國立政治大學風險管理與保險學研究所

博士學位論文

具傳染效果的隨機死亡風險模型之建立及其應用

Modeling Infectious Mortality Risk and Its Application

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Modeling Infectious Mortality Risk and Its Application

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謝辭

這是我的第二個博士論文，當我寫下謝辭時，代表自己也拿到了第二個博士學位。從第一個博士到第二個博士，我何其有幸能順著自己的志向，突破層層的困難和考驗，來到了現在。這條路並不好走，幸好有指導教授黃泓智老師的關懷和體諒，總在我沮喪難過時，又再次給了我信心和勇氣。黃老師的仁慈，適時溫暖學生的心，老師豐富的學識帶領我進入保險精算的殿堂，開拓了我的研究視野。謝謝楊曉文老師的醍醐灌頂，讓我這個駑鈍的學生在迷惑而不知所措時看見了曙光。我何其幸運在這段期間深受這兩位老師的照顧，讓我一路走來不覺得徬徨無助。這份恩情將永遠銘誌於心。

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人生是一長串的奮鬥，未來的路還很長，加油，陳芬英，謝謝你始終與我同行，一起努力。

陳芬英　　謹誌於

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Abstract

This thesis examines the valuation of mortality-linked bonds in two infectious mortality models in two main parts:

(1) Valuation and Analysis of the Swiss Re Bond without Coupons in an Infectious Mortality Model

(2) Valuation and Analysis of Fixed-Coupon and Floating-Coupon Mortality Bonds in an Infectious Mortality Model

The two main parts of this dissertation focus on infectious mortality risk, and two infectious models are developed to analyze the impacts of infectious mortality risk on mortality-linked bonds. This approach is different from that in the literature. To capture the infectious mortality dynamics across countries, two mortality jumps are considered in the mortality modeling: infectious jumps and specific country jumps. An infectious jump occurs only when there is a catastrophic event that causes considerable mortality. Furthermore, the mortality experience in France, the United Kingdom, the United States, Italy, and Switzerland is employed to fit the proposed infectious mortality model.

Using the two infectious mortality models, this dissertation explores the impacts of infectious mortality risk on the two types of mortality-linked bonds: zero-coupon mortality bonds and coupon mortality bonds. The first part demonstrates the structure of a zero-coupon mortality bond, namely Vital Capital I, which is a type of Swiss Re bond without coupons and was first issued as a 3-year catastrophic mortality bond in 2003. Under the infectious mortality framework, the closed-form solution of Vital Capital I is derived using Wang's transform (2000). An empirical analysis reveals that the fair price of Vital Capital I in the model is lower than face value (market price). Sensitivity analyses illustrate that the sensitivity of the volatilities of the magnitudes of infectious mortality is the largest among the model parameters, whereas that of

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threshold values is the smallest.

In the second part, coupon mortality bonds, namely fixed-coupon and floating-coupon bonds, are examined. These bonds are similar to the Swiss Re bond. The closed-form solution of a fixed-coupon mortality bond is derived, and it is assumed that the coupons of floating-coupon mortality bonds are linked to a stochastic interest rate, which follows the Cox–Ingersoll–Ross interest rate model. Monte Carlo simulation is employed to evaluate the sensitivities of fair prices of floating-coupon bonds. The empirical results show the fair spreads of these two types of bonds are also higher than the spreads of 0.45% indicated by Cox et al. (2006) and closer to the market prices of 1.35% of the Swiss Re bond.

A common phenomenon is revealed in the first and second parts, which specifies that the fair prices of mortality-linked securities in high-infectious mortality model are fewer than those of mortality-linked securities in low-infectious mortality model. Therefore, ignoring the effects of infectious mortality rates significantly overestimates the par spread of mortality bonds; by contrast, considering this phenomenon provides a par spread of the mortality security that is closer to real-world values. This is helpful for pricing mortality securities and for managing catastrophic mortality risk for reinsurers.

Keywords: Infectious mortality risk, Mortality-linked bond, Wang transform, Jump model, Floating-coupon bond.

DOI:10.6814/DIS.NCCU.RMI.001.2018.F08

Contents

DOI:10.6814/DIS.NCCU.RMI.001.2018.F08

DOI:10.6814/DIS.NCCU.RMI.001.2018.F08

Tables and Figures

DOI:10.6814/DIS.NCCU.RMI.001.2018.F08

DOI:10.6814/DIS.NCCU.RMI.001.2018.F08

DOI:10.6814/DIS.NCCU.RMI.001.2018.F08

Chapter 1 Foreword

Mortality uncertainty is the primary source of risk for life insurance and annuity providers. Mortality uncertainty can appear as contrasting longevity and mortality risks. Insurers endure longevity risk for their annuity products if future mortality improves relative to current expectations because they have to pay annuity benefits longer than expected. Conversely, if mortality deteriorates or a catastrophic event occurs, insurers endure mortality risk for their life insurance products because the insurance benefits paid out are higher than expected. Therefore, modeling mortality risk is essential.

In the past two decades, numerous researchers propose and discuss various mortality models for modeling the dynamics of mortality over time. For example, Lee and Carter (1992) pioneer the modeling of central mortality rates as log-linearly correlated with a time-dependent mortality factor, and they adjust for age-specific effects by using two sets of age-dependent coefficients. However, earlier mortality models do not consider catastrophic mortality risk and cannot explicitly capture structural changes and catastrophic shocks that may cause mortality jumps such as the 2004 Indian Ocean earthquake and tsunami that killed 182,340 people or comovement trends such as the 1918 flu pandemic (the Spanish flu). Although recent studies examine mortality jumps, they do not consider the impacts of catastrophic shocks across

DOI:10.6814/DIS.NCCU.RMI.001.2018.F08

countries, except for studies by Cox et al. (2006) and Lin et al. (2013).

