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Seller's Optimal Credit Period and Delivery Number in EPQ Models when Production Costs have the Learning-by-Experience Effects

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To stimulate sales and remain competitive, the seller usually offers the buyer a credit period to settle the purchase amount with no interest charges. In addition, the more quantity produced and sold, the cheaper the unit production cost due to the learning-by-experience effect. Therefore, from the seller's perspective, offering trade credit increases sales volume, resulting in lower unit production cost. On the other hand, granting trade credit increases not only interest loss during credit period but also default risk. However, relatively little attention has been paid to the fact that trade credit increases sales volume and reduces the production cost due to the learning-by-experience effect. In this paper, we develop the seller's optimal credit period and number of deliveries in an Economic Production Quantity model in which trade credit has positive impacts on sales and learning production cost while it has negative impacts on interest loss and default risk. We then formulate the problem as a mixed integer programming problem, and solve it by computer software. For simplicity, we propose a remarkably good heuristic algorithm. Finally, we use sensitivity analysis to show several managerial insights, and that the learning-by-experience effect can significantly increase the seller's credit period and total profit.

Key Words: Inventory management, Trade credit, Learning production cost, Economic production quantity.

Introduction

To increase sales and profits, sellers usually offer their buyers a credit period to settle the purchase amount without interest charges. During the credit period, buyers can earn the interest from sales revenue; meanwhile sellers lose the interest earned during the same time frame. However, if the payment is not paid in full by the end of the credit period, then sellers charge buyers an interest on the outstanding amount. From the buyer's prospective, trade credit reduces the buyer's inventory holding cost, and thus affects the buyer's order quantity. From the seller's prospective, granting trade credit enables buyers to increase their purchases because the short-term free financing. It is a well-known fact as in Arrow (1962) or Teng and Thompson (1983, 1996) that the unit production cost declines each time the accumulated production volume doubles, due to learning-by-doing effect. Consequently, granting trade credit not only increases sales volume but also reduces learning production cost simultaneously. On the other hand, offering credit period also increases interest loss during the credit period and the probability that the buyer will not be able to pay off its debt obligations. In short, trade credit has positive effects on both sales volume and production cost while has negative effects on both opportunity cost and default risk.

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As a result, how to determine the optimal credit period becomes an important and relevant strategy for the seller to maximize his/her profit.

Trade credit financing is increasingly recognized as an important strategy to increase profitability in the Inventory Management (Chen, Cárdenas-Barrón, & Teng, 2014; Chen & Teng, 2014). In today's competitive markets, most companies grant buyers varied credit terms to stimulate sales and reduce inventory (Chen & Teng, 2015). In addition, a supplier frequently offers her/his retailers a permissible delay in payments in order to stimulate sales and reduce inventory (Chen, Teng, & Skouri, 2014). Trade credit is vital to today's business tractions, and calculated based on discounted cash flow analysis on the purchase cost. Trade credit is important to both seller and buyer, then discounted cash flow analysis should also be used on the revenue and the other costs (Chen, Chou, & Wu, 2012; Sua & Linb, 2009; Guo, Chiang, & Yang, 2008; Fang, Hsieh, & Deng, 2008; Ting, 2008; Ku, 2003; Chou, Ho, & Lu, 2013; Chou, Lin, & Yu, 2003).

Harris (1913) proposed the Economic Order Quantity (thereafter, EOQ) model with constant demand and cost. Then Beranek (1967) first introduced trade credit into inventory model. Goyal (1985) proposed an EOQ model with permissible delay in payments. Shah (1993) considered an exponential decaying inventory model when delay in payment is permissible. Hwang and Shinn (1997) added the pricing strategy to the model, and derived the optimal price and lot sizing for a retailer under the condition of permissible delay in payments. Teng (2002) proved that it makes economic sense for a well-established buyer to order less quantity and take the benefits of the permissible delay more frequently. Shinn and Hwang (2003) considered both pricing and ordering policies under order-size dependent delay in payments. Chang, Ouyang, and Teng (2003) developed an EOQ model for deteriorating items under supplier credits linked to ordering quantity. Ouyang, Chang, and Teng (2005) discussed an EOQ model for deteriorating items under trade credits, and then extended the model to allow for partial backlogging in Ouyang, Teng, and Chen (2006). Goyal, Teng, and Chang (2007) established optimal ordering policies when the supplier provides a progressive interest-payable scheme. Chang, Teng, and Goyal (2008) provided a review on inventory lot-size models under trade credits. Teng (2009) established an EOQ model for a retailer who offers distinct trade credits to its good and bad credit customers. Chang, Ouyang, Teng, and Cheng (2010) presented an economic production quantity (thereafter, EPQ) model for deteriorating items with discounted cash-flow analysis. Teng, Krommyda, Skouri, and Lou (2011) obtained the retailer's optimal ordering policy when the supplier offers a progressive permissible delay in payments. Skouri, Konstantaras, Papachristos, and Teng (2011) considered inventory models with ramp-type demand rate under trade credit financing. Su (2012) built up an optimal ordering policy for an integrated inventory system with defective items and allowable shortage under trade credit financing. Ouyang and Chang (2013) proposed an EPQ model with imperfect production process and complete backlogging. Recently, Chen et al. (2014a) developed an EOQ model when the supplier offers partial trade credit link to order quantity. Then Chen et al. (2014b) further extended to an EPQ model for deteriorating items. Concurrently, Tsao (2014) expanded an EPQ model to limited warehouse capacity. In all articles described above, the inventory models are studied only from the perspective of the buyer. How to determine the credit period for the seller has received relatively little attention by the researchers.

