lattice in [6].*

3. *For $x = 1$, our model corresponds to a Domany-Kinzel model on the $2N_\alpha \times 2N$ triangular lattice. (2.2) and (2.6) lead to $\alpha_c = (1-y)/(d+y-dy)$, $\sigma^2 = 2(1-y)(1-d+dy)/(d+y-dy)^2$ for the triangular lattice with diagonal edges from lower-left to upper-right. (2.3) and (2.7) lead to $\alpha_c = (1-2d-y+dy)/(d+y-dy)$, $\sigma^2 = 2(1-d)(1-y+dy)/(d+y-dy)^2$ for the triangular lattice with diagonal edges from lower-right to upper-left.*

4. *Our result gives that $\tau(N_\alpha, N)$ with $\alpha < \alpha_c$ and $1-\tau(N_\alpha, N)$ with $\alpha > \alpha_c$ both decay exponentially to zero. Furthermore, we obtain $B = 1/\sigma^2$ and the critical exponent $\nu = 2$ in (2.1) for $\alpha < \alpha_c$.*

5. *We obtain the similar result on the honeycomb lattice (c.f. [7]).*

Finally, we investigate the asymptotic phenomena of $\tau(N_{\alpha_N^-}, N)$ and $\tau(N_{\alpha_N^+}, N)$ where $\alpha_N^+ \downarrow \alpha_c$ and $\alpha_N^- \uparrow \alpha_c$ as $N \uparrow \infty$. A sequence $\{\ell_n\}_{n=1}^\infty$ is called a regularly varying sequence if for any $\lambda \in (0,\infty)$, $\lim_{n\to\infty} \ell_{\lfloor \lambda n \rfloor}/\ell_n = 1$. For example, $\ell_n = \log n$ or $\ell_n = c \in (0,\infty)$ for all n. For convenience, we denote $\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^x e^{-\frac{u^2}{2}}\, du$ as the standard cumulative distribution function of *Gaussian* distribution with mean 0, variance 1 and let $\Psi(x) = 1 - \Phi(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-\frac{u^2}{2}}\, du$. It is not difficult to see that

$$\Psi(x) = \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}x}\big(1 + O(x^{-2})\big) \quad \text{when } x \text{ is large.}$$

Theorem 2.4. *Given $x \in [0,1]$, $y \in [0,1)$, $d \in [0,1)$ with $(1-y)(1-d) \neq 1$, $\rho \in (0,\infty)$ and a positive regularly varying sequence $\{\ell_n\}_{n=1}^\infty$. Denote $\alpha_N^- = \alpha_c - \sigma N^{-\rho}\ell_N/\sqrt{2}$ and $\alpha_N^+ = \alpha_c + \sigma N^{-\rho}\ell_N/\sqrt{2}$, then both*

$$\tau(N_{\alpha_N^-}, N)\ , \qquad 1 - \tau(N_{\alpha_N^+}, N)$$
$$\begin{cases} \approx \exp(-N^{-2\rho+1}\ell_N^2) & \text{if} \quad \rho \in (0,\frac{1}{2}) \\ \approx \exp(-\ell_N^2) & \text{if} \quad \rho = \frac{1}{2},\ \ell_N \to \infty \\ = \Psi(\ell) + O(1)\max\{\frac{1}{\sqrt{N}}, |\ell - \ell_N|\} & \text{if} \quad \rho = \frac{1}{2},\ \ell_N \to \ell \in [0,\infty) \\ = \frac{1}{2} + O(1)N^{-\rho+\frac{1}{2}}\ell_N & \text{if} \quad \rho \in (\frac{1}{2}, 1) \\ = \frac{1}{2} + O(\frac{1}{\sqrt{N}}) & \text{if} \quad \rho \in [1,\infty) \end{cases} .$$

Note that $\rho = \frac{1}{2}$ is a critical value and we have the following corollary.

