

科技部補助專題研究計畫成果報告

期末報告

多變量複合卜瓦松跳躍擴散模型與高頻資料下之選擇權評價與
投資組合策略之研究(第2年)

計畫類別：個別型計畫

計畫編號：MOST 102-2410-H-004-042-MY2

執行期間：103年08月01日至104年10月31日

執行單位：國立政治大學金融系

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處理方式：

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中華民國 104 年 12 月 30 日

中文摘要 : (第一年)

本文利用一個多變量複合卜瓦松模型(Multivariate compound Poisson diffusion model)來描述資產價格的動態過程，此模型不僅能解釋資產跳躍亦能解釋資產間共同跳躍情況，並利用Esscher測度轉換得到一個風險中利的資產動態過程，並將此模型應用到互換選擇權評價來探討共同跳躍對於選擇權評價之影響。研究發現當資產間共同跳躍次數愈高，選擇權價值愈高。

(第二年)

隨著資產間的共同移動與共同跳躍的現象加劇，過去建構在資產動態服從幾何布朗運動的情況將無法描繪。本文提出一個多變量複合卜瓦松跳躍擴散進一步捕捉資產間的共同移動現象，同時也將共同跳躍的現象納入到模型中，並透過Markowitz的平均數-變異數法則來建構投資組合。研究結果發現當共同跳躍次數的增加，將增加資產間的相關係數，這使得投資組合的風險分散的效果遞減，此外，當有重大的系統性風險發生時，共同跳躍的次數也會增加，且當共同跳躍的頻率增加至某一程度時，資產間的相關係數將達到1，此時，投資組合的建構將無法達到風險分散的效果。

中文關鍵詞 : 共同跳躍、平均數-變異數法則、多變量複合卜瓦松跳躍擴散

英文摘要 : (First Year)

In this study, we investigate the valuation of European exchange options under a two-asset jump-diffusion process with correlations, where both individual jumps and cojumps in the underlying stock price dynamics are modeled by two independent compound Poisson processes with log-normal jump sizes. The Esscher transform technique is applied to provide an efficient way for exchange option valuation under an incomplete market setting. The estimated results and numerical examples are provided to illustrate the impact of cojumps on option prices.

(Second Year)

The phenomena of co-movement and co-jump among assets become more and more frequent and result in the dynamic process built basing on the geometric Brownian motion cannot depict these anymore. For the sake of taking co-movement and co-jump among assets, we propose a new process called multivariate compound poisson diffusion model to more accurately model the dynamics of asset price. In addition, we use the mean-variance method proposed by Markowitz to construct the portfolio and investigate the impacts of the co-jump on the portfolio construction. We find the increasing of co-jump intensity will also increase the correlation between assets and result in decreasing the effect of risk diversification through portfolio construction. Further, we also find the major systematic risk occurs, such as subprime crisis, the intensity of co-jump will also increase.

英文關鍵詞：Co-jump, mean-variance, multivariate compound poisson jump diffusion, Esscher transform

(多變量複合卜瓦松跳躍擴散模型與高頻資料下之選擇權評價與投資組合策略之研究第一年)

Cojump phenomena and their impacts on exchange option pricing[☆]

Abstract

In this study, we investigate the valuation of European exchange options under a two-asset jump-diffusion process with correlations, where both individual jumps and cojumps in the underlying stock price dynamics are modeled by two independent compound Poisson processes with log-normal jump sizes. The Esscher transform technique is applied to provide an efficient way for exchange option valuation under an incomplete market setting. The estimated results and numerical examples are provided to illustrate the impact of cojumps on option prices.

JEL classification: C58; G12

Keywords: European exchange option; Two-asset jump-diffusion process; Cojump; Esscher transform

1. Introduction

Many derivative contracts traded in exchanges and in over-the-counter markets, such as exchange options, have payoffs depending on more than one asset and are most commonly used in stock markets and foreign exchange markets. For pricing exchange options explicitly and practically, a necessary step is to establish a dynamic model for the asset fluctuations. Margrabe (1978) initially introduces the pricing formula for an exchange option, in which the pure diffusion dynamics of both stock prices are log-normal with correlated Wiener processes. In addition, Bjerksund and Stensland (1993), Broadie and Detemple (1997), and Lindset (2007) show the pricing formulas for the American exchange options. Nevertheless, increasing empirical evidences have revealed that the geometric Brownian motion is not completely consistent with the reality. In other words, there exist empirically observed jumps or extreme events in stock prices (Eraker et al., 2003; Eraker, 2004; Maheu and McCurdy, 2004). Although the price behavior for a single asset is already known to jump individually, the occurrence of cojumps for both asset prices is also possibly existed (Chan, 2003; Lu et al., 2010; Lahaye et al., 2011; Dungey and Hvozdyk, 2012). Cojump refers to the presence of simultaneous discontinuities in two asset price series. In view of the aforementioned literature, this study proposes a dynamic model for two asset prices with illustrating such a cojump phenomenon, in which the

cojump event is governed by a bivariate compound Poisson process with a log-normal jump size. The model captures not only the existence of jump phenomena but seems to generally describe the cojumping behavior of two asset prices as well.

The datasets used in the descriptive analysis of Table 1 and Table 2 consist of the multiple daily stock prices. Analyzing stock returns makes us to investigate the potential effects of different jump behaviors. Table 1 and Table 2 show several significant jumps of 30 components of the Down Jones Industrial Average (DJIA) in the daily data¹ after the U.S. subprime financial crisis and before this turmoil. Furthermore, we also find evidences of cojumps between the 30 components in Table 3 and Table 4. A further implication of Table 3 and Table 4 is that in the sample data the jumping and cojumping behavior of two stock prices could happen over time. More specifically, the global financial turmoil does really have influence of changing the jump relation between two asset prices.

[Insert **Table 1–4**]

A bivariate Poisson distribution is a well-known bivariate discrete distribution (Holgate, 1964). It is appropriate for modeling paired counting data with correlation. In the case of a cojumping, the simultaneous jumps of two stock prices cannot be

¹ The empirical data are from Yahoo Finance and cover the period from 3 January 2005 to 31 December 2010.

considered independently. In this situation, a bivariate compound Poisson distribution is more appropriate when the counting distribution is bivariate Poisson and the size distribution is bivariate for the claim (Hesselager, 1996). In this study, instead of using constant arrival intensity under a pure Poisson process used in the Merton-type jump-diffusion process (Merton, 1976), we set the jump components of both stock prices to be a bivariate compound Poisson process whereby the intensity processes of individual jumps and cojumps are modeled at the same time.

Motivated by our empirical evidences in Table 1 and Table 3, in order to capture various ways of individual jumps and cojumps in stock prices generated by the actual market simultaneously, we model the price dynamics of two stocks by identifying a two-asset jump-diffusion process with both individual jumps and cojumps. Under such a dynamic process, the stock price can be decomposed into two parts, a continuous diffusion part driven by a two-dimensional geometric Brownian motion and a jump part with jump events modeled by a bivariate compound Poisson process where there are both individual jumps and cojumps. More exactly, if a jump event corresponds to an individual jump of the i^{th} stock price, then only the i^{th} stock price will jump; otherwise, if a jump event corresponds to a cojump, then both stock prices will jump with a positive probability. This dynamic model therefore provides a flexible framework to study the individual jumps and cojumps for both stock prices.

The market is incomplete in such a two-asset jump-diffusion economy, and we therefore make use of Esscher transform technique adopted from Gerber and Shiu (1994). By relaxing the assumption of a non-systematic jump risk (Merton, 1976), we select a pricing kernel and determine the Esscher parameters (risk premiums) for option valuation. In this study, we show that even in the presence of individual jumps and cojumps for both stock prices, the price of an European exchange option can be derived using the specific approach of Esscher transform.

This study makes three major contributions. First, it extends the relevant empirical literature by identifying the existence of individual jumps and cojumps in stock prices. The empirical evidences are significant for considering such phenomena in paired stock price modeling. Next, it extends the growing empirical literature on the jump behaviors of stock prices in the actual market. Analyzing these behaviors is critical for understanding the operation of the stock market and the risks involved. Finally, it extends the pricing literature on the valuation of European exchange options for an economy with non-systematic jump risks and takes the systematic risks of both individual jumps and cojumps into consideration in pricing European exchange options.

This article is organized as follows. The dynamic model is presented in Section 2.

Section 3 illustrates the risk-neutral pricing measure and the generalized exchange option pricing formula. In Section 4, we provide the empirical and numerical results. Section 5 concludes this study.

2. Model framework

As shown in Table 1 and Table 3, the individual jumps and cojumps do exist in stock price realizations. Therefore, we use a bivariate jump-diffusion process to model the stock prices. Let (Ω, \mathcal{F}, P) be a complete probability space, where P is the physical probability measure. A two-asset jump-diffusion process for the stock prices at time t , $S_i(t)$, can be set as

$$\frac{dS_i(t)}{S_i(t-)} = \mu_i - \lambda_i \kappa_i dt + \sigma_i dW_i(t) + d \left(\sum_{j=1}^{N_i(t)} \exp Y_{i,j} - 1 \right), \quad i = 1, 2, \quad (1)$$

where the appreciation rate μ_i and the volatility σ_i of the stock i are constants, $W_i(t)$ is a two-dimensional Wiener process with correlation ρ under P . The jump risk components are indicated by a bivariate Poisson process $N_i(t)$ with the constant arrival intensity λ_i , and the jump sizes are supposed to follow a log-normal distribution as in Merton (1976). $\{Y_{i,j} : j = 1, 2, \dots\}$ are the jump sizes of the stock i which are assumed to be independently identically distributed random variables. If a jump event occurs at time j , the jump size $Y_{i,j}$ is normally distributed with mean

u_i and variance δ_i^2 . Therefore, the mean percentage jump size of the i th stock price is $\kappa_i = E[\exp Y_{i,j} - 1] = \exp\left(u_i + \frac{1}{2}\delta_i^2\right) - 1 = \phi_{Y_i}(1) - 1$. In Eq. (1), we also assume that all of the random variables $W_i(t)$, $N_i(t)$, and $Y_{i,j}$ are mutually independent.

Considering the case of two correlated stock prices, we suppose that the two jump terms $N_1(t)$ and $N_2(t)$ are partially correlated, meaning that if one stock price jumps then the other will jump with a probability p . We construct them in such a manner using three independent Poisson processes denoted by $n_1(t)$, $n_2(t)$, and $n_c(t)$. The independent Poisson process $n_i(t)$ has intensity λ_i with discrete probability density function as follows:

$$P[n_i(t)=g]=\frac{\lambda_i t^g}{g!} \exp^{-\lambda_i t}, \quad (2)$$

for $i=\{1,2\}$. We define $N_i(t)=n_i(t)+n_c(t)$ and $N_i(t) \square \text{Poisson}(\lambda_i + \lambda_c)$. In other words, there are two types of jumps, individual jumps only for the i^{th} stock price with the arrival intensity λ_i and cojumps with the arrival intensity λ_c , for both stock prices. The Poisson processes $N_1(t)$ and $N_2(t)$ are able of producing the individual jumps through $n_1(t)$ and $n_2(t)$, as well as the cojumps through $n_c(t)$. A change of variables and integrating out $n_c(t)$ result in the joint probability density

function for $N_1(t)$ and $N_2(t)$, which is given by

$$P \ N_1(t) = m, N_2(t) = n$$

$$= \sum_{v=0}^{\min(m,n)} \exp - \lambda_l + \lambda_2 + \lambda_c t \frac{\lambda_l t^{m-v} \lambda_2 t^{n-v} \lambda_c t^v}{(m-v)! (n-v)! v!}, \quad (3)$$

Hence, Eq. (1) can be rewritten as

$$\frac{dS_i(t)}{S_i(t-)} = \mu_i - \lambda_i \kappa_i - \lambda_c \kappa_i dt + \sigma_i dW_i(t) + d \left(\sum_{j=1}^{n_i(t)+n_c(t)} \exp Y_{i,j} - 1 \right), \quad (4)$$

for $i = 1, 2$.

3. Change of measures and option pricing

The security economy defined by Eq. (4) is incomplete, this means that, under the assumption of no arbitrage opportunities in this market, there are infinitely many equivalent martingale measures with which to price options. We therefore need to determine a risk-neutral pricing measure Q . This implies changing the probability measure linked to the two-asset jump-diffusion process so that both stock prices discounted at the risk-free rate are Q -martingales. In such a two-asset jump-diffusion economy, we employ the Esscher transform developed by Gerber and Shiu (1994; 1996) to select the martingale pricing measure for pricing European exchange options.

3.1. Esscher transform for the two-asset jump-diffusion process

Relaxing the model assumptions of Merton (1976), and applying the Esscher transform for the two-asset jump-diffusion process we then determine a risk-neutral pricing measure. For $i \in \{1, 2\}$ and all $t \in [0, T]$, we decompose the two-asset jump-diffusion log-return process $Z_i(t) = \log S_i(t)/S_i(0) = C_i(t) + J_i(t) + J_c(t)$ into a continuous diffusion part $C_i(t) = \left(\mu_i - \frac{1}{2} \sigma_i^2 - \lambda_i \kappa_i - \lambda_c \kappa_i \right) t + \sigma_i W_i(t)$, an individual jump part $J_i(t) = \sum_{j=1}^{n_i(t)} Y_{i,j}$, and a cojump part $J_c(t) = \sum_{j=1}^{n_c(t)} Y_{i,j}$. Here, let $F_t^{W_i}$ and $F_t^{N_i}$ be the P -augmentation of the natural filtrations generated by $W_i(t)$ and $N_i(t)$, respectively, and define $F_t = F_t^{W_i} \vee F_t^{N_i} = F_t^{W_i} \vee F_t^{n_i} \vee F_t^{n_c}$ as the σ -algebra for each $t \in [0, T]$. Under a filtered probability space $(\Omega, \mathcal{F}, P, \{F_t\}_{t \in [0, T]})$, the Radon-Nikodym derivative of the Esscher transform is formally given by

$$\begin{aligned} \xi^{h^\theta}(t) &= \frac{dQ^{h^\theta}}{dP} \Big|_{F_t} = \frac{\exp h^{C_i} \sigma_i W_i(t)}{E \left[\exp h^{C_i} \sigma_i W_i(t) \Big| F_0^{W_i} \right]} \\ &\cdot \frac{\exp \left(h^{J_i} \sum_{j=1}^{n_i(t)} Y_{i,j} \right)}{E \left[\exp \left(h^{J_i} \sum_{j=1}^{n_i(t)} Y_{i,j} \right) \Big| F_0^{n_i} \right]} \cdot \frac{\exp \left(h^{J_c} \sum_{j=1}^{n_c(t)} Y_{i,j} \right)}{E \left[\exp \left(h^{J_c} \sum_{j=1}^{n_c(t)} Y_{i,j} \right) \Big| F_0^{n_c} \right]} \\ &= \text{ex} \left\{ h^{C_i} \sigma_i W_i(t) - \frac{1}{2} h^{C_i} (\sigma_i^2 t) \right\} \end{aligned}$$

$$\cdot \exp\left(h^{J_i} \sum_{j=1}^{n_i(t)} Y_{i,j} - \lambda_i \kappa_i^{h^{J_i}} t\right) \cdot \exp\left(h^{J_c} \sum_{j=1}^{n_c(t)} Y_{c,j} - \lambda_c \kappa_c^{h^{J_c}} t\right), \quad (5)$$

where Q^{h^θ} is called the Esscher measure and $h^\theta \in R^n$ for $\theta \in \{C_i, J_i, J_c\}$, where

h^{C_i} , h^{J_i} , and h^{J_c} are the Esscher parameters of $C_i(t)$, $J_i(t)$, and $J_c(t)$,

respectively. As a consequence, the mean percentage jump size becomes

$$\kappa_i^{h^{J_i}} = E\left[\exp h^{J_i} Y_{i,j} - 1\right] = \exp\left(h^{J_i} u_i + \frac{1}{2} (h^{J_i} \delta_i)^2\right) - 1 = \phi_{Y_i}(h^{J_i}) - 1. \text{ Furthermore, the}$$

Esscher transform density process $\xi^{h^\theta}(t)$ is an exponential F_t -martingale.

In line with the general option pricing theory, the existence of a risk-neutral pricing measure is equivalent to the non-existence of arbitrage trading strategies that replicate option payouts (Harrison and Pliska, 1981; 1983). Thus, under a risk-neutral pricing measure, the driving two-asset jump-diffusion process for two correlated stock prices is an F_t -martingale. It is possible to select the risk-neutral Esscher measure as the measure Q^{h^θ} such that the discounted stock price processes are Q^{h^θ} -martingales. This is obtained by determining the Esscher parameters h^{C_i} , h^{J_i} , and h^{J_c} as solutions of $E^{h^\theta}\left[\exp -rt S_i(t) | F_0\right] = S_i(0)$. Let the Esscher transform be defined by Eq. (5), then the martingale condition is satisfied if and only if

$$h^{C_i} = \frac{r - \mu_i + \lambda_i \kappa_i + \lambda_c \kappa_i}{\sigma_i^2} \quad (6)$$

$$h^{J_i} = -\frac{u_i}{\delta_i^2} - \frac{1}{2} \quad (7)$$

and

$$h^{J_c} = -\frac{u_i}{\delta_i^2} - \frac{1}{2}. \quad (8)$$

Appendix A shows the detailed proof.