Arrow (1962) observed that the unit cost to produce a product declines by a factor of from 10 to 50 percent each time the accumulated production volume doubles, due to learning-by-experience effect. In other words, when production cost vs. production volume is plotted on a log-log scale, the graph is approximately a straight line with negative slope –l, where $0.1 \le l \le 0.5$. As noted the learning coefficient l in this learning-by-experience phenomenon can be estimated by plotting cost vs. volume on a log-log scale. Many researchers have applied this learning-by-doing phenomenon into production-marketing model to obtain optimal pricing, advertising, quality, and other strategies, such as Teng and Thompson (1983, 1996), Thompson and Teng (1984), Tsai (2012), Tsao (2013), and others. However, a few researchers in the field of inventory control with trade credit financing implemented this well-known learning production cost into EOQ/EPQ models.

In this paper, we will establish the seller's lot-sizing and trade-credit policies in EPQ models by taking the following relevant and important facts into consideration: (i) granting trade credit has positive effects on both sales volume and learning-by-experience production cost meanwhile has negative effects on both interest loss and default risk, and (ii) the unit production cost declines when the accumulated production volume increases due to the learning-by-experience effect. Then we will derive the necessary and sufficient conditions to obtain the optimal production lot size and trade credit period for the seller. Finally, we will use some numerical examples to show that (1) learning-by-experience production cost may significantly increase trade credit and improve total profit for the seller, and (2) the sensitivity analysis on the optimal solution with respect to each parameter reveals some managerial insights.

Notations and assumptions

Notations

t	the buyer's replenishment time in years.
$D=D(m)$	the buyer's annual demand rate in units as a function of the trade credit period m.
Dt	the buyer's order quantity per order in units.
m	the seller's trade credit period to his/her buyers in years (the seller's decision variable).
n	the seller's number of deliveries to the buyer per production cycle, which is a positive integer (the seller's decision variable).
$Q=nDt$	the seller's production lot size in units.
S	the seller's setup cost in dollars per production run.
F	the seller's fixed process cost in dollars to deal with each buyer's order.
R	the seller's annual production rate in units, with $R > D$.
$c(Q)$	the seller's learning production cost in dollars for making Q units.
P	the seller's unit price in dollars. We assume without loss of generality (WLOG) that the unit price is greater than the average unit production cost (i.e., $P > c(Q) / Q$).
H	the seller's holding cost per unit per year in dollars.
r	the seller's annual compounded interest rate on opportunity cost.
$\Pi(m,n)$	the seller's total profit function per year in dollars.
m^*	the seller's optimal trade credit in years.
n^*	the seller's optimal number of deliveries to the buyer per production cycle.
Π^*	the seller's optimal profit per year in dollars.

Next, we present the necessary assumptions to establish the mathematical inventory model with trade credit financing and learning-by-doing production cost.

Assumptions

1. The buyer orders Dt units every t years. The seller produces Q (i.e., nDt) units with a finite annual production rate R ($R > D$) in one production run but delivers in Dt units to the buyer over n (i.e., a positive integer) times.
2. Due to learning by experience, the unit production cost declines when the accumulated production volume increases (e.g., see Arrow, 1962; Teng, Lou, & Wang, 2014). For simplicity, we may assume that the learning-by-experience production cost for making Q units is as follows:

$$c(Q) = C_s Q^u, \tag{1}$$

where C_s (i.e., the production cost of the first unit) and $u \le 1$ are positive constants, and the learning effect $l = 1 - u \ge 0$. Note that if $u = 1$ (i.e., $l = 0$) then the total unit production cost is constant and there is no learning-by-experience effect.

