Managerial Finance

Validity of the short- and long-run Fisher relationships: an empirical analysis

Keshab Shrestha, Sheng-Syan Chen,

Volume 24 Number 8 1998

Validity of the Short- and Long-Run Fisher Relationships: An Empirical Analysis

by *Keshab Shrestha* and *Sheng-Syan Chen,* Schulich School of Business, York University, Toronto, Canada and Department of Finance, College of Management, Yuan-Ze University, Taoyuan, Taiwan respectively.

Abstract

In this paper we analyze the short- and long-run Fisher relationships for US, UK, Canada and Japan. using monthly data on Eurocurrency interest rates and inflation rates. We show that the long-run Fisher relationship holds for all four countries. We also show that the short-run Fisher relationship holds for UK and Japan, but not for Canada. As for the US, the short-run Fisher relationship is significant only at the 10 percent level.

1. Introduction

Nominal interest rates play an important role in finance. This is why there are numerous studies devoted to the analysis of the behavior of interest rates over time. One of the questions raised concerns whether the nominal interest rates on financial securities (fully) incorporate the inflation premium as required by efficient markets. The theoretical relationship between the nominal interest rate and the expected inflation rate is expressed by the well-known Fisher hypothesis.

The Fisher hypothesis states that the nominal interest rate on financial securities must be equal to the expected inflation rate plus the expected real interest rate. In other words, if the expected real interest rate is constant or reasonably stable over time, there should be a one-for-one relationship between the nominal interest rate and the expected inflation rate. It is important to note that the Fisher hypothesis expresses the equilibrium relationship between the expected nominal return and the expected inflation rate, and this equilibrium relationship should hold in an efficient market.

There are numerous studies devoted to the analysis of the Fisher hypothesis (e.g., Fama (1975), Huizinga and Mishkin (1984), Rose (1988), Atkins (1989), Groenewold (1989), MacDonald and Murphy (1989), Bonham (1991), Moazzami (1991), Evans and Wachtel (1992), Mishkin (1992), Owen (1993), Boudoukh and Richardson (1993), Evans and Lewis (1995), etc.). Most of the previous studies concentrate only on the long-run. Fisher relationship by looking at the relationship between the nominal interest rate and the inflation rate. Some researchers (e.g., Atkins (1989), Mishkin and Simon (1995), Crowder and Hoffman (1996), etc.) have used cointegration technique to test the Fisher equation. Such cointegration-based tests represent only the long-run tests of the Fisher hypothesis.

On the other hand, the short-run test of the Fisher hypothesis has been to some extent neglected. One can examine the short-run relationship by analyzing the relationship between the changes in inflation and nominal interest rates. Mishkin (1992, 1995) analyzes the relationship between the changes, rather than the level of inflation rate and nominal rate, in order to examine the short-run Fisher relationship. He uses T-bills (for US) and Treasury notes (for Australia) as the financial instruments in testing the Fisher hypothesis. Mishkin finds no support for short-run Fisher relationship in both US and Australia.

In this paper, the short- and the long-run Fisher equations are estimated for four of the industrialized economies, namely US, UK, Canada and Japan. The analysis performed in this paper differs from previous studies in four ways. Firstly, in this paper, monthly data, instead of quarterly, semi-annual or annual data, are used. This provides an opportunity to incorporate the higher frequency components of the series. which are averaged out when the quarterly or annual data are used. Secondly, London one-month Eurocurrency interest rates, instead of domestic interest rates, are used in the analysis. This avoids risk, tax and regulatory differences between the interest rates series. Thirdly. the data include more recent observations (from July 1978 up to September 1997). This allows us to capture some recent phenomenon in the analysis. Finally, in addition to the long-run analyses, short-run analyses are performed for four countries, some of which have not been examined in the earlier studies.

The paper is divided into four sections. Section 2 is devoted to the discussion of the model. The empirical results and their implications are discussed in Section 3. A Summary and conclusion are presented in Section 4.