An equivalent martingale measure can be treated as the Esscher measure Q^{h^θ} with respect to the measure P . We begin with identifying the dynamic process for two correlated stock prices under the risk-neutral pricing measure Q^{h^θ} . Let h^{C_i} , h^{J_i} , and h^{J_c} be the Esscher parameters of the risk-neutral Esscher measure, then under Q^{h^θ} and conditional on $F_t^{W_i}$,

$$W_t^{h^\theta}(t) = W_i(t) - h^{C_i} \sigma_i t, \quad (9)$$

is a Wiener process. Furthermore, under Q^{h^θ} , the arrival intensities $\lambda_i^{h^\theta}$ and $\lambda_c^{h^\theta}$ of the Poisson processes $n_i^{h^\theta}(t)$ and $n_c^{h^\theta}(t)$, and the jump size $Y_{i,j}^{h^\theta}$ are respectively given by

$$\lambda_i^{h^\theta} = \lambda_i \phi_{Y_i}(h^{J_i}) = \lambda_i \exp\left(h^{J_i} u_i + \frac{1}{2} |h^{J_i} \delta_i|^2\right), \quad (10)$$

$$\lambda_c^{h^\theta} = \lambda_c \phi_{Y_i}(h^{J_i}) = \lambda_c \exp\left(h^{J_i} u_i + \frac{1}{2} |h^{J_i} \delta_i|^2\right), \quad (11)$$

and

$$Y_{i,j}^{h^\theta} \stackrel{i.i.d.}{\square} N(u_i + h^{J_i} \delta_i^2, \delta_i^2), \quad (12)$$

where $W_i^{h^\theta}(t) = W_i(t) - h^{C_i} \sigma_i t$ is changed by the Esscher transform, which means that the investors receive a premium $-h^{C_i} \sigma_i$ for the continuous diffusion risk at time t , and the Wiener process is affected by the measure change. Through the change of measures, the jump risk can be formulated by the Esscher transform intensities $\lambda_i^{h^\theta}$ and $\lambda_c^{h^\theta}$ of the Poisson processes $n_i^{h^\theta}(t)$ and $n_c^{h^\theta}(t)$. For $i = \{1, 2\}$, the arrival intensities $\lambda_i^{h^\theta} = \lambda_i \phi_{Y_i}(h^{J_i})$ and $\lambda_c^{h^\theta} = \lambda_c \phi_{Y_i}(h^{J_i})$ are altered by the Esscher transform, which means that the investors receive a premium $\phi_{Y_i}(h^{J_i})$ for the jump risk at time t , and thus the arrival intensity is affected by the measure change. If $\phi_{Y_i}(h^{J_i}) = 1$, the jump risk is not priced as in Merton (1976), and the arrival intensity and distribution are unaffected by the measure change. Under Q^{h^θ} , if a jump event occurs at time j , the jump size $Y_{i,j}^{h^\theta}$ is normally distributed with mean $u_i + h^{J_i} \delta_i^2$ and variance δ_i^2 . Appendix B presents the detailed proof.

Using the solutions of Esscher parameters given by Eqs. (6)–(8), we have

$$W_i^{h^\theta}(t) = W_i(t) + \left\{ \frac{\mu_i - r - \lambda_i \kappa_i - \lambda_c \kappa_i}{\sigma_i} \right\}_t, \quad (13)$$

$$\lambda_i^{h^\theta} = \lambda_i \exp\left(-\frac{u_i^2}{2\delta_i^2} + \frac{\delta_i^2}{8}\right), \quad (14)$$

$$\lambda_c^{h^\theta} = \lambda_c \exp\left(-\frac{u_i^2}{2\delta_i^2} + \frac{\delta_i^2}{8}\right), \quad (15)$$

and

$$Y_{i,j}^{h^\theta} \stackrel{i.i.d.}{\square} N\left(-\frac{1}{2}\delta_i^2, \delta_i^2\right), \quad (16)$$

where $\left(\frac{\mu_i - r - \lambda_i \kappa_i - \lambda_c \kappa_i}{\sigma_i}\right)$ and $\exp\left(-\frac{u_i^2}{2\delta_i^2} + \frac{\delta_i^2}{8}\right)$ are the market prices of the continuous diffusion risk and jump risk at time t , respectively. In addition, under

Q^{h^θ} , the jump size $Y_{i,j}^{h^\theta}$ is normally distributed with mean $-\frac{1}{2}\delta_i^2$ and variance δ_i^2 .

Consequently, the two-asset jump-diffusion process for both stock prices under Q^{h^θ} is

$$\frac{dS_i(t)}{S_i(t-)} = rdt + \sigma_i dW_i^{h^\theta}(t) + d\left(\sum_{j=1}^{n_i^{h^\theta}(t)+n_c^{h^\theta}(t)} \exp Y_{i,j}^{h^\theta} - 1\right), \quad i=1,2, \quad (17)$$

where $r = \mu_i - \lambda_i \kappa_i - \lambda_c \kappa_i + h^{C_i} \sigma_i^2 + \lambda_i^{h^\theta} \kappa_i^{h^{J_i}} + \lambda_c^{h^\theta} \kappa_i^{h^{J_i}}$ for all $t \in [0, T]$.

3.2. Pricing European exchange options in a two-asset jump-diffusion economy

In the stock market, an exchange option gives the holder the right to exchange

one stock to another stock. More precisely, the payoff of an exchange option is $(S_1(T) - KS_2(T))^+$, where K is the ratio of the shares to be exchanged. Under the assumption that there are no arbitrage opportunities in the market, we price European exchange options under the risk-neutral pricing measure Q^{h^θ} . In a two-asset jump-diffusion economy, the price of an European exchange option at time zero is given by

$$C^{\lambda_i^{h^\theta}}(0) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{v=0}^{\min(m,n)} \exp(-\lambda_1^{h^\theta} + \lambda_2^{h^\theta} + \lambda_c^{h^\theta}) T \frac{\lambda_1^{h^\theta} T^{m-v} \lambda_2^{h^\theta} T^{n-v} \lambda_c^{h^\theta} T^v}{m-v! n-v! v!} \\ \cdot S_1(0)N(d_{1,m,n}) - KS_2(0)N(d_{2,m,n}), \quad (18)$$

where $\lambda_i^{h^\theta}$ denotes the arrival intensity of the Poisson process $n_i^{h^\theta}(T)$, m and n denote the numbers of jumps for $N_1^{h^\theta}(T)$ and $N_2^{h^\theta}(T)$ in the time interval $[0, T]$.

In addition, $N(\cdot)$ denotes the cumulative distribution function of a standard normal random variable and

$$d_{1,m,n} = \frac{\ln\left(\frac{S_1(0)}{KS_2(0)}\right) + \frac{1}{2}\sigma^2 T + \delta_1^2 m + \delta_2^2 n - \delta_1^2 m}{\sqrt{\sigma^2 T + \delta_1^2 m + \delta_2^2 n}} \quad (19)$$

$$d_{2,(m,n)} = d_{1,(m,n)} - \sqrt{\sigma^2 T + \delta_1^2 m + \delta_2^2 n} \quad (20)$$

and

$$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho_{12}\sigma_1\sigma_2} . \quad (21)$$

Eq. (18) can be viewed as the weighted sum of an expected European exchange option with weights being the joint probability of Poisson jumps. It is clear that the correlations between the two stocks also have impacts on the option prices. Appendix C gives the detailed proof.

We further illustrate the properties of the generalized exchange option pricing formula by considering several special cases with their specific formulas in the following examples. If $\phi_{Y_i}(h^{J_i})=1$, which implies that no premium is paid for the jump risk, as in Merton (1976). Hence, Eq. (18) can be reduced to

$$C^{\lambda_i}(0) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{v=0}^{\min(m,n)} \exp(-\lambda_1 + \lambda_2 + \lambda_c T) \frac{\lambda_1 T^{m-v} \lambda_2 T^{n-v} \lambda_c T^v}{m-v! n-v! v!} \\ \cdot S_1(0)N(d_{1,m,n}) - K S_2(0)N(d_{2,m,n}) , \quad (22)$$

If $K=1$, Eqs. (18) and (22) are the solutions of a regular exchange option under the different jump risk considerations (systematic and non-systematic jump risks), respectively. If $K=1$ with the absence of jumps, Eq. (18) reduces to the pricing formula of Margrabe (1978) in the Black–Scholes framework, which is given by

$$C_{BS}(0) = S_1(0)N(d_1) - S_2(0)N(d_2) , \quad (23)$$

where

$$d_1 = \frac{\ln\left(\frac{S_1(0)}{S_2(0)}\right) + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}} \quad (24)$$

$$d_2 = d_1 - \sigma\sqrt{T} \quad (25)$$

and

$$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho_{12}\sigma_1\sigma_2}. \quad (26)$$

4. Empirical results and numerical illustrations

In this section, we take two financial institution of American bank (denoted as BAC) and J.P. Morgan Chase company (denoted as JPM) of the 30 components of the Down Jones Industrial Average (DJIA)² as example to investigate the suitability of traditional geometric Brownian motion model and the proposed cojump model to the market data and the impact of cojump on exchange option and its difference during the period of pre- and post- subprime crisis.

4.1. Empirical results

The summary statistics of the return of the underlying stock price listed in Table 1 and Table 2 shows the distribution of return of the 30 component of DJIA is not

² The purpose of taking these two financial institution as example lies in the financial institution suffering the most during the subprime crisis.

normal in the point of view of the coefficients of skewness and kurtosis. Furthermore, taking the three standard deviation of individual return as a criterion of jump, we can tell the jump and cojump phenomena exists and the jump and cojump is more frequent in the period post- crisis than it in the pre-crisis from Table 3 and Table 4, such as the the number of jump of BAC in post-crisis is 11 is larger than 6 in the pre-crisis and number of cojump between BAC and JMP in the post-crisis is 7 is also larger than 4 in pre-crisis. The discussions about the Table 1, Table 2, Table 3 and Table 4 shed doubts on the validity of the geometric Brownian motion and the jump-diffusion model introduced by Merton (1976). Motivated by these findings we investigate the capability of a two-asset jump-diffusion process for modeling two correlated stock prices. In the empirical analyses, we take the American bank (denoted as BAC) and J.P. Morgan Chase company (denoted as JPM) as an example for illustrating the impact of cojump on exchange option pricing during and before the subprime financial crisis and denote s_1 and s_2 being JPM and BAC, respectively. We show the estimated results using actual market data from the Yahoo Finance and use the traditional Black–Scholes model as a benchmark for the actual data analyzed.

The estimated parameters and corresponding standard errors (the latter in parentheses) after and before the subprime financial crisis are respectively reported in Table 5 and Table 6. These tables provide several interesting results. After the turmoil,

the individual jump intensity λ_i is 0.0053 larger than 0.0027 before the crisis for BAC, while JMP's intensity behaves just in the opposite way. However, the cojump intensity λ_c after the turmoil is 0.0093 larger than 0.0053 before the crisis. Furthermore, the volatilities of Brownian motion σ (0.0378 for BAC and 0.0342 for JPM) and jump δ (0.1915 for BAC and 0.1498 for JPM) after the turmoil are also respectively larger than those (σ : 0.0089 for BAC and 0.0109 for JPM, δ : 0.0356 for BAC and 0.0425 for JPM) before the crisis. It reveals that after the crisis the assets seem to be more volatile. Furthermore, we can find the likelihood ratio test³(Hereafter denoted as LRT) show the proposed cojump dominates the BSM in the both post- and pre- crisis period in Table 5 and Table 6 under the 95% confidence level.

[Insert **Table 5–6**]

4.2. Numerical illustrations

In this section, we evaluate the exchange option prices for pre-crisis and post-crisis periods. As a benchmark, we apply the Black–Scholes model (BSM) to

³ $LRT = -2[\ln(L_R(\theta_0) - L_U(\theta_1)) \xrightarrow{asy} \chi^2_{r,1-\alpha}$ is for test the null hypothesis and alternative hypothesis H_0 : BSM Model Holds H_1 : The proposed cojump model Holds and $L_R(\theta_0)$ and $L_U(\theta_1)$ are likelihood function of the restricted and unrestricted model, respectively. The r is the degree of freedom and equals to the number of parameters difference between the restricted and the unrestricted model. If LRT is significant large by comparing it with the $100(1-\alpha)$ percentile point of the Chi-Square with degrees of freedom $\chi^2_{r,1-\alpha}$, then the null hypothesis is rejected and the alternative hypothesis holds.

price European exchange options. Here we calculate the exchange option prices using the estimated parameters in Table 5 and Table 6 and the initial stock prices of the BAC and JPM after the subprime financial crisis are 13.34 and 42.42 at 31 December 2010, respectively. In addition, the initial stock prices of BAC and JPM before the subprime financial crisis are 41.26 and 43.65 at 31 December 2007, respectively. In addition, we use one-year U.S Treasury yield rate as a proxy for risk free rate (the risk free rate for post-crisis and pre-crisis are 0.0029 and 0.0334, respectively). According to Table 7, we can find the difference between closed-form solution and simulation is very trivial, and this verifies the validity of the derived closed-form formula of cojump-diffusions applied in exchange option pricing. In addition, there is no big difference between the Merton measure and the Esscher measure. This may due to the jump premium simultaneously considered in the esscher measure for both asset and offset the jump premium effect on exchange option price. The last, we conduct scenario analyses to analyze the influences of the changing cojump intensity on the exchange option prices. The increasing cojump intensity would increase the exchange option prices as in Figure 1. This result is also match the original conclusion of equation (23), the larger correlation coefficient between two assets, the higher the exchange option price.

[Insert **Table 7**]

[Insert **Fig. 1**]

5. Conclusions

In this study, we empirically investigate the presence of individual jumps and cojumps in stock prices. The empirical data show that the two correlated stock prices are better approximated by two-asset jump-diffusion processes with both individual jumps and cojumps. Compared with the existing jump-diffusion processes (Merton, 1976; Kou, 2002), the main contribution of this article is that we incorporate cojump risks into the continuous-time stochastic process. According to the empirically favored two-asset jump-diffusion processes for the underlying stock prices under the incomplete market setting, we use the Esscher transform to identify a risk-neutral pricing measure for valuing European exchange options. After determining the risk premiums, we further derive the generalized exchange option pricing formula and demonstrate that the solution under the non-systematic jump risk assumptions of Merton (1976) and the pricing formula of Margrabe (1978) are special cases of the generalized pricing formula. We conclude that a bivariate Poisson process can explain more the cojump phenomena than a single Poisson process when pricing European exchange options and the relationship between cojump intensity and exchange option price is positive.

Appendix A: Solving the Esscher Parameters

Proof. Let E^{h^θ} denote the mathematical expectation operator with respect to the Esscher measure Q^{h^θ} equivalent to P . Applying Eq. (5), we have

$$\begin{aligned}
S_i(0) &= \exp -rt E^{h^\theta} \left[S_i(t) \middle| F_0 \right] = \exp -rt E \left[\frac{dQ^{h^\theta}}{dP} S_i(t) \middle| F_0 \right] \\
&= S_i(0) E \left[\exp \left(\left(\mu_i - r - \frac{1}{2} \sigma_i^2 - \lambda_i \kappa_i - \lambda_c \kappa_i \right) t + \sigma_i W_i(t) + \sum_{j=1}^{n_i(t)+n_c(t)} Y_{i,j} \right) \frac{dQ^{h^\theta}}{dP} \right] \\
&= S_i(0) \exp \left(\left(\mu_i - r - \frac{1}{2} \sigma_i^2 - \lambda_i \kappa_i - \lambda_c \kappa_i \right) t + \frac{1}{2} (1+h^{C_i}) \sigma_i^2 t - \frac{1}{2} h^{C_i} \sigma_i^2 t \right) \\
&\quad \cdot \exp \left(\lambda_i \left(\kappa_i^{1+h^{J_i}} - \kappa_i^{h^{J_i}} \right) t \right) \cdot \exp \left(\lambda_c \left(\kappa_i^{1+h^{J_c}} - \kappa_i^{h^{J_c}} \right) t \right), \tag{A.1}
\end{aligned}$$

From the mutual independence of random shocks $W_i(t)$, $N_i(t)$, and $Y_{i,j}$, and then the martingale condition $E^{h^\theta} [\exp -rt S_i(t) | F_0] = S_i(0)$ holds if and only if the Esscher parameters h^{C_i} , h^{J_i} , and h^{J_c} satisfy

$$\mu_i - r - \lambda_i \kappa_i - \lambda_c \kappa_i + h^{C_i} \sigma_i^2 = 0 \tag{A.2}$$

$$u_i + \frac{1}{2} \delta_i^2 + h^{J_i} \delta_i^2 = 0 \tag{A.3}$$

and

$$u_i + \frac{1}{2}\delta_i^2 + h^{J_c}\delta_i^2 = 0, \quad (\text{A.4})$$

for all $t \in [0, T]$. Therefore, we can define the solutions of Esscher parameters for the martingale condition by Eqs. (6)–(8).