Corollary 2.5. *Under the same assumptions of Theorem 2.2, we have*

$$\lim_{N\to\infty} \tau(N_{\alpha_N^-}, N) = \lim_{N\to\infty}\Big(1 - \tau(N_{\alpha_N^+}, N)\Big) = \begin{cases} 0 & \text{if} \quad \rho \in (0,\frac{1}{2})\ , \\ \frac{1}{2} & \text{if} \quad \rho \in (\frac{1}{2},\infty)\ . \end{cases}$$

When $\rho = 1/2$ and $\ell_N \to \ell \in [0,\infty]$, we have

$$\lim_{N\to\infty} \tau(N_{\alpha_N^-}, N) = \exp(-\ell^2)\ , \quad \lim_{N\to\infty} \tau(N_{\alpha_N^+}, N) = 1 - \exp(-\ell^2)\ .$$

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科技部補助專題研究計畫執行國際合作與移地研究心得報告（一）

日期 104 年 12 月 3 日

計畫編號	MOST 102 － 2115 － M － 004 － 005 － MY2		
計畫名稱	高維度下具長域自我相斥隨機漫步,滲流與 Ising 模型的兩點函數之臨界行為		
出國人員姓名	陳隆奇	服務機構及職稱	政治大學應用數學系 副教授
出國時間	103 年 3 月 20 日至 103 年 3 月 29 日	出國地點	Mathematisch Instituut, Leiden 大學，荷蘭
出國研究目的	□實驗 □田野調查 □採集樣本 X 國際合作研究 □使用國外研究設施		

一、執行國際合作與移地研究過程

由於到香港參與世界統計會議後我這學年度的計劃參與國際會議還剩有兩萬八千多元，正巧於今年年初荷蘭 Mathematisch Instituut, Leiden 大學 Markus Heydenreich 教授要請我到他任職的學校訪問，因為 Markus Heydenreich，阪井哲和我想一起合作一個計劃。最初我告訴 Heydenreich 教授因我經費不足且時間正好時學期間，所以不便前往，但 Heydenreich 教授表達非常希望我能前往的意願，同時他說他可補助我交通費與住宿費，故我答應他的邀請且把我剩下的參與國際會議的經費轉成移地研究費（當做我的生活費），於三月 20-29 日訪問 Mathematisch Instituut, Leiden 大學並且在機率 seminar 上給個五十分鐘的報告。在此次訪問的期間，我認識許多荷蘭的機率學家或是在 Leiden 大學訪問的學者，且與 Heydenreich 教授和阪井哲教授有愉快且有效率的學術交流。可說成果豐富。

二、研究成果

由於數學的研究並不是馬上就會有結果，此次訪問最大的成果是擴展本人研究的領域與認識同領與的專家。

三、建議

無。

四、本次出國若屬國際合作研究，雙方合作性質係屬：(可複選)

X 分工收集研究資料

X 交換分析實驗或調查結果

X 共同執行理論建立模式並驗証

X 共同執行歸納與比較分析

☐元件或產品分工研發

☐其他 (請填寫) _______

五、其他

無。

科技部補助專題研究計畫執行國際合作與移地研究心得報告（二）

日期 104 年 12 月 3 日

計畫編號	MOST 102 － 2115 － M － 004 － 005 － MY2		
計畫名稱	高維度下具長域自我相斥隨機漫步,滲流與 Ising 模型的兩點函數之臨界行為		
出國人員姓名	陳隆奇	服務機構及職稱	政治大學應用數學系 副教授
出國時間	104 年 1 月 20 日至 104 年 1 月 26 日	出國地點	中國北京師範大學，數學系
出國研究目的	☐實驗 ☐田野調查 ☐採集樣本 X 國際合作研究 ☐使用國外研究設施		

一、執行國際合作與移地研究過程

103 年六月 30 日到七月 3 日在臺北舉辦第三屆泛太平洋數學統計會議，此次會議我與北師大的何輝教授相談甚歡，由於時間短促，他邀請我於 104 年一月 20 日到一月 26 日訪問北師大與他進行更進一步的學術交流，於是促成此次訪問。在北師大訪期間，我不但與何輝教授有學術交流，且認識許多北師大的機率領與的教授與在那期間訪問北師大的機率專家，成果豐盛。