2. **The Model**

One of the most general forms of the Fisher equation can be written as follows (see Lucas (1978) and Crowder and Hoffman (1996)):

$$(1-\tau)R_t = r_t + I_t^e + 0.5Var(I_t) - \gamma Cov(\Delta Log(C_t), I_t) \qquad (1)$$

where R_t, = nominal interest rate for period t (known at the beginning of the period), r_t = expected real interest rate for the period t, I_t, = inflation rate for the period t, I_t^e = $E_t(I_t)$ = the expected inflation rate, C_t = level of consumption in period t, and τ the marginal tax rate of the marginal investor which is assumed to be constant. In equation (1), γ represents the coefficient of relative risk aversion. All the rates are continuously compounded rates. If it is assumed that the real rate is stationary with fixed mean real interest rate (ρ), one can replace the expected real rate by ρ without affecting the nature of the relationship. Furthermore, if the inflation expectation is assumed to be rational, then the following relationship between the expected inflation rate (I_t^e) and the actual inflation rate (I_t) can be established as follows:

$$I_t = I_t^e + u_t \qquad (2)$$

where u_t is the stationary zero mean random process. Then, the Fisher equation can be rewritten as follows:

$$I_t^e = (1-\tau)R_t - \rho - \phi + u_t \qquad (3)$$

Or,

$$I_t^e = (1-\tau)R_t - \theta + u_t, \theta = \rho + \phi \tag{4}$$

where $\phi = 0.5Var(I_t) - \gamma Cov(\Delta Log(C_t), I_t)$.

Therefore, the long-run relationship is usually estimated using the regression of the observed inflation against the observed nominal interest rate as follows:

$$I_t = \beta R_t + \alpha + u_t \tag{5}$$

where the estimate of β provides the estimate of (I - τ) and the estimate of α gives the estimate of $-\theta$.

A. Long-Run Fisher Relationship

Equation (5) can be used as a basis for the analysis of the long-run Fisher relationship. As explained by Mishkin (1992, 1995), the long-run Fisher relationship is the relationship between the nominal interest rate and the inflation rate. On the other hand, the short-run Fisher relationship is the relationship between the change in nominal interest rate and the change in inflation rate. In this sense, equation (5) can be used to test the long-run relationship.

Whether an Ordinary Least-Square (OLS) estimator can be applied to equation (5) depends on the nature of the two series (nominal interest rate and inflation rate). If both series are stationary, then one can apply the OLS estimation technique in testing the long-run Fisher equation. However, if one or both series are non-stationary (i.e., if one or both series consist of unit root), then OLS estimation can not be applied without further tests.

If only one of the series is stationary, then there cannot be a long-run (stationary) relationship between the two series. In this case, the long-run Fisher relationship does not exist. However, if both nominal interest rate and inflation rate consist of single unit root, then there may or may not be a long-run stationary relationship between the nominal interest rate and the inflation rate. If the two unit root series have a stationary relationship, then the two series are said to be cointegrated. Therefore, the existence of the long-run relationship depends on whether or not the two series are cointegrated.

In this case, we need to use a cointegration test in order to determine if the nominal interest rate is cointegrated with the inflation rate. A few different tests are available that can be used to test for cointegration. In this paper, we use the maximum likelihood estimation technique developed by Johansen and Juselius (1990). This method is considered to be more efficient compared to the residual-based two-step procedure developed by Engle and Granger (1987) (see Enders (1995), Chap. 6, Sec. 7). Furthermore, this test will lead to the so-called cointegrating vector, which can be used to obtain the long-run relationship between the two series in the form of an equation.

B. Short-Run Fisher Relationship

As explained above (see Mishkin (1992, 1995)), the short-term relationship is given by the regression of the change in the inflation rate against the change in the nominal interest rate. In other words, the short-run Fisher relationship is given by:

$$\Delta I_t = \delta_0 + \delta_1 \Delta R_t + \nu_t \tag{6}$$

where Δ represents the first difference operator (i.e., $\Delta I = (1 — L)I_t = I_t — I_{t-1}$) and ν_t represents the residual term. This is the equation used by Mishkin (1992, 1995). Strictly speaking, the short-run Fisher relation will hold if $\delta_0 = 0$ and $\delta = (1 - \tau)$. In other words, any short-run change in inflation will be fully reflected in the interest rate.

As far as the estimation of equation (6) is concerned, the OLS estimation technique will be sufficient. In general, there will be no need to use the cointegration test. This is because, even if the two series are nonstationary, they generally consist of single unit root. Therefore, the first differenced series will be stationary, and hence the OLS estimation technique will be valid and can be used to make inferences about the short-run relationship between the nominal interest rate and the inflation rate.

3. Empirical Results

In this paper, the short- and long-run Fisher equations are estimated for US, UK, Canada and Japan. The monthly data on 1-month Eurocurrency interest rates are used to represent the nominal interest rates. All the interest rates are middle rates. The monthly inflation rate is calculated based on the CPI for each country. All the rates are annualized continuously compounded rates. The data are obtained from Datastream. The sample period covers from July 1978 to September 1997 (i.e., sample size N = 231).