Appendix B: Distributional Properties under \mathcal{Q}^{h^θ}

Proof. By Eq. (5), apply the Girsanov theorem and from the mutual independence of random shocks $W_i(t)$, $N_i(t)$, and $Y_{i,j}$, we can obtain that $W_i^{h^\theta}(t) = W_i(t) - h^{C_i}\sigma_i t$ is a Wiener process under \mathcal{Q}^{h^θ} . Next, we denote the moment-generating function of $Y_{i,j}$ by $\phi_{Y_i}(h^{J_i}) = E\left[\exp h^{J_i} Y_{i,j}\right] = \kappa_i^{h^{J_i}} + 1$. This does not depend on the index j because $\{Y_{i,j} : j = 1, 2, \dots\}$ all have the same distribution. Then we have

$$\begin{aligned} E\left[\exp\left(h^{J_i} \sum_{j=1}^{n_i(t)} Y_{i,j}\right)\right] &= P(n_i(t) = 0) + \sum_{g=1}^{\infty} E\left[\exp\left(h^{J_i} \sum_{j=1}^g Y_{i,j}\right) \middle| n_i(t) = g\right] P(n_i(t) = g) \\ &= \sum_{g=0}^{\infty} E\left[\exp h^{J_i} Y_{i,j}\right]^g P(n_i(t) = g) = \sum_{g=0}^{\infty} \phi_{Y_i}(h^{J_i})^g P(n_i(t) = g) \\ &= \exp \lambda_i \kappa_i^{h^{J_i}} t, \end{aligned} \quad (\text{B.1})$$

Therefore, we note that $h^{J_i} \sum_{j=1}^{n_i(t)} Y_{i,j} - \lambda_i \kappa_i^{h^{J_i}} t$ is a martingale at time t . Given $n_i(t) = g$, the Radon-Nikodym derivative of the probability density function can be set as

$$\frac{dQ_{n_i}^{h^\theta}}{dP_{n_i}} \Big|_{n_i(t)=g} = \phi_{Y_i}(h^{J_i})^g \exp -\lambda_i \kappa_i^{h^{J_i}} t , \quad (\text{B.2})$$

then we get $dP^{h^\theta} n_i(t)=g = dP n_i(t)=g \phi_{Y_i}(h^{J_i})^g \exp -\lambda_i \kappa_i^{h^{J_i}} t$, where

$P^{h^\theta} n_i(t)=g$ denotes the probability density function under Q^{h^θ} . Letting

$$P^{h^\theta} n_i(t)=g = P n_i(t)=g \phi_{Y_i}(h^{J_i})^g \exp -\lambda_i \kappa_i^{h^{J_i}} t , \text{ we can get}$$

$$\begin{aligned} P^{h^\theta} n_i(t)=g &= \frac{\lambda_i \phi_{Y_i}(h^{J_i}) t^g}{g!} \exp -\lambda_i \phi_{Y_i}(h^{J_i}) t \\ &= \frac{\left(\lambda_i \exp \left(h^{J_i} u_i + \frac{1}{2} (h^{J_i} \delta_i)^2 \right) t \right)^g}{g!} \exp \left(-\lambda_i \exp \left(h^{J_i} u_i + \frac{1}{2} (h^{J_i} \delta_i)^2 \right) t \right), \end{aligned} \quad (\text{B.3})$$

Under Q^{h^θ} , the jump risk can be formulated by the Esscher transform intensities

$\lambda_i \exp \left(h^{J_i} u_i + \frac{1}{2} h^{J_i} \delta_i^2 \right)$ and $\lambda_c \exp \left(h^{J_i} u_i + \frac{1}{2} h^{J_i} \delta_i^2 \right)$. Finally, we investigate the jump size, where $Y_{i,1}, Y_{i,2}, \dots, Y_{i,g}$ are independently identically distributed random

variables. Hence, the Radon-Nikodym derivative of each specific jump size can be written as

$$\frac{dQ_Y^{h^\theta}}{dP_Y} \Big|_{F_t^{n_i}} = \frac{\exp h^{J_i} Y_{i,j}}{E \left[\exp h^{J_i} Y_{i,j} \Big|_{F_0^{n_i}} \right]}, \quad (\text{B.4})$$

then we obtain $dQ_Y^{h^\theta} = \frac{1}{\sqrt{2\pi\delta_i^2}} \exp\left(-\frac{Y_{i,j} - u_i + h^{J_i}\delta_i^2}{2\delta_i^2}\right)$.

Appendix C: Derivation of the generalized exchange option pricing formula

Proof. Let $C^{\lambda_i^{h^\theta}}(0)$ represent the value of an European exchange option at time zero

with K (the ratio of the shares to be exchanged) and matured at time T . Since

$\left(\frac{S_1(t)}{S_2(t)}\right)$ is a martingale under the risk-neutral pricing measure $Q_2^{h^\theta}$ associated

with the numeraire $S_2(t)$, then we have the following equation:

$$\begin{aligned}
\frac{C^{\lambda_i^{h^\theta}}(0)}{S_2(0)} &= E_2^{h^\theta} \left[\frac{(S_1(T) - KS_2(T))^+}{S_2(T)} \middle| F_0 \right] \\
&= \frac{S_1(0)}{S_2(0)} E_2^{h^\theta} \left[\frac{S_1(T)}{S_2(T)} \mathbf{1}_{S_1(T) \geq KS_2(T)} \right] - KE_2^{h^\theta} \left[\mathbf{1}_{S_1(T) \geq KS_2(T)} \right] \\
&= \frac{S_1(0)}{S_2(0)} E_2^{h^\theta} \left[\frac{dQ_1^{h^\theta}}{dQ_2^{h^\theta}} \mathbf{1}_{S_1(T) \geq KS_2(T)} \right] - KE_2^{h^\theta} \left[\mathbf{1}_{S_1(T) \geq KS_2(T)} \right] \\
&= \frac{S_1(0)}{S_2(0)} E_1^{h^\theta} \left[\mathbf{1}_{S_1(T) \geq KS_2(T)} \right] - KP_2^{h^\theta} \quad S_1(T) \geq KS_2(T) \\
&= \frac{S_1(0)}{S_2(0)} \cdot A - K \cdot B,
\end{aligned} \tag{C.1}$$

Using Eq. (17) and Ito's rule with jumps (Protter, 1990), we obtain the derivatives of

the logarithm function $\log\left(\frac{S_1(t)}{S_2(t)}\right)$ under $\mathcal{Q}_2^{h^\theta}$ as follows:

$$d \log\left(\frac{S_1(t)}{S_2(t)}\right) = -\frac{1}{2}\sigma^2 dt + \sigma dW^{h^\theta}(t) \\ + d \left(\sum_{k=1}^{n_1^{h^\theta}(t)+n_c^{h^\theta}(t)} \exp Y_{1,k}^{h^\theta} - 1 \right) - d \left(\sum_{l=1}^{n_2^{h^\theta}(t)+n_c^{h^\theta}(t)} \exp Y_{2,l}^{h^\theta} - 1 \right), \quad (\text{C.2})$$

where $E_2^{h^\theta}[dW^{h^\theta}(t)] = 0$ and $\text{Var}[dW^{h^\theta}(t)] = \sigma_1^2 + \sigma_2^2 - 2\rho_{12}\sigma_1\sigma_2 dt = \sigma^2 dt$. The

correlation between the two stocks under this model is defined as

$$\text{Corr}\left(\frac{dS_1(t)}{S_1(t)}, \frac{dS_2(t)}{S_2(t)}\right) = \frac{\text{Cov}\left(\frac{dS_1(t)}{S_1(t)}, \frac{dS_2(t)}{S_2(t)}\right)}{\sqrt{\text{Var}\left(\frac{dS_1(t)}{S_1(t)}\right)} \sqrt{\text{Var}\left(\frac{dS_2(t)}{S_2(t)}\right)}} \\ = \frac{\rho_{12}\sigma_1\sigma_2 + \lambda_c^{h^\theta} \kappa_1^{h^{J_1}} \kappa_2^{h^{J_2}}}{\sqrt{\sigma_1^2 + \kappa_1^{h^{J_1}}^2} \sqrt{\lambda_1^{h^\theta} + \lambda_c^{h^\theta}} \sqrt{\sigma_2^2 + \kappa_2^{h^{J_2}}^2} \sqrt{\lambda_2^{h^\theta} + \lambda_c^{h^\theta}}}, \quad (\text{C.3})$$

First, we calculate B as follows:

$$B = P_2^{h^\theta} S_1(T) \geq K S_2(T)$$

$$= P_2^{h^\theta} \left(\ln \left(\frac{S_1(0)}{S_2(0)} \right) - \frac{1}{2}\sigma^2 T + \sigma W^{h^\theta}(T) + \sum_{k=1}^{N_1^{h^\theta}(T)} Y_{1,k}^{h^\theta} - \sum_{l=1}^{N_2^{h^\theta}(T)} Y_{2,l}^{h^\theta} \right) \geq \ln K$$

$$\left| N_1^{h^\theta}(T) = m, N_2^{h^\theta}(T) = n \right. \cdot \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} P^{h^\theta} \quad N_1^{h^\theta}(T) = m, N_2^{h^\theta}(T) = n$$

$$= \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} P^{h^\theta} \quad N_1^{h^\theta}(T) = m, N_2^{h^\theta}(T) = n$$

$$\cdot P_2^{h^\theta} \left(-\frac{N(0, \sigma^2 T + \delta_1^2 m + \delta_2^2 n)}{\sqrt{\sigma^2 T + \delta_1^2 m + \delta_2^2 n}} \leq d_{2, m, n} \mid N_1^{h^\theta}(T) = m, N_2^{h^\theta}(T) = n \right)$$

$$= \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} P^{h^\theta} \quad N_1^{h^\theta}(T) = m, N_2^{h^\theta}(T) = n \quad N(d_{2, m, n}) , \quad (C.4)$$

$$\text{where } d_{2, m, n} = \frac{\ln\left(\frac{S_1(0)}{KS_2(0)}\right) - \frac{1}{2} \sigma^2 T + \delta_1^2 m + \delta_2^2 n - \delta_1^2 m}{\sqrt{\sigma^2 T + \delta_1^2 m + \delta_2^2 n}} .$$

Since $\left(\frac{S_2(t)}{S_1(t)}\right)$ is a martingale under the risk-neutral pricing measure $Q_1^{h^\theta}$

associated with the numeraire $S_1(t)$. Applying the results of Eq. (C.2), we also obtain

the derivatives of the logarithm function $\log\left(\frac{S_2(t)}{S_1(t)}\right)$ under $Q_1^{h^\theta}$ as follows:

$$d \log\left(\frac{S_2(t)}{S_1(t)}\right) = -\frac{1}{2} \sigma^2 dt + \sigma dW^{h_1^\theta}(t)$$

$$+ d \left(\sum_{l=1}^{n_2^{h^\theta}(t) + n_c^{h^\theta}(t)} \exp Y_{2,l}^{h^\theta} - 1 \right) - d \left(\sum_{k=1}^{n_1^{h^\theta}(t) + n_c^{h^\theta}(t)} \exp Y_{1,k}^{h^\theta} - 1 \right) , \quad (C.5)$$

where $E_1^{h^\theta}[dW^{h_1^\theta}(t)] = E_2^{h^\theta}[dW^{h_2^\theta}(t)] = 0$ and $Var[dW^{h_1^\theta}(t)] = Var[dW^{h_2^\theta}(t)] = \sigma^2 dt$.

Second, we calculate A :

$$A = P_1^{h^\theta} \quad S_1(T) \geq K S_2(T)$$

$$= P_1^{h^\theta} \left(\ln \left(\frac{S_2(0)}{S_1(0)} \right) - \frac{1}{2} \sigma^2 T + \sigma W^{h^\theta}(T) + \sum_{l=1}^{N_2^{h^\theta}(T)} Y_{2,l}^{h^\theta} - \sum_{k=1}^{N_1^{h^\theta}(T)} Y_{1,k}^{h^\theta} \leq \ln \frac{1}{K} \right)$$

$$\left| N_1^{h^\theta}(T) = m, N_2^{h^\theta}(T) = n \right. \cdot \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} P^{h^\theta} \quad N_1^{h^\theta}(T) = m, N_2^{h^\theta}(T) = n$$

$$= \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} P^{h^\theta} \quad N_1^{h^\theta}(T) = m, N_2^{h^\theta}(T) = n$$

$$\cdot P_1^{h^\theta} \left(\frac{N_1^{h^\theta}(T) = m, N_2^{h^\theta}(T) = n}{\sqrt{\sigma^2 T + \delta_1^2 m + \delta_2^2 n}} \leq d_{1,m,n} \right)$$

$$= \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} P^{h^\theta} \quad N_1^{h^\theta}(T) = m, N_2^{h^\theta}(T) = n \quad , \quad (C.6)$$

$$\text{where } d_{1,m,n} = \frac{\ln \left(\frac{S_1(0)}{KS_2(0)} \right) + \frac{1}{2} \sigma^2 T + \delta_1^2 m + \delta_2^2 n - \delta_1^2 m}{\sqrt{\sigma^2 T + \delta_1^2 m + \delta_2^2 n}} .$$

Combining Eqs. (C.4) and (C.6), we obtain the following equation:

$$C^{\lambda_i^{h^\theta}}(0) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} P^{h^\theta} \quad N_1^{h^\theta}(T) = m, N_2^{h^\theta}(T) = n \\ \cdot S_1(0) N \quad d_{1,m,n} - KS_2(0) N \quad d_{2,m,n} \quad , \quad (C.7)$$

where $P^{h^0} N_1^{h^0}(T) = m, N_2^{h^0}(T) = n$

$$= \sum_{v=0}^{\min(m, n)} \exp -\lambda_1^{h^0} + \lambda_2^{h^0} + \lambda_c^{h^0} T \frac{\lambda_1^{h^0} T^{m-v} \lambda_2^{h^0} T^{n-v} \lambda_c^{h^0} T^v}{(m-v)! (n-v)! v!}, \quad (\text{C.8})$$

This proves the generalized exchange option pricing formula.

References

- Bjerksund, P., Stensland, G., 1993. American exchange options and a put-call transformation: A note. *Journal of Business Finance and Accounting* 20, 761–764.
- Broadie, M., Detemple, J., 1997. The valuation of American options on multiple assets. *Mathematical Finance* 7, 241–286.
- Chan, W. H., 2003. A correlated bivariate Poisson jump model for foreign exchange. *Empirical Economics* 28, 669–685.
- Dungey, M., Hvozdyk, L., 2012. Cojumping: Evidence from the US Treasury bond and futures markets. *Journal of Banking and Finance* 36, 1563–1575.
- Eraker, B., Johannes, M., Polson, N., 2003. The impact of jumps in volatility and returns. *Journal of Finance* 58, 1269–1300.
- Eraker, B., 2004. Do stock market and volatility jump? Reconciling evidence from spot and option prices. *Journal of Finance* 59, 1367–1403.
- Gerber, H. U., Shiu, E. S. W., 1994. Option pricing by Esscher transforms (with discussions). *Transactions of Society of Actuaries* 46, 99–191.
- Gerber, H. U., Shiu, E. S. W., 1996. Actuarial bridges to dynamic hedging and option pricing. *Insurance: Mathematics and Economics* 18, 183–218.

- Harrison, J. M., Pliska, S. R., 1981. Martingales and stochastic integrals in the theory of continuous trading. *Stochastic Processes and their Applications* 11, 215–280.
- Harrison, J. M., Pliska, S. R., 1983. A stochastic calculus model of continuous trading: Complete markets. *Stochastic Processes and their Applications* 15, 313–316.
- Hesselager, O., 1996. Recursions for certain bivariate counting distributions and their compound distributions. *ASTIN Bulletin* 26, 35–52.
- Holgate, P., 1964. Estimation for the bivariate Poisson distribution. *Biometrika* 51, 241–245.
- Kou, S. G., 2002. A jump-diffusion model for option pricing. *Management Science* 48, 1086–1101.
- Lahaye, L., Laurent, S., Neely, C., 2011. Jumps, cojumps and macro announcements. *Journal of Applied Econometrics* 26, 893–921.
- Lindset, S., 2007. Pricing American exchange options in a jump-diffusion model. *Journal of Futures Markets* 27, 257–273.
- Lu, X., Kawai, K. I., Maekawa, K., 2010. Estimating bivariate GARCH-JUMP model based on high frequency data: The case of revaluation of the Chinese Yuan in July 2005. *Asia-Pacific Journal of Operational Research* 27, 287–300.
- Maheu, J. M., McCurdy, T. H., 2004. News arrival, jump dynamics and volatility components for individual stock returns. *Journal of Finance* 59, 755–793.
- Margrabe, W., 1978. The value of an option to exchange one asset for another. *Journal of Finance* 33, 177–186.
- Merton, R. C., 1976. Option pricing when underlying stock returns are discontinuous. *Journal of Financial Economics* 3, 125–144.
- Protter, P., 1990. Stochastic integration and differential equations. Springer-Verlag, Berlin.

Table 1. Descriptive statistics of the Down Jones 30 components after the global financial crisis from 2 January 2008 to 31 December 2010.

Company Name	3M Company	AT&T, Inc.	Alcoa Inc.	American Express Company	Bank of America Corp	Boeing Company	Caterpillar Inc.	Chevron Corporation	Cisco Systems, Inc.	Coca-Cola Company	E.I. du Pont de Nemours & Company	ExxonMobil Corporation	General Electric Company	Hewlett-Packard Company	Home Depot, Inc.
Observations	756	756	756	756	756	756	756	756	756	756	756	756	756	756	756
Mean	5.6E-05	-0.0004	-0.0011	-0.0002	-0.0015	-0.0004	0.0004	-3E-05	-0.0004	9.8E-05	0.0002	-0.0003	-0.0009	-0.0002	0.0004
SD	0.0189	0.0199	0.0420	0.0397	0.0590	0.0252	0.0289	0.0239	0.0247	0.0162	0.0257	0.0219	0.0305	0.0229	0.0244
Skewness	0.1044	0.6796	-0.1342	0.1104	-0.1323	0.2314	0.1161	0.2869	-0.4822	0.7137	-0.2894	0.2278	0.0557	0.3350	0.4971
Kurtosis	6.3149	11.1067	7.1080	6.8904	10.7147	5.7704	5.7287	14.2124	9.2884	12.7676	6.0765	14.9061	7.7086	7.7081	5.8720
Num. of Jumps	7	8	6	7	11	7	7	6	4	7	5	7	7	7	6
Company Name	Intel Corporation	International Business Machines Corp	J.P. Morgan Chase & Co.	Johnson & Johnson	Kraft Foods, Inc.	McDonald's Corporation	Merck & Co Inc	Microsoft Corporation	Pfizer Inc.	Procter & Gamble Company	The Travelers Companies, Inc.	United Technologies	Verizon Communications Inc.	Wal-Mart Stores, Inc.	Walt Disney Company
Observations	756	756	756	756	756	756	756	756	756	756	756	756	756	756	756
Mean	-0.0002	0.0004	7.8E-06	-8E-05	-2E-05	0.0004	-0.0006	-0.0003	-0.0004	-0.0002	8.2E-05	6E-05	-0.0002	0.0002	0.0002
SD	0.0252	0.0177	0.0435	0.0133	0.0162	0.0159	0.0236	0.0236	0.0197	0.0151	0.0287	0.0209	0.0194	0.0155	0.0245
Skewness	-0.1080	0.2508	0.3016	0.6443	-0.3380	0.0027	-0.4636	0.3123	-0.0743	-0.1597	0.3508	0.5089	0.4019	0.2253	0.4313
Kurtosis	6.1699	6.9804	8.8599	15.7737	7.3922	6.9104	9.9087	9.7193	7.1836	8.7528	15.5679	7.8996	8.9475	10.0008	7.9443
Num. of Jumps	5	5	7	5	5	6	6	6	4	5	9	7	8	5	7

Note: The descriptive statistics are reported for the 30 components of the Down Jones Industrial Average (DJIA) after the financial turmoil from 2008 to 2010. This table shows the mean, standard deviation, skewness, and kurtosis of the logarithm returns of the 30 components of the DJIA. The number of jumps is calculated by counting the number of the individual return over three standard deviation of the return.