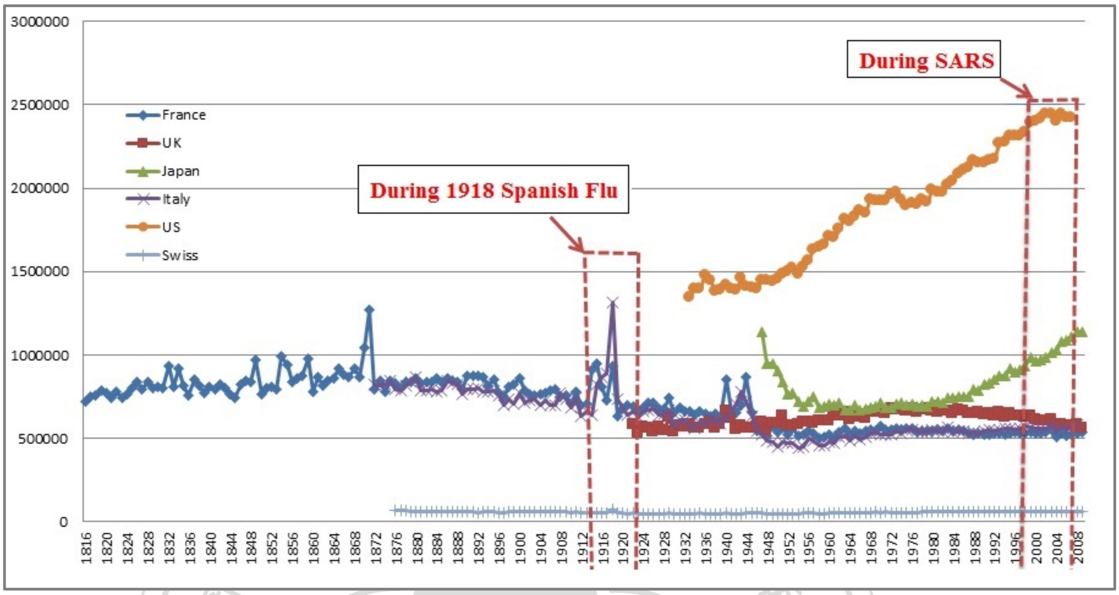

Figure 1.1 Worldwide Deaths from 1816 to 2008

The comovement trend of the mortality rates in different countries may not be properly modeled as a common factor. A mortality jump occurs only when there is a catastrophic event that causes considerable mortality. Figure 1.1 shows the mortality rates in different countries from 1816 to 2008. In 2002, severe acute respiratory syndrome killed 775 people in Europe, Asia, and North America, but deaths in France, the United Kingdom, Italy, Switzerland, and the United States did not show a significant comovement trend. Conversely, a comovement phenomenon was present in France, the United Kingdom, Italy, Switzerland, and the United States during the Spanish flu, which killed at least 20 million people.

DOI:10.6814/DIS.NCCU.RMI.001.2018.F08

Figure 1.1 shows that mortality rates across countries have a significant comovement phenomenon when the common factor leads to substantially higher deaths, such as deaths during the Spanish flu, which is referred to as infectious mortality risk in this study.

Following Forbes and Rigobon's (2002) definition of contagion[1], this dissertation defines infectious mortality risk as a considerable increase in cross-country (or cross-regional) linkages after catastrophic shock in one county (or region) or a group of countries. By definition, infection only occurs when cross-country comovement increases considerably after mortality shocks; infection does not occur if the comovement does not increase considerably after mortality shocks.

Globalization and transportation may facilitate the spread of infectious diseases across countries, causing catastrophic losses. A recent example is the Ebola virus outbreak in early 2014, whose severe effects are often fatal to humans. The abovementioned examples demonstrate that we cannot afford to ignore infectious mortality risks or their impacts when pricing catastrophic mortality securities. Although infectious mortality risk clearly exists, the literature has not addressed the challenge of modeling this risk.

Therefore, this study conducts infectious mortality modeling and considers

[1] See page 2224 in Forbes and Rigobon (2002).

DOI:10.6814/DIS.NCCU.RMI.001.2018.F08

infectious mortality risk when pricing a catastrophic mortality security. The main contributions of this dissertation are twofold. First, this study presents two multicountry infectious mortality models that capture the comovement phenomenon during a catastrophic event. The proposed models take a general form that can be reduced to the morality model introduced by Lin and Cox (2008). Second, using two types of catastrophic mortality securities, namely the Swiss Re bond without coupons (Vital Capital I) and coupon bonds (i.e., fixed-coupon bonds and floating-coupon bonds), as examples, we obtain a closed-form solution by using Wang's transform (2000) (hereafter the Wang transform) and thereby investigate the effects of infectious mortality risk on catastrophic mortality bonds.

The remainder of this dissertation is organized as follows. Chapter 2 presents a literature review of mortality-linked bonds. Chapter 3 presents a valuation and analysis of the Swiss Re bond without coupons in an infectious mortality model. The closed-form solution of the fair prices of the bond is derived, and the sensitivities of bond prices are also derived in this chapter. Chapter 4 presents a valuation and analysis of fixed-coupon and floating-coupon mortality bonds in the infectious mortality model. Moreover, the closed-form solution of a fixed-coupon bond is derived, and the coupons of floating-coupon mortality bonds are assumed to be linked to a stochastic interest rate. The dynamic process of the stochastic interest rate follows the Cox–Ingersoll–Ross

DOI:10.6814/DIS.NCCU.RMI.001.2018.F08

(CIR) model. Monte Carlo simulation is employed to explore the sensitivities of the fair prices of floating-coupon bonds. Chapter 5 presents comparison of the two infectious mortality models. Chapter 6 is the conclusion.

DOI:10.6814/DIS.NCCU.RMI.001.2018.F08

Chapter 2 Literature Review

2.1 Introduction of Securitization of Mortality Risk

Securitization of mortality risk is an innovative capital solution to infectious mortality risk. Jaffee and Russell (1997) and Froot (2001) describe that insurance securitization potentially offers a more efficient mechanism for financing catastrophic losses than traditional reinsurance does. Cummins and Lewis (2002) demonstrate that securitization is the repackaging and trading of cash flows that traditionally would have been held on-balance-sheet by financial institutions. Securitization brings more capital and enhances the capacity of the life insurance industry to manage catastrophic losses from epidemics, hurricanes, earthquakes, and other natural or manmade disasters. The advantages of securitization may be lower costs in the long run, more favorable contracts, and elimination of default risk.

Using capital market solutions to manage mortality risk such as mortality-linked securities is rapidly increasing in recent years. The Swiss Reinsurance Company, the world's second-largest reinsurance company, first issued a 3-year catastrophic mortality bond in 2003 (Vital Capital I), with a face value of $400 million in coverage from institutional investors. The second bond (Vital Capital II) was issued in 2008. Both mortality securities aim to transfer mortality risk from the insurer by using a combined

DOI:10.6814/DIS.NCCU.RMI.001.2018.F08

mortality index that measures annual population mortality in five countries and applies predetermined weights to each nation's publicly reported mortality data. Vital Capital I uses the annual population death rates for France, the United Kingdom, the United States, Italy, and Switzerland, whereas Vital Capital II uses the annual population death rates for the United States, the United Kingdom, Canada, and Germany. Moreover, through its Mythen Re program in 2012, Swiss Re obtained USD 200 million in coverage against North Atlantic hurricanes and against extreme mortality risk in the United Kingdom. The issuance comprises two tranches of notes. The first tranche is class A notes (USD 120 million), rated as B+ by Standard & Poor (S&P), which combines PCS North Atlantic hurricane risk with extreme mortality risk in the United Kingdom. The second tranche, rated as B- by S&P, provides USD 80 million in protection for North Atlantic hurricane risk. This is the first time hurricane and mortality risks have been combined into one bond offering. Thus, mortality securitization, in which catastrophic losses are transferred to financial markets, is gaining much popularity among life insurers.

