ARTICLE IN PRESS

Insurance: Mathematics and Economics ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Insurance: Mathematics and Economics

journal homepage: www.elsevier.com/locate/ime

The choice of trigger in an insurance linked security: The mortality risk case[☆]

Richard MacMinn[a,b,*], Andreas Richter[c]

[a] National Chengchi University, Taipei, Taiwan
[b] The University of Texas, Austin, TX, United States
[c] Ludwig–Maximilians-Universität, Munich, Germany

ARTICLE INFO

Article history:
Available online xxxx

JEL classification:
G22
G32
D82

Keywords:
Alternative risk transfer
Insurance
Default risk
Mortality based security
Index trigger
Indemnity trigger

ABSTRACT

In 2003, Swiss Re introduced a mortality-based security designed to hedge excessive mortality changes for its life book of business. The concern was mortality risk, i.e., the risk of premature death. The mortality risk due to a pandemic is similar to the property risk associated with catastrophic events such as earthquakes and hurricanes and the security used to hedge the risk is similar to a CAT bond. This work looks at the incentives associated with insurance linked securities. It considers the trade-offs an insurer or reinsurer faces in selecting a hedging strategy. We compare index and indemnity-based hedging as alternative design choices and ask which is capable of creating the greater value for stakeholders. Additionally, we model an insurer or reinsurer that is subject to insolvency risk, which creates an incentive problem known as the judgment proof problem. The corporate manager is assumed to act in the interests of shareholders and so the judgment proof problem yields a conflict of interest between shareholders and other stakeholders. Given the fact that hedging may improve the situation, the analysis addresses what type of hedging tool would be best. We show that an indemnity-based security tends to worsen the situation, as it introduces an additional incentive problem. Index-based hedging, on the other hand, under certain conditions turns out to be beneficial and therefore dominates indemnity-based strategies. This result is further supported by showing that for the same sufficiently small strike price the current shareholder value is greater with the index-based security than the indemnity-based security.

1. Introduction

The threat of SARS in 2003 and avian flu in 2004 subsequently have provided reminders that life insurers face correlated mortality risks on a large scale when pandemics occur. In December 2003, Swiss Re introduced a mortality-based security designed to hedge excessive mortality changes for its life books of business.[1] The motivating concern was mortality risk, i.e., the risk of premature death. Mortality risk can be managed with the standard tools as long as there are no correlated mortality surprises. Such would not be the case with a recurrence of the 1918 flu or more generally with the occurrence of a new avian flu. The potential for pandemics introduces correlated risks on a large scale and so the potential for mortality surprises. The mortality risk due to a pandemic is similar to the property risk associated with catastrophic events such as earthquakes and hurricanes and the security used to hedge the risk is similar to a catastrophe (CAT) bond that pays the insurer or reinsurer when the option component of the bond is triggered by a catastrophe (Dubinsky and Laster, 2003). These instruments help hedge risk when the catastrophe generates correlated risks in the tails of the distribution.

The model constructed here is designed to analyze the potential usefulness of mortality-based securities in hedging risk. A publicly held and traded corporation with a book of life business is constructed or equivalently a portfolio of life risks. The corporation may be an insurer or reinsurer; it will be referred to as a reinsurer throughout this article. The organization is structured so that it faces mortality risk in addition to other risks such as credit and interest rate risk. Under these conditions, a reinsurer facing a capital constraint may find a mortality-based security to be a natural risk management tool and therefore turn to the capital markets to hedge the risk. It may also retrocede its book of business. The model employed here is sufficiently general to allow for both types of instruments to be considered. The focus, however, is highlighting

[☆] Paper presented at Longevity 11: The Eleventh International Longevity Risk and Capital Markets Solutions Conference, 7–9 September 2015, Universit Lyon 1, Lyon, France.

* Corresponding author at: National Chengchi University, Taipei, Taiwan.
E-mail address: richard@macminn.org (R. MacMinn).

[1] A similar mortality-based instrument was introduced by Swiss Re in April 2005 and there have been subsequent issues including Vita Capital V Ltd in 2012. There have also been a number of other mortality bond issues; in 2013 SCOR's Atlas Capital IX Limited issued a mortality bond. See Artemis—the alternative risk transfer portal.

https://doi.org/10.1016/j.insmatheco.2017.09.018
0167-6687/

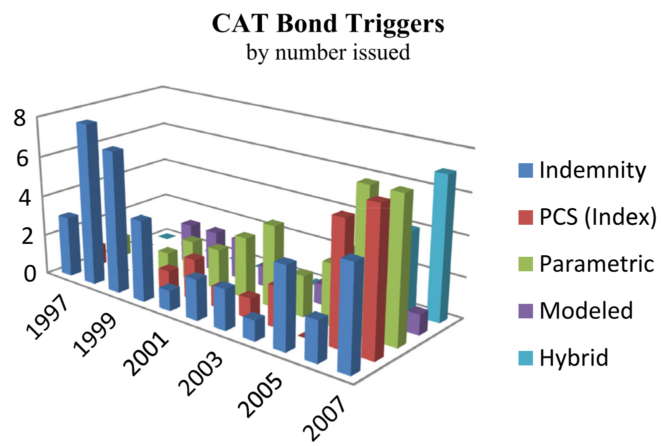

Fig. 1. CAT bond triggers by number issued.

a design choice that is particularly important in catastrophe bond issues; the question is whether an index or indemnity trigger should be used as the underlying for such a transaction.

The literature on Alternative Risk Transfer (ART) explains how the securitization of catastrophic exposures can create value. Some articles have identified the trade-offs involved in the design of optimal risk management programs integrating traditional insurance, reinsurance and ART instruments, i.e., see (Doherty, 1997; Froot, 1997; Croson and Kunreuther, 2000); also see (Cummins, 2008; Bouriaux and MacMinn, 2009; Cummins and Weiss, 2009). On the one hand, securitization of insurance risk offers advantages over traditional reinsurance arrangements, such as the potential to substantially reduce moral hazard, credit risk and transaction costs. On the other hand, possible improvements typically come at a cost of the basis risk incurred by an index-linked transaction; this is true since an index cannot perfectly represent the individual risk and would therefore only provide an imperfect hedge.

This recent literature focuses on transactions based upon index triggers. This approach seems justified in light of empirical observations in the CAT bond market: While earlier CAT bond issues were mainly based upon indemnity triggers (which have also traditionally been used in insurance and reinsurance coverage), transactions in the available data show a greater use of indexed instruments, e.g., see McGhee et al. (2005) and Fig. 1 which is based on data obtained from Goldman-Sachs.[2] As index-based solutions create the problem of basis risk, their recent popularity naturally raises the question of why the industry prefers index over indemnity triggers. The straightforward answer is that, besides potentially reducing transaction cost, an appropriately constructed index reduces or eliminates moral hazard. The introduction of a catastrophe index in a CAT bond issue or the use of a population's average life expectancy in a mortality-based security solves the moral hazard problem inherent in almost any insurance transaction.