3. Similar to Teng and Lou (2012) and Chern, Pan, Teng, Chan, and Chen (2013), we assume that the demand rate $D(m)$ is a positive exponential function of the

credit period m as

$$D(m) = Ke^{am}, \tag{2}$$

where K (i.e., the constant demand rate if no trade credit) and a are positive constants. For convenience, $D(m)$ and D will be used interchangeably.

4. The longer the trade credit period to the buyer, the higher the default risk to the seller. In practice, there are three simple ways to represent an increasing of default risk with respect to the credit period m: linear, polynomial, or exponential. It has been shown that these three functions lead to similar conclusions such as in Chern, Chan, Teng, and Goyal (2014) and Lau and Lau (2003), For simplicity, we may assume that the rate of default risk giving the credit period m is assumed here to be

$$f(m) = 1 - e^{-bm}, \tag{3}$$

where b is the coefficient of the default risk, which is a positive constant.

5. Replenishment rate is instantaneous.
6. Shortages are not allowed to occur.

Mathematical models and solutions

Since the seller offers a permissible delay of m years, and hence receives the buyer's purchase amount at time m with the rate of not receiving debt obligations is $f(m) = 1 - e^{-bm}$, the seller's annual revenue is the product of the sales revenue, the present value factor, and the rate of receiving debt obligations as follow:

$$PD(m)e^{-rm}[1 - f(m)] = PKe^{am}e^{-rm}e^{-bm} = PK e^{[a-(b+r)]m}. \tag{4}$$

The seller's production cycle time is nt while the buyer's replenishment time is t. Therefore, the seller's learning production cost for making $D(m)$ units per year is

$$c(D(m)) = C_s(Ke^{am})^u. \tag{5}$$

The seller's annual setup cost is

$$\frac{S}{nt}. \tag{6}$$

The seller's annual process cost is

$$\frac{nF}{nt} = \frac{F}{t}. \tag{7}$$

From Figure 1, we know that the seller's on-hand inventory is equal to the area of the shaded region OEFGDO.

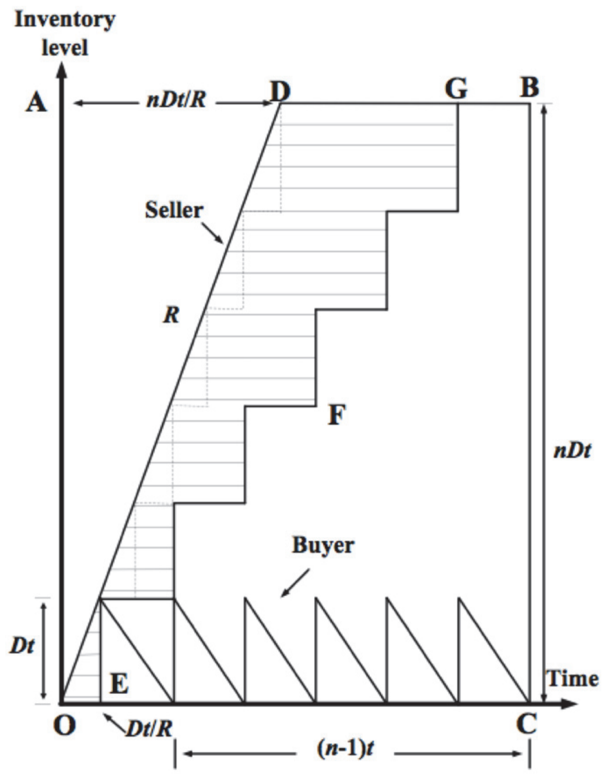

Figure 1 The on-hand inventory levels for both the seller and the buyer

Area of region OEFGDO = the area of the rectangle OCBA – the area of the triangle ODA – the area of the stair-shaped graph EFGBCE

$$= \left[(n-1)t + \frac{Dt}{R}\right]nDt - \frac{nDt}{R} \times nDt \times \frac{1}{2} - [(n-1)t + (n-2)t + \ldots + t]Dt$$

which can be seen in Chang, Ho, Ouyang, and Su (2009), and Chern et al. (2013). As a result, the seller's annual inventory holding cost is

$$\frac{H}{nt}\left\{\left[(n-1)t + \frac{Dt}{R}\right]nDt - \left(\frac{nDt}{R}\right)\left(\frac{nDt}{2}\right) - [(n-1)t + (n-2)t + \ldots + t]Dt\right\} = \frac{HDt}{2}\left[(n-1)\left(1 - \frac{D}{R}\right) + \frac{D}{R}\right]. \tag{8}$$