2.2 Literature related to Stochastic Mortality Models without Jumps

Pricing a catastrophic mortality bond requires an understanding of the catastrophic event for mortality uncertainty. Modeling catastrophic mortality risk is essential. Numerous mortality models are proposed and discussed to model the dynamics of

DOI:10.6814/DIS.NCCU.RMI.001.2018.F08

mortality over time (Lee and Carter, 1992; Renshaw and Haberman, 2006; Cairns et al., 2006). Lee and Carter (1992) pioneer the modeling of central mortality rates as log-linearly correlated with a time-dependent mortality factor, and they adjust for age-specific effects by using two sets of age-dependent coefficients. Cairns et al. (2006) examine the pricing of longevity bonds in a two-factor stochastic mortality model (the Cairns–Blake–Dowd [CBD] model) for high ages. The Lee–Carter and CBD models both project mortality rates based on age and period effects. Renshaw and Haberman (2006) extend the Lee–Carter model by considering cohort effects in mortality modeling.

2.3 Literature related to Stochastic Mortality Models with Jumps

Early mortality models do not consider catastrophic mortality risk and cannot explicitly capture structural changes or catastrophic shocks that can cause mortality jumps, such as the 2004 Indian Ocean earthquake and tsunami that killed 182,340 people or comovement trends such as the Spanish flu. Recent research examines mortality jumps, such as research by Cox et al. (2006), Lin and Cox (2008), Chen and Cox (2009), Wang et al. (2013), Deng et al. (2012), Zhou et al. (2013), Lin et al. (2013), and Chen (2014). Yang et al. (2009) use a principal component analysis and Milidonis et al. (2011) employ a Markov regime-switching model to describe structural changes in mortality rates. However, most of these studies ignore the potential impacts of mortality shocks

DOI:10.6814/DIS.NCCU.RMI.001.2018.F08

across countries, except for studies by Cox et al. (2006), Lin et al. (2013), and Zhou et al. (2013). To model transitory mortality jumps, Zhou et al. (2013) propose a two-population generalization of the model developed by Chen and Cox (2009). Cox et al. (2006) decompose mortality shocks into a specific factor and a common factor. The common factor appears more substantial, in that it causes the comovement of mortality indices in all countries. Lin et al. (2013) extend the model of Cox et al. (2006) to a general setting and disentangle transient jumps from persistent volatilities. In contrast to Cox et al. (2006), who model unanticipated mortality jumps as permanent shocks, Lin et al. (2013) model them as transient jumps using a double-jump process. Cox et al. (2006) and Lin et al. (2013) anticipate that the comovement of the jump effect is a common factor in all countries. Their models imply that mortality jumps occur simultaneously in all countries.

In addition, some multi-country mortality models have been developed such as studies by Zhou et al. (2014), Chen et al. (2015), Wang et al. (2015), and Zhu et al. (2017). Allowing to visualize the cross-correlations and the long-term equilibrium relation between two countries, Zhou et al. (2014) use a vector error correction model to discuss how the modeling of the stochastic factors may be improved. Chen et al. (2015) apply factor copula to model multipopulation mortality. They employ a two-stage procedure and a factor copula approach. Wang et al. (2015) use a dynamic copula

DOI:10.6814/DIS.NCCU.RMI.001.2018.F08

framework to model multicountry mortality. Zhu et al. (2017) propose the Lévy subordinated hierarchical Archimedean copulas approach to model multicountry mortality dependence. They show that there is an association between geographical locations and dependence of the overall mortality improvement. These literature concentrates on multicounty mortality dependence.

However, in some cases, comovement trends or dependence of the mortality rates in different countries might not be properly modeled as a common factor. In this scenario, the jump occurs only when there is a catastrophic event that causes considerable mortality. Although the phenomenon of infectious mortality undoubtedly exists, it is not modeled in the literature.

Therefore, this dissertation considers two types of mortality jumps in mortality modeling: infectious jumps and specific country jumps, and first proposes two models to capture the infectious mortality dynamics across countries.

DOI:10.6814/DIS.NCCU.RMI.001.2018.F08

Chapter 3 Valuation and Analysis of the Swiss Re Bond Without Coupons in an Infectious Mortality Model

This chapter examines the structure of a zero-coupon mortality bond, namely Vital Capital I, which is a type of Swiss Re bond without coupons. Under the infectious mortality framework, the closed-form solution of Vital Capital I is derived by using the Wang transform. Furthermore, sensitivity analyses are conducted.

3.1 Modeling Infectious Mortality Risk

To capture the effect of infectious mortality rates across countries, we assume that there are m countries, and each country has n_i people, with $i = 1, 2, 3, \ldots, m$. Let $\tau_{i,j}$ denote the time of death for the j^{th} person in the i^{th} country, with $j = 1, 2, 3, \ldots, n_i$, and the corresponding number of deaths in each country at time t is denoted as $D_{i,j}(t)$. Thus, the total number of deaths in all countries at time t, N_t, is calculated as

$$N_t \triangleq \sum_{i=1}^{m} \sum_{j=1}^{n_i} D_{i,j}(t)$$

and $D_{i,j}(t) = \begin{cases} 1, & \text{if } t-1 < \tau_{i,j} \le t \\ 0, & \text{o.w} \end{cases}$, $i = 1, 2, 3, \ldots, m;\ j=1, 2, 3, \ldots, n_i$.

Infectious mortality risk is modeled using the mortality comovement that occurs only when there is a catastrophic event that causes considerable mortality in all countries; that is, whether an infectious mortality jump in the i^{th} country is affected by mortality

DOI:10.6814/DIS.NCCU.RMI.001.2018.F08

shocks in the world. To express the infectious effect, let N_t be the total number of deaths in all countries, and let $N_t^{\tilde{i}}$ denote the total number of deaths in other countries except for those in the i^{th} country, equivalently, $N_t^{\tilde{i}} \triangleq \sum_{k\neq i=1}^{m}\sum_{j=1}^{n_i} D_{k,j}(t)$. Furthermore, we define $V_t^{\tilde{i}}$ as the ratio of the total deaths in all countries except for those in the i^{th} country relative to the total deaths in all countries. That is, $V_t^{\tilde{i}} \triangleq \frac{N_t^{\tilde{i}}}{N_t}$ for $i = 1, 2, 3, ..., m$. Assume that $V_t^{\tilde{i}}$ follows a geometric Brownian motion, which is expressed as

$$\frac{dV_t^{\tilde{i}}}{V_t^{\tilde{i}}} = \mu_{v^{\tilde{i}}}\, dt + \sigma_{v^{\tilde{i}}}\, dW_{v,t}, \tag{3.1}$$

where $\mu_{v^{\tilde{i}}}$ and $\sigma_{v^{\tilde{i}}}$ denote the drift term and volatility, respectively, and $W_{v,t}$ is a one-dimensional standard Brownian motion under the original probability measure P.