An index trigger is a new device for addressing moral hazard. If compensation from a reinsurance contract or any other hedging instrument is based upon an index beyond the hedging party's control, this party will still reap the entire benefit of loss control in addition to the hedge. The other party, e.g., a reinsurer or the investors in an insurance linked security, do not need to be concerned about monitoring the cedent's or issuer's risk selection or loss-handling practices. A trade-off results between these benefits and the basis risk that is incurred due to the index.

A few papers have addressed the trade-offs analytically: Cummins and Mahul (2000) consider an insurance product that is subject to credit risk as well as basis risk,[3] as the insurer's payment is tied to an exogenous index. The interaction between these two factors is also analyzed by Richter (2004) albeit with two different instruments: On the one hand, insurance is subject to credit risk but can be used to generate a perfect hedge while, on the other hand, risk securitization comes without credit risk but incurs basis risk. The analysis shows that under these conditions the indexed security is beneficial whenever the credit risk on the reinsurance exists. As a tool that mainly counteracts reinsurance credit risk, securitization primarily replaces reinsurance for high levels of the loss. The latter result is confirmed by Nell and Richter (2004) who study the trade-off between the implicit transaction cost incurred by a reinsurer's risk aversion and the basis risk of a CAT bond.

The trade-off between moral hazard and basis risk has been discussed analytically by Doherty and Mahul (2001) and Doherty and Richter (2002), who investigate the interaction of these two problems, when insurance can be used to cover the basis risk of an index-linked transaction. It is shown that combining the two hedging tools might extend the possibility set and therefore lead to efficiency gains.

This analysis is constructed to examine the choice of the insurance linked security that best hedges corporate value. Like Doherty and Mahul and Doherty and Richter we consider index and indemnity triggers; the focus here, however, is on a publicly held and traded corporation acting in the interests of shareholders rather than on a risk averse manager maximizing expected utility. Rather than considering a mix of hedging instruments, we compare index and indemnity-based hedging as alternative design choices and ask which is capable of creating the greater value for corporate stakeholders. Additionally, and quite importantly, we model a reinsurer that is subject to insolvency risk[4] ; this risk of insolvency creates an additional incentive problem known as the judgment proof problem. The corporate manager is assumed to act in the interests of shareholders and so the judgment proof problem yields a conflict of interest between shareholders and other stakeholders. The judgment proof problem then yields a situation in which management does not have an incentive to select the socially optimal level of care.

A solution for the underinvestment problem suggested in the risk management literature is that potential creditors demand that the corporation hedge insolvency risk, e.g., (Jensen and Meckling, 1976; Smith and Stulz, 1985; Mayers and Smith Jr., 1987; Froot et al., 1993; Garven and MacMinn, 1993; MacMinn, 2005). This requirement can be enforced, for instance, by adding a covenant to the debt that requires the company to hedge. Given the fact that hedging improves the situation, the following analysis will address, in light of the new financial instruments described above, what type of hedging tool would be best to use. We ask whether one of

[2] A recent study by Guy Carpenter & Company (Guy Carpenter 2005), for instance identifies new risk capital in the amount of $915.3 million ($1.47 billion) that was provided through index-linked CAT bonds in 2004 (2003), while new indemnity-based transactions only amounted to 227.5 (260) million. Contrasting this, indemnity-based transactions in 1998 (1997) amounted to $846.1 ($431) million while index-based CAT bonds generated risk capital in the amount of $0 ($202 million).

[3] We refer to one of the risks as credit rather than default risk since the organization that is the object of analysis is not subject to default but rather owns a contract that is subject to default. The recently published version of (Cummins and Mahul, 2000), however, does not include the basis risk.

[4] Insolvency risk is difficult to quantify for life reinsurers. One indication of the importance of insolvency risk is represented in the impairment due to catastrophe losses of eight reinsurers between 2000 and 2011, i.e., see (2012). Credit Risk of Property Catastrophe Reinsuers. Chicago, Illinois, AON Benfield. Another indication of its importance came in 2009 when Swiss Re found it necessary to seek an injection of capital from Berkshire Hathaway, i.e., see (2009). "Swiss Re seeks injection from Berkshire Hathaway". *The Actuary* Retrieved 01/16/2017, from http://www.theactuary.com/archive/old-articles/part-4/swiss-re-seeks-injection-from-berkshire-hathaway/.

the two types of hedging discussed earlier is better than the other as a solution for the incentive distortions created by insolvency risk. Thus, the primary interest here is in the incentives associated with index versus indemnity-based insurance linked securities (or other forms of hedging) in a framework in which the issuer faces the risk of insolvency. We consider the impact such securities have on the company's actions, e.g., monitoring claims.

The analysis here extends previous work by incorporating basis risk and moral hazard in modeling a reinsurer facing insolvency risk in its books of business. In a financial market setting, we show that an indemnity-based security tends to worsen the situation, as it introduces additional incentive problems. Index-based hedging, on the other hand, under certain conditions turns out to be beneficial and dominates indemnity-based strategies. This result is further supported by showing that for the same sufficiently small strike price the current shareholder value is greater with the index-based security than the indemnity-based security.

The financial market model with a life reinsurer's mortality risk is introduced in the next section. The socially efficient operating decisions are also derived there. The following section on triggers and incentives introduces the indemnity and index instruments; the incentive effects of each are analyzed. The penultimate section compares the current shareholder values for the indemnity and index triggers given the same strike prices on those options.

2. A financial market model with mortality and insolvency risk

Consider a corporation in a competitive economy operating between the dates $t = 0$ and 1. The dates $t = 0$ and 1 are subsequently referred to as *now* and *then*, respectively. Decisions are made *now* and payoffs on those decisions are received *then*. The economy is composed of corporations and risk averse investors. Investors make portfolio decisions on personal account to maximize expected utility subject to a budget constraint.[5] The corporation will be assumed to act on behalf of its principals, i.e., the investors who are shareholders.[6] The corporation of interest here is the reinsurer.

In a standard reinsurance transaction, the profitability of a contract depends on the cedent's as well as the reinsurer's loss control effort. A primary insurer selects a portfolio of risks and negotiates the contract terms with the insured. This includes required safety and loss reduction operations as well as aspects of product design such as deductibles, retention levels or coinsurance arrangements. When claims arise, the primary settles those claims with its policyholders. Each of these activities and considerations is costly and each activity can affect the frequency and severity of claims. Obviously, this implies an incentive problem in a reinsurance relationship: If the primary is heavily reinsured, it still bears the cost of loss reduction, but the other contracting party reaps the benefit. In order to limit this incentive conflict, the reinsurer will monitor the cedent and also make use of various contractual controls. Contracts may be experience rated or retrospectively priced. Additionally, long term and brokered relationships are common in reinsurance and provide further incentive to undertake loss control. In what follows we address a reinsurer's hedging decisions.[7] We abstract from the plethora of methods available to the reinsurer to manage the risks underwritten and focus on the single activity of monitoring the claims process and we call it the level of care[8]; we suppose that increasing the level of care reduces the reinsurer's expected loss and risk.[9]