In general, the raw prices in the CPI are sampled at different times over the course of a month. Thus, the CPI inflation rate measured from month *t* to month (*t*+1 does not correspond to the 1-month Eurocurrency interest rate observed at the beginning of month ***t*****.** In this paper, 1-month Eurocurrency interest rates in the middle of the month are used in the analysis. This will minimize the problem associated with non-overlapping periods.

It is important to note the characteristics of the data set used in the analysis. Firstly, monthly data on the 1-month nominal interest and inflation rates are used. This avoids the overlapping problem associated with the use of the monthly data on 3-month or 1-year interest rates and inflation rates. Furthermore, the monthly data provides an opportunity to incorporate the higher frequency components of the series in the analysis. When the quarterly or annual data are used, these higher frequency components are averaged out. Secondly, London one-month Eurocurrency interest rates are used to avoid the risk, tax and regulatory differences between the interest rates series. Thirdly, the data include more recent observations (the 7/1978 — 9/1997 period.). This allows us to capture some recent phenomenon in the analysis.

Figure 1
Annualized Monthly Eurocurrency Interest Rates and CPI Inflation Rates
(July 1978-September 1997)

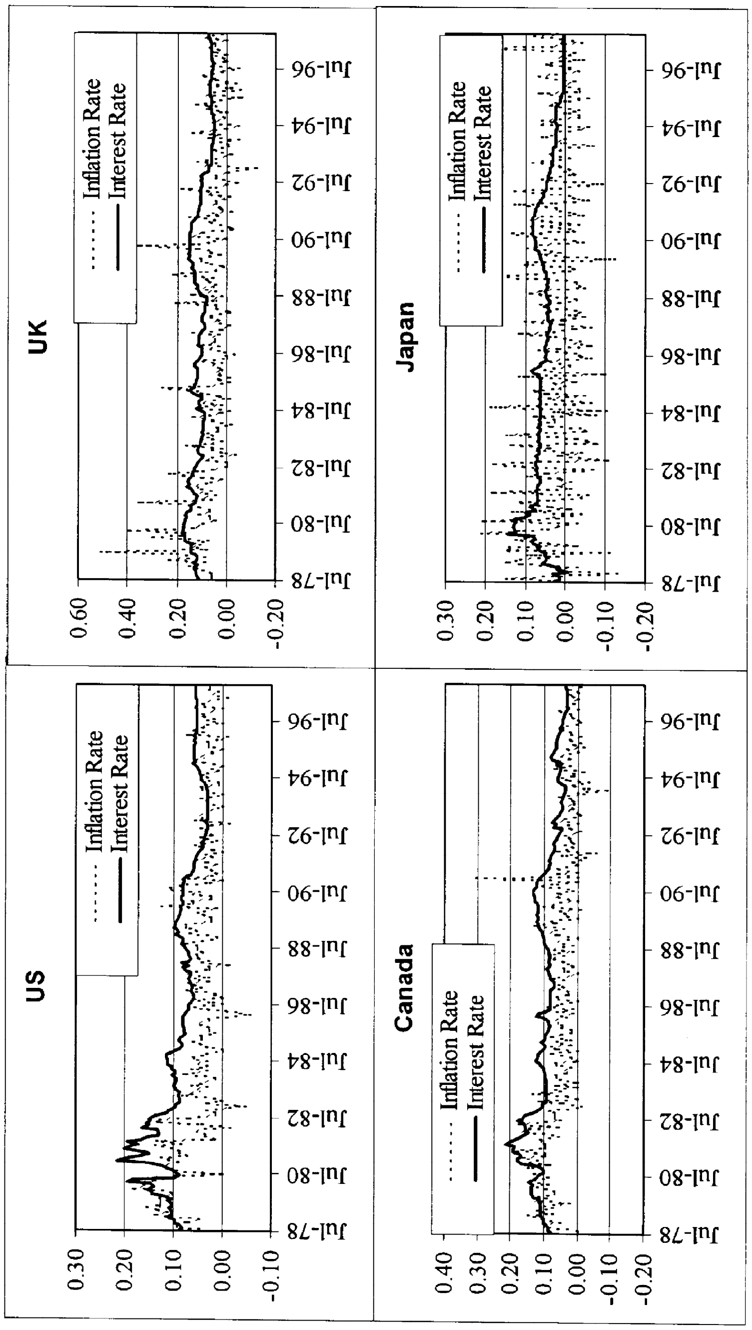

Table 1
Summary Statistics and
Correlation Coefficients between Inflation Rates and Nominal Interest Rates