$V_t^{\tilde{i}}$ follows a geometric Brownian motion because is it will be confirmed as positive. Additionally, using the raw mortality data of five countries (i.e., the United States, the United Kingdom, France, Italy, and Switzerland) obtained from the Human Mortality Database (HMD) from 1933 to 2007, we calibrate the model with the HMD through the initial values of $\mu_{v^{\tilde{i}}}$ and $\sigma_{v^{\tilde{i}}}$ set as the mean and volatility of the ratio of the deaths except for those in the United States, the United Kingdom, France, Italy, and Switzerland relative to total deaths in the five countries, respectively. The initial values of $\mu_{v^{\tilde{i}}}$ and $\sigma_{v^{\tilde{i}}}$ are shown in Table 3.1. Therefore, the estimated parameters of

DOI:10.6814/DIS.NCCU.RMI.001.2018.F08

Equation (3.1) are illustrated in Table 3.2.

Table 3.1 Initial Values of the Calibrated Parameters for Five Countries

	US	UK	France	Italy	Switzerland
$\mu_{\tilde{v}^i}$	-0.0019634	0.000294152	0.000574653	0.000290888	0.00000063
$\sigma_{\tilde{v}^i}$	0.007780363	0.00280517	0.006080277	0.003269581	0.000177765
$V_0^{\tilde{i}}$	0.493798630	0.829089975	0.843320034	0.829089975	0.984227223

Note that the initial values of $\mu_{\tilde{v}^i}$ and $\sigma_{\tilde{v}^i}$ are set as the mean and volatility of logarithm of the ratios of the deaths except for those in the United Sates, the United Kingdom, France, Italy, and Switzerland relative to total deaths of the five countries, respectively. $V_0^{\tilde{i}}$ is set as the average value of the deaths except for those in the United States, the United Kingdom, France, Italy, and Switzerland relative to total deaths of the five countries, respectively.

Table 3.2 Parameter Estimation of Dynamic Processes of $V_t^{\tilde{i}}$

	US	UK	France	Italy	Switzerland
$\mu_{\tilde{v}^i}$	-0.001933133 (0.000012)	0.000298086 (0.000011)	0.000593138 (0.000012)	0.000296233 (0.000010)	0.000000651 (0.000011)
$\sigma_{\tilde{v}^i}$	0.00778125 (0.000034)	0.00280918 (0.000032)	0.006080413 (0.000042)	0.003269487 (0.000035)	0.000177777 (0.000039)

Notes: Parameter estimates in Equation (3.1) for i = the United States, the United Kingdom, France, Italy, and Switzerland. Standard errors are shown in parentheses.

The deaths except for those in the i^{th} country relative to total deaths of the other countries for $i = $ US, UK, France, Italy and Swiss are plotted in Figure 2. Figure 2 illustrates that $V_t^{\tilde{i}}$ is more than 0 and less than 1 for $i = $ US, UK, France, Italy and Swiss .

DOI:10.6814/DIS.NCCU.RMI.001.2018.F08

Furthermore, the simulated ratios of $\tilde{V}_t^i$ are provided in Figures 3.3 to 3.7. The ratios (the deaths in other countries except for those in the i^{th} country relative to total deaths for $i =$ US, UK, France, Italy and Swiss) are obviously larger than 0 but smaller than 1. Generally, $\tilde{V}_t^i$ is neither 0 nor 1 for $i =$ US, UK, France, Italy and Swiss in the model setting. Therefore, it is reasonable that $\tilde{V}_t^i$ follows a lognormal distribution.

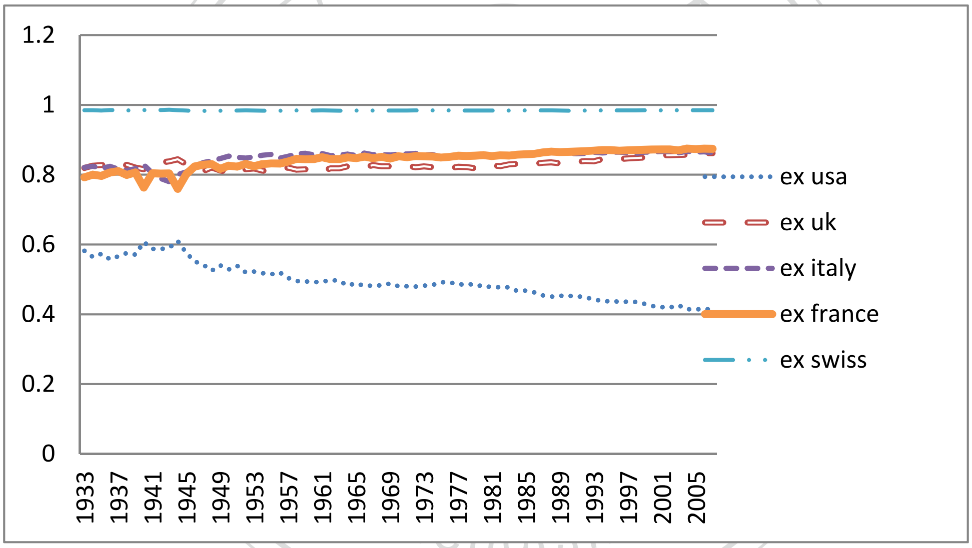

Figure 3.2 Deaths Except for Those in the i^{th} Country Relative to Total Deaths in All Five Countries

DOI:10.6814/DIS.NCCU.RMI.001.2018.F08

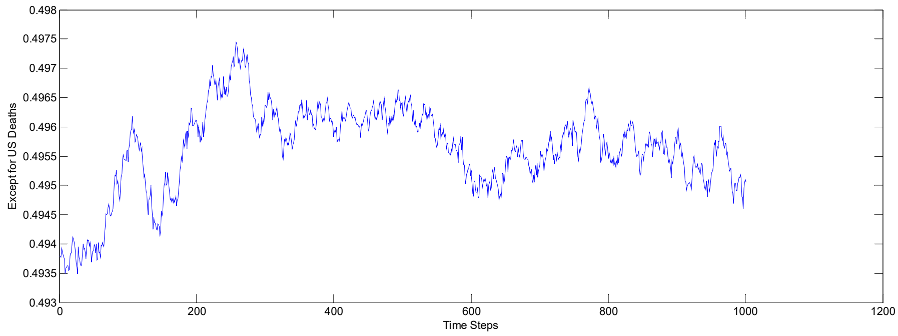

Figure 3.3 Ratio of the Deaths in Other Countries Except for Those in the United States Relative to Total Deaths in All Five Countries.