The reinsurer also faces the standard capital market risks such as interest rate and insolvency risks in addition to the mortality risk on its books of life business. The premium income will be generated *now* and invested in an asset portfolio. The losses on the books of business occur *then* and depend on the state of nature revealed as well as the care exercised. The following partially summarizes the notation used in the development of the model:

ω	State of nature
$\Omega = [0, \zeta]$	Set of states
$p(\omega)$	Basis stock price *now* in state ω
$P(\omega)$	Sum of basis stock prices $\varepsilon \le \omega$; $P(\omega) = \int_0^{\omega} p(\varepsilon)\, d\varepsilon$[a]
$\Gamma(\omega)$	Premium income *then* on the book of business; $\Gamma' > 0$[b]
a	Level of care expended on the book of business measured in dollars
$L(a, \omega)$	Mortality on book of business, $D_1 L < 0$[c]
$\Pi(\alpha, \omega)$	Payoff on book of business, i.e., $\Pi(a, \omega) = \Gamma(\omega) - L(a, \omega) - a$; $D_2 \Pi > 0$
$I(\omega)$	Population mortality index
i	Exercise price for mortality based security
S	Stock value *now*

[a] The sum of basis stock prices is not a distribution function; one can interpret the sum of all basis stock prices as the discount factor of a safe asset.
[b] As the economy improves in state so does the premium income *then* since the premium income is invested.
[c] $D_1 L$ is standard notation for the partial derivative of L with respect to the first argument, i.e., the care level a. The losses are assumed to decline with state since mortality and income are inversely related, e.g., see Table 2 in Feinstein (1993). "The Relationship between Socioeconomic Status and Health: A Review of the Literature". *The Milbank Quarterly* **71**(2): 279–322. and Preston (2007). "The changing relation between mortality and level of economic development". *International Journal of Epidemiology* **36**(3): 484–490.

Suppose the financial markets are competitive. In the absence of any insurance linked security and any insolvency risk, the stock market value of the reinsurer may be expressed as the value of its books of business as follows:

$$S(a) = \int_{\Omega} \max\{0, \Pi\}\, dP. \tag{1}$$

The reinsurer has the payoff $\max\{0, \Pi\}$ in the absence of hedging instruments such as a mortality-based security. The reinsurer may create a mortality-based security for its life book by forming a special purpose entity (SPE) similar to that for a CAT bond; the essence of the security from the perspective of the insurer, however, is the creation of an option that yields a payoff of $L(a, \omega)$ dollars in state ω for losses on its life book in excess of a trigger amount i; equivalently, the security pays $\max\{0, L(a, \omega) - i\}$ if the reinsurer uses an indemnity trigger. Alternatively, if the reinsurer uses an index trigger then the essence of the SPE is the creation of a security that yields an indexed payoff of $I(\omega)$ in state ω for losses on its life book in excess of a trigger amount i; hence, the index security pays $\max\{0, I(\omega) - i\}$. Both mortality based securities provide the reinsurer with a hedge and will be considered in the next section. Here, we analyze the behavior of the reinsurer without a hedge and compare it to socially efficient behavior.

From the reinsurer's perspective the life book of business exposes the corporation to the risk that an insured's life is briefer than expected and so we refer to it as mortality risk. The mortality risk

5 The investor portfolio decisions yield the demands for all the stock and so determine the basis for the stock prices which in turn form the means to value other financial instruments.

6 The assumption is only for convenience. The corporate objective function can be derived; for example, see MacMinn (2005). *The Fisher Model and Financial Markets*. Singapore, World Scientific Publishing.

7 The analysis (within in the same model framework) can also be carried out from the perspective of a primary insurer.

8 Please note that the same type of incentive problem, as was just discussed for the primary's loss control effort, results with respect to the reinsurer's care choice when the reinsurer is covered by a significant retrocession or other means of indemnity-based hedging. This constitutes the moral hazard issue that will be analyzed in part of this work.

9 The risk reduction is specified more carefully in the assumption.

may yield insolvency risk if the return on the premium income is not sufficient to cover the losses on the life book. This insolvency risk introduces the judgment proof problem with its associated incentive problems.

Consider the value of the reinsurer without the mortality-based security. The mortality based security is a hedging instrument and so this is the unhedged case. The unhedged reinsurer has a stock value S^u. If there is insolvency risk in an event such as a pandemic then let the state δ be the boundary of the insolvency event and let it be implicitly defined by $\Pi(a,\delta)=0$. The unhedged stock value may then be expressed as

$$\begin{aligned} S^u(a) &= \int_\Omega \max\{0, \Pi(a,\omega)\}\,dP(\omega) \\ &= \int_\delta^\zeta \Pi(a,\omega)\,dP(\omega). \end{aligned} \tag{2}$$

The reinsurer selects the level of care to maximize the current shareholder value. The first order condition (FOC) is

$$\begin{aligned} \frac{dS^u}{da} &= \int_\delta^\zeta D_1\Pi\left(a^u,\omega\right)dP(\omega) \\ &= \int_\delta^\zeta \left(-1 - D_1L(a,\omega)\right)dP(\omega)\text{ [10]} \\ &= 0. \end{aligned} \tag{3}$$

Eq. (3) implicitly defines the optimal level of care a^u. Reinsurer care is assumed to reduce the mortality risk. This assumption is formalized in the following:

Assumption. The reinsurer's payoff $L(a,\omega)$ satisfies the principle of decreasing uncertainty (PDU) and the PDU is defined by the following derivative properties: $D_2L<0$ and $D_{12}L>0$.[11]

After compensating for the change in the mean, the PDU provides a decrease in the risk of the payoff in the Rothschild–Stiglitz sense (Rothschild and Stiglitz, 1970; MacMinn and Holtmann, 1983).

Next, consider the second order condition. Observe that

$$\begin{aligned} \frac{d^2S^u}{da^2} &= \int_\delta^\zeta D_{11}\Pi\left(a^u,\omega\right)dP(\omega) \\ &\quad - D_1\Pi\left(a^u,\delta\right)p(\delta)\frac{d\delta}{da} < 0.\text{[12]} \end{aligned} \tag{4}$$

The concavity of Π or equivalently the convexity of L suffices to make the first term on the right hand side (RHS) of (4) negative. $D_1L<0$ and the PDU suffice to show that $D_1\Pi(a^u,\delta)>0$ and

$$\frac{d\delta}{da} = -\frac{D_1\Pi(a^u,\delta)}{D_2\Pi(a^u,\delta)} = -\frac{-D_1L(a^u,\delta)-1}{\Gamma'-D_2L} < 0. \tag{5}$$

Hence, the second order condition holds if the second term on the RHS of (4) is less than the first. It may also be noted that the second order condition reduces to just the first term in the absence of insolvency risk and so the concavity of the payoff suffices to show that the condition holds. We will assume that the second order condition is satisfied in the remaining analysis.

[10] $D_1\Pi$ is standard notation for the partial derivative of the function Π with respect to its first argument. Similarly $D_{12}\Pi$ is standard notation for the partial derivative of the function $D_2\Pi$ with respect to its first argument.