Based on the above assumptions, the proposed inventory system here is as follows. The seller must decide his/her trade credit period m and number of deliveries n of a single product simultaneously in order to maximize his/her total profit per year. From (4) – (8), we know the seller's annual profit can be expressed as:

$\Pi(m, n)$ = annual net revenue after default risk and opportunity cost – annual learning production cost – annual set-up cost – annual process cost – annual holding cost

$$\Pi(m, n) = PK\,e^{[a-(b+r)]m} - C_s(Ke^{am})^u - \frac{S}{nt} - \frac{F}{t} - \frac{Ht}{2}Ke^{am}\left[(n-1)-(n-2)\frac{Ke^{am}}{R}\right]. \quad (9)$$

Consequently, the production and finance problem to be solved is:

$$\underset{(m,n)}{Maximize}\,\Pi(m, n) = PK\,e^{[a-(b+r)]m} - C_s(Ke^{am})^u - \frac{S}{nt} - \frac{F}{t} - \frac{Ht}{2}Ke^{am}\left[(n-1)-(n-2)\frac{Ke^{am}}{R}\right], \quad (10)$$

subject to: *m* is a non-negative real number (i.e., $m \geq 0$), and *n* is a positive integer (i.e., *n* = 1, 2, 3,…). Hence, Problem (10) is a mixed integer programming problem, and can be solved by computer software such as LINGO 12.0. However, to obtain an easy-to-use near-optimal solution, we try to solve (10) by treating *n* as a real number.

Taking the first-order and second-order partial derivatives of $\Pi(m,n)$ with respect to *n*, we get:

$$\frac{\partial\Pi(m,n)}{\partial n} = \frac{S}{tn^2} - \frac{Ht}{2}Ke^{am}\left(1-\frac{Ke^{am}}{R}\right), \quad (11)$$

and

$$\frac{\partial^2\Pi(m,n)}{\partial n^2} = -\frac{2S}{tn^3} < 0. \quad (12)$$

Since $\Pi(m,n)$ is concave in *n* by (12), we know from (11) that the optimal real number of deliveries per production cycle is

$$n = \frac{1}{t}\sqrt{\frac{2S}{HKe^{am}(1-Ke^{am}/R)}}. \quad (13)$$

From (13) one can get the following results easily.

Theorem 1

(1) A higher value of S causes a higher value of n^*.

(2) A higher value of *t*, *H*, or *R* causes a lower value of n^*.

Proof. See Appendix A.

A simple economic interpretation of Theorem 1 is as follows:

(1) If the set-up cost S is more expensive then the seller produces more quantity per production run Q = nDt, which reduces the number of production runs while increases the number of deliveries n.

(2) If the buyer's replenishment time t is longer than the buyer orders more quantity Dt, which implies that the seller's annual number of deliveries is less frequent.

(3) If the inventory holding cost is more costly, then the seller cannot afford to build up larger inventory units, and hence delivers more frequently to the buyer with smaller batches.

(4) A higher value of production rate R causes a shorter production run time *nDt* / *R*, which in turn implies a larger on-hand inventory. Consequently, the seller would deliver larger batches (i.e. less number of deliveries) to the buyer in order to reduce larger inventory level.

Substituting (13) into (9), and simplifying terms, we reduce (9) to a single decision variable of *m* as follow:

$$\Pi(m) = PK\,e^{[a-(b+r)]m} - C_s(Ke^{am})^u - \sqrt{2SHKe^{am}\left(1-\frac{Ke^{am}}{R}\right)} - \frac{F}{t} - \frac{tH}{R}(Ke^{am})^2 + \frac{tHKe^{am}}{2}. \quad (14)$$

In order to find the optimal solution m^* of $\Pi(m)$, we derive the first-order necessary condition for $\Pi(m)$ in (14) to be maximized as

$$\frac{d\Pi(m)}{dm} = [a-(b+r)]PKe^{[a-(b+r)]m} - uaC_s(Ke^{am})^u - \frac{2atH}{R}(Ke^{am})^2 + \frac{atHKe^{am}}{2} - aSHKe^{am}\left(1-\frac{2Ke^{am}}{R}\right)\Big/\sqrt{2SHKe^{am}\left(1-\frac{Ke^{am}}{R}\right)} = 0. \quad (15)$$