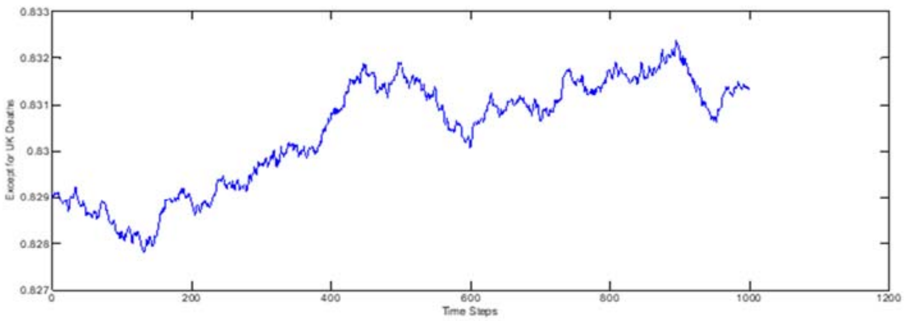

Figure 3.4 Ratio of the Deaths in Other Countries Except for Those in the United Kingdom Relative to Total Deaths in all Five Countries.

DOI:10.6814/DIS.NCCU.RMI.001.2018.F08

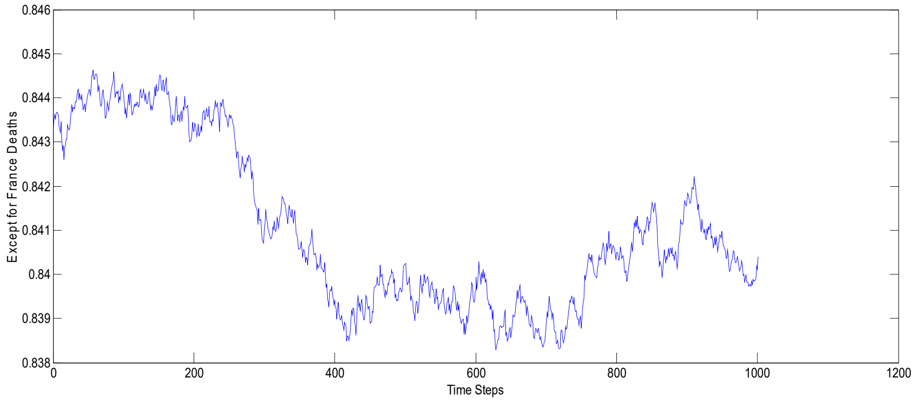

Figure 3.5 Ratio of the Deaths in Other Countries Except for Those in France Relative to Total Deaths in all Five Countries.

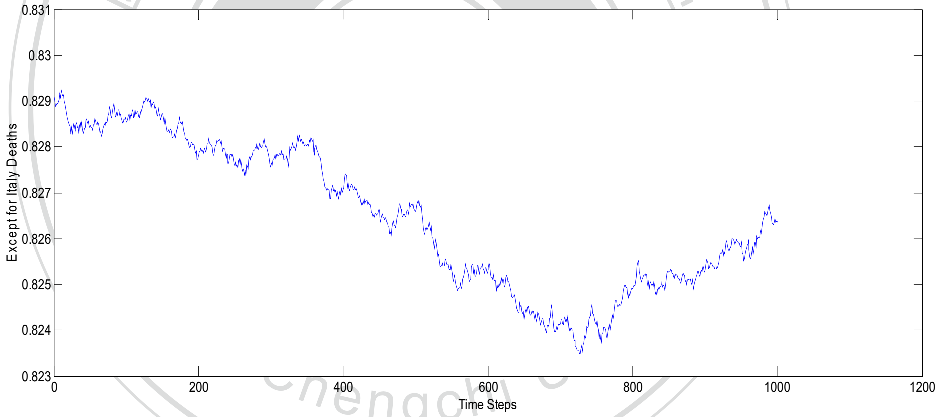

Figure 3.6 Ratio of the Deaths in Other Countries Except for Those in Italy Relative to Total Deaths in all Five Countries.

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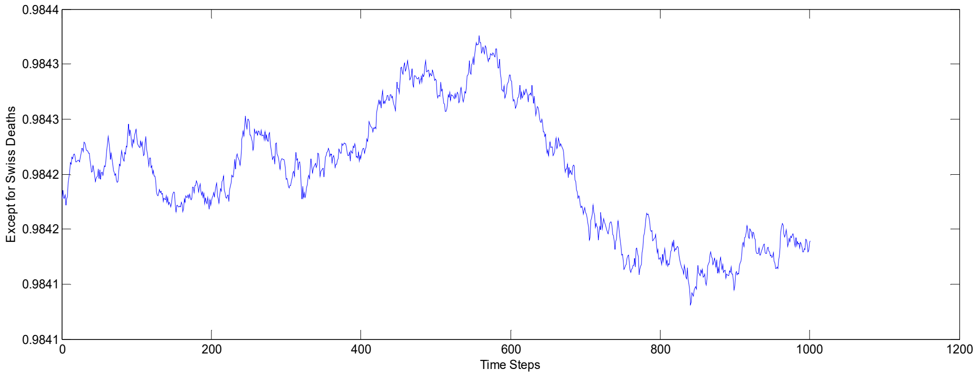

Figure 3.7 Ratio of the Deaths in Other Countries Except for Those in Switzerland Relative to Total Deaths in all Five Countries.

In addition, the mean squared errors of $\tilde{V}_t^i$ in the United States, the United Kingdom, France, Italy and Switzerland are 0.00085, 0.00071, 0.00075, 0.00091 and 0.00058, respectively. In average, the mean squared errors are 0.00076 for all five countries.

To capture the comovement phenomenon when a catastrophic event occurs, we set a threshold for $\tilde{V}_t^i$. If $\tilde{V}_t^i$ is higher than the threshold, say α, the mortality rate of the i^{th} country can be affected by the mortality rate of other countries. Thus, the mortality rate of the i^{th} country at time t is modeled based on two types of jumps: infectious jumps and specific country jumps. Let $\tilde{I}_t^i$ denote the jump number of the i^{th} country at time t infected by other countries when a catastrophic event occurs, such as infectious diseases, and let $\Gamma_{i,t}$ represent the jump frequency resulting from the

DOI:10.6814/DIS.NCCU.RMI.001.2018.F08

mortality shock in the i^{th} country at time t. Assume that both $I_t^{\tilde{i}}$ and $\Gamma_{i,t}$ follow a Poisson distribution with the intensities of $\lambda_{I_t^{\tilde{i}}}$ and $\lambda_{\Gamma_{i,t}}$, respectively. $I_t^{\tilde{i}}$ can be expressed as

$$I_t^{\tilde{i}} \triangleq \int_0^t D_u^{\tilde{i}}\, du \sim \text{Poisson}(\lambda_{I_t^{\tilde{i}}}), \tag{3.2}$$

where $D_t^{\tilde{i}} = \begin{bmatrix} 1, & \text{if } V_t^{\tilde{i}} \geq \alpha \\ 0, & \text{o.w} \end{bmatrix}$.