[11] See (MacMinn and Holtmann, 1983) for a description of this principle. It is a mirror image of the principle of increasing uncertainty introduced by Leland, i.e., see Leland (1972). "Theory of the Firm Facing Uncertain Demand". *American Economic Review* **62**: 278–291.

[12] Since the lower limit of integration is implicitly defined as a Bartle, function of care, Leibniz's rule for differentiating integrals is used here. See Bartle (1964). *The Elements of Real Analysis*. New York, John Wiley & Sons, Inc.

It is useful to compare the care decisions of the firms with and without insolvency risk. Recall that Shavell (1986) has described the situation in which an individual does not possess the resources to cover all losses with certainty as the judgment proof problem. A reinsurer facing the judgment proof problem does not have the incentive to select the socially efficient level of care as noted in the following claim and proof. Let a^e denote the level of care selected by the reinsurer given no insolvency risk.[13]

Claim. *The level of care selected by the reinsurer is greater in the absence of insolvency risk, i.e.,* $a^e>a^u$.

Proof. In the absence of insolvency risk the value is S^e where

$$S^e(a) = \int_0^\zeta \Pi(a,\omega)\,dP(\omega) \tag{6}$$

and the FOC for a socially optimal level of care is

$$\begin{aligned} \frac{dS^e}{da} &= \int_0^\zeta D_1\Pi\left(a^e,\omega\right)dP(\omega) \\ &= \int_0^\zeta \left(-1 - D_1L\left(a^e,\omega\right)\right)dP(\omega) \\ &= 0. \end{aligned} \tag{7}$$

Hence, the claim follows by noting that

$$\begin{aligned} \left.\left(\frac{dS^e}{da}-\frac{dS^u}{da}\right)\right|_{a=a^u} &= \int_0^\zeta D_1\Pi\left(a^u,\omega\right)dP(\omega) \\ &\quad - \int_\delta^\zeta D_1\Pi\left(a^u,\omega\right)dP(\omega) \\ &= \int_0^\delta D_1\Pi\left(a^u,\omega\right)dP(\omega) \\ &> 0. \end{aligned} \tag{8}$$

The inequality in (8) follows by the PDU since $D_1\Pi(a^u,\delta)$ is positive and $D_{12}\Pi<0$ yields $D_1\Pi(a^u,\omega)>0$ for all $\omega\le\delta$. □

It may also be noted that the social optimum implicitly defined in Eq. (7) is, with appropriate discounting, equivalent to the optimum noted in the literature by Shavell, 1986, Kahan (1989), MacMinn (2002). The social optimum is the level of care such that the present value of the marginal benefit equals that of the marginal cost, as seen in the following rewrite of Eq. (7)

$$\begin{aligned} \frac{dS^e}{da} &= \int_0^\zeta \left(-D_1L\left(a^e,\omega\right)-1\right)dP(\omega) \\ &= -\int_0^\zeta D_1L\left(a^e,\omega\right)dP(\omega) - \int_0^\zeta dP(\omega) \\ &= 0. \end{aligned} \tag{9}$$

The first term on the RHS of the second equality is the marginal benefit or equivalently the present value of the marginal loss reduction and the second term is the marginal cost or equivalently the present value of the last dollar spent on care.

In the next section the incentive effect of the triggers is analyzed. The analysis in this section will allow us to compare the incentive effects and see whether they move the care level in the direction of the socially efficient level; that socially efficient level is important because it maximizes the interests of all stakeholders in the corporation.

[13] The socially efficient care is that level that maximizes the value for all stakeholders in the enterprise and so can also be described in situations with insolvency risk as well. Eq. (6) would still apply.

3. Triggers and incentives

Next, consider the introduction of insurance linked securities. The security considered here is a mortality-based bond issued by a special purpose entity (SPE). The instrument is designed to pay the reinsurer in the event of mortality surprise, e.g., if the mortality rate is 130% or more of the mortality rate on the date of issue. Such an instrument may be constructed with an indemnity, index or parametric trigger. In the indemnity case the payoff from the perspective of the reinsurer would be an option payoff like $\max\{0, L(a,\omega) - i\}$ where i is the strike price. In the index trigger the payoff from the perspective of the reinsurer would be $\max\{0, I(\omega) - i\}$ where $I(\omega)$ is the population loss index.

4. Indemnity trigger

The indemnity trigger case of an insurance linked security costs C^m dollars now where C^m is the call option price for the coverage. Then

$$\begin{aligned} C^m(a,i) &= \int_{\Omega} \max\{0, L(a,\omega) - i\}\, dP \\ &= \int_0^{\gamma} (L(a,\omega) - i)\, dP \end{aligned} \tag{10}$$

where γ is the boundary of the in the money event as shown in figure one.[14] The risk adjusted present value of the area shown in Fig. 1(a) represents the option value or equivalently the cost of the indemnity trigger.

The stock value *now* of the corporation with this indemnity triggered ILS is

$$\begin{aligned} S^m(a,i) &= \int_{\Omega} \max\{0, \Pi(a,\omega) + \max\{0, L(a,\omega) - i\}\}\, dP \\ &= \int_{\alpha}^{\zeta} (\Pi(a,\omega) + \max\{0, L(a,\omega) - i\})\, dP \end{aligned} \tag{11}$$

where α is the boundary of the insolvency event as shown in Fig. 1(b).

4.1. Incentive effects of the indemnity trigger

The ILS with an indemnity trigger will have an impact on the incentive to take care. The indemnity trigger has the effect of full loss coverage in some states and that in turn impacts the care choice; equivalently, a well-known moral hazard problem (Shavell, 1979) occurs with this form of the ILS. The relationship between the exercise price and the care will be specified by the function $a^m(i)$where i is the exercise price of the option. The next theorem shows that the care level is a non-decreasing function of the strike price.

Indemnity trigger theorem: Given an indemnity trigger on an issued ILS, the care level is a non-decreasing function of the exercise price of the associated option.