By using L'Hospital's rule, we get

$$\lim_{m\to\infty}\frac{[a-(b+r)]PKe^{[a-(b+r)]m}}{e^{am}}=0, \tag{16}$$

and hence

$$\lim_{m\to\infty}\frac{d\Pi(m)}{dm}=-\infty. \tag{17}$$

In addition, substituting $m = 0$ into (15), we obtain

$$\frac{d\Pi(0)}{dm}=[a-(b+r)]PK-uaC_sK^u-\frac{2atHK^2}{R}+\frac{atHK}{2}-aSHK\left(1-\frac{2K}{R}\right)/\sqrt{2SHK\left(1-\frac{K}{R}\right)}. \tag{18}$$

discuss the second-order sufficient condition. Taking the derivative of (15) with respect to m again, and re-arranging terms, we get

$$\frac{d^2\Pi(m)}{dm^2}=[a-(b+r)]^2PKe^{[a-(b+r)]m}-(ua)^2C_s(Ke^{am})^u-\frac{4a^2tH}{R}(Ke^{am})^2+\frac{1}{2}a^2tHKe^{am}-(aSHKe^{am})^2\left[\left(\frac{2Ke^{am}}{R}\right)^2-\frac{6Ke^{am}}{R}+1\right]\left[2SHKe^{am}\left(1-\frac{Ke^{am}}{R}\right)\right]^{-3/2} \tag{19}$$

Consequently, if the following two conditions hold:

$$[a-(b+r)]^2PKe^{[a-(b+r)]m}-(ua)^2C_s(Ke^{am})^u-\frac{4a^2tH}{R}(Ke^{am})^2+\frac{1}{2}a^2tHKe^{am}\le 0, \tag{20}$$

and

$$\left(\frac{2Ke^{am}}{R}\right)^2-\frac{6Ke^{am}}{R}+1>0, \tag{21}$$

then we know that $\frac{d^2\Pi(m)}{dm^2}<0$ in (19), and hence $\Pi(m)$ in (14) is a strictly concave function of m. From Equations (14) - (19), we can obtain the following theoretical results.

Theorem 2

If Conditions (20) and (21) hold, then we obtain the following results:

(1) If $[a-(b+r)]PK-uaC_sK^u-\frac{2atHK^2}{R}+\frac{atHK}{2}-aSHK\left(1-\frac{2K}{R}\right)/\sqrt{2SHK\left(1-\frac{K}{R}\right)}>0,$ then $\Pi(m)$ in (14) has a unique optimal solution $m^*>0$ such that $\frac{d\Pi(m^*)}{dm}=0$ as in (15).

(2) If $[a-(b+r)]PK-uaC_sK^u-\frac{2atHK^2}{R}+\frac{atHK}{2}-aSHK\left(1-\frac{2K}{R}\right)/\sqrt{2SHK\left(1-\frac{K}{R}\right)}\le 0,$ then $\Pi(m)$ in (14) has a unique optimal solution $m^*=0$.

Proof. See Appendix B.

Due to the complexity of the problem, we are unable to fully understand economic interpretations of Conditions (20) and (21), and the following third condition:

$$[a-(b+r)]PK-uaC_sK^u-\frac{2atHK^2}{R}+\frac{atHK}{2}-aSHK\left(1-\frac{2K}{R}\right)/\sqrt{2SHK\left(1-\frac{K}{R}\right)}>0. \tag{22}$$

Consequently, we are unable to fully understand when and why the optimal credit period is positive (i.e., $m^*>0$). However, we can use computer software to check those three conditions easily. Next, we discuss the other case in which $\frac{d^2\Pi(m)}{dm^2}>0$.

Theorem 3

If $\frac{d^2\Pi(m)}{dm^2}>0$, then $\Pi(m)$ in (14) has a unique optimal solution $m^*=0$.

Proof. See Appendix C.

Now, we propose a simple and easy heuristic algorithm to obtain a near-optimal solution ($m^\wedge, n^\wedge, \Pi^\wedge$) as below.

Algorithm for obtaining a near-optimal solution ($m^\wedge, n^\wedge, \Pi^\wedge$)

Step 1.

If $[a-(b+r)]PK-uaC_sK^u-\frac{2atHK^2}{R}+\frac{atHK}{2}$

$$-aSHK\left(1-\frac{2K}{R}\right)/\sqrt{2SHK\left(1-\frac{K}{R}\right)} \le 0,$$

then we get:

$m^\wedge = 0$, $n^\wedge$ = the rounded integer of

$$\frac{1}{t}\sqrt{\frac{2S}{HK(1-K/R)}}, \ \Pi^\wedge \text{ in (9)}, \qquad (23)$$

and stop.

Step 2.