Furthermore, $\lambda_{I_t^{\tilde{i}}}$ is the expectation of infectious jump frequency, which can be calculated as

$$\begin{aligned} \lambda_{I_t^{\tilde{i}}} &= E\left[I_t^{\tilde{i}} \right] \\ &= \int_0^t \Phi\left(\frac{\ln\left(\dfrac{V_0^{\tilde{i}}}{\alpha}\right) + (\mu_{V^{\tilde{i}}} - \dfrac{1}{2}\sigma^2_{V^{\tilde{i}}})\delta}{\sigma_{V^{\tilde{i}}}\sqrt{\delta}} \right) d\delta. \end{aligned} \tag{3.3}$$

Next, let $q_{i,t}$ represent the mortality rate of the i^{th} country at time t. The multicountry mortality dynamics can be modeled as

$$\frac{dq_{1,t}}{q_{1,t}} = \mu_1\, dt + \sigma_1\, dW_{1,t} + (\Lambda_1 - 1)\, d\Gamma_{1,t} + (\pi_1 - 1)\, dI_t^{\tilde{1}}, \tag{3.4}$$

$$\frac{dq_{2,t}}{q_{2,t}} = \mu_2\, dt + \sigma_2\, dW_{2,t} + (\Lambda_2 - 1)\, d\Gamma_{2,t} + (\pi_2 - 1)\, dI_t^{\tilde{2}},$$

....

$$\frac{dq_{m,t}}{q_{m,t}} = \mu_m\, dt + \sigma_m\, dW_{m,t} + (\Lambda_m - 1)\, d\Gamma_{m,t} + (\pi_m - 1)\, dI_t^{\tilde{m}},$$

where μ_i and σ_i are constants, and $W_{i,t}$ is a one-dimensional standard Brownian

DOI:10.6814/DIS.NCCU.RMI.001.2018.F08

motion under the original probability measure P. Moreover, the correlation coefficient between $W_{i,t}$ and $W_{v,t}$ is $corr(dW_{v,t}, dW_{i,t}) = \rho_{v,i}$. Both $\Gamma_{i,t}$ and $\tilde{I}_t^i$ are independent Poisson-jump processes driven by different risks at time t. Furthermore, $d\tilde{I}_t^i$ is independent of $d\Gamma_{i,t}$, and $\pi_i - 1$ is the random variable percentage in the mortality index of the i^{th} country that results from common jumps of deaths in other countries. We assume that the natural logarithm of π_i, the jump amplitude driven by deaths in other countries, follows a normal distribution with a mean of u_{π_i} and a variance of $\sigma^2_{\pi_i}$, which is also denoted as $\ln\pi_i \sim N(u_{\pi_i}, \sigma^2_{\pi_i})$, $\pi_i > 0$, $i = 1, 2, 3, ..., m$. By contrast, $\Lambda_i - 1$ denotes the percentage in the mortality index of the i^{th} country resulting from specific jumps in deaths of the i^{th} country, and the specific jump size distributes a normality, namely $\ln\Lambda_i \sim N(u_{\Lambda_i}, \sigma^2_{\Lambda_i})$, $\Lambda_i > 0$, $i = 1, 2, 3, ..., m$. Finally, π_i is independent of Λ_i.

From Equation (3.4), $\ln\pi_i$ can represent the impact magnitude of infectious mortality of the i^{th} country driven by deaths in other countries. When the threshold (α) is infinite, mortality rates do not exert any infectious effects, and the proposed model can be reduced to the morality model introduced by Lin and Cox (2008).

3.2 Structure of Vital Capital I

The infectious mortality index of each country is modeled in Section 3.1. Using the obtained infectious mortality indices, the effect of infectious mortality risk on the

DOI:10.6814/DIS.NCCU.RMI.001.2018.F08

Swiss Re mortality bond is further analyzed by using Vital Capital I as an example. The Swiss Reinsurance Company issued 3-year Vital Capital I in 2003, with a face value of $400 million in coverage from institutional investors; this bond matured on January 1, 2007. The principal was exposed to mortality risk, and this mortality risk was defined in terms of an index based on the average annual population death rates in the United States, the United Kingdom, France, Italy, and Switzerland. If the index exceeded 130% of the actual 2002 level, investors had a percentage loss. The percentage loss of principal in year t is as follows:

$$L_t = \begin{cases} 0, & \text{if } Y_t < 1.3Y_{t_0^-} \\ \dfrac{Y_t - 1.3Y_{t_0^-}}{0.2Y_{t_0^-}}, & \text{if } 1.3Y_{t_0^-} \le Y_t \le 1.5Y_{t_0^-} \\ 1, & \text{if } Y_t > 1.5Y_{t_0^-} \end{cases}$$

Y_t denotes the geometric average population death rates in the United States, the United Kingdom, France, Italy, and Switzerland in year t. Again, the properties of the bonds can be written as Equation (3.5). Let B_T denote the principal payment at maturity time T, which is expressed as

$$B_T = \mathrm{Max}(1 - \mathrm{Loss},\ 0), \tag{3.5}$$

with $\mathrm{Loss} = \dfrac{\mathrm{Max}(Y_{Max} - 1.3Y_{t_0^-},\ 0) - \mathrm{Max}(Y_{Max} - 1.5Y_{t_0^-},\ 0)}{0.2Y_{t_0^-}}$, $Y_{Max} = \mathrm{Max}(Y_{t_1}, Y_{t_2}, Y_{t_3})$,

and $Y_{t_i} = (q_{1,t_i}^{a_1} q_{2,t_i}^{a_2} \ldots\ldots q_{5,t_i}^{a_5})^{\frac{1}{a_1+a_2+\ldots+a_5}}$, for all $i = 0,\ 1,\ 2,\ 3$ with $t_0 = 0$ and $t_3 = T$ for the bond; where $Y_{t_0^-}$, Y_{t_0}, Y_{t_1}, Y_{t_2}, and Y_{t_3} denote the geometric average population

DOI:10.6814/DIS.NCCU.RMI.001.2018.F08

death rates in the United States, the United Kingdom, France, Italy, and Switzerland in 2002, 2003, 2004, 2005, and 2006, respectively. $q_{1,t}, q_{2,t}, \ldots$ and $q_{5,t}$ represent the mortality indices of the United States, the United Kingdom, France, Italy, and Switzerland, respectively. $a_1, a_2, \ldots a_4$ and a_5 denote the weights of population mortality indices for the United States, the United Kingdom, France, Italy, and Switzerland, respectively.