Proof. Consider three cases: (a)$i \le i_1$; (b) $i_1 < i < i_2$ and (c) $i_2 \le i$.
Case (a): $i \le i_1$. Let $i_1$denote the exercise price such that$\gamma_1 > \delta$ and $\alpha_1 = 0$; for this exercise price the probability of exercise is greater than the probability of insolvency in the unhedged case and the probability of insolvency is equal to zero. Any smaller exercise price leaves the probability of insolvency equal to zero and increases the probability of exercise. It follows that the stock value of the reinsurer is

$$S^m(a) = \int_0^{\zeta} \Pi(a,\omega)\, dP + \int_0^{\gamma_1} (L(a,\omega) - i)\, dP. \tag{12}$$

The FOC is

$$\frac{dS^m}{da} = \int_0^{\zeta} D_1\Pi(a,\omega)\, dP + \int_0^{\gamma_1} D_1 L(a,\omega)\, dP = 0. \tag{13}$$

Since (13) implicitly defines a^m as a function of i and the SOC holds, it follows that

$$\frac{da^m}{di} = -\frac{\frac{\partial^2 S^m}{\partial a\, \partial i}}{\frac{\partial^2 S^m}{\partial^2 a}} \ge 0 \tag{14}$$

if the numerator in (14) is non-negative. Note that

$$\begin{aligned} \frac{\partial^2 S^m}{\partial a\, \partial i} &= \frac{\partial}{\partial i}\left(\int_0^{\zeta}\left(-\frac{\partial L}{\partial a} - 1\right) dP + \int_0^{\gamma} \frac{\partial L}{\partial a} dP\right) \\ &= \frac{\partial L}{\partial a} p(\gamma)\ \frac{\partial \gamma}{\partial i} \\ &> 0. \end{aligned} \tag{15}$$

Since L is decreasing in a and γ is decreasing in i (see Fig. 2).
Case (b): $i_1 < i < i_2$. This case is depicted in figure one and the stock value in Eq. (11). The FOC is

$$\begin{aligned} \frac{\partial S^m}{\partial a} &= \int_{\alpha}^{\zeta} \frac{\partial \Pi}{\partial a} dP + \int_{\alpha}^{\gamma} \frac{\partial L}{\partial a} dP \\ &= \int_{\alpha}^{\zeta}\left(-\frac{\partial L}{\partial a} - 1\right) dP + \int_{\alpha}^{\gamma} \frac{\partial L}{\partial a} dP \\ &= 0. \end{aligned} \tag{16}$$

Since (16) implicitly defines a^m as a function of I and the SOC holds, it follows by the Implicit Function Theorem that

$$\frac{da^m}{di} = -\frac{\frac{\partial^2 S^m}{\partial a\, \partial i}}{\frac{\partial^2 S^m}{\partial^2 a}} \ge 0 \tag{17}$$

if the numerator is non-negative. To see that the numerator in (17) is non-negative observe that

$$\begin{aligned} \frac{\partial^2 S^m}{\partial a\, \partial i} &= \frac{\partial}{\partial i}\left(\int_{\alpha}^{\zeta}\left(-\frac{\partial L}{\partial a} - 1\right) dP + \int_{\alpha}^{\gamma} \frac{\partial L}{\partial a} dP\right) \\ &= -\left(-\frac{\partial L}{\partial a} - 1\right) p(\alpha)\ \frac{\partial \alpha}{\partial i} - \frac{\partial L}{\partial a} p(\alpha)\ \frac{\partial \alpha}{\partial i} \\ &\quad + \frac{\partial L}{\partial a} p(\gamma)\ \frac{\partial \gamma}{\partial i} \\ &= p(\alpha)\ \frac{\partial \alpha}{\partial i} + \frac{\partial L}{\partial a} p(\gamma)\ \frac{\partial \gamma}{\partial i} \\ &> 0. \end{aligned} \tag{18}$$

The inequality in (18) follows because α is increasing in i, γ is decreasing in i and L is decreasing in a.[15] Therefore, the inequality in (17) is a strict inequality and $a^m(i)$ is increasing in the strike price i.
Case (c): $i_2 \le i$. This case is depicted in figure three and the stock value in Eq. (2). The FOC is in Eq. (3) .

[14] The payoffs in the figures will be represented as linear only due to the authors' limited drawing ability; the analysis does not depend on the linear functions represented in the figures.

[15] Note that α is implicitly defined by the condition $\Pi(a,\alpha) + (L(a,\alpha) - i) = 0$ or equivalently by $\Gamma(\alpha) - a - i = 0$ and so

$$\frac{\partial \alpha}{\partial i} = \frac{1}{\Gamma'(\alpha)} > 0.$$

Similarly, Γ is implicitly defined by the condition $L(a,\Gamma) - i = 0$ and so

$$\frac{\partial \Gamma}{\partial i} = \frac{1}{\frac{\partial L}{\partial \Gamma}} < 0.$$

Please cite this article in press as: MacMinn R., Richter A., The choice of trigger in an insurance linked security: The mortality risk case. Insurance: Mathematics and Economics (2017), https://doi.org/10.1016/j.insmatheco.2017.09.018.

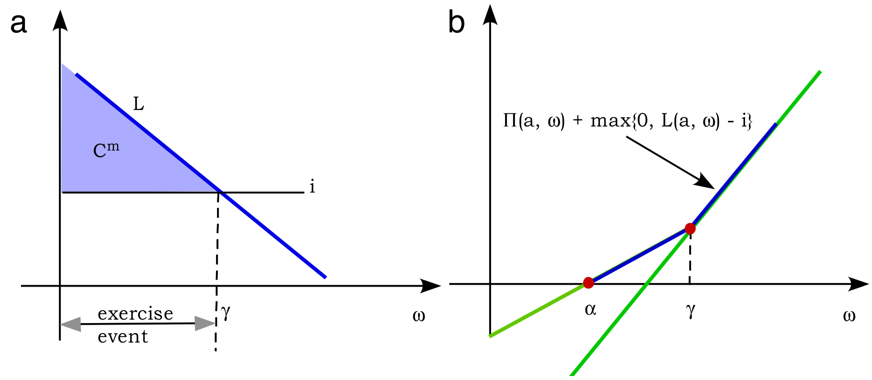

Fig. 2. The indemnity trigger option value in (a) and insolvency event boundary in (b).

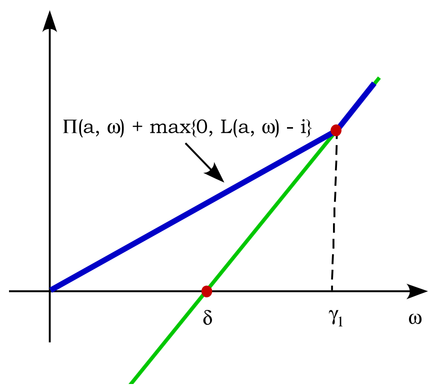

Fig. 3. The indemnity trigger Case (a).

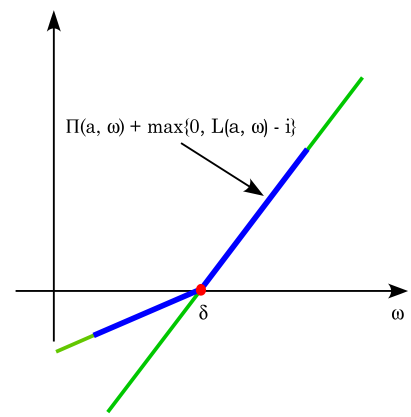

Fig. 4. The indemnity trigger Case (c).