Table 2. Descriptive statistics of Down Jones 30 components before the global financial crisis from 3 January 2005 to 31 December 2007.

Company Name	3M Company	AT&T, Inc.	Alcoa Inc.	American Express	Bank of America Corp	Boeing Company	Caterpillar Inc.	Chevron Corporation	Cisco Systems, Inc.	Coca-Cola Company	E.I. du Pont de Nemours & Company	ExxonMobil Corporation	General Electric Company	Hewlett-Packard Company	Home Depot, Inc.
Observations	753	753	753	753	753	753	753	753	753	753	753	753	753	753	753
Mean	3.06E-05	0.0006	0.0002	-0.0001	-0.0002	0.0007	-0.0004	0.0008	0.0004	0.0005	-0.0001	0.0008	1.73E-05	0.0012	-0.0006
SD	0.0114	0.0114	0.0175	0.0143	0.0103	0.0136	0.0294	0.0140	0.0161	0.0077	0.0120	0.0140	0.0095	0.0157	0.0138
Skewness	-1.7683	0.0544	-0.0257	-0.8355	-0.1983	0.2795	-16.0415	-0.3647	0.3559	0.4136	-0.3855	-0.3970	0.0560	1.0663	-0.1016
Kurtosis	16.9813	4.7989	4.7342	12.3762	7.0569	4.7062	362.3247	3.0110	12.5879	4.8627	5.6866	3.7720	4.5133	10.4417	4.3990
Num. of Jumps	2	5	4	7	6	4	0	0	4	6	3	2	5	6	4
Company Name	Intel Corporation	International Business Machines Corp	J.P. Morgan Chase & Co.	Johnson & Johnson	Kraft Foods, Inc.	McDonald's Corporation	Merck & Co Inc	Microsoft Corporation	Pfizer Inc.	Procter & Gamble Company	The Travelers Companies, Inc.	United Technologies	Verizon Communications Inc.	Wal-Mart Stores, Inc.	Walt Disney Company
Observations	753	753	753	753	753	753	753	753	753	753	753	753	753	753	753
Mean	0.0002	0.0001	0.0001	7.79E-05	-8.86E-05	0.0008	0.0008	0.0004	-0.0002	0.0004	0.0005	-0.0004	0.0001	-0.0002	0.0002
SD	0.0156	0.0112	0.0125	0.0078	0.0111	0.0126	0.0144	0.0124	0.0129	0.0089	0.0127	0.0275	0.0108	0.0110	0.0121
Skewness	-0.7903	-0.8659	0.2545	0.5021	0.2941	0.4013	0.3587	-0.4797	-0.5852	-0.1450	0.0124	-20.8786	-0.2079	0.2757	0.0153
Kurtosis	8.7519	9.4701	6.6576	6.1341	6.6414	4.8727	16.9317	19.0833	16.1495	6.3854	5.3273	524.3804	4.0898	5.5677	5.3438
Num. of Jumps	2	4	6	10	3	7	4	6	6	6	7	0	3	6	3

Note: The descriptive statistics are reported for the 30 components of the Down Jones Industrial Average (DJIA) before the financial turmoil from 2005 to 2007. This table shows the mean, standard deviation, skewness, and kurtosis of the logarithm returns of the 30 components of the DJIA. The number of jumps is calculated by counting the number of the individual return over three standard deviation of the return.

Table 3. Number of jumps and cojumps in Down Jones 30 components after the global financial crisis from 2 January 2008 to 31 December 2010.

Name	MMM	T	AA	AXP	BAC	BA	CAT	CVX	CSCO	KO	DD	XOM	GE	HPQ	HD	INTC	IBM	JPM	JNJ	KFT	MCD	MRK	MSFT	PFE	PG	TRV	UTX	VZ	WMT	DIS			
MMM	7	4	4	3	2	3	4	4	2	4	4	4	2	4	4	2	4	3	3	3	2	4	2	2	2	5	3	5	3	6	4	2	4
T	4	8	3	3	2	5	4	5	4	4	4	4	5	1	3	4	2	4	2	3	1	3	3	3	2	4	3	4	2	4			
AA	4	3	6	2	1	3	3	3	3	3	4	3	2	3	2	3	1	1	3	2	2	2	4	3	3	2	4	3	2	4			
AXP	3	3	2	7	4	2	4	1	2	1	3	1	1	2	2	2	2	4	1	0	1	1	4	1	2	2	3	1	1	2			
BAC	2	2	1	4	11	1	3	0	1	0	2	0	1	1	2	1	3	7	0	0	0	0	2	0	1	0	2	0	0	1			
BA	3	5	3	2	1	7	3	3	3	2	4	3	1	3	2	2	2	1	2	1	2	2	3	2	3	3	3	2	2	4			
CAT	4	4	3	4	3	3	7	2	2	2	4	2	1	2	2	2	2	3	2	0	1	1	3	1	3	2	4	2	1	3			
CVX	4	5	3	1	0	3	2	6	2	5	2	6	1	3	3	2	2	0	4	2	3	3	3	3	4	4	4	5	2	3			
CSCO	2	4	3	2	1	3	2	2	4	2	3	2	1	2	3	2	2	1	2	1	2	2	2	2	1	2	2	2	3				
KO	4	4	3	1	0	2	2	5	2	7	2	5	1	3	3	2	2	0	4	2	3	3	3	4	3	4	5	2	3				
DD	4	4	4	3	2	4	4	2	3	2	5	2	2	3	2	3	2	2	2	1	2	2	4	2	3	1	4	2	2	4			
XOM	4	5	3	1	0	3	2	6	2	5	2	7	1	3	3	2	2	0	5	2	4	3	3	3	4	4	5	5	3	3			
GE	2	1	2	1	1	1	1	1	1	1	2	1	7	1	1	3	1	1	1	1	1	3	2	2	1	1	2	1	1	1			
HPQ	4	3	3	2	1	3	2	3	2	3	3	3	1	7	2	2	2	1	3	2	2	2	4	3	4	2	4	3	2	4			
HD	3	4	2	2	2	2	2	3	3	3	2	3	1	2	6	2	3	1	3	1	3	2	2	2	3	2	3	3	2	2			
INTC	3	2	3	2	1	2	2	2	2	2	3	2	3	2	2	5	1	1	2	1	2	2	3	2	2	2	3	2	2	2			
IBM	3	4	1	2	3	2	2	2	2	2	2	2	1	2	3	1	5	3	2	1	1	1	2	1	3	1	3	2	1	2			
JPM	2	2	1	4	7	1	3	0	1	0	2	0	1	1	1	1	3	7	0	0	0	0	2	0	1	0	2	0	0	1			
JNJ	4	3	3	1	0	2	2	4	2	4	2	5	1	3	3	2	2	0	5	2	3	2	3	3	4	3	5	4	3	3			
KFT	2	1	2	0	0	1	0	2	1	2	1	2	1	2	1	1	1	0	2	5	1	1	2	2	2	2	2	2	2				

Table 3. (continued)

Name	MMM	T	AA	AXP	BAC	BA	CAT	CVX	CSCO	KO	DD	XOM	GE	HPQ	HD	INTC	IBM	JPM	JNJ	KFT	MCD	MRK	MSFT	PFE	PG	TRV	UTX	VZ	WMT	DIS
MCD	2	3	2	1	0	2	1	3	2	3	2	4	1	2	3	2	1	0	3	1	6	3	2	2	2	1	3	3	3	2
MRK	2	3	2	1	0	2	1	3	2	3	2	3	3	2	2	2	1	0	2	1	3	6	2	3	2	1	2	3	2	2
MSFT	5	3	4	4	2	3	3	3	2	3	4	3	2	4	2	3	2	2	3	2	2	6	3	4	2	5	3	2	4	
PFE	3	2	3	1	0	2	1	3	2	3	2	3	2	3	2	2	1	0	3	2	2	3	3	4	3	2	3	3	2	3
PG	5	4	3	2	1	3	3	4	2	4	3	4	1	4	3	2	3	1	4	2	2	2	4	3	5	3	5	4	2	4
TRV	3	3	2	2	0	3	2	4	1	3	1	4	1	2	2	2	1	0	3	2	1	1	2	2	3	9	3	3	2	2
UTX	6	4	4	3	2	3	4	4	2	4	4	5	2	4	3	3	3	2	5	2	3	2	5	3	5	3	7	4	3	4
VZ	4	4	3	1	0	2	2	5	2	5	2	5	1	3	3	2	2	0	4	2	3	3	3	4	3	4	8	2	3	
WMT	2	2	2	1	0	2	1	2	2	2	2	3	1	2	2	2	1	0	3	2	3	2	2	2	2	3	2	5	2	
DIS	4	4	4	2	1	4	3	3	3	3	4	3	1	4	2	2	2	1	3	2	2	2	4	3	4	2	4	3	2	7

Note: This table shows the number of jumps and cojumps between the 30 components in DJIA after the financial turmoil from 2008 to 2010. The number of jumps is calculated by counting the number of the individual return over three standard deviation of the return. Meanwhile, if the returns of two components of DJIA are simultaneous over three standard deviation of their own return, then we define the phenomenon as a cojump and calculate the number of cojumps.

Table 4. Number of jumps and cojumps in Down Jones 30 components before the global financial crisis from 3 January 2005 to 31 December 2007.

Name	MMM	T	AA	AXP	BAC	BA	CAT	CVX	CSCO	KO	DD	XOM	GE	HPQ	HD	INTC	IBM	JPM	JNJ	KFT	MCD	MRK	MSFT	PFE	PG	TRV	UTX	VZ	WMT	DIS	
MMM	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
T	0	5	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	1	0	0	0		
AA	0	0	4	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
AXP	0	0	1	7	2	0	0	0	0	0	0	0	1	0	0	1	1	2	1	0	0	0	0	0	1	1	0	0	1	0	
BAC	0	0	0	2	6	0	0	0	0	0	0	1	2	0	0	0	2	4	0	0	0	0	0	0	0	2	0	0	1	0	
BA	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
CAT	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
CVX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
CSCO	0	0	0	0	0	0	0	0	4	1	1	0	0	1	0	0	0	1	0	0	0	0	1	0	0	0	1	0	0	0	
KO	0	0	0	0	0	0	0	0	1	6	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	
DD	0	0	0	0	0	0	0	0	1	1	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
XOM	0	0	0	0	1	0	0	0	0	0	0	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
GE	0	0	0	1	2	0	0	0	0	0	0	1	5	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	
HPQ	0	0	0	0	0	0	0	0	1	0	0	0	0	6	0	0	0	0	1	0	0	0	0	1	0	0	0	1	0	0	
HD	0	1	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	
INTC	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
IBM	0	0	0	1	2	0	0	0	0	0	0	0	0	0	0	0	4	1	0	0	0	0	0	0	0	0	0	0	1	0	0
JPM	0	0	0	2	4	0	0	0	0	0	0	0	1	0	0	0	1	6	0	0	0	0	0	0	0	2	0	0	1	0	0
JNJ	0	0	0	1	0	0	0	0	1	1	0	0	0	1	0	0	0	0	10	0	0	0	0	1	1	0	0	1	0	0	
KFT	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	

Table 4. (continued)

Name	MMM	T	AA	AXP	BAC	BA	CAT	CVX	CSCO	KO	DD	XOM	GE	HPQ	HD	INTC	IBM	JPM	JNJ	KFT	MCD	MRK	MSFT	PFE	PG	TRV	UTX	VZ	WMT	DIS
MCD	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
MRK	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	2	0	0	0	0	0	
MSFT	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	6	0	0	0	0	0	0	1	
PFE	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	2	0	6	1	0	0	1	0	1	
PG	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	6	0	0	0	0	0	
TRV	0	1	0	1	2	0	0	0	0	0	0	0	1	0	1	0	0	2	0	0	0	0	0	0	0	7	0	0	0	
UTX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
VZ	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	3	0	0	
WMT	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	6	0	
DIS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	3	

Note: This table shows the number of jumps and cojumps between the 30 components in DJIA before the financial turmoil from 2005 to 2007. The number of jumps is calculated by counting the number of the individual return over three standard deviation of the return. Meanwhile, if the returns of two components of DJIA are simultaneous over three standard deviation of their own return, then we define the phenomenon as a cojump and calculate the number of cojumps.

Table 5. Parameter estimation after the subprime financial crisis.

Parameter	BAC		JPM	
	BSM	COJDM	BSM	COJDM
μ	-0.0015 (0.0021)	-0.0024 (0.0015)	7.60E-06 (0.0016)	-0.0008 (0.0013)
σ	0.0590 (0.0015)***	0.0378 (0.0014)***	0.0435 (0.0011)***	0.0342 (0.0012)***
u		0.0171 (0.0313)		0.0248 (0.0239)
δ		0.1915 (0.0226)***		0.1498 (0.0341)***
λ_l		0.0053 (0.1200)		0. (0.0958)
λ_c		0.0093 (0.0958)		0.0093 (0.0958)
LRT		236.0803***		120.4079***

Note: Table 5 presents the empirical results of dynamic models, reporting the estimated parameters and corresponding standard errors and *** means the significant level of 5%. The notation of BSM and COJDM are the traditional Black–Scholes model and the Merton-type jump-diffusion model with consideration of cojump phenomena, respectively. Data for estimation in Table 5 cover the period from 2 January 2008 to 31 December 2010. Estimation settings for the BSM and COJDM are determined via the maximum likelihood (ML) approach. First, we use the Table 3 and Table 4 to calculate the individual jump intensity denoted λ_l and the cojump intensity denoted λ_c ; Second, we put the derived jump intensity parameters into the Merton-type jump-diffusions, and use the MLE method to estimate the rest four parameters. From the LRT, we can tell the proposed cojump model dominates the BSM model in both JPM and BAC cases during the post-crisis period.

Table 6. Parameter estimation before the subprime financial crisis.

Parameter	BAC		JPM	
	BSM	COJDM	BSM	COJDM
μ	-0.0002 (0.0004)	-5.67E-05 (0.0003)	0.0001 (0.0005)	-1.98E-05 (0.0004)
σ	0.0103 (0.0003)***	0.0089 (0.0003)***	0.0125 (0.0003)***	0.0109 (0.0003)***
u		-0.0025 (0.0072)		0.0081 (0.0086)
δ		0.0356 (0.0105)***		0.0425 (0.0125)***
λ_l		0.0027 (0.0890)		0.0027 (0.0890)
λ_c		0.0053 (0.0727)		0.0053 (0.0727)
LRT		64.7725***		59.6422***

Note: Table 6 presents the empirical results of dynamic models, reporting the estimated parameters and corresponding standard errors and *** means the significant level of 5%. The notation of BSM and COJDM are the traditional Black–Scholes model and the Merton-type jump-diffusion model with consideration of cojump phenomena, respectively. Data for estimation in Table 6 cover the period from

3 January 2005 to 31 December 2007. Estimation settings for the BSM and COJDM are determined via the maximum likelihood (ML) approach. First, we use the Table 3 and Table 4 to calculate the individual jump intensity denoted λ_1 and the cojump intensity denoted λ_C ; Second, we put the derived jump intensity parameters into the Merton-type jump-diffusions, and use the MLE method to estimate the rest four parameters. From the LRT, we can tell the proposed cojump model dominates the BSM model in both JPM and BAC cases during the pre-crisis period.

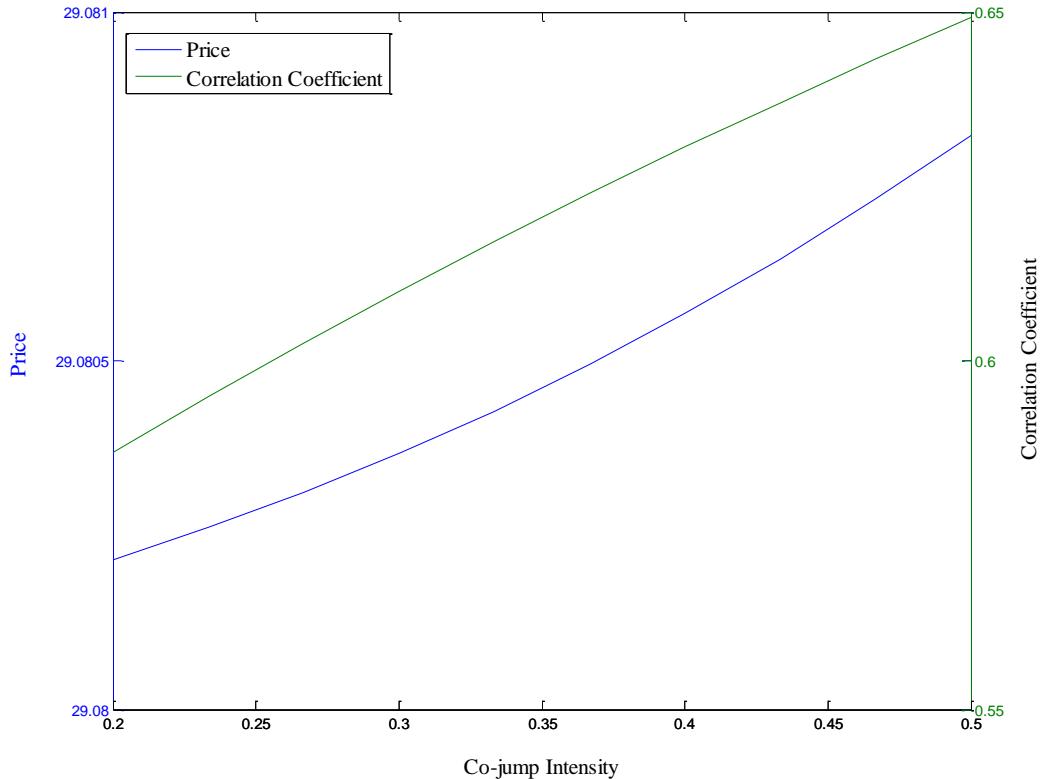
Table 7. Pre-crisis and post-crisis pricing results of European exchange option pricing models.