Panel A: Summary Statistics

		Mean	Median	Std. Dev.	Sample Size
US:	Inflation Rate	0.0467	0.0397	0.0405	231
	Nominal Interest Rate	0.0827	0.0804	0.0379	231
UK:	Inflation Rate	0.0601	0.0481	0.0741	231
	Nominal Interest Rate	0.1055	0.1070	0.0335	231
Canada:	Inflation Rate	0.0464	0.0424	0.0469	231
	Nominal Interest Rate	0.0937	0.0909	0.0379	231
Japan:	Inflation Rate	0.0213	0.0128	0.0640	231
	Nominal Interest Rate	0.0503	0.0518	0.0278	231

Panel B: Correlation Coefficients between Inflation Rates and Nominal Interest Rate

		US		UK		Canada		Japan	
		Inflation Rate	Nominal Interest Rate	Inflation Rate	Nominal Interest Rate	Inflation Rate	Nominal Interest Rate	Inflation Rate	Nominal Interest Rate
US:	Inflation Rate	1.000	0.546	0.462	0.484	0.493	0.422	0.338	0.311
	Nominal Interest Rate		1.000	0.402	0.677	0.629	0.877	0.213	0.579
UK:	Inflation Rate			1.000	0.431	0.228	0.348	0.401	0.286
	Nominal Interest Rate				1.000	0.547	0.787	0.207	0.795
Canada:	Inflation Rate					1.000	0.585	0.105	0.454
	Nominal Interest Rate						1.000	0.170	0.693
Japan:	Inflation Rate							1.000	0.214
	Nominal Interest Rate								1.000

The historical series used in the analysis are plotted in Figure 1. From the figure, it is clear that the monthly inflation rates are more volatile compared to the monthly interest rates. This is also clear from the summary statistics given in Panel A of Table I.

For the sample period considered, UK has the highest average inflation rate and Japan has the lowest inflation rate. Similarly, UK has the highest average interest rate and Japan has the lowest average interest rate. UK seems to have the most volatile inflation rate and US seems to have the least volatile inflation rate.

The correlation matrix is given in Panel B of Table 1. The correlation coefficient between the nominal interest rate and the inflation rate seems to be quite high for each country except for Japan. This is partly because for Japan the inflation rate is quite volatile whereas the nominal interest rate is quite stable.

However, it is important to note that the above descriptions of the series are very rough. Now, we proceed to perform a more formal analysis of the relationship between the nominal interest rate and the inflation rate. We first look at the long-run relationship. As mentioned before, the long-run analysis can be based on regression equation (5).

Before proceeding, we must first decide whether we can simply use the OLS estimation technique or we need to use the more advanced cointegration technique. Therefore, the Augmented Dickey-Fuller (ADF) unit root tests are performed on each series. The results are given in Table 2. From the table, it is clear that all the eight series consist of unit roots. A further unit root test on the first differenced series (not reported in the table) confirms that each of the series consists of a single unit root. This calls for the use of cointegration tests.

Table 2
Unit Root and Cointegration Tests

	ADF Test For Unit Root		Johansen's Likelihood Ratio Test	
			No. of Cointegration	
	Nominal Interest Rate	Inflation Rate	None	At Most 1
US	-1.1784	-2.6338	20.598	3.032
UK	-2.0462	-2.5156	49.136	3.376
Canada	-1.3251	-1.2440	27.095	2.015
Japan	-1.6515	-1.7657	72.657	4.311
Critical Values (5%)			19.96	9.24

Note: The critical value for the ADF test statistic is -2.88 at the 5% level of significance (see Fuller (1976), p373.). The critical values for the Johansen's test are obtained from Osterwald-Lenum (1992).

The cointegration test results are also reported in Table 2. The cointegration tests indicate that there is a single cointegrating relationship between the inflation rate and the nominal interest rate for each country. The cointegrating vectors are reported in Table 3. For example, according to the cointegration analysis, the long-run relationship between the US inflation rate and nominal interest rate is given by:

$$I_t = 0.5104R_t + 0.0047.$$

Similarly, the long-run relationship for other countries can be obtained from the information given in Table 3.