As in the previous two cases, the function $a^m(i)$ is non-decreasing if the cross partial of the stock value is non-negative. In this case that cross partial is

$$\frac{\partial^2 S^m}{\partial a\,\partial i} = \frac{\partial}{\partial i}\left(\int_\delta^\zeta (-1 - D_1 L(a,\omega))\, dP(\omega)\right) = 0. \quad \square \tag{19}$$

The indemnity trigger theorem results are shown in Fig. 4. The care level is increasing up to the point at which the probability of insolvency equals that of the unhedged reinsurer; at that point any further increase in the exercise price has no further impact on the insolvency event or choice of care. The non-decreasing structure of $a^m(i)$ is confirmation of a moral hazard problem since an increase in the strike price i is equivalent to less insurance coverage. Fig. 4 also suggests that the indemnity triggered ILS cannot provide the incentive for adequate care since the maximum care is that for the unhedged case. Rather than improving or solving the incentive problem the introduction of the indemnity hedge aggravates the problem.

5. Index trigger

Next suppose the ILS has an index trigger; the embedded option costs C^b dollars *now* where C^b is the call option price for the coverage. Then

$$C^b(i) = \int_\omega \max\{0, I(\omega) - i\}\, dP = \int_0^\eta (I(\omega) - i)\, dP \tag{20}$$

where η is the boundary of the in the money event or equivalently the exercise event for this option as shown in Fig. 5. Unlike the indemnity trigger, this trigger is determined by the index of losses for the population. Hence the stock value for the reinsurer with this index triggered ILS is

$$\begin{aligned} S^b(a) &= \int_\omega \max\{0, \Pi(a,\omega) + \max\{0, I(\omega) - i\}\}\, dP \\ &= \int_\beta^\eta (\Pi(a,\omega) + I(\omega) - i)\, dP + \int_\eta^\zeta \Pi(a,\omega)\, dP \\ &= \int_\beta^\eta (\Gamma(\omega) - a + (I(\omega) - L(a,\omega)) - i)\, dP \\ &\quad + \int_\eta^\zeta \Pi(a,\omega)\, dP \end{aligned} \tag{21}$$

where β is the boundary of the insolvency event as shown in Fig. 3(b). The basis risk $(I - L)$ is represented in the last expression for the stock value.

5.1. Incentive effects of the index trigger

The ILS with an index trigger will have an impact on the incentive to take care. Unlike the indemnity trigger, this instrument does not generate a moral hazard problem but it does generate basis risk. The relationship between the option coverage and the underwriting care will be specified in the function $a^b(i)$ where i is the exercise price of the option. The next theorem shows that the care level is a non-increasing function of the strike price.

Index trigger theorem: Given an index trigger on an issued ILS and a loss function $L(a,\omega)$such that$I' - D_2 L \geq 0$, the care level is a non-increasing function of the exercise price of the associated option.

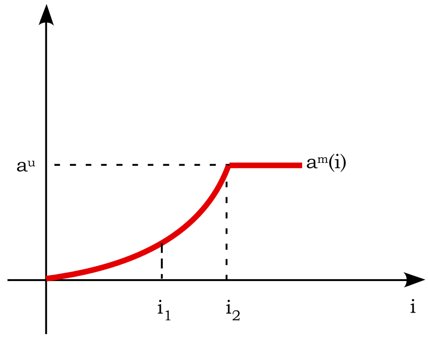

Fig. 5. The indemnity trigger and the optimal care level as a function of the exercise price.

Proof. Consider three cases: (a) $i \leq i_3$; (b) $i_3 < i < i_4$ and (c) $i_4 \leq i$. Let i_3 denote the exercise price such that insolvency risk is eliminated and the probability of exercising the option embedded in the ILS is greater than the probability of insolvency for the unhedged reinsurer, i.e., $\eta_3 > \delta$ and $\beta_3 = 0$; let i_4 denote the exercise price such that the probability of exercising the option embedded in the ILS is equal to the probability of insolvency for the unhedged reinsurer, i.e., $\eta_4 = \delta$.

Case (a): $i \leq i_3$. Let $i_3$denote the exercise price such that$\eta_3 > \delta$ and $\beta_3 = 0$. Any smaller exercise price leaves the probability of insolvency equal to zero and increases the probability of exercise. It follows that the stock value of the reinsurer is

$$S^b(a) = \int_0^{\zeta} \Pi(a, \omega)\, dP + \int_0^{\eta} (I(\omega) - i)\, dP \tag{22}$$

and the FOC is

$$\frac{\partial S^b}{\partial a} = \int_0^{\zeta} \frac{\partial \Pi}{\partial a} dP = \int_0^{\zeta} \left(-\frac{\partial L}{\partial a} - 1 \right) dP = 0. \tag{23}$$

This is the same FOC as in Eq. (7) which implicitly defines the socially efficient level of care a^e. Since the SOC holds, the function $a^b(i)$ exists and its derivative has the same sign as the cross partial. In this case the cross partial is zero and so the function $a^b(i)$ is constant at the value a^e on this interval.

Case (b): $i_3 < i < i_4$. This case is depicted in Fig. 5 and the stock value in Eq. (21). Then the FOC is

$$\frac{dS^b}{da} = \int_{\beta}^{\zeta} D_1 \Pi(a, \omega)\, dP = 0 \tag{24}$$

and the cross partial is

$$\begin{aligned} \frac{\partial^2 S^b}{\partial a\, \partial i} &= -D_1 \Pi(a,\ \beta)\ p(\beta)\ \frac{\partial \beta}{\partial i} \\ &= -(-D_1 L - 1)\, p(\beta)\ \frac{\partial \beta}{\partial i} \\ &< 0. \end{aligned} \tag{25}$$

The sign of the inequality in (25) follows because the $(-D_1 L(a,\ \beta) - 1)$ is positive by the PDU and β is an increasing function of the strike price. To verify that β is increasing note that β is implicitly defined by the condition $\Pi(a,\ \beta) + I(\beta) - i = 0$, and so it follows by implicit differentiation that

$$\frac{\partial \Pi}{\partial \beta} \frac{\partial \beta}{\partial i} + \frac{\partial I}{\partial \beta} \frac{\partial \beta}{\partial i} - 1 = 0 \tag{26}$$

or equivalently, that

$$\begin{aligned} \frac{\partial \beta}{\partial i} &= \frac{1}{\frac{\partial \Pi}{\partial \beta} + \frac{\partial I}{\partial \beta}} \\ &= \frac{1}{\Gamma' + \frac{\partial I}{\partial \beta} - \frac{\partial L}{\partial \beta}} \\ &> 0. \end{aligned} \tag{27}$$

Hence, the inequality in (25) holds and a^b is decreasing in this interval.

Case (c): $i_4 \leq i$. Let i_4 denote the exercise price such that $\eta_4 = \delta = \beta_4$. This case is depicted in Fig. 6. The stock value is that provided in Eq. (2) and the FOC is that provided in Eq. (3). Similarly the cross partial is provided in (19) and therefore the function $a^b(i)$ is a constant equal to a^u on this interval.[16] □

The results are collected in Fig. 7. Note that $a^b(i)$ is non-increasing and this is confirmation that the moral hazard problem can be eliminated and the incentive distortions due to insolvency risk can be mitigated or eliminated. The theorem also shows that the socially efficient care can be achieved by structuring the index trigger in the ILS to provide sufficient protection. The analysis also shows that the index trigger dominates the indemnity trigger in the sense that it reduces the insolvency without creating an incentive problem; the indemnity trigger, on the other hand, reduces the insolvency risk but engenders an incentive that tends to increase the insolvency risk. The dominance is investigated in the next section by comparing current shareholder values (see Fig. 8).