二、研究成果

雖然與何輝教授沒有談到彼此可合作的問題，但彼此有更進一步的認識，期待有與他合作的機會。此次訪問最大的成果是擴展本人研究的領域與認識同領與的專家。

三、建議

無。

四、本次出國若屬國際合作研究，雙方合作性質係屬：(可複選)

X 分工收集研究資料
X 交換分析實驗或調查結果
☐共同執行理論建立模式並驗証
☐共同執行歸納與比較分析
☐103 年六月 30 日到七月 3 日元件或產品分工研發
☐其他 (請填寫) _______

五、其他

無。

科技部補助專題研究計畫執行國際合作與移地研究心得報告（三）

日期 104 年 12 月 3 日

計畫編號	MOST 102－2115－M－004－005－MY2		
計畫名稱	高維度下具長域自我相斥隨機漫步,滲流與 Ising 模型的兩點函數之臨界行為		
出國人員姓名	陳隆奇	服務機構及職稱	政治大學應用數學系 副教授
出國時間	104 年 9 月 6 日至 104 年 9 月 13 日	出國地點	日本北海道大學，數學系
出國研究目的	□實驗 □田野調查 □採集樣本 X 國際合作研究 □使用國外研究設施		

一、執行國際合作與移地研究過程

此次出訪，主要是想完成與 Akira Sakai 教授合作的問題，由於我和 Akira Sakai 教授暑假都有空的時間就剩下九月，故我利用開學前到北海道大學訪問他，因本人經費有限同時也，非常感謝 Akira Sakai 教授幫我出機票錢。雖我們在我訪問前間，努力交換意見設法解決困難的部分，但很可惜，我們目前還沒有辦法解決此問題，Akira Sakai 教授也將於十一月底來台與我面對面討論如何解決之。

二、研究成果

雖然與 Akira Sakai 教授還沒有完成我們合作的問題，但我們相信我們終究可以解決它，透過彼此面對面地討論，才真的明白問題的困難與關鍵點在哪，著就是此次出訪的收獲。

三、建議

無。

四、本次出國若屬國際合作研究，雙方合作性質係屬：(可複選)

X 分工收集研究資料

X 交換分析實驗或調查結果

X共同執行理論建立模式並驗証

X 共同執行歸納與比較分析

□103 年六月 30 日到七月 3 日元件或產品分工研發

□其他 (請填寫) ________

五、其他

無。

科技部補助專題研究計畫出席國際學術會議心得報告（一）

日期：104 年 12 月 3 日

計畫編號	MOST 102 － 2115 － M － 004 － 005 － MY2		
計畫名稱	高維度下具長域自我相斥隨機漫步,滲流與 Ising 模型的兩點函數之臨界行為		
出國人員姓名	陳隆奇	服務機構及職稱	政治大學應用數學系 副教授
會議時間	102 年 8 月 25 日至 102 年 8 月 30 日	會議地點	Grand Hall of the Hong Kong Convention, Hong Kong, China
會議名稱	(中文) ５９屆世界統計會議 (英文) **The 59th World Statistics Congress**		
發表題目	(中文) (英文) Critical two-point functions for long-range self-avoiding walk in high dimensions		

一、參加會議經過

參與此會議是因為英國 Brunel University 徐禮虎教授邀請我在世界統計會議上 Recent advance in stochastic processes and their applications 給個二十五分鐘報告。因我重未在此會議上給個報告，且我在新加坡大學的朋友孫嶸楓教授和香港科技大學的朋友鄭新華教授都在此會議上給個報告，故我欣然接受徐教授的邀請。

二、與會心得

由於參與此會議的學者太多，且我們沒有合適的地方討論問題，會議好像一場聯誼會一樣，所以感覺有點浪費時間。下次我可能不再參與此會議。

三、發表論文全文或摘要

We consider long-range self-avoiding walk on Z^d that is defined by power-law decaying pair potentials of the form $D(x) \asymp |x|^{-d-\alpha}$ with $\alpha > 0$. The upper-critical dimension d_c is $2(\alpha \wedge 2)$. In this talk, I present that for $d > d_c$ (and the spread-out parameter sufficiently large), and $\alpha \neq 2$, the Green function G(x) is asymptotically $C|x|^{\alpha \wedge 2-d}$, where the constant C $\in$ $(0,\infty)$ is expressed in terms of lace-expansion coefficients and exhibits crossover between $\alpha < 2$ and $\alpha > 2$.