The fair price of the Swiss Re mortality bond without coupons is shown in Equation (3.6).

$$
\begin{aligned}
B_0 &= 400000000 \times e^{-rT} E^Q\left[B_T\right] \\
&= 400000000 \times e^{-rT} E^Q\left[\operatorname{Max}\left(1-\frac{\operatorname{Max}(Y_{Max}-K_1,0)-\operatorname{Max}(Y_{Max}-K_2,0)}{K_2-K_1},0\right)\right],
\end{aligned}
\tag{3.6}
$$

where $E_t^Q(.)$ denotes the expectation value under the risk-neutral probability measure Q at time t, r is the riskless rate, and $K_1=1.3Y_{t_0}$ and $K_2=1.5Y_{t_0}$.

3.3 Valuation Formula for Vital Capital I

Pricing derivative securities in a complete market involves replicating portfolios. If a traded bond and stock index exist, options on the stock index can be replicated by holding the bond and index, which are priced. Vital Capital I is a mortality derivative, but no efficiently traded mortality index exists that can be used to create a replicating hedge. For pricing in such an incomplete market, the Wang transform is a popular

DOI:10.6814/DIS.NCCU.RMI.001.2018.F08

method that relies on the following transformation: for a risk with a cumulative density function (CDF) $F(x)$ under the original probability measure P, the risk-adjusted CDF $F^{*}(x)$ under the risk-neutral probability measure Q for pricing risk is given by

$$F^{*}(x)=\Phi(\Phi^{-1}(F(x))+\theta), \tag{3.7}$$

where θ is a constant risk premium, and $\Phi(.)$ is a cumulative standard normal probability.

For pricing Vital Capital I, we apply the Wang transform to solve Equation (3.6). We denote the total risks at time t in the i^{th} country as $X_{i,t} \triangleq \Gamma_{i,t} + \tilde{I}_t^{i}$, which follows a Poisson-jump process with the intensity of $\lambda_{X_{i,t}} = \lambda_{\tilde{I}_t^{i}} + \lambda_{\Gamma_{i,t}}$. The proof is given in Appendix A.

Assume that $x_i - 1$ is the percentage of the mortality index of the i^{th} country resulting from total risks, and x_i follows normal distributions with a mean of u_{x_i} and a variance of $\sigma_{x_i}^2$. Moreover,

$$(x_i - 1)dX_{i,t} \overset{d}{=} (\Lambda_i - 1)\, d\Gamma_{i,t} + (\pi_i - 1)\, d\tilde{I}_t^{i}. \tag{3.8}$$

Thus, we can obtain Equations (3.9) and (3.10).

$$E\left[(x_i - 1)dX_{i,t}\right] = E\left[(\Lambda_i - 1)\, d\Gamma_{i,t} + (\pi_i - 1)\, d\tilde{I}_t^{i}\right], \tag{3.9}$$

$$Var\left[(x_i - 1)dX_{i,t}\right] = Var\left[(\Lambda_i - 1)\, d\Gamma_{i,t} + (\pi_i - 1)\, d\tilde{I}_t^{i}\right]. \tag{3.10}$$

Subsequently, using Equations (3.9) and (3.10), we can obtain the following:

DOI:10.6814/DIS.NCCU.RMI.001.2018.F08

$$E\left[x_i-1\right]=\frac{(e^{u_{\pi_i}+\frac{1}{2}\sigma_{\pi_i}^2}-1)\lambda_{\tilde{I}_t^i}+(e^{u_{\Lambda_i}+\frac{1}{2}\sigma_{\Lambda_i}^2}-1)\lambda_{\Gamma_i}}{\lambda_{\Gamma_i}+\lambda_{\tilde{I}_t^i}},$$

$$\Rightarrow u_{x_i}=\frac{(e^{u_{\pi_i}+\frac{1}{2}\sigma_{\pi_i}^2}-1)\lambda_{\tilde{I}_t^i}+(e^{u_{\Lambda_i}+\frac{1}{2}\sigma_{\Lambda_i}^2}-1)\lambda_{\Gamma_i}}{\lambda_{\Gamma_i}+\lambda_{\tilde{I}_t^i}}+1, \tag{3.11}$$

$$\sigma_{x_i}^2=\frac{A+\left[(e^{u_{\pi_i}+\frac{1}{2}\sigma_{\pi_i}^2}-1)\lambda_{\tilde{I}_t^i}+(e^{u_{\Lambda_i}+\frac{1}{2}\sigma_{\Lambda_i}^2}-1)\lambda_{\Gamma_i}\right]^2}{(\lambda_{\Gamma_i}+\lambda_{\tilde{I}_t^i}+1)(\lambda_{\Gamma_i}+\lambda_{\tilde{I}_t^i})}-\left[\frac{(e^{u_{\pi_i}+\frac{1}{2}\sigma_{\pi_i}^2}-1)\lambda_{\tilde{I}_t^i}+(e^{u_{\Lambda_i}+\frac{1}{2}\sigma_{\Lambda_i}^2}-1)\lambda_{\Gamma_i}}{\lambda_{\Gamma_i}+\lambda_{\tilde{I}_t^i}}\right]^2, \tag{3.12}$$

with

$$\begin{aligned}A=&\left[e^{2u_{\Lambda_i}+\sigma_{\Lambda_i}^2}(e^{\sigma_{\Lambda_i}}-2e^{-u_{\Lambda_i}-\frac{1}{2}\sigma_{\Lambda_i}^2}+2)+1\right](\lambda_{\Gamma_i}+\lambda_{\Gamma_i}^2)-\lambda_{\Gamma_i}^2\left(e^{u_{\Lambda_i}+\frac{1}{2}\sigma_{\Lambda_i}^2}-1\right)^2\\&+\left[e^{u_{\pi_i}+\sigma_{\pi_i}^2}(e^{\sigma_{\pi_i}}+1)+\left(e^{u_{\pi_i}+\frac{1}{2}\sigma_{\pi_i}^2}-1\right)^2\right]\left[\lambda_{\tilde{I}_t^i}+\lambda_{\tilde{I}_t^i}^2\right]-\left[(e^{u_{\pi_i}+\frac{1}{2}\sigma_{\pi_i}^2}-1)\lambda_{\tilde{I}_t^i}\right]^2.\end{aligned}$$