6. Comparison of shareholder values

The analysis shows that the insurance linked security with an index trigger can under certain conditions provide the corporate manager, *ceteris paribus*, with the incentive to take additional care as the level of protection is increased. The security with an indemnity trigger, however, does not align incentives and in fact additional protection provides the corporate manager, *ceteris paribus*, with an incentive to reduce rather than increase care. Indeed, in the case of the indemnity trigger, the care level taken by an unhedged firm provides an upper bound on the care that the manager with this instrument will take. The two triggers provide different shareholder values first because of the difference in the cost of the protection and second because of the difference in incentive effects.

The goal of this section, therefore, is to make the simplest comparison possible between the shareholder values with the index versus the indemnity trigger, taking into account the incentives associated with these instruments. Consider an exercise price that suffices to generate the socially efficient level of care when an ILS with an index trigger is used. The shareholder value $S^b(a^e)$ is expressed in Eq. (22). The option cost is C^b *now* and so the current shareholder value is $S^b - C^b$, i.e.,

$$\begin{aligned} S^b(a) &= \int_0^{\zeta} (\Pi(a, \omega) + \max\{0, I(\omega) - i\})\, dP \\ &\quad - \int_0^{\eta} (I() - i)\, dP \\ &= \int_0^{\zeta} \Pi(a, \omega)\, dP. \end{aligned} \tag{28}$$

The optimal care choice in this case is a^e.

[16] The proof is similar to that provided in the previous theorem's case (c) and so is omitted here.

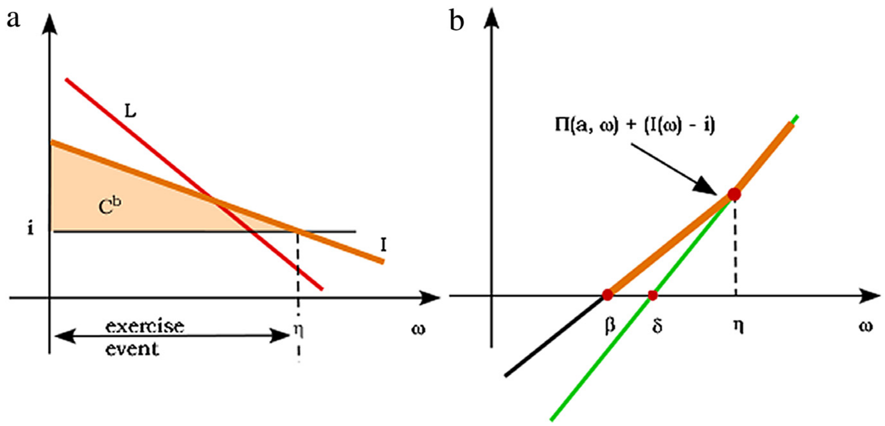

Fig. 6. The index trigger option value in (a) and insolvency event boundary in (b).

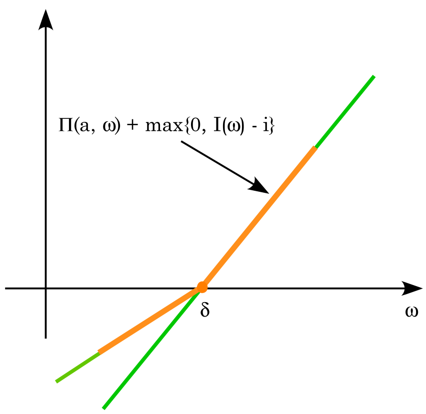

Fig. 7. The index trigger Case (c).

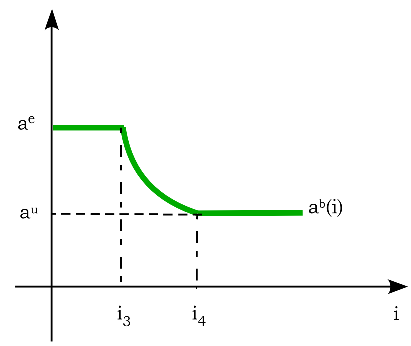

Fig. 8. The index trigger and the optimal care level as a function of the exercise price.

If an ILS with an indemnity trigger is issued then shareholder value $S^m(a)$ is expressed in Eq. (12). The option value is C^m *now* and so the current shareholder value is $S^m - C^m$, i.e.,

$$\begin{aligned} S^m(a) - C^m(a) &= \int_0^{\zeta} \Pi(a,\omega)\,dP + \int_0^{\gamma} (L(a,\omega) - i)\,dP \\ &\quad - \int_0^{\gamma} (L(a,\omega) - i)\,dP \\ &= \int_0^{\zeta} \Pi(a,\omega)\,dP. \end{aligned} \tag{29}$$

The optimal choice of care is $a < a^u < a^e$. The difference in current shareholder values is

$$\begin{aligned} &\left(S^b\left(a^e\right) - C^b\left(a^e\right)\right) - \left(S^m(a) - C^m(a)\right) \\ &= \int_0^{\zeta} \Pi\left(a^e,\omega\right)dP - \int_0^{\zeta} \Pi(a,\omega)\,dP \\ &> 0. \end{aligned} \tag{30}$$

Since a^e maximizes $\int_0^{\zeta} \Pi(a,\omega)\,dP$. Hence, for protection that suffices to eliminate insolvency risk, the ILS with the index trigger tends to dominate that instrument with the indemnity trigger.

7. Concluding remarks

The analysis begins by noting that insolvency risk in conjunction with limited liability creates an incentive problem known as the judgment proof problem. The manager of a publicly held and traded reinsurance corporation represents the stockholders interest and the judgment proof problem puts those interests in conflict with those of other stakeholders. Not surprisingly we show that such a manager selects a level of care less than the socially efficient level. The conflict of interest described here also generates an underinvestment problem; while not our focus, that problem does motivate the analysis of some new capital market instruments that have been designed to manage reinsurer or insurer insolvency risk. The new capital market instruments considered here are similar to the CAT bonds discussed in the literature in the sense that they may be designed with triggers that are either indemnity or index based so that the options attached to the bonds are in the money if the reinsurer suffers a sufficiently large loss or if the index of losses, i.e., mortality, is sufficiently large. We study the incentive effects associated with each instrument and show that the index based instrument dominates the indemnity based instrument in the sense that it reduces insolvency risk and provides the corporate manager with the incentive to take more rather than less care. We go on to show that, given the same sufficiently small strike price that eliminates the insolvency risk, the current shareholder value of the index based instrument exceeds that of the indemnity based instrument.