科技部補助專題研究計畫出席國際學術會議心得報告（二）

日期：104 年 12 月 3 日

計畫編號	MOST 102－2115－M－004－005－MY2		
計畫名稱	高維度下具長域自我相斥隨機漫步,滲流與 Ising 模型的兩點函數之臨界行為		
出國人員姓名	陳隆奇	服務機構及職稱	政治大學應用數學系 副教授
會議時間	104 年 8 月 10 日 至 104 年 8 月 14 日	會議地點	National convention center, Beijing, China
會議名稱	(中文) 第八屆國際工業與應用數學會議 (英文) The 8-th International Congress on Industrial and Applied mathematics		
發表題目	(中文) (英文) Asymptotic Behavior for Long-Range Self-Avoiding Walks in high dimensions		

一、參加會議經過

參與此會議是因為北京首都師範大學吳憲遠教授邀請我在第八屆國際工業與應用數學會議上給個三十分鐘報告，順道訪問首都師範大學與吳憲遠教授與他作學術交流。
吳教授與我研究領域有些雷同，故我們彼此交談歡甚歡，雖我們目前沒有合作的問題，但彼此有更深入的交流，期待我們以後有合作文章的機會。

二、與會心得

因我轉到政治大學剛滿一年，學校鼓勵老師參與國際會議與跨國學術交流，所以補助我此次出訪的機票與生活費，但我不知此會議的註冊費如此貴且參與人數帶多，超乎我想像，故轉移部分科技部雜費作為註冊費。下次若有類似註冊費如此高且參與人數眾多的會議，考慮不參加。

三、發表論文全文或摘要

We consider long-range self-avoiding walk on Z^d where 1-step distribution is given by D. Suppose D(x) decays as $|x|^{-d-\alpha}$ with $\alpha > 0$. The upper-critical dimension d_c is $2(\alpha \wedge 2)$ for self-avoiding walk. Assume certain heat-kernel bounds on r=the n-step distribution of the underlying random walk. In this talk, I present that the critical two-point function obeys various critical exponents take on their respective mean-filed values if the $d > d_c$.

科技部補助計畫衍生研發成果推廣資料表

日期:2015/11/06

科技部補助計畫	計畫名稱: 高維度下具長域自我相斥隨機漫步，滲流與 Ising模型的兩點函數之臨界行為
	計畫主持人: 陳隆奇
	計畫編號: 102-2115-M-004-005-MY2　　　學門領域: 財務數學