If $[a-(b+r)]PK - uaC_sK^u - \dfrac{2atHK^2}{R} + \dfrac{atHK}{2}$

$$-aSHK\left(1-\frac{2K}{R}\right)/\sqrt{2SHK\left(1-\frac{K}{R}\right)} > 0,$$

then we solve (15) to obtain

$m^\wedge$, $n^\wedge$ = the rounded integer of

$$\frac{1}{t}\sqrt{\frac{2S}{HKe^{am^\wedge}(1-Ke^{am^\wedge}/R)}}, \ \Pi^\wedge \text{ in (9)}, \qquad (24)$$

and stop.

Numerical examples

In order to illustrate the previous results, let us apply the theoretical results to solve the following examples.

Example 1.

Let a = 0.2, b = 0.1, r = 0.05, u = 0.9, t = 0.05 years, P = \$15 per unit, C_s = \$8 for the first production unit, S = \$20 per production run, F = \$1 per order, H = \$1 per unit per year, K = 1,000 units per year, and R = 10,000 units per year. We first check the condition:

$$[a-(b+r)]PK - uaC_sK^u - \frac{2atHK^2}{R} + \frac{atHK}{2}$$

$$-aSHK\left(1-\frac{2K}{R}\right)/\sqrt{2SHK\left(1-\frac{K}{R}\right)} = 14.425 > 0.$$

From Step 2 of the proposed algorithm, by using computer software such as MATHEMATICA 9.0, MAPLE 16.0, and others to solve (15), we get:

the proposed near-optimal trade credit period $m^\wedge$ = .1520,

the proposed near-optimal number of deliveries as

$n^\wedge$ = the rounded integer of $\dfrac{1}{t}\sqrt{\dfrac{2S}{HKe^{am^\wedge}(1-Ke^{am^\wedge}/R)}} = 4,$

and the proposed seller's near-optimal annual profit as shown in (9)

$\Pi^\wedge$ = \$10,801.72.

Checking the concavity conditions with $m^\wedge$ = 0.1520, we get:

$$[a-(b+r)]^2 PKe^{[a-(b+r)]m} - (ua)^2 C_s (Ke^{am})^u$$

$$-\frac{4a^2tH}{R}(Ke^{am})^2 + \frac{1}{2}a^2tHKe^{am} =$$

= - 95.5447 < 0,

and

$$\left(\frac{2Ke^{am}}{R}\right)^2 - \frac{6Ke^{am}}{R} + 1 = 0.4240 > 0.$$

Using computer software LINGO 12.0, we get the optimal solution as follow:

m^* = 0.1587, n^* = 4, and Π^* = \$10,801.72.

Comparing the optimal solution and the proposed near-optimal solution, we know that both are identical in the number of deliveries n^*, and the optimal annual profit Π^*.

Example 2.

For simplicity, we use the same data as in Example 1 except P = \$12. Then we have:

$$[a-(b+r)]PK - uaC_sK^u - \frac{2atHK^2}{R} + \frac{atHK}{2}$$

$$-aSHK\left(1-\frac{2K}{R}\right)/\sqrt{2SHK\left(1-\frac{K}{R}\right)} = -135.5751 < 0.$$

By applying Step 1 of the proposed algorithm, we obtain: the proposed near-optimal trade credit period $m^\wedge$ = 0.00, the proposed near-optimal number of deliveries

$n^\wedge$ = the rounded integer of $\dfrac{1}{t}\sqrt{\dfrac{2S}{HK(1-K/R)}} = 4,$

and the proposed seller's near-optimal annual total profit as $\Pi^\wedge$ = \$7,800.50.

Checking the concavity conditions with $m^\wedge$ = 0.00, we have:

$$[a-(b+r)]^2 PKe^{[a-(b+r)]m} - (ua)^2 C_s (Ke^{am})^u - \frac{4a^2 tH}{R}(Ke^{am})^2 + \frac{1}{2}a^2 tHKe^{am} = -99.7077 < 0,$$

and

$$\left(\frac{2Ke^{am}}{R}\right)^2 - \frac{6Ke^{am}}{R} + 1 = 0.44 > 0.$$

Using computer software, we get the optimal solution as follow:
$m^* = 0.00$, $n^* = 4$, and $\Pi^* = \$7{,}800.50$.
Comparing the optimal solution and the proposed near-optimal solution, we know that both solutions are exactly identical.

Example 3.

Using the same data as in Example 1, we study the sensitivity analysis on the optimal solution with respect to each parameter. The computational results are shown in Table 1 below.