Table 3
Cointegrating Vectors Obtained from Johansen's Maximum Likelihood Estimation Technique

Country	Coefficients of Inflation Rate	Coefficients of Interest Rate	Coefficients of Intercept
US	1.0	-0.5104 (0.1507)	-0.0047 (0.0136)
UK	1.0	-0.9546 (0.1475)	0.0389 (0.0164)
Canada	1.0	-0.8149 (0.1070)	0.0295 (0.0108)
Japan	1.0	-0.4175 (0.0811)	0.0001 (0.0047)

Note: Standard errors are given in parentheses.

Once it is established that there is a single cointegrating relationship between the two series, we can estimate equation (5) using the OLS estimation technique. The results of the OLS estimation are summarized in Table 4. In table 4, the marginal tax rates are also reported. Looking at *p*-values for β, it is clear that there exists a long-run relationship between the inflation rate and the nominal interest rate. This is also confirmed by the cointegration analysis. Table *5* presents the long-run relationships obtained from the OLS technique and the cointegration technique. The results are quite similar.

We will now analyze if the long-run relationship strictly confirms to the Fisher relationship. It is clear that there exists a strong positive relationship between the inflation rate and the nominal interest rate. This is true for each of the countries analyzed. The implied marginal tax rates are different for different currencies. It is as low as about 5% for the investors who invest in the British pound and as high as about 50% for those who invest in the Japanese yen. Although the

Table 4
Estimation of Equation (5)

		Coefficient	Std. Error	p-value	Marginal Tax Rate	Adj. R^2
US:	α	-0.0015	0.0054	0.7836		0.2947
	β	0.5834	0.0592	0.0000	0.4166	
UK:	α	-0.0405	0.0146	0.0059		0.1825
	β	0.9535	0.1318	0.0000	0.0465	
Canada:	α	-0.0215	0.0067	0.0015		0.3395
	β	0.7242	0.0663	0.0000	0.2758	
Japan:	α	-0.0034	0.0085	0.6873		0.0416
	β	0.4920	0.1484	0.0011	0.5080	

Note: The marginal tax rate (τ) is obtained from the relationship $\beta = (1-\tau)$.

Table 5
Estimation of the Long-Run Relationship $I_t = \beta R_t + \alpha + u_t$
Obtained from OLS Regressions and Cointegrating Vectors

From (OLS/Cointegrating Vector)		β	α	Marginal Tax Rates
US:	Cointegrating Vector	0.5104	0.0047	0.4896
	OLS	0.5834	-0.0015	0.4166
UK:	Cointegrating Vector	0.9546	-0.0389	0.0454
	OLS	0.9535	-0.0405	0.0465
Canada:	Cointegrating Vector	0.8149	-0.0295	0.1851
	OLS	0.7242	-0.0215	0.2758
Japan:	Cointegrating Vector	0.4175	-0.0001	0.5825
	OLS	0.4920	-0.0034	0.5080

range of marginal rate rates is wide, it is reasonable to conclude that there exists strong evidence in support of the long-run Fisher hypothesis.

The short-run relationship can be obtained by estimating the regression equation (6). Since all the series are found to consist of single unit root, the OLS estimation can be used in estimating equation (6). There is no need to use the cointegration test.

The results of the estimation of equation (6) are summarized in Table 6. The results indicate that there exists a significant positive short-run relationship between the inflation rate and the interest rate for the British pound and Japanese yen. This is evident from the *p*-values for δ_1. However, the positive short-run relationship is significant only at the 10% level for the US dollar, and it is insignificant for the Canadian dollar. This result is a little different from the one obtained by Mishkin (1992), where he does not obtain any significant positive short-run relationship between the T-bills rate and the inflation rate for US.

As for the strict short-run Fisher relationship, we need to look at the implied

Table 6
Estimation of Equation (6)

		Coefficient	Std. Error	*p*-value	Marginal Tax Rate	Adj. R^2
US:	δ_0	0.0000	0.0021	0.987		0.0081
	δ_1	0.3287	0.1944	0.092	0.6713	
UK:	δ_0	0.0002	0.0055	0.966		0.0323
	δ_1	2.4678	0.8399	0.004	-1.4678	
Canada:	δ_0	0.0001	0.0035	0.966		0.0017
	δ_1	0.5300	0.4497	0.240	0.4700	
Japan:	δ_0	0.0003	0.0056	0.957		0.0349
	δ_1	2.2055	0.7241	0.003	-1.2055	

Note: The marginal tax rate (τ) is obtained from the relationship $\delta_1 = (1-\tau)$.

marginal tax rates as before. Although the marginal tax rates for UK and Japan are negative, they are not significantly negative. Therefore, we can conclude that the short-run Fisher hypothesis holds for UK and Japan. Similarly, it holds to a lesser extent for US. The results indicate that the positive short-run relationship between the interest rate and the inflation rate is not as universal as the long-run relationship.