無研發成果推廣資料

102年度專題研究計畫研究成果彙整表

<table>
<tr><td colspan="3">**計畫主持人：**陳隆奇</td><td colspan="5">**計畫編號：**102-2115-M-004-005-MY2</td></tr>
<tr><td colspan="8">**計畫名稱：**高維度下具長域自我相斥隨機漫步，滲流與 Ising模型的兩點函數之臨界行為</td></tr>
<tr><td colspan="3" rowspan="2">成果項目</td><td colspan="3">量化</td><td rowspan="2">單位</td><td rowspan="2">備註（質化說明：如數個計畫共同成果、成果列為該期刊之封面故事...等）</td></tr>
<tr><td>實際已達成數（被接受或已發表）</td><td>預期總達成數（含實際已達成數）</td><td>本計畫實際貢獻百分比</td></tr>
<tr><td rowspan="12">國內</td><td rowspan="4">論文著作</td><td>期刊論文</td><td>0</td><td>0</td><td>100%</td><td rowspan="3">篇</td><td></td></tr>
<tr><td>研究報告/技術報告</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
<tr><td>研討會論文</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
<tr><td>專書</td><td>0</td><td>0</td><td>100%</td><td>章/本</td><td></td></tr>
<tr><td rowspan="2">專利</td><td>申請中件數</td><td>0</td><td>0</td><td>100%</td><td rowspan="2">件</td><td></td></tr>
<tr><td>已獲得件數</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
<tr><td rowspan="2">技術移轉</td><td>件數</td><td>0</td><td>0</td><td>100%</td><td>件</td><td></td></tr>
<tr><td>權利金</td><td>0</td><td>0</td><td>100%</td><td>千元</td><td></td></tr>
<tr><td rowspan="4">參與計畫人力（本國籍）</td><td>碩士生</td><td>1</td><td>0</td><td>100%</td><td rowspan="4">人次</td><td></td></tr>
<tr><td>博士生</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
<tr><td>博士後研究員</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
<tr><td>專任助理</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
<tr><td rowspan="12">國外</td><td rowspan="4">論文著作</td><td>期刊論文</td><td>3</td><td>3</td><td>100%</td><td rowspan="3">篇</td><td></td></tr>
<tr><td>研究報告/技術報告</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
<tr><td>研討會論文</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
<tr><td>專書</td><td>0</td><td>0</td><td>100%</td><td>章/本</td><td></td></tr>
<tr><td rowspan="2">專利</td><td>申請中件數</td><td>0</td><td>0</td><td>100%</td><td rowspan="2">件</td><td></td></tr>
<tr><td>已獲得件數</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
<tr><td rowspan="2">技術移轉</td><td>件數</td><td>0</td><td>0</td><td>100%</td><td>件</td><td></td></tr>
<tr><td>權利金</td><td>0</td><td>0</td><td>100%</td><td>千元</td><td></td></tr>
<tr><td rowspan="4">參與計畫人力（外國籍）</td><td>碩士生</td><td>0</td><td>0</td><td>100%</td><td rowspan="4">人次</td><td></td></tr>
<tr><td>博士生</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
<tr><td>博士後研究員</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
<tr><td>專任助理</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
<tr><td colspan="2">其他成果
（無法以量化表達之成果如辦理學術活動、獲得獎項、重要國際合作、研究成果國際影響力及其他協助產業技術發展之具體效益事項等，請以文字敘述填列。）</td><td colspan="6">無。</td></tr>
</table>

	成果項目	量化	名稱或內容性質簡述
科教處計畫加填項目	測驗工具(含質性與量性)	0	
	課程/模組	0	
	電腦及網路系統或工具	0	
	教材	0	
	舉辦之活動/競賽	0	
	研討會/工作坊	0	
	電子報、網站	0	
	計畫成果推廣之參與（閱聽）人數	0	

科技部補助專題研究計畫成果報告自評表

請就研究內容與原計畫相符程度、達成預期目標情況、研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）、是否適合在學術期刊發表或申請專利、主要發現或其他有關價值等，作一綜合評估。

1. 請就研究內容與原計畫相符程度、達成預期目標情況作一綜合評估

■達成目標

□未達成目標（請說明，以100字為限）

□實驗失敗

□因故實驗中斷

□其他原因

說明：

2. 研究成果在學術期刊發表或申請專利等情形：

論文：■已發表 □未發表之文稿 □撰寫中 □無

專利：□已獲得 □申請中 ■無

技轉：□已技轉 □洽談中 ■無

其他：（以100字為限）

3. 請依學術成就、技術創新、社會影響等方面，評估研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）（以500字為限）

此次計劃，最大的收穫是與日本北海道大學Akira Sakai教授完成的統計力學上一些具長域模型（Ising model, percolation 和 Self-avoiding walk）的兩點函數的漸進行為與收斂速度。兩點函數是一個重要的物理量，透過兩點函數我們可獲的許多重要的資訊，因此此結果不單是數學上的重要結果，物理學家也十分重視此量，所以很順利於２０１５年初刊出在Ann. Probab. 並且我們持續研究後續的問題。此外本人與成大物理系張書銓教授探討在二維三角晶格與蜂窩狀晶格上特殊的定向滲流之兩點函數的臨界行為與收斂數度也有兩篇文章分別發表于 J. Stat. Phys. 和 Physica A， 二維的定向滲流，幾乎沒有任何結果，所以我們探討在特殊的模型下兩點函數之收斂速度的上界估計與下界估計的結果，並且我們也持續研究此類的問題。