Table 1 Sensitivity analysis on parameters

Parameter	m*	n*	Π*
u = 0.80	7.4917	3	$14,825.87
u = 0.90	0.1587	4	$10,801.72
u = 1.00	0.0000	4	$ 6,810.00
P = 12	0.0000	4	$ 7,800.50
P = 15	0.1587	4	$10,801.72
P = 18	1.5654	4	$13,937.75
C = 6	2.3847	3	$12,052.88
C = 8	0.1587	4	$10,801.72
C = 10	0.0000	4	$ 9,798.13
S = 10	0.2005	3	$10,858.28
S = 20	0.1587	4	$10,801.72
S = 40	0.0754	6	$10,722.44
t = 0.03	0.1380	7	$10,780.35
t = 0.05	0.1587	4	$10,801.72
t = 0.07	0.1627	3	$10,815.76
H = 1	0.1587	4	$10,801.72
H = 4	0.0851	2	$10,670.86
H = 7	0.0000	2	$10,595.50
F = 1	0.1587	4	$10,801.72
F = 5	0.1587	4	$10,721.72
F = 10	0.1587	4	$10,621.72
a = 0.19	0.0000	4	$10,800.50
a = 0.20	0.1587	4	$10,801.72
a = 0.21	1.1975	4	$10,881.38
b = 0.09	1.6954	4	$10,950.42
b = 0.10	0.1587	4	$10,801.72
b = 0.11	0.0000	4	$10,800.50
r = 0.04	1.6954	4	$10,950.42
r = 0.05	0.1587	4	$10,801.72
r = 0.06	0.0000	4	$10,800.50
K =1,000	0.1587	4	$10,801.72
K =2,000	0.7404	3	$22,326.94
K =3,000	1.0648	3	$34,102.41
R = 8,000	0.1641	4	$10,803.06
R = 9,000	0.1611	4	$10,802.31
R=10,000	0.1587	4	$10,801.72

The sensitivity analysis reveals the following managerial insights:

(1) A higher value of u, b, r, C_s, S, or R causes lower values of m*, and Π*(m*, n*),

(2) In contrast, a higher value of a, P, or K causes higher values of m*, and Π*(m*, n*),

(3) A higher value of t causes a lower value of n* while a higher value of m*,

(4) A higher value of H causes lower values of m*, n* and Π*(m*, n*),

(5) Judging from the computational results in Table 1, we know that the seller's trade credit and annual profit are significantly affected by the selling price and the learning curve effect (i.e., $l = 1-u$), and

(6) By contrast, the higher the default risk rate *b* (as well as the set-up cost *S*), the lower the trade credit *m**, and the annual profit Π*(*m**, *n**).

Example 4.

Using those cases with varying n* in Table 1, we compare the proposed solution and the optimal solution as shown in Table 2 below.

Table 2 reveals that the proposed heuristic solutions attain the optimal number of deliveries and the optimal annual profit in all of our numerical examples. Hence, the proposed heuristic algorithm is remarkably good.

Table 2 Comparisons between optimal and proposed solutions

Solutions / Parameters	Optimal solution			Proposed solution		
	*m**	*n**	Π*	*m*^	*n*^	Π^
$S = 10$	0.2005	3	\$10,858.28	0.2035	3	\$10,858.28
$S = 40$	0.0754	6	\$10,722.44	0.0787	6	\$10,722.44
$t = 0.03$	0.1380	7	\$10,780.35	0.1395	7	\$10,780.35
$t = 0.07$	0.1627	3	\$10,815.76	0.1645	3	\$10,815.76

Conclusions

In this paper, we have obtained the seller's optimal trade credit and number of deliveries in an EPQ model which captures the facts that (1) granting trade credit has positive impacts on demand and learning production cost, while negative impacts on both interest loss and default risk, and (2) the unit learning production cost declines when the accumulated production volume increases. Since the number of deliveries is an integer, the proposed inventory problem becomes a mixed integer programming problem, which can be solved by computer software such as LINGO 12.0. However, for simplicity, we have proposed a simple-in-concept and easy-to-apply heuristic algorithm which attains the optimal solution most of time in our numerical examples. Finally, we have provided several numerical examples to illustrate the algorithm and obtain some managerial insights. From the sensitivity analysis, we know that the production learning-by-experience effect improves the seller's annual profit significantly.

For further research, the paper could be extended in several ways. For instance, one could take pricing, advertising and quality into consideration as other decision variables (Chou, Chung, Hsiao, & Wang, 2011). Also, one may generalize the model to allow for shortages, partial backlogging, and deteriorating items. Furthermore, one might consider the effect of time value of money on the order quantity, trade credit, and total profit. Finally, one should expand a single-player optimal solution for the seller to a non-cooperative Nash or Stackelberg equilibrium solution or a cooperative Pareto equilibrium solution for multiplayers (e.g., both the seller and the buyer) in a supply chain.