There is a growing literature that is concerned with hedging longevity risk, i.e., the risk of outliving ones wealth, e.g., see MacMinn et al. (2006). For further research we note that to date there has been no similar comparison of mortality-based securities to hedge excessive mortality changes for annuity books of business; such mortality-based securities could, of course, be designed to cover excessive mortality changes in the opposite direction. The concern here would be longevity risk, i.e., the risk of living too long. Mortality improvements are being reported;there has been acceleration in the mortality improvements at older ages in Sweden (Wilmoth et al., 2000). There has also been some evidence that genetics plays a major role in the ability to survive to extremely old ages and hence that genetic research may yield insights into how to slow the aging process (Strauss, 2001). The mortality improvements do yield correlated risks for insurers with life annuity books. To the extent that the improvements can be predicted accurately over the horizon of the life annuities, the longevity risk can be managed by insurers with the standard tools. Life annuities, however, have tails that are quite long and so although the mortality improvements may seem less surprising, the correlated risks are just as problematic.

Survivor bonds (Blake and Burrows, 2001) have been suggested as an effective means of managing longevity risk. The survivor bond is essentially a reverse tontine; the bond pays a coupon that is proportional to the number of survivors in a cohort. A basis risk problem might remain depending on how the cohort is structured. An instrument similar to that issued by Swiss Re for mortality risk could also be structured for longevity risk. The Swiss Re instrument is in the money if the mortality rate becomes too large but one could also write a security that would be in the money if the mortality rate became too small. In 2010, Swiss Re did sponsor Kortis Capital Ltd in the issue of a fifty million dollar note to hedge its longevity risk, i.e., see (Panteli, 2010; Stapleton, 2010).

References

Bartle, R.G., 1964. The Elements of Real Analysis. John Wiley & Sons Inc., New York.
Blake, D., Burrows, W., 2001. Survivor bonds: Helping to hedge mortality risk. J. Risk Insurance 68 (2), 339–348.
Bouriaux, S., MacMinn, R., 2009. Securitization of catastrophe risk: New developments in insurance-linked securities and derivatives. J. Insur. Issues.
Credit Risk of Property Catastrophe Reinsuers. 2012. Chicago, Illinois, AON Benfield.
Croson, D.C., Kunreuther, H.C., 2000. Customizing indemnity contracts and indexed cat bonds for natural hazard risks. J. Risk Finance 1 (3), 24–41.
Cummins, D.J., 2008. CAT bonds and other risk-linked securities: State of the market and recent developments. Risk Manag. Insur. Rev. 11 (1), 23–47.
Cummins, J.D., Mahul, O., 2000. Managing Catastrophic Risk with Insurance Contracts Subject to Default Risk. Working Paper. Wharton.
Cummins, J.D., Weiss, M.A., 2009. Convergence of insurance and financial markes: Hybred and securitized risk-transfer solutions. J. Risk Insurance 76 (3), 493–545.
Doherty, N., 1997. Corporate insurance: Competition from capital markets and financial institutions. Assurances 65 (1), 63–94.
Doherty, N.A., Mahul, O., 2001. Mickey Mouse and Moral Hazard: Uninformative but Correlated Triggers. Working Paper. Wharton School.
Doherty, N.A., Richter, A., 2002. Moral hazard, basis risk and gap insurance. J. Risk Insurance 69 (1), 9–24.
Dubinsky, W., Laster, D., 2003. Insurance-linked securities, Swiss Re Capital Markets Corporation.
Feinstein, J.S., 1993. The relationship between socioeconomic status and health: A review of the literature. Milbank Q. 71 (2), 279–322.
Froot, K., 1997. The Limited Financing of Catastrophe Risk : An Overview. National Bureau of Economic Research, Cambridge, MA.
Froot, K.A., Scharfstein, D.S., Stein, J.C., 1993. Risk management: Coordinating corporate investment and financing policies. J. Finance 48 (5), 1629–1658.
Garven, J.R., MacMinn, R.D., 1993. The underinvestment problem, bond covenants and insurance. J. Risk Insurance.
Jensen, M., Meckling, W., 1976. Theory of the firm: Managerial behavior, agency costs and ownership structure. J. Financ. Econ. 3, 305–360.
Kahan, M., 1989. Causation and incentives to take care under the negligence rule. J. Legal Stud. 18, 427–447.
Leland, H., 1972. Theory of the firm facing uncertain demand. Amer. Econ. Rev. 62, 278–291.
MacMinn, R.D., 2002. On the judgment proof problem. Geneva Pap. Risk Insur. Theory 27, 143–152.
MacMinn, R.D., 2005. The Fisher Model and Financial Markets. World Scientific Publishing, Singapore.
MacMinn, R., Brockett, P., Blake, D., 2006. Longevity risk and capital markets. J. Risk Insur. 73 (4), 551–557.
MacMinn, R.D., Holtmann, A., 1983. Technological uncertainty and the theory of the firm. South. Econ. J. 50, 120–136.
Mayers, D., Smith Jr., C.W., 1987. Corporate insurance and the underinvestment problem. J. Risk Insurance 54 (1), 45–54.
McGhee, C., Faust, J., Clarke, R., 2005. The Growing Appetite for Catastrophe Risk: The Catastrophe Bond Market At Year-End 2004. Guy Carpenter & Company, Inc..
Nell, M., Richter, A., 2004. Improving risk allocation through CAT bonds. Geneva Pap. Risk Insur. 29 (2), 183–204.
Panteli, C., 2010. Swiss Re completes first longevity trend bond. Global Pensions.
Preston, S.H., 2007. The changing relation between mortality and level of economic development. Int. J. Epidemiol. 36 (3), 484–490.
Richter, A., 2004. Moderne Finanzinstrumente im Rahmen des Katastrophen-Risk-Managements - Basisrisiko versus Ausfallrisiko. Z. Betriebswirtsch. Forsch. 56, 99–121.
Rothschild, M., Stiglitz, J.E., 1970. Increasing risk: I. A definition. J. Econom. Theory 2, 225–243.
Shavell, S., 1979. On moral hazard and insurance. Quart. J. Econ. 93 (4), 541–562.
Shavell, S., 1986. The judgement proof problem. Int. Rev. Law Econ. 6, 45–58.
Smith, C.W., Stulz, R.M., 1985. The determinants of firms' hedging policies. J. Finan. Quant. Anal. 20 (4), 391–405.
Stapleton, J., 2010. Swiss Re completes first longevity trend bond. Professional Pensions.
Strauss, E., 2001. Longevity: hints of a 'master gene' for extreme old age. Science 293 (5535), 1570–1571.
Swiss Re seeks injection from Berkshire Hathaway. 2009. The Actuary Retrieved 01/16/2017, from http://www.theactuary.com/archive/old-articles/part-4/swiss-re-seeks-injection-from-berkshire-hathaway/..
Wilmoth, J.R., Deegan, L.J., Lundstrom, H., Horiuchi, S., 2000. Increase of maximum life-span in sweden, 1861-1999. Science 289 (5488), 2366–2368.

Please cite this article in press as: MacMinn R., Richter A., The choice of trigger in an insurance linked security: The mortality risk case. Insurance: Mathematics and Economics (2017), https://doi.org/10.1016/j.insmatheco.2017.09.018.