Pre-crisis							
BSM	Co-jump						
	Esscher Measure			Merton Measure			
	Closed Form	Simulation	Relative Error	Closed Form	Simulation	Relative Error	
K=1	2.3900	2.3912	2.3911	-2.85E-05	2.3912	2.3924	4.85E-04
K=0.5	23.0200	23.0189	23.0200	5.07E-05	23.0189	23.0193	1.99E-05
K=0.2	35.3980	35.3963	35.3983	5.88E-05	35.3962	35.3985	6.48E-05

Post-crisis							
BSM	Co-jump						
	Esscher Measure			Merton Measure			
	Closed Form	Simulation	Relative Error	Closed Form	Simulation	Relative Error	
K=1	29.080	28.8103	29.0785	9.31E-03	28.8104	29.0766	9.24E-03
K=0.5	35.7500	35.4184	35.7538	9.47E-03	35.4186	35.7562	9.53E-03
K=0.2	39.7520	39.3833	39.7501	9.31E-03	39.3835	39.7534	9.39E-03

Note: This table shows the pricing results of applying the traditional BSM and the cojump-diffusion model (COJDM) to the exchange option pricing in two sample periods. In this table, we employ not only the derived closed form of the proposed model in Eq. (18) but also apply the Monte Carlo simulation technique to price exchange options. The model parameters used in Table 7 for pre-crisis and post-crisis periods are based on Table 5 and Table 6 with $T = 1$. In addition, the correlation coefficients between two assets' Brownian motions computed by Eq. (C.3) for pre-crisis and post-crisis periods are respectively 0.7981 and 0.8042. The relative error is calculated in the form of (Simulation prices–Closed-from prices) / (Closed-from prices). From the relative error results, we can verify validity of the closed from formula of the dynamic model for exchange option pricing, in further, it can be significant for practitioners of the stock markets

Fig. 1. The impact of cojump intensity (λ_c) after the subprime financial crisis of 2008.



Note: Fig. 1 is basing on the parameters in Table 5, we set $\lambda_1^{\text{JPM}} = \lambda_2^{\text{BAC}} = 0.5$, $K = 1$ and vary the λ_c in the range [0.2, 0.5]. According to Eq. (C.3), the rising of the cojump intensity (λ_c) will increase the correlation coefficient. As expected as in equation (23), the higher the correlation between two assets, the larger the exchange price.

(多變量複合卜瓦松跳躍擴散模型與高頻資料下之選擇權評價與投資組合策略之研究第二年)

多變量複合卜瓦松跳躍擴散過程下投資組合建構

摘要

隨著資產間的共同移動與共同跳躍的現象加劇，過去建構在資產動態服從幾何布朗運動的情況將無法描繪。本文提出一個多變量複合卜瓦松跳躍擴散進一步捕捉資產間的共同移動現象，同時也將共同跳躍的現象納入到模型中，並透過Markowitz 的平均數-變異數法則來建構投資組合。研究結果發現當共同跳躍次數的增加，將增加資產間的相關係數，這使得投資組合的風險分散的效果遞減，此外，當有重大的系統性風險發生時，共同跳躍的次數也會增加，且當共同跳躍的頻率增加至某一程度時，資產間的相關係數將達到 1，此時，投資組合的建構將無法達到風險分散的效果。

關鍵字:共同跳躍、平均數-變異數法則、多變量複合卜瓦松跳躍擴散。

1. 緒論

由於全球金融市場的整合與法規對於財務創新的鬆綁造成金融市場有重大的結構型轉變，主要是因為跨國境的資本投資機會變多與資本的流動速度增加，同時也造成了彼此市場間的相依程度更加密切。如：2007 年至 2010 年的全球的次貸風暴(Subprime Crisis)造成全球金融市場存在著一個傳染風險(Contagion Risk)。此傳染現象會影響全球資產動態的相關係數，進而造成投資組合的風險比原先的預期更加的集中與嚴重，反而沒有因投資組合而達到風險分散的效果，此外，資產間的共同移動(Co-move)也因為傳染的現象速度加劇。投資組合是指由一種以上的證券、衍生性商品或資產所建構而成的集合，而如何將有限的資金有效的配置在投資標的，在傳統的投資組合理論中，最有名的莫過於 Markowitz(1952,1959)提出平均數-變異數分析投資組合模型(Mean-Variance Portfolio Model, M-V 模型)，此模型利用平均數-變異數分析投資組合選擇與最適化的問題，進而推導出由報酬率與標準差所建構出的效率前緣(Efficient Frontier)。投資者可利用在此架構下的資產配置符合固定報酬率下風險達到最小或是固定風險之下報酬率達到最大來制定投資決策。Sharp(1964), Lintner(1965)及 Black (1972)根據投資組合理論發展出以一種線性關係描述在投資組合中報酬率與風險的關係的資本資產訂價理論(Capital Asset Pricing Model, CAPM)。

在 Markowitz 的 M-V 模型下，都已事先假設投資標的預期報酬率與共變異數為已知，然而，在實際的應用下，預期報酬率與變異數都是未知且必須由歷史資料來進行估計，再將這些估計值代入效率前緣。過去文獻指出 Markowitz 的投資組合對於預期報酬率與共變異數的估計相當敏感，如：Brianson(1991)指出，效率前緣對於微小變動的資產的預期報酬率、變異數與共變異數相當敏感；Chopra and Ziemba (1993)實證指出這些輸入參數的微小變動將使得投資組合產生巨大的改變；Koshroaidis and Duarte(1997)提出利用所選取的歷史報酬率的選取期間的不同也將導致結論的差異；此外，Laloux et al.(1999)發現對於共變異數估

計誤差對於投資組合的配置有重大的影響。而上述的問題，在投資組合規模愈大的情況下會愈為嚴重，例如：當投資組合中必須配置與管理 2000 檔股票時，此時的共變異數矩陣將會有 2,001,000 個未知的參數，然而，在一般的樣本資料下，其樣本數最多不會超過 1000 個(大約四年的資料，或是二十年的周資料)，因此，在共變異數中每個元素的估計準確度(Accuracy of order)大約為 $O(n^{-\frac{1}{2}})$ (在此例子下大約為 0.032)，若將共變異數裡這兩百多萬個的估計應用到投資組合中，將會導致破壞性的影響。事實上，Jagannathan 和 Ma (2003)指出當投資組合規模很大的情況下，最適的不能賣空的投資組合績效會比 Markowitz 有賣空投資組合績效好。因此，利用 M-V 法則所建構的最適投資組合，在根據歷史資料所 M-V 投資組合中所建構的投資組合其績效是不好的(Lai et al, 2011)。

所以如何建構穩健而不會受到共變異數矩陣估計與估計的預期報酬率所影響的投資組合成為許多文獻研究的議題。過去文獻利用修正預期報酬率的共變異數矩陣的估計來降低 Markowitz 最適投資組合對於這些輸入參數的不確定性的影響，如：Chopra and Ziemba (1993)提出 James-Stein 平均數的估計方法；Ledoit and Wolf(2003,2004)提出將共變異數矩陣縮減到一個單位矩陣或是由因子模型推導出來的共變異數矩陣的方法；Fan et al.(2008)利用因子模型來估計共變異數矩陣，雖然上述文獻的方法可以降低預期報酬率與共變異數矩陣估計對於 M-V 投資組合的敏感性，但這些方法並無法解決估計的累計誤差的逆向影響，尤其是當投資組合規模很大的時候(Fan et al. 2008)。然而，一些研究從修正 Markowitz 未受限的 M-V 投資組合作為出發點，來降低資產配置對於輸入參數(預期報酬率與共變異數)的敏感度(De Roon et al,2002; Goldfarb and Iyengar,2003; Jagannathan and Ma,2003; Fan et al, 2008,2012)。若將 Markowitz 的 M-V 最適投資法則應用在高頻交易的投資組合上，同樣也會遇到共變異數矩陣估計的問題。

然而在實證上，伴隨抽樣頻率的增加 RV 對於累計波動的而言並不是穩健的估計值，因為 Jocod and Shiryaev(2003) 指出 RV 是抽樣頻率的函數且並不會隨著

抽樣頻率的增加而收斂。關於此現象的主要解釋認為此為市場微結構(Market microstructure)所造成，如：價格的不連續性(Price-discreteness)、買賣價差的存在(Bid-ask spread)。因此，抽樣的頻率的增加，因為市場微結構的影響並無法達成累計波動度的一致性估計，雖然此問題可以利用拉長抽樣樣本的時間來解決(如：抽樣的長度五分鐘，而原本的資料是以每秒抽樣一次，但現在只保留第五分鐘的資料而將其他 299 個資料丟棄)，但丟棄這些資料將減少這些內含的有用訊息對於共變異數的影響。此外，Zhang et al.(2005)也認為拉長抽樣樣本的時間，只能降低這些市場微結構的影響，但並不能量化與修正這些因素對於波動度估計的影響。因此，Zhang et al.(2005)提出一個 Two-Scale 的估計方法來決定最適的抽樣頻率。

過去文獻雖然提供了改善資產間的共變異矩陣的估計，或是調整 Markowitz 的均數變異數法則的限制條件，以更符合現實情況與改善投資績效。然而，文獻中投資組合的動態資產過程都假設服從幾何布朗運動(Zhang et al., 2005; Lai et al., 2011; Fan et al., 2012)，這與現實的情況，資產價格存在跳躍與資產間因為一些總體因素而存在共同跳躍的現象(Lahay, Laurent and Neely , 2011)，造成資產報酬率為非常態而是呈現非對稱高狹峰(Asymmetric Leptokurtic)、厚尾(Heavy-tail)的情況不符。因此，本研究提出並提出一個多變量複合卜瓦松跳躍擴散做為標的資產的動態行為來分析資產間共同跳的傳染現象對於投資組合的影響。

2. 投資組合建構與資產動態過程

本節將利用 Markowitz 所提出的平均數-變異數模型與所提出多變量複合卜瓦松跳躍擴散模型做為標的資產的動態過程來建構投資組合。在 2.1 節將介紹平均數-變異數投資組合的建構，在 2.2 節為本文所提出的資產動態過程模型。

2.1 平均數-變異數投資組合建構

本研究利用 Markowitz 的平均數-變異數模型來建構一個有 m 個資產的投資組合，其資產的預期報酬率序列為 $U = [u_1, u_2, \dots, u_m]^T$ ，而投資在各個資產的權重為 $W = [w_1, w_2, \dots, w_m]^T$ ，資產報酬率的共變異數矩陣為 Σ ，因此，投資組合的平均報酬率為 $W^T U$ ，與變異數為 $W^T \Sigma W$ 。在 Markowitz 的 M-V 模型下給定投資組合的預期報酬率為 u^* 下，所建構出來出來的效率前緣投資權重可由下列方程式的最佳解求得：

$$W = \arg \min_W W^T \Sigma W \text{ subject to } W^T U = u^*, W^T I = 1 \quad (1)$$

$$W = \frac{\left[B\Sigma^{-1}I - A\Sigma^{-1}U + u^* (C\Sigma^{-1}U - A\Sigma^{-1}I) \right]}{D} \quad (2)$$

其中， $I = [1, 1, \dots, 1]^T$, $A = U^T \Sigma^{-1} I$, $B = U^T \Sigma^{-1} U$, $C = I^T \Sigma^{-1} I$ 且 $D = BC - A^2$ 。而效率前緣的曲度，會伴隨的投資組合中的資產的報酬率標準差與資產間的相關性不同而有所變化，因此，本文將探討考量資產間的共同跳躍的傳染現象對於資產配置的效率前緣的影響為何。

2.2 資產動態過程

過去的投資組合文獻的資產動態都假設服從幾何布朗運動(Zhang et al., 2005; Lai et al., 2011; Fan et al., 2012)，然而，在此假設資產動態服從幾何布朗運動的情況下，並無法與過去文獻中 Merton (1976) 提出資產價格存在跳躍 (Jump) 不連續的現象，同時，不同資產資產間可能因為外在的因素變化而產生跳躍與共同跳躍(Co-jump)的現象(Andersen et al., 2007a; Dungey et al., Dungey et al., 2012; 2009; Lahaye et al., 2011)吻合。

$$dX_{it} / X_{it} = u_i dt + \sigma_i dB_t^i \quad (3)$$

其中， u_i 為第 i 資產的平均瞬時報酬率， σ_i 為第 i 資產的瞬時報酬率的波動度， B_t^i 為第 i 資產所服從的標準布朗運動(Standard Brownian Motion)。在

Markowitz 理論的下都假設 U 與 Σ 都為已知，然而在實際情況下，兩個變數都是未知的。因此，在報酬率為獨立與同質的假設下，利用離散模型分析資產的價格過程可得第 i 資產在 t 時間點的報酬率為 $r_{it} = (X_{i,t} - X_{i,t-1}) / X_{i,t-1}$ 而對數的報酬為 $\log(X_{i,t} / X_{i,t-1}) = \log(1 + r_{it}) \approx r_{it}$ ，且 $r_{it} \sim N(u_i - \sigma_i^2 / 2, \sigma_i^2)$ ，再利用最大概似法或是動差法來估計 U 與 Σ 。

此外，過去研究指出不同資產間的聯合跳躍的情形對於投資組合避險部位有重大的影響(Stein, 1961; Lien and Tse, 2002; Lee, 2010)，因此，本研究提出一個將跳躍與共同跳躍的納入到資產動態過程而提出一個多變量複合卜瓦松跳躍擴散過程(Multivariate compound Poisson diffusion process)進一步描繪資產間共同跳躍的傳染現象。

$$dX_{it} / X_{it} = u_i dt + \sigma_i dB_t^i + d(\sum_{i=0}^{N(t)} Y_i - 1) \quad (4)$$

其中， u_i 為第 i 資產的平均瞬時報酬率， σ_i 為第 i 資產的瞬時報酬率的波動度， B_t^i 為第 i 資產所服從的標準布朗運動(Standard Brownian Motion)，而 $N(t)$ 為一個多變量卜瓦松過程， $Y_i \sim N(\mu_i, \delta_i^2)$ ，且 $B_t, N(t)$ 與 Y 三者間彼此獨立。例如：當投資組合中有兩個資產 E 和 F 時，這兩個資產動態過程都存在著跳躍過程，且彼此間也存在的共同跳躍的過程。因此，

$$dX_{Et} / X_{Et} = u_E dt + \sigma_E dB_t^E + d(\sum_{i=0}^{N_E(t)} Y_{Ei} - 1) \quad (5)$$

$$dX_{Ft} / X_{Ft} = u_F dt + \sigma_F (\rho_{EF} dB_t^E + \sqrt{1 - \rho_{EF}^2} dB_t^G) + d(\sum_{i=0}^{N_F(t)} Y_{Fi} - 1) \quad (6)$$

其中， $dB_t^F = \rho_{EF} dB_t^E + \sqrt{1 - \rho_{EF}^2} dB_t^G$ 且 $B_t^E \perp B_t^G$ ， ρ_{EF} 為兩資產間的相關係數，而 $N_E(t) = \lambda_E + \lambda_c$ ， $N_F(t) = \lambda_F + \lambda_c$ ， $Y_i \sim N(\mu_i, \delta_i^2)$ ，而 $N_E(t)$ 與 $N_F(t)$ 的聯合機率分配為：

$$\begin{aligned}
P_{N_E, N_F}(n_1, n_2) &= P(N_E = n_1, N_F = n_2) \\
&= e^{-(\lambda_E + \lambda_F + \lambda_c)} \sum_{i=0}^{\min(n_1, n_2)} \frac{\lambda_E^{n_1-i} \lambda_F^{n_2-i} \lambda_c^i}{(n_1-i)!(n_2-i)!i!}, \quad n_1, n_2 = 0, 1, 2, \dots
\end{aligned} \tag{7}$$

此外， $N_E(t)$ 與 $N_F(t)$ 的共變異數為 $Cov(N_E(t), N_F(t)) = \lambda_{EF}$ ，而其相關係數為：

$$\rho_{EF} = \frac{Cov(N_E(t), N_F(t))}{\sqrt{Var(N_E(t))}\sqrt{Var(N_F(t))}} = \frac{\lambda_{EF}}{\sqrt{(\lambda_E + \lambda_{EF})(\lambda_F + \lambda_{EF})}} \tag{8}$$

因此，可以利用 λ_c 來衡量兩個跳躍過程的相依性，藉此，來衡量兩個資產之間共同跳躍的傳染情況。進一步利用所提出的動態過程來計算彼此間的共變異數來建構投資組合。在所提出的模型之下，資產間相關係數如方程式(9)所示：

$$\begin{aligned}
Corr\left(\frac{dS_1(t)}{S_1(t)}, \frac{dS_2(t)}{S_2(t)}\right) &= \frac{Cov\left(\frac{dS_1(t)}{S_1(t)}, \frac{dS_2(t)}{S_2(t)}\right)}{\sqrt{Var\left(\frac{dS_1(t)}{S_1(t)}\right)}\sqrt{Var\left(\frac{dS_2(t)}{S_2(t)}\right)}} \\
&= \frac{\rho_{12}\sigma_1\sigma_2 + \lambda_c^{h^\theta} \kappa_1^{h^{J_1}} \kappa_2^{h^{J_2}}}{\sqrt{\sigma_1^2 + \kappa_1^{h^{J_1}}^2} \sqrt{\sigma_2^2 + \kappa_2^{h^{J_2}}^2} \lambda_1^{h^\theta} + \lambda_c^{h^\theta}} \tag{9}
\end{aligned}$$