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Appendix

Appendix A. Proof of Theorem 1

Taking the partial derivative of (13) with respect to S, we get

$$\frac{\partial n}{\partial S}=\frac{1}{tHKe^{am}(1-Ke^{am}/R)}\left[\frac{2S}{HKe^{am}(1-Ke^{am}/R)}\right]^{-0.5}>0. \quad \text{(A1)}$$

Consequently, a higher value of S causes a higher value of n^*. Similarly, taking the partial derivative of (13) with respect to t, K, and R, we obtain the following results

$$\frac{\partial n}{\partial t}=-\frac{1}{t^2}\sqrt{\frac{2S}{HKe^{am}(1-Ke^{am}/R)}}<0, \quad \text{(A2)}$$

$$\frac{\partial n}{\partial H}=-\frac{1}{t}\left[\frac{2S}{HKe^{am}(1-Ke^{am}/R)}\right]^{-0.5}\frac{S}{H^2Ke^{am}(1-Ke^{am}/R)}<0, \quad \text{(A3)}$$

and

$$\frac{\partial n}{\partial R}=-\frac{1}{t}\left[\frac{2S}{HKe^{am}(1-Ke^{am}/R)}\right]^{-0.5}\frac{S}{H^2(R-Ke^{am})^2}<0, \quad \text{(A4)}$$

respectively. This completes the proof of Theorem 1.

Appendix B. Proof of Theorem 2

From (19), we know that $\Pi(m)$ is a strictly concave function of m (i.e., $\frac{d^2\Pi(m)}{dm^2}<0$) if $[a-(b+r)]^2PKe^{[a-(b+r)]m}-(ua)^2C_s(Ke^{am})^u-\frac{4a^2tH}{R}(Ke^{am})^2+\frac{1}{2}a^2tHKe^{am}\le 0$, and $\left(\frac{2Ke^{am}}{R}\right)^2-\frac{6Ke^{am}}{R}+1>0.$ Since $\lim_{m\to\infty}\frac{d\Pi(m)}{dm}=-\infty$ as in (17), if $\frac{d\Pi(0)}{dm}>0$ *then* applying the Mean-Value Theorem we know that there exists a unique optimal trade credit period $m^*>0$ such that $\frac{d\Pi(m^*)}{dm}=0$. This proves Part (1) of the theorem. However, if $\frac{d\Pi(0)}{dm}\le 0$, then $\frac{d\Pi(m)}{dm}<0$ for all $m>0$, which implies that $\Pi(m)$ in (14) is a strictly decreasing function of m. Hence, if $\frac{d\Pi(0)}{dm}\le 0$ then $m^*=0$ is the unique optimal solution to $\Pi(m)$ in (14). This proves Part (2) of the theorem, and thus completes the proof of Theorem 2.

Appendix C. Proof of Theorem 3

$\lim_{m\to\infty}\frac{d\Pi(m)}{dm}=-\infty$ as shown in (17). If $\frac{d^2\Pi(m)}{dm^2}>0$, then $\frac{d\Pi(m)}{dm}<\lim_{m\to\infty}\frac{d\Pi(m)}{dm}=-\infty$ for all real number m. Consequently, $\Pi(m)$ in (14) is decreasing in m (≥ 0) which implies its optimal solution $m^*=0$. This completes the proof.

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當賣家之生產成本具有經驗學習效應下最佳的信用交易期間和交貨次數

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實務上賣家經常給買家固定的信用交易期間，以無息付款來刺激銷售量和市場的競爭力。此外，從經驗學習效應來看，銷售量愈大則生產量也愈大，而生產量增加則學習效應高，促使每單位生產成本跟著降低。因此，從賣家觀點，提供信用交易，不但增加銷售量，也帶來具有學習效應的單位生產成本降低。另一方面，給予信用交易不只增加信用期間的利息損失，還有增加違約風險率。現有文獻鮮少注意到這個事實。本研究配合信用交易對銷售和學習生產成本有正面影響，但對利息損失和違約風險卻有負面衝擊，建立生產系統中賣家最佳的信用交易期間和交貨次數模型來達到利潤最大化，這是混合整數規劃問題，本研究透過電腦軟體去解決。為簡化，論文中提出一優化啟發演算法，最後，用敏感度分析顯示一些觀點洞見和可顯著增加賣家的信用期間和總利潤的經驗學習效應。

關鍵字：庫存管理、信用交易、學習性生產成本、經濟生產量。

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