4. Summary and Conclusion

In this paper we have analyzed the short- and long-run Fisher relationships for four different currencies using 1-month Eurocurrency interest rates and 1-month inflation rates. For each of the currencies considered, the evidence suggests a significant positive long-run relationship between the nominal interest rate and the inflation rate. This is consistent with the long-run Fisher hypothesis. Although the range of the implied marginal tax rates for different currencies is found to be wide, it is reasonable to conclude that the long-run Fisher hypothesis holds in its strict sense.

As for the short-run Fisher relationship, the positive relationship between the nominal interest rate and the inflation rate seems to hold for UK and Japan. However, this positive short-run relationship is only significant at the 10% level for US and not significant for Canada. Therefore, the evidence regarding the short-run Fisher relationship is not as universal as the evidence for the long-run relationship.

References

Atkins, F. J., "Co-Integration, Error Correction and the Fisher Effect." *Applied Economics* 21, 1611-1620, (1989).

Bonham, C. S., "Correct Cointegration Tests of the Long-Run Relationship between Nominal Interest and Inflation." *Applied Economics* 23, 1487-1492, (1991).

Boudoukh, Jacob and Matthew Richardson, "Stock Returns and Inflation: A Long-Horizon Perspective." *American Economic Review* 83, 1347-1355, (1993).

Crowder, William J. and Dennis L. Hoffman, "The Long-Run Relationship between Nominal Interest Rates and Inflation: The Fisher Equation Revisited." *Journal of Money, Credit, and Banking* 28, 102-118, (1996).

Enders, Walter, *Applied Econometric Time Series* John Wiley & Sons, New York, (1995).

Engle, R.F. and C.W.J. Granger, "Cointegration and Error Correction; Representation, Estimation, and Testing," *Econometrica* 55, 251-276. (1987).

Evans, Martin and Paul Wachtel, "Interpreting the Movement in Short-Term Interest Rates." *Journal of Business* 65, 395-429, (1992).

Evans, Martin D. D. and Karen K. Lewis, "Do Expected Shifts in Inflation Affect Estimates of the Long-Run Fisher Relation?" *Journal of Finance* 50. 225-253. (1995).

Fama, Eugene F., "Short-Term Interest Rates as Predictors of Inflation." *American Economic Review* 65, 265-282, (1975).

Fuller, W. A., *Introduction to Statistical Time Series,* John Wiley & Sons, New York, (1976).

Groenewold, N., "The Adjustment of the Real Interest Rate to Inflation." *Applied Economics* 21, 947-956, (1989).

Huizinga, John and Frederic S. Mishkin, "Inflation and Real Interest Rates on Assets with Different Risk Characteristics." *Journal of Finance* 39, 699-714, (1984).

Johansen, Soren, "Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models." *Econometrica* 59. 1551-1580, (1991).

Johansen, Soren and Katarina Juselius, "Maximum Likelihood Estimation and Inference on Cointegration—With Applications to the Demand for Money." *Oxford Bulletin of Economies and Statistics* 52, 169-121, (1990).

Lucas, Robert J., "Asset Prices in an Exchange Economy." *Econometrica* 68, 1426-1445, (1978).

MacDonald, R. and P. D. Murphy, "Testing for the Long Run Relationship Between Nominal Interest Rates and Inflation Using Cointegration Techniques." *Applied Economics* 21, 439-447, (1989).

Mishkin, Frederic S., “Is the Fisher Effect for Real? A Reexamination of the Relationship between Inflation and Interest Rates.” *Journal of Monetary Economics* 30. 195-215, (1992).

________ and J. Simon, “An Empirical Examination of the Fisher Effect in Australia.” *The Economic Record* 71, 217-229, (1995).

Moazzami, B., “The Fisher Equation Controversy Re-Examined.” *Applied Financial Economics* 1, 129-133, (1991).

Osterwald-Lenum, M., “A Note with Quantiles of the Asymptotic Distribution of the Maximum Likelihood Cointegration Rank Test Statistics.” *Oxford Bulletin of Economics and Statistics* 54, 461-472, (1992).

Owen, P. Dorian, “Cointegration Analysis of the Fisher Hypothesis: The Role of the Real Rate and the Fisher Identify.” *Applied Financial Economics* 3,21-26, (1993).

Rose, Andrew K., “Is the Real Interest Rate Stable’?” *Journal of Finance* 43, 1095-1112, (1988),