其中， $\kappa_i = E\left[\exp\left(\sum_{i=1}^{N_i(t)} Y_i\right)\right]$ 。

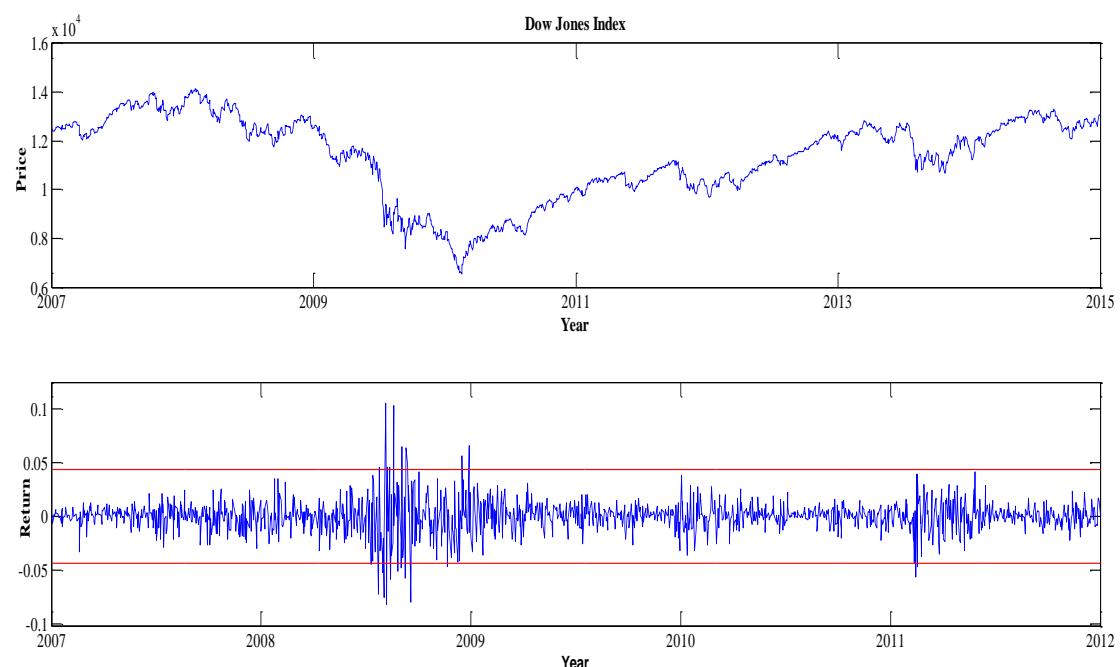
3. 資料與實證結果

本節將利用道瓊三十檔成分股來建構投資組合，並利用所提出的多變量複合卜瓦松跳躍擴散做為標的資產的動態過程並進行參數估計，並將參數估結果代入方程式(9)來計算資產間的共變異數，進而利用共變異數來建構效率前緣，並比較次貸風暴期間與次貸後資產配置的差異。3.1 節為資料的描述；3.2 節為參數的估計與效率前緣建構。

3.1 資料

本研究採用道瓊三十檔成分股來建構投資組合¹，並將研究時間區間分為次貸風暴期間與次貸風暴後的期間，而次貸風暴期間為 2007 年 1 月 3 號至 2010 年 12 月 31 日；次貸風暴後期間為 2011 年 1 月 3 日至 2014 年 12 月 31 日。道瓊三十檔成分股股價資料來源為從 YAHOO FINACE 取得。從圖一可以發現在次帶風暴期間道瓊指數報酬率與次貸風暴後的報酬率波動有明顯的不同，同時，在表一與表二的敘述統計比較也顯著發現有些成分股發生跳躍的次數²在次貸風暴期間是相對較多的，此外，在表三、表四與圖二的共同跳躍³統計表可以發現，在次貸風暴期間資三十檔成分股的共同跳躍次數相對於次貸風暴後的次數是比較多的。

圖一 道瓊指數價格與報酬率



註:圖一可以發現在道瓊指數在次貸風暴期間報酬率的波動相對於風暴後期間的波動大。同時，次貸風暴期間報瓊指數的跳躍情況也比較多。

¹ 由於道瓊 30 檔成分股中的 Visa 成分股為 2008 年 3 月才納入，因此，為了與其他成分股的研究區間一樣，本文的投資組合建構並無納入此檔成分股。

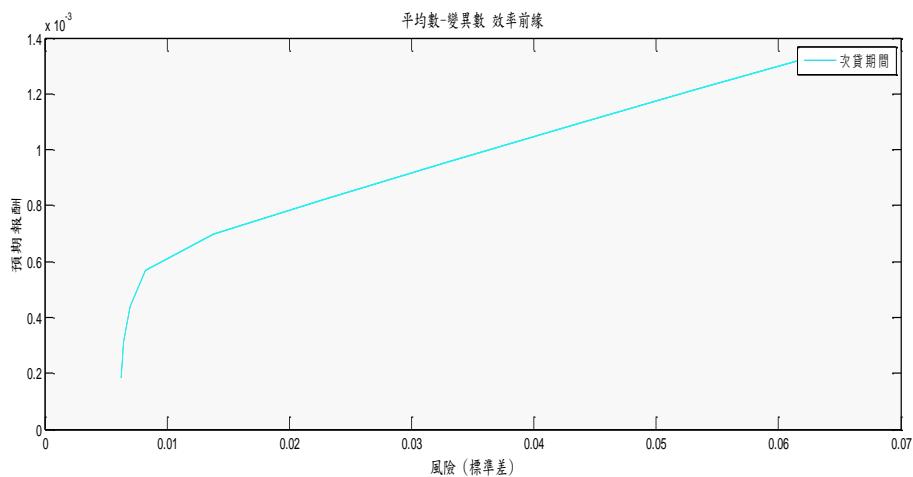
² 本研究對於跳躍次數的定義為，當個別成分股的報酬率大於其本身報酬率的 3 倍標準差即定義為跳躍。

³ 本文對於共同跳躍的定義為，當成分股 A 與成分股 B 在研究樣本第 i 天同時被定義為有跳躍現象時，即成分股 A 與成分股 B 在這第 i 天有共同跳躍的現象。

3.2 參數估計與效率前緣建構

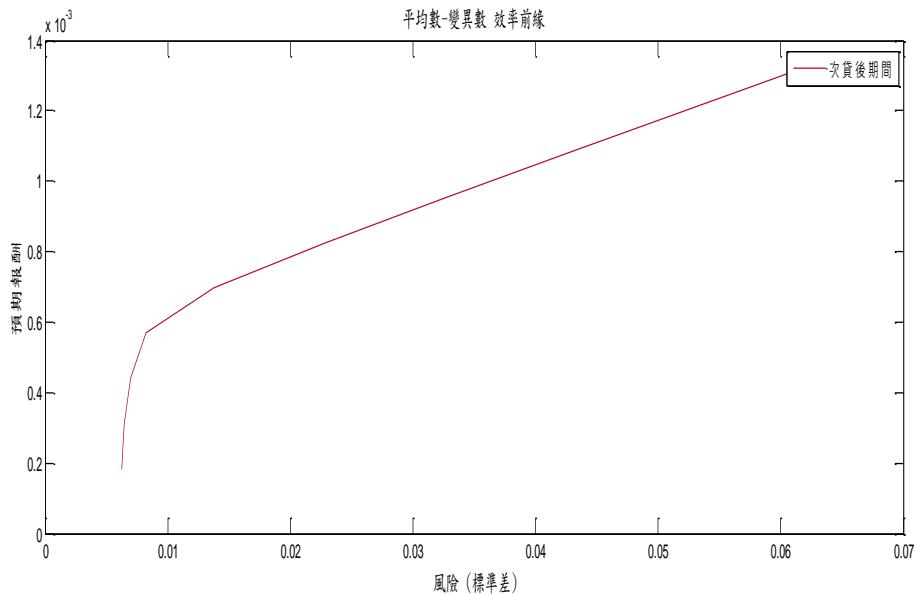
在建構投資組合前，我們必須先對於參數進行估計，本研究採兩階段估計方式，第一階段先利用報酬率超過三倍標準的方式來估計跳躍次數與共同跳躍次數，第二階段為利用最大概似法估計方程式(5)中的漂移項、波動度、跳躍波動度與跳躍幅度。根據表五與表六參數估計的結果，將其代入方程式(9)可計算出資產間的共變異數，進一步可以計算 M-V 法則下的資產配置效率前緣。從圖三與圖四可以看出次貸期間與次貸過後的效率前緣並沒有明顯的差異，雖然次貸期間的跳躍與共同跳躍次數有些微增加，也因此增加了資產間的相關係數，但增加幅度不足以影響到資產的配置情況。本文進一步分析，在次貸期間若提高共同跳躍的次數，效率前緣則會有明顯的不同，如圖五。隨著共同跳躍次數愈高，效率前緣曲線會趨近於一條直線，這顯示資產間的相關係數會因為共同跳躍的產生而提高風險分散的效果遞減，且在共同跳躍次數增加到一定程度時，資產間的相關係數趨近 1，此時，投資組合再也無法達到風險分散的效果，即當系統性風險發生時，資產間的共同跳躍現象也會顯著的增加至一定的程度，此時，就會造成風險無法分散而造成損失。

圖三 次貸期間平均數-變異數 效率前緣



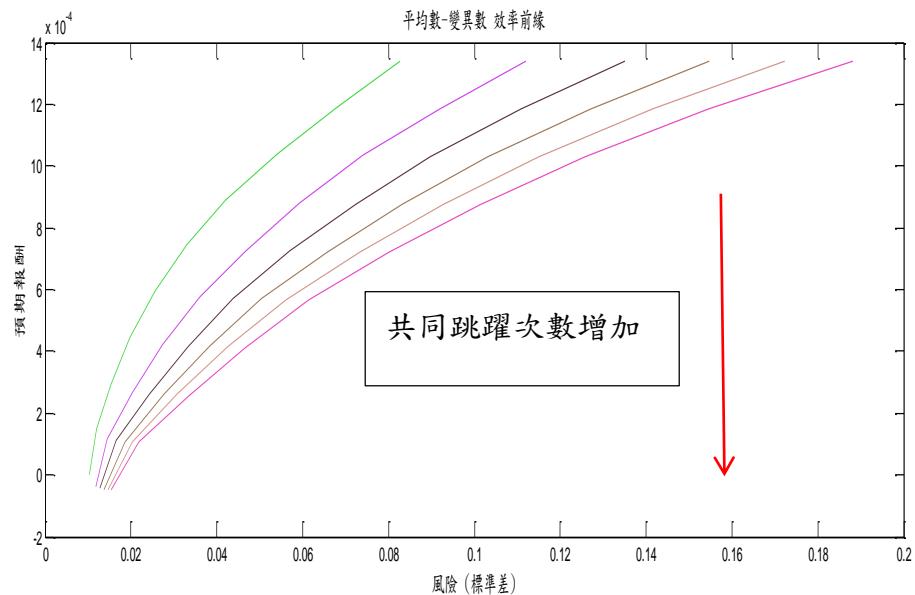
註:此圖示利用次貸期間樣本所估計的參數，所畫出來的效率前緣。

圖四 次貸後期間平均數-變異數 效率前緣



註:此圖示利用次貸後期間樣本所估計的參數，所畫出來的效率前緣。

圖五 共同跳躍次數增加情境分析平均數-變異數 效率前緣



註:圖五是利用次貸期間樣本所估計的參數並增加共同跳躍次數進行情境分析所畫出來的效率前緣。顯示出當共同跳躍次數增加，資產間的相關係數亦會增加，但投資組合的風險分散的效果會遞減，且當共同跳躍次數增加至某一程度時，資產間的相關係數會達到 1，此時，建構投資組合就無法達到風險分散的效果。

4. 結論

本文將 Markowitz 所提出的平均數-變異數法與本文所提出的多變量複合卜瓦松跳躍擴散來描繪資產的動態過程結合，並探討共同跳躍現象對於投資組合效率前緣的影響，並以道瓊三十檔成分股為研究標的。首先，研究結果發現次貸風暴期間與次貸後期間成分股存在著跳躍與共同跳躍的現象，而這些現象在次貸風暴期間更為顯著。此外，就投資組合的效率前緣而言，兩個樣本期間並無明顯的不同，雖然次貸風暴期間跳躍與共同跳躍現象比次貸後期間明顯，但增加的幅度不足以影響到投資組合的配置，然而，本文進一步分析，若在次貸期間持續增加共同跳躍的頻率，則發現效率前緣會有明顯的不同，且共同跳躍頻率愈高則在一定的風險(報酬的標準差)之下預期報酬率愈低，主要是因為共同跳躍頻率的增加，將進一步增加資產間的相關係數，進而使得風險分擔的效果遞減，且當共同跳躍次數增加至某一程度時，資產間的相關係數將趨近於 1，此時，投資組合的建構將無法達到風險分散的效果。

參考文獻

- Andersen, T. G., Bollerslev, T. and Diebold, F. X. (2007) "Roughing it up: Including jump components in the measurement, modeling, and forecasting of return volatility," *The review of economics and statistics*, Vol. 89, Issue 4, p.701-720.
- Black, F. (1972) "Capital market equilibrium with restricted borrowing," *Journal of Business*, p. 444-455.
- Brinson, G. P., Singer, B. D. and Beebower, G. L., (1991) " Determinants of portfolio performance II: An update," *Financial Analysts Journal*, Vol. 47, Issue 3, p. 40-48.
- Chopra, V. K. and Ziemba, W. T. (1993) "The effect of errors in means, variances, and covariances on optimal portfolio choice," *The journal of portfolio management*, Vol. 19, Issue 2, p. 6-11.
- De Roon, F. A., Nijman, T. E., and Werker, B. J. M. (2002) "Testing for Mean-Variance Spanning with Short Sales Constraints and Transaction Costs: The Case of Emerging Markets," *The Journal of Finance*, Vol. 56, Issue 2, p. 721-742.
- Dungey, M., McKenzie, M. and Smith, L., (2009) "News, no-news and jumps in the U.S. treasury market," *Journal of empirical finance*, Vol. 16, p. 430-445.
- Dungey, M. and Hvozdyk, L. (2012) "Cojumping: evidence from the U.S. treasury bond and futures Markets," *Journal of Banking and Finance*, Vol. 36, Issue 5, p. 1563-1575.
- Fan, J., Zhang, J. and Yu, K. (2008) "Asset allocation and risk assessment with gross exposure constraints for vast portfolios," *The Annals of Statistics*, Vol. 25, p. 1425–1432.
- Fan, J., Li, Y. and Yu, K. (2012) "Vast volatility matrix estimation using high-frequency data for portfolio selection," *Journal of the American Statistical Association*, Vol. 107, Issue 497, p. 412-428.
- Goldfarb, D. and Iyengar, G. (2003) "Robust portfolio selection problems," *Mathematics of Operations Research*, Vol. 28, Issue 1, p.1-38.
- Jacod, J. and Shiryaev, A. N. (2003) "Limit theorems for stochastic processes" New York: Springer-Verlag.
- Jagannathan, R., and Ma, T. (2003) "Risk reduction in large portfolios: Why imposing the wrong constraints helps," *The Journal of Finance*, Vol. 58, Issue 4, p.1651-1684.

- Koskosidis, Y. A. and Duarte, A. M. (1997), "A scenario-based approach to active asset allocation," *The journal of portfolio management*, Vol.23, Issue 2, p.74-85.
- Lai, T. L., Xing, H. and Chen, Z. (2011) "Mean-variance portfolio optimization when means and covariances are unknown," *The Annals of Applied Statistics*, Vol. 5, Issue 2, p.798-823.
- Lahaye, J., Laurent, S. and Neely C. J. (2011) "Jumps, cojumps and macro announcements," *Journal of Applied Econometrics*, Vol. 26, Issue 6, p.893-921.
- Laloux, L., Cizeau, P., Bouchaud, J. P. and Potters, M. (1999) "Noise dressing of financial correlation matrices," *Physical Review Letters*, Vol. 83, Issue 7, p.1467-1470.
- Ledoit, O. and Wolf, M. (2003) "Improved estimation of the covariance matrix of stock returns with an application to portfolio selection," *Journal of empirical finance*, Vol. 10, Issue 5, p. 603-621.
- Ledoit, O. and Wolf, M. (2004) "A well-conditioned estimator for large-dimensional covariance matrices," *Journal of multivariate analysis*, Vol. 88, Issue 2, p.365-411.
- Lee, H. T. (2010) "Regime switching correlation hedging," *Journal of Banking & Finance*, Vol. 34, Issue 11, p.2728-2741.
- Lien, D. and Tse, Y. K. (2002) "Some recent developments in futures hedging," *Journal of Economic Surveys*, Vol. 16, Issue 3, p.357-396.
- Lintner, J. (1965) "The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets," *The review of economics and statistics*, Vol. 47, Issue 1, p.13-37.
- Markowitz, H. M. (1952) "Portfolio selection," *Journal of Finance*, Vol. 7, p.77-91.
- Markowitz, H. M. (1959) "Portfolio selection: Efficient diversification of investments," Cowles Foundation Monograph.
- Merton, R. C. (1973) "Theory of rational option pricing," *The Bell Journal of Economics and Management Science*, Vol. 4, Issue 1, p. 141-183.
- Sharpe, W. F. (1964) "Capital asset prices: a theory of market equilibrium under Conditions of Risk," *Journal of Finance*, Vol. 19, p. 425-442.
- Stein, J. L. (1961) "The simultaneous determination of spot and futures prices," *The American Economic Review*, Vol. 51, Issue 5, p. 1012-1025.
- Zhang, L., Mykland, P. A. and Aït -Sahalia, Y. (2005) "A tale of two time scales,"

Journal of the American Statistical Association, Vol. 100, Issue 472, p.
1394-1411.

表一 次貸期間敘述統計量 (2007/01/03~2010/12/31)

公司	AAPL	AXP	BA	CAT	CSCO	CVX	DD	DIS	GE	GS	HD	IBM	INTC	JNJ	JPM
觀察個數	1007	1007	1007	1007	1007	1007	1007	1007	1007	1007	1007	1007	1007	1007	1007
平均數	0.0013	-0.0003	-0.0003	0.0004	-0.0003	0.0002	0.0000	0.0001	-0.0007	-0.0002	-0.0002	0.0004	0.0000	-0.0001	-0.0001
標準差	0.0259	0.0356	0.0227	0.0261	0.0231	0.0219	0.0232	0.0222	0.0270	0.0344	0.0226	0.0167	0.0234	0.0121	0.0386
偏態系數	-0.4722	0.1139	0.2190	0.1042	-0.5512	0.2226	-0.2970	0.4203	0.0419	0.3477	0.4771	0.1621	-0.1334	0.6364	0.3347
峰態系數	8.2840	8.0644	6.6179	6.5683	9.7740	15.0810	6.9466	9.0235	9.3881	12.4291	6.3200	7.1467	6.5316	17.5827	10.7506
跳躍個數	6	12	8	10	5	7	7	10	10	10	7	7	6	6	14

公司	KO	MCD	MMM	MRK	MSFT	NKE	PFE	PG	TRV	UNH	UTX	VZ	WMT	XOM	
觀察個數	1007	1007	1007	1007	1007	1007	1007	1007	1007	1007	1007	1007	1007	1007	1007
平均數	0.0003	0.0006	0.0001	-0.0002	-0.0001	-0.0001	-0.0004	0.0000	0.0000	-0.0004	0.0002	-0.0001	0.0001	0.0000	
標準差	0.0147	0.0149	0.0174	0.0216	0.0216	0.0308	0.0179	0.0139	0.0260	0.0291	0.0190	0.0179	0.0148	0.0204	
偏態系數	0.6942	0.0170	-0.0578	-0.4126	0.3527	-11.2710	-0.0830	-0.2150	0.3425	0.5451	0.4880	0.3348	0.2330	0.1371	
峰態系數	14.2434	7.0405	7.4827	11.0434	10.6887	258.2040	8.0482	9.7230	17.5242	21.0881	8.7202	9.4760	9.7929	15.1223	
跳躍個數	8	7	8	9	9	5	6	6	10	8	10	8	7	8	

註:表一為次貸期間 2007/01/03~2010/12/31 道瓊三十檔成分股的敘述統計量，跳躍個數的計算是採用當個別分股的報酬率高於其報酬率的三倍標準差時，即定義為跳躍。

表二 次貸後期間敘述統計量 (2011/01/03~2014/12/31)

公司	AAPL	AXP	BA	CAT	CSCO	CVX	DD	DIS	GE	GS	HD	IBM	INTC	JNJ	JPM
觀察個數	1005	1005	1005	1005	1005	1005	1005	1005	1005	1005	1005	1005	1005	1005	1005
平均數	-0.001	0.001	0.001	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.001	0.001	0.000
標準差	0.063	0.014	0.015	0.017	0.017	0.013	0.014	0.014	0.013	0.017	0.013	0.012	0.014	0.009	0.018
偏態系數	-28.358	-0.307	-0.365	-0.271	-0.322	-0.471	-0.557	-0.544	-0.111	-0.193	0.011	-0.834	0.182	0.125	-0.199
峰態系數	866.201	6.811	5.917	5.986	23.574	6.481	8.233	7.670	6.303	6.994	5.428	9.796	6.310	5.334	7.163
跳躍個數	0	6	6	6	6	7	7	6	5	8	8	6	5	6	10

公司	KO	MCD	MMM	MRK	MSFT	NKE	PFE	PG	TRV	UNH	UTX	VZ	WMT	XOM	
觀察個數	1005	1005	1005	1005	1005	1005	1005	1005	1005	1005	1005	1005	1005	1005	1005
平均數	-0.0004	0.0002	0.0006	0.0005	0.0005	0.0001	0.0006	0.0003	0.0006	0.0010	0.0004	0.0002	0.0005	0.0002	
標準差	0.0240	0.0089	0.0119	0.0117	0.0140	0.0273	0.0113	0.0089	0.0116	0.0147	0.0128	0.0105	0.0093	0.0115	
偏態系數	-24.2256	-0.2648	-0.5774	-0.1376	-0.4174	-18.2240	-0.0533	-0.2585	-0.1416	-0.1240	-0.6311	-0.2309	-0.2628	-0.3442	
峰態系數	702.1708	6.1756	7.3249	6.6150	9.9591	484.8927	5.5468	8.2200	8.5860	6.8717	7.8083	4.6235	6.8496	6.3467	
跳躍個數	0	4	6	10	5	3	6	10	12	9	4	6	6	6	

註:表二為次貸後期間 2011/01/03~2014/12/31 道瓊三十檔成分股的敘述統計量，跳躍個數的計算是採用當個別分股的報酬率高於其報酬率的三倍標準差時，即定義為跳躍。

表三 次貸期間共同跳躍次數 (2007/01/03~2010/12/31)

Name	AAPL	AXP	BA	CAT	CSCO	CVX	DD	DIS	GE	GS	HD	IBM	INTC	JNJ	JPM	KO	MCD	MMM	MRK	MSFT	NKE	PFE	PG	TRV	UNH	UTX	VZ	WMT	XOM
AAPL	6	2	2	1	3	2	2	2	3	2	3	2	2	2	2	2	2	2	2	2	1	2	2	1	3	2	2	3	2
AXP	2	12	3	5	4	2	4	3	3	5	3	3	2	1	5	1	1	3	1	4	0	1	2	2	2	4	1	1	2
BA	2	3	8	3	3	4	4	4	2	3	2	3	2	2	2	2	2	4	2	3	1	2	3	3	2	3	2	2	4
CAT	1	5	3	10	3	3	6	5	2	3	2	3	2	2	3	2	1	4	2	3	1	2	3	2	2	5	3	1	3
CSCO	3	4	3	3	5	2	4	3	3	3	3	3	3	2	2	2	2	3	2	3	1	2	2	1	3	3	2	2	2
CVX	2	2	4	3	2	7	4	6	3	2	3	3	2	4	1	5	3	6	4	4	3	4	5	4	3	5	5	2	7
DD	2	4	4	6	4	4	7	6	3	3	3	4	3	3	2	3	2	5	3	4	2	3	4	2	3	5	3	2	4
DIS	2	3	4	5	3	6	6	10	3	2	3	4	2	4	1	5	3	5	4	4	3	4	5	3	3	5	5	2	6
GE	3	3	2	2	3	3	3	3	10	3	2	3	3	2	6	2	1	4	3	5	2	3	4	3	3	5	2	1	3
GS	2	5	3	3	3	2	3	2	3	10	3	3	2	1	6	1	1	3	1	3	0	1	2	2	2	3	1	1	2
HD	3	3	2	2	3	3	3	3	2	3	7	3	2	3	1	3	3	3	3	2	2	3	3	2	4	3	3	2	3
IBM	2	3	3	3	3	3	4	4	3	3	3	7	1	2	3	2	1	3	2	2	2	2	3	1	3	3	2	1	3
INTC	2	2	2	2	3	2	3	2	3	2	2	1	6	2	2	2	2	3	2	4	1	2	3	2	3	4	2	2	2
JNJ	2	1	2	2	2	4	3	4	2	1	3	2	2	6	0	4	3	4	3	4	4	4	3	3	5	4	3	5	
JPM	2	5	2	3	2	1	2	1	6	6	1	3	2	0	14	0	0	2	1	3	0	1	2	3	2	5	0	0	1
KO	2	1	2	2	2	5	3	5	2	1	3	2	2	4	0	8	3	4	4	3	3	4	4	3	3	4	5	2	5
MCD	2	1	2	1	2	3	2	3	1	1	3	1	2	3	0	3	7	2	3	3	2	2	2	1	2	3	3	3	4
MMM	2	3	4	4	3	6	5	5	4	3	3	3	3	4	2	4	2	8	3	5	3	4	5	4	3	6	4	2	6
MRK	2	1	2	2	2	4	3	4	3	1	3	2	2	3	1	4	3	3	9	2	2	4	3	3	3	3	4	2	4
MSFT	2	4	3	3	3	4	4	4	5	3	2	2	4	4	3	3	3	5	2	9	3	3	5	3	3	7	3	3	5
NKE	1	0	1	1	1	3	2	3	2	0	2	2	1	4	0	3	2	3	2	3	5	3	3	2	2	4	3	2	4
PFE	2	1	2	2	2	4	3	4	3	1	3	2	2	4	1	4	2	4	4	3	3	6	4	3	3	4	4	2	4
PG	2	2	3	3	2	5	4	5	4	2	3	3	3	4	2	4	2	5	3	5	3	4	6	4	4	6	4	2	5
TRV	1	2	3	2	1	4	2	3	3	2	2	1	2	3	3	3	1	4	3	3	2	3	4	10	3	4	3	2	4

UNH	3	2	2	2	3	3	3	3	3	2	4	3	3	3	2	3	2	3	3	3	2	3	4	3	8	4	3	2	3
UTX	2	4	3	5	3	5	5	5	5	3	3	3	4	5	5	4	3	6	3	7	4	4	6	4	4	10	4	3	6
VZ	2	1	2	3	2	5	3	5	2	1	3	2	2	4	0	5	3	4	4	3	3	4	4	3	3	4	8	2	5
WMT	3	1	2	1	2	2	2	2	1	1	2	1	2	3	0	2	3	2	2	3	2	2	2	2	2	3	2	7	3
XOM	2	2	4	3	2	7	4	6	3	2	3	3	2	5	1	5	4	6	4	5	4	4	5	4	3	6	5	3	8

註:表三為次貸期間共同跳躍的統計，共同跳躍的定義為，當成分股 A 與成分股 B 在研究樣本第 i 天同時被定義為有跳躍現象時，即成分股 A 與成分股 B 在這第 i 天有共同跳躍的現象。

表四 次貸後期間共同跳躍次數 (2011/01/03~2014/12/31)

Name	AAPL	AXP	BA	CAT	CSCO	CVX	DD	DIS	GE	GS	HD	IBM	INTC	JNJ	JPM	KO	MCD	MMM	MRK	MSFT	NKE	PFE	PG	TRV	UNH	UTX	VZ	WMT	XOM
AAPL	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
AXP	0	6	2	2	2	3	3	2	2	2	2	1	1	2	3	0	2	4	3	1	0	2	2	5	3	1	2	1	3
BA	0	2	6	2	1	1	2	2	1	2	1	1	1	1	2	0	1	2	2	1	0	1	1	2	2	1	1	1	
CAT	0	2	2	6	1	2	3	2	2	2	2	1	1	1	3	0	1	3	2	1	0	2	1	2	2	2	1	1	1
CSCO	0	2	1	1	6	2	2	1	1	2	1	1	1	2	2	0	0	2	2	0	0	1	2	2	2	1	0	0	2
CVX	0	3	1	2	2	7	2	1	2	3	2	1	1	3	3	0	1	3	2	0	0	1	2	4	2	1	1	0	4
DD	0	3	2	3	2	2	7	2	2	3	2	1	1	2	5	0	1	4	3	1	0	2	2	3	3	2	1	1	2
DIS	0	2	2	2	1	1	2	6	1	1	1	1	1	1	2	0	1	2	2	1	0	1	1	2	2	1	1	1	
GE	0	2	1	2	1	2	2	1	5	2	0	1	1	1	2	0	0	3	1	0	0	0	1	2	1	2	0	0	2
GS	0	2	2	2	2	3	3	1	2	8	1	1	1	2	5	0	0	3	2	0	0	1	2	3	2	2	0	0	2
HD	0	2	1	2	1	2	2	1	0	1	8	0	0	1	2	0	1	2	2	1	0	2	1	2	2	0	1	2	
IBM	0	1	1	1	1	1	1	1	1	0	6	1	1	1	0	0	1	1	1	0	0	0	1	1	1	1	0	0	1
INTC	0	1	1	1	1	1	1	1	1	0	1	5	2	1	0	0	1	1	0	0	0	0	1	1	1	2	0	0	1
JNJ	0	2	1	1	2	3	2	1	1	2	1	1	2	6	2	0	0	2	2	0	0	1	2	2	2	2	1	0	3
JPM	0	3	2	3	2	3	5	2	2	5	2	1	1	2	10	0	1	4	3	1	0	2	2	4	3	2	1	1	2
KO	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
MCD	0	2	1	1	0	1	1	1	0	0	1	0	0	0	1	0	4	1	1	1	0	1	0	2	1	0	1	1	0

MMM	0	4	2	3	2	3	4	2	3	3	2	1	1	2	4	0	1	6	3	1	0	2	2	4	3	2	1	1	1	3
MRK	0	3	2	2	2	2	3	2	1	2	2	1	1	2	3	0	1	3	10	1	0	2	3	3	3	1	1	1	1	2
MSFT	0	1	1	1	0	0	1	1	0	0	1	1	0	0	1	0	1	1	5	0	1	0	1	1	1	0	1	1	1	0
NKE	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0
PFE	0	2	1	2	1	1	2	1	0	1	2	0	0	1	2	0	1	2	2	1	0	6	1	3	2	0	1	1	1	2
PG	0	2	1	1	2	2	2	1	1	2	1	1	1	2	2	0	0	2	3	0	0	1	10	2	2	1	0	0	0	2
TRV	0	5	2	2	2	4	3	2	2	3	2	1	1	2	4	0	2	4	3	1	0	3	2	12	4	1	1	1	1	3
UNH	0	3	2	2	2	2	3	2	1	2	2	1	1	2	3	0	1	3	3	1	0	2	2	4	9	1	1	1	1	2
UTX	0	1	1	2	1	1	2	1	2	2	0	1	2	2	2	0	0	2	1	0	0	0	1	1	1	1	4	0	0	1
VZ	0	2	1	1	0	1	1	1	0	0	1	0	0	1	1	1	0	1	1	1	0	1	0	1	1	1	0	6	1	1
WMT	0	1	1	1	0	0	1	1	0	0	2	0	0	0	1	0	1	1	1	1	0	1	0	1	1	0	1	6	0	0
XOM	0	3	1	1	2	4	2	1	2	2	1	1	1	3	2	0	0	3	2	0	0	2	2	3	2	1	1	0	6	0

註：表四為次貸後期間共同跳躍的統計，共同跳躍的定義為，當成分股 A 與成分股 B 在研究樣本第 i 天同時被定義為有跳躍現象時，即成分股 A 與成分股 B 在這第 i 天有共同跳躍的現象。

表五 次貸期間參數估計 (2007/01/03~2010/12/31)

公司名稱	u	σ	δ	μ	λ	公司名稱	u	σ	δ	μ	λ
AAPL	0.0015 (0.0008)	0.0235 (0.0006)	-0.0163 (0.0224)	0.0980 (0.032)	0.0060 (0.077)	KO	0.0001 (0.0004)	0.0122 (0.0004)	0.0117 (0.0115)	0.0615 (0.016)	0.0079 (0.0889)
AXP	-0.0005 (0.001)	0.0287 (0.0009)	0.0061 (0.0164)	0.1183 (0.0235)	0.0119 (0.1086)	MCD	0.0005 (0.0005)	0.0132 (0.0004)	0.0022 (0.0103)	0.0538 (0.0151)	0.0070 (0.0832)
BA	-0.0008 (0.0008)	0.0208 (0.0006)	0.0321 (0.0167)	0.0684 (0.0282)	0.0079 (0.0889)	MMM	0.0002 (0.0005)	0.0147 (0.0004)	-0.0039 (0.01)	0.0616 (0.0143)	0.0079 (0.0889)
CAT	0.0001 (0.0008)	0.0228 (0.0006)	0.0143 (0.0167)	0.0857 (0.0282)	0.0099 (0.0889)	MRK	0.0002 (0.0005)	0.0165 (0.0004)	-0.0129 (0.01)	0.0782 (0.0143)	0.0089 (0.0889)

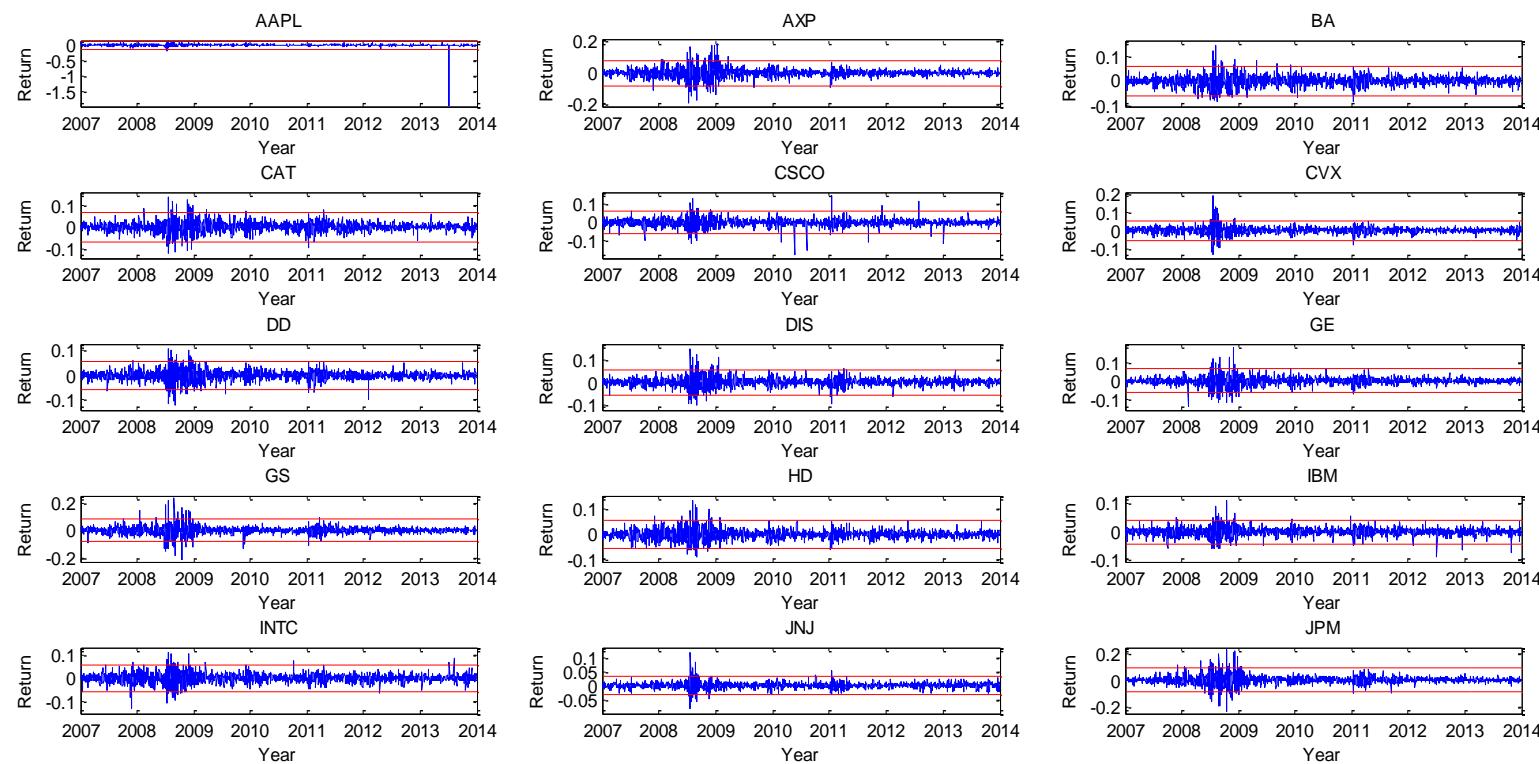
	(0.0008)	(0.0006)	(0.0148)	(0.0214)	(0.0993)		(0.0006)	(0.0005)	(0.0108)	(0.0153)	(0.0942)
CSCO	0.0002	0.0200	-0.0345	0.0891	0.0050	MSFT	0.0000	0.0178	-0.0020	0.0798	0.0089
	(0.0007)	(0.0005)	(0.018)	(0.0262)	(0.0704)		(0.0006)	(0.0005)	(0.0128)	(0.0182)	(0.0942)
CVX	0.0003	0.0171	-0.0043	0.0978	0.0070	NKE	0.0000	0.0183	-0.0085	0.1789	0.0050
	(0.0006)	(0.0005)	(0.0168)	(0.023)	(0.0832)		(0.0006)	(0.0005)	(0.0328)	(0.0422)	(0.0704)
DD	0.0004	0.0203	-0.0187	0.0787	0.0070	PFE	-0.0003	0.0155	-0.0096	0.0704	0.0060
	(0.0007)	(0.0006)	(0.0144)	(0.021)	(0.0832)		(0.0006)	(0.0004)	(0.0138)	(0.0193)	(0.077)
DIS	-0.0001	0.0182	0.0070	0.0792	0.0099	PG	0.0002	0.0117	-0.0124	0.0534	0.0060
	(0.0007)	(0.0005)	(0.0123)	(0.0174)	(0.0993)		(0.0004)	(0.0004)	(0.0097)	(0.0138)	(0.077)
GE	-0.0001	0.0206	-0.0164	0.0906	0.0099	TRV	-0.0001	0.0192	0.0065	0.1059	0.0099
	(0.0007)	(0.0007)	(0.0117)	(0.0164)	(0.0993)		(0.0007)	(0.0006)	(0.0156)	(0.0214)	(0.0993)
GS	-0.0001	0.0252	-0.0014	0.1319	0.0099	UNH	-0.0002	0.0213	-0.0053	0.1172	0.0079
	(0.0009)	(0.0007)	(0.018)	(0.0247)	(0.0993)		(0.0008)	(0.0007)	(0.0168)	(0.0234)	(0.0889)
HD	-0.0009	0.0209	0.0900	0.0167	0.0070	UTX	0.0000	0.0161	0.0090	0.0653	0.0099
	(0.0007)	(0.0005)	(0.014)	(0.014)	(0.0832)		(0.0006)	(0.0005)	(0.0107)	(0.0156)	(0.0993)
IBM	0.0004	0.0150	-0.0013	0.0571	0.0070	VZ	-0.0002	0.0151	0.0063	0.0659	0.0079
	(0.0006)	(0.0005)	(0.0111)	(0.0175)	(0.0832)		(0.0005)	(0.0005)	(0.0113)	(0.0161)	(0.0889)
INTC	0.0002	0.0211	-0.0095	0.0828	0.0060	WMT	0.0001	0.0125	-0.0002	0.0586	0.0070
	(0.0007)	(0.0006)	(0.0171)	(0.0252)	(0.077)		(0.0005)	(0.0004)	(0.0107)	(0.015)	(0.0832)
JNJ	0.0000	0.0094	-0.0046	0.0513	0.0060	XOM	-0.0001	0.0157	0.0022	0.0899	0.0079
	(0.0004)	(0.0003)	(0.0084)	(0.0117)	(0.077)		(0.0006)	(0.0004)	(0.0151)	(0.0206)	(0.0889)
JPM	-0.0010	0.0276	0.0207	0.1285	0.0139						
	(0.001)	(0.0009)	(0.0152)	(0.0211)	(0.1172)						

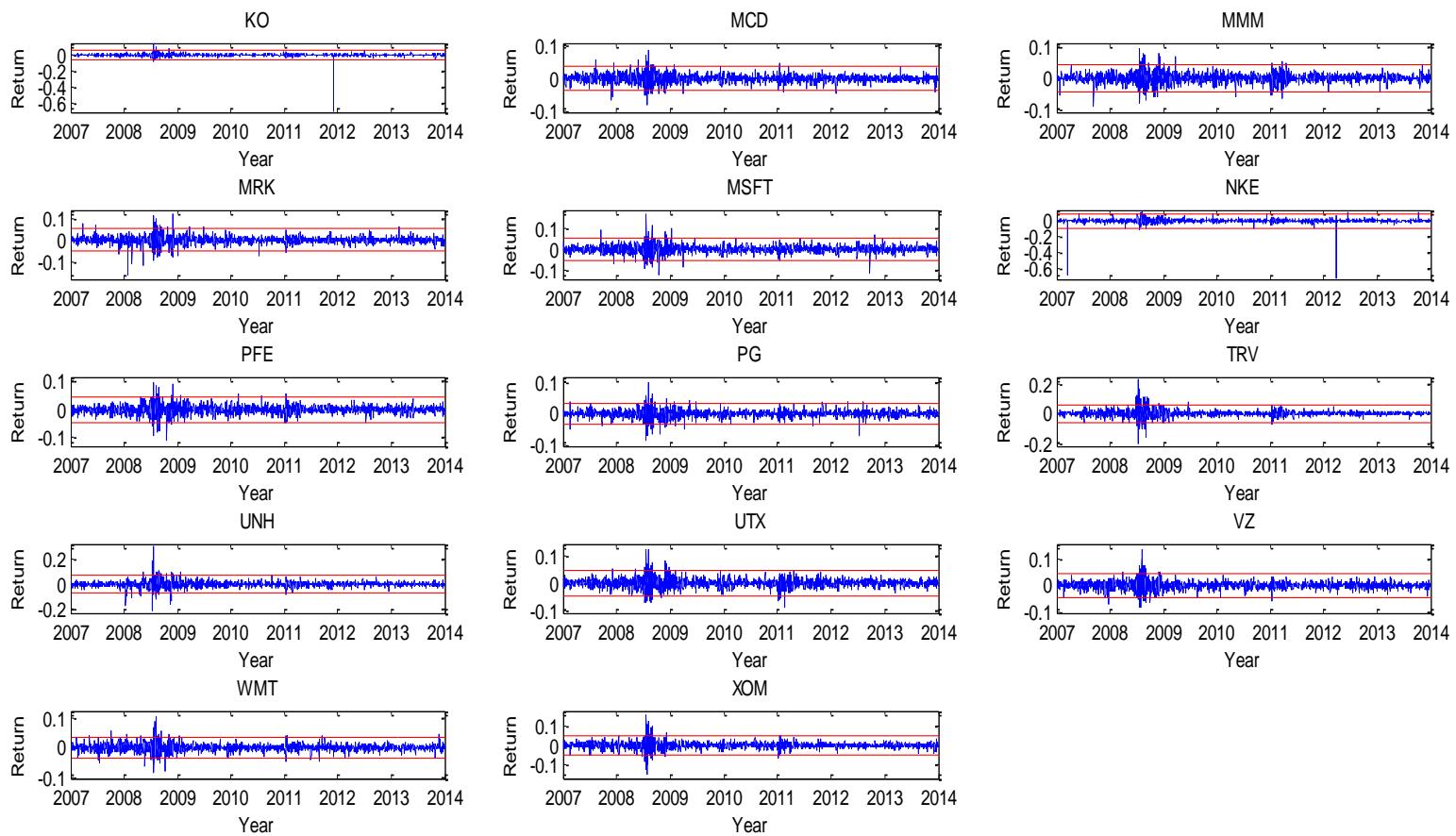
表六 次貸後期間參數估計 (20011/01/03~2014/12/31)

公司名稱						公司名稱					
AAPL	-0.0011 (0.002)	0.0632 (0.0015)	-0.0086 (443.4051)	0.0521 (443.4051)	0.0000 (0)	KO	-0.0005 (0.0008)	0.0240 (0.0006)	-0.0035 (443.4051)	0.0197 (443.4051)	0.0000 (0)
AXP	0.0008 (0.0005)	0.0130 (0.0004)	-0.0031 (0.0128)	0.0519 (0.0188)	0.0060 (0.0771)	MCD	0.0004 (0.0003)	0.0081 (0.0003)	-0.0085 (0.0074)	0.0326 (0.0105)	0.0040 (0.063)
BA	0.0010 (0.0005)	0.0135 (0.0004)	-0.0229 (0.0118)	0.0449 (0.0179)	0.0060 (0.0771)	MMM	0.0012 (0.0004)	0.0101 (0.0003)	-0.0212 (0.0064)	0.0368 (0.0091)	0.0060 (0.0771)
CAT	0.0003 (0.0006)	0.0155 (0.0004)	-0.0232 (0.0132)	0.0526 (0.0199)	0.0060 (0.0771)	MRK	0.0005 (0.0004)	0.0105 (0.0003)	0.0001 (0.0072)	0.0389 (0.0106)	0.0100 (0.0994)
CSCO	0.0005 (0.0005)	0.0124 (0.0004)	-0.0085 (0.0139)	0.0806 (0.0191)	0.0060 (0.0771)	MSFT	0.0006 (0.0005)	0.0128 (0.0004)	-0.0049 (0.0148)	0.0580 (0.0209)	0.0050 (0.0704)
CVX	0.0006 (0.0004)	0.0115 (0.0004)	-0.0231 (0.0093)	0.0368 (0.0138)	0.0070 (0.0833)	NKE	0.0008 (0.0005)	0.0127 (0.0004)	-0.0558 (0.047)	0.2142 (0.0631)	0.0030 (0.0546)
DD	0.0005 (0.0004)	0.0119 (0.0003)	-0.0063 (0.0101)	0.0522 (0.0142)	0.0070 (0.0833)	PFE	0.0007 (0.0004)	0.0105 (0.0003)	-0.0049 (0.0084)	0.0381 (0.0124)	0.0060 (0.0771)
DIS	0.0012 (0.0004)	0.0122 (0.0004)	-0.0128 (0.0099)	0.0502 (0.0142)	0.0060 (0.0771)	PG	0.0004 (0.0003)	0.0078 (0.0003)	0.0013 (0.0054)	0.0306 (0.0077)	0.0100 (0.0994)
GE	0.0005 (0.0004)	0.0122 (0.0003)	-0.0079 (0.0112)	0.0495 (0.0158)	0.0050 (0.0704)	TRV	0.0007 (0.0004)	0.0095 (0.0003)	0.0012 (0.0058)	0.0399 (0.0082)	0.0120 (0.1087)
GS	0.0002 (0.0006)	0.0154 (0.0005)	-0.0031 (0.0116)	0.0606 (0.0169)	0.0080 (0.089)	UNH	0.0011 (0.0005)	0.0131 (0.0004)	-0.0004 (0.0097)	0.0512 (0.0139)	0.0090 (0.0943)
HD	0.0010	0.0117	0.0074	0.0399	0.0080	UTX	0.0008	0.0118	-0.0610	0.0118	0.0040

	(0.0004)	(0.0003)	(0.0085)	(0.0127)	(0.089)		(0.0004)	(0.0003)	(0.0082)	(0.0087)	(0.063)
IBM	0.0006	0.0101	-0.0247	0.0394	0.0060	VZ	0.0006	0.0101	-0.0362	0.0029	0.0060
	(0.0004)	(0.0003)	(0.0076)	(0.0111)	(0.0771)		(0.0004)	(0.0003)	(0.0131)	(0.0053)	(0.0771)
INTC	0.0005	0.0134	0.0125	0.0535	0.0050	WMT	0.0006	0.0082	-0.0083	0.0334	0.0060
	(0.0005)	(0.0004)	(0.0135)	(0.0199)	(0.0704)		(0.0003)	(0.0003)	(0.0062)	(0.0088)	(0.0771)
JNJ	0.0005	0.0084	0.0113	0.0252	0.0060	XOM	-0.0005	-0.0004	-0.0135	-0.0199	-0.0704
	(0.0003)	(0.0003)	(0.0084)	(0.0138)	(0.0771)		(0.0005)	(0.0084)	(0.0113)	(0.0252)	(0.006)
JPM	0.0004	0.0151	0.0004	0.0588	0.0100						
	(0.0006)	(0.0005)	(0.0095)	(0.0139)	(0.0994)						

圖二 道瓊三十檔成分股報酬率圖





科技部補助計畫衍生研發成果推廣資料表

日期:2015/10/24

科技部補助計畫	計畫名稱: 多變量複合卜瓦松跳躍擴散模型與高頻資料下之選擇權評價與投資組合策略之研究
	計畫主持人: 廖四郎
	計畫編號: 102-2410-H-004-042-MY2 學門領域: 財務

無研發成果推廣資料

102年度專題研究計畫研究成果彙整表

計畫主持人：廖四郎		計畫編號：102-2410-H-004-042-MY2				
計畫名稱：多變量複合卜瓦松跳躍擴散模型與高頻資料下之選擇權評價與投資組合策略之研究						
成果項目		量化			備註（質化說明 ：如數個計畫共 同成果、成果列 為該期刊之封面 故事...等）	
		實際已達成 數（被接受 或已發表）	預期總達成 數（含實際 已達成數）	本計畫實 際貢獻百 分比		
國內	論文著作	期刊論文	0	0	100%	篇
		研究報告/技術報告	0	0	100%	
		研討會論文	0	0	100%	
		專書	0	0	100%	
	專利	申請中件數	0	0	100%	件
		已獲得件數	0	0	100%	
	技術移轉	件數	0	0	100%	件
		權利金	0	0	100%	千元
	參與計畫人力 (本國籍)	碩士生	0	0	100%	人次
		博士生	2	2	100%	
		博士後研究員	0	0	100%	
		專任助理	0	0	100%	
國外	論文著作	期刊論文	0	0	100%	篇
		研究報告/技術報告	0	0	100%	
		研討會論文	0	0	100%	
		專書	0	0	100%	
	專利	申請中件數	0	0	100%	件
		已獲得件數	0	0	100%	
	技術移轉	件數	0	0	100%	件
		權利金	0	0	100%	千元
	參與計畫人力 (外國籍)	碩士生	0	0	100%	人次
		博士生	0	0	100%	
		博士後研究員	0	0	100%	
		專任助理	0	0	100%	
其他成果 (無法以量化表達之 成果如辦理學術活動 、獲得獎項、重要國際 合作、研究成果國際 影響力及其他協助 產業技術發展之具體 效益事項等，請以文 字敘述填列。)		無。				

	成果項目	量化	名稱或內容性質簡述
科教處 計畫 加填 項目	測驗工具(含質性與量性)	0	
	課程/模組	0	
	電腦及網路系統或工具	0	
	教材	0	
	舉辦之活動/競賽	0	
	研討會/工作坊	0	
	電子報、網站	0	
	計畫成果推廣之參與（閱聽）人數	0	

科技部補助專題研究計畫成果報告自評表

請就研究內容與原計畫相符程度、達成預期目標情況、研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）、是否適合在學術期刊發表或申請專利、主要發現或其他有關價值等，作一綜合評估。

1. 請就研究內容與原計畫相符程度、達成預期目標情況作一綜合評估

達成目標

未達成目標（請說明，以100字為限）

實驗失敗

因故實驗中斷

其他原因

說明：

2. 研究成果在學術期刊發表或申請專利等情形：

論文：已發表 未發表之文稿 撰寫中 無

專利：已獲得 申請中 無

技轉：已技轉 洽談中 無

其他：（以100字為限）

3. 請依學術成就、技術創新、社會影響等方面，評估研究成果之學術或應用價值 (簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性)（以500字為限）