Under the original probability measure P, using Ito's lemma, Equation (3.4) can be rewritten as $q_{i,T=}q_{i,t_0}e^{(\mu_i-\frac{1}{2}\sigma_i^2)(T-t_0)+\sigma_i W_{i,T-t_0}}\prod_{l=1}^{X_{i,T}}x_{i,l}$, i=1,2,.....,5. The numbers for i indicate the United States, the United Kingdom, France, Italy, and Switzerland, respectively. In addition,

$$\ln q_{i,T}=\ln q_{i,t_0}+(\mu_i-\frac{1}{2}\sigma_i^2)(T-t_0)+\sigma_i W_{i,T-t_0}+\sum_{l=1}^{X_{i,T}}\ln x_{i,l} \tag{3.13}$$

Next, let X_t represent the sum of the total risks for the United States, the United Kingdom, France, Italy, and Switzerland, namely $X_t\triangleq X_{1,t}+X_{2,t}+....+X_{5,t}$, which follows a Poisson distribution with the intensity of λ_t and $\lambda_t=\sum_{i=1}^{5}(\lambda_{\tilde{I}_t^i}+\lambda_{\Gamma_{i,t}})$. To derive the closed-form solution of the fair price of the mortality bond, we adopt

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Proposition 1.

Proposition 1. Let Z_t be a random variable, and assume that $\ln Z$ follows a normal distribution with a mean of u_z and a variance of σ_z^2. Given the respective logarithm of the mortality indices for the United States, the United Kingdom, France, Italy, and Switzerland, as shown in Equation (3.14), the logarithm of the geometric average population mortality rates of the five countries takes the following form:

$$\ln Y_T = \ln Y_{t_0} + \mu_y (T - t_0) + \sigma_y W_{T-t_0} + a \sum_{l=1}^{X_T} \ln Z_l \,, \tag{3.14}$$

where $\mu_y = \frac{1}{a_1 + a_2 + ... + a_5} \sum_{i=1}^{5} a_i (\mu_i - \frac{1}{2}\sigma_i^2)$; $a = \frac{1}{a_1 + a_2 + ... + a_5}$;

$\sum_{l=1}^{X_T} \ln Z_l = a_1 \sum_{l=1}^{X_{1,T}} \ln x_{1,l} + a_2 \sum_{l=1}^{X_{2,T}} \ln x_{2,l} + ... + a_5 \sum_{l=1}^{X_{5,T}} \ln x_{5,l}$; and

$$\sigma_y = \frac{1}{a_1 + a_2 + ... + a_5} \sqrt{\begin{bmatrix} a_1\sigma_1 & a_2\sigma_2 & a_3\sigma_3 & a_4\sigma_4 & a_5\sigma_5 \end{bmatrix} \begin{pmatrix} 1 & \cdots & \rho_{15} \\ \vdots & \ddots & \vdots \\ \rho_{51} & \cdots & 1 \end{pmatrix} \begin{bmatrix} a_1\sigma_1 & a_2\sigma_2 & a_3\sigma_3 & a_4\sigma_4 & a_5\sigma_5 \end{bmatrix}'} .$$

Proof. See Appendix B.

From Proposition 1, if X_t is any constant ($X_t = s$), $\ln Z_t | X_t = s$ has a normal distribution with a mean of u_z and a variance of σ_z^2. When $X_t = X_{1,t} + X_{2,t} + ... + X_{5,t}$, and $X_{i,t} = s_i$, such that s_i is any constant i = 1, 2, …, 5, we can obtain $u_z = \frac{\sum_{i=1}^{5} s_i a_i u_{x_i}}{s}$ and $\sigma_z^2 = \frac{\sum_{i=1}^{5} s_i a_i^2 \sigma_{x_i}^2}{s}$. Additionally, we suppose that $S_T \triangleq \frac{Max(Y_{Max} - K_1, 0) - Max(Y_{Max} - K_2, 0)}{K_2 - K_1}$.

Conditional on $Y_{Max} = Y_{t_i}$, we can obtain

DOI:10.6814/DIS.NCCU.RMI.001.2018.F08

$$S_T=\frac{(Y_{t_i}-K_1)1_{\{Y_{t_i}>K_1\}}-(Y_{t_i}-K_2)1_{\{Y_{t_i}>K_2\}}}{K_2-K_1}, \text{ for } i=1,2,3. \tag{3.15}$$

Therefore, Equation (3.6) can be rewritten as

$$B_0=400000000\times e^{-rT}\times$$
$$\left\{\begin{array}{l} E^Q\left[(1-S_T)1_{\{S_T<1\}}\middle|Y_{Max}=Y_{t_1}\right]P_r^Q(Y_{Max}=Y_{t_1})+E^Q\left[(1-S_T)1_{\{S_T<1\}}\middle|Y_{Max}=Y_{t_2}\right]P_r^Q(Y_{Max}=Y_{t_2}) \\ +E^Q\left[(1-S_T)1_{\{S_T<1\}}\middle|Y_{Max}=Y_{t_3}\right]P_r^Q(Y_{Max}=Y_{t_3}) \end{array}\right\}. \tag{3.16}$$

The Wang transform is used to obtain the closed-form solution. Using Equation (3.16), we solve the probability of $Y_{Max}=Y_{t_i}$ under the risk-neutral probability measure. Therefore, Proposition 2 is necessary.

Proposition 2. Given $Y_{Max}=Max(Y_{t_1},Y_{t_2},Y_{t_3})$, the probability of $Y_{Max}=Y_{t_i}$ under the risk-neutral measure Q is as follows:

$$P_r^Q(Y_{max}=Y_{t_i})=\Phi(\Phi^{-1}(P_r^p(Y_{max}=Y_{t_i}))+\theta_i)=\Phi(\Phi^{-1}(\Phi(d_{2i-1},d_{2i},\rho_{2i-1,2i}))+\theta_i), \tag{3.17}$$

i=1, 2, 3. θ_i is the risk premiums of Y_{t_i}; and

$$d_1=\frac{\mu_y(t_2-t_1)-a\,u_z s}{\sqrt{\sigma_y|t_2-t_1|+a^2\sigma_z^2 s}};\ d_2=\frac{\mu_y(t_3-t_1)-a\,u_z s}{\sqrt{\sigma_y|t_3-t_1|+a^2\sigma_z^2 s}};\ \rho_{1,2}=corr(\frac{W_{t_2-t_1}}{\sqrt{|t_2-t_1|}},\frac{W_{t_3-t_1}}{\sqrt{|t_3-t_1|}});$$

$$d_3=\frac{\mu_y(t_2-t_1)-a\,u_z s}{\sqrt{\sigma_y|t_1-t_2|+a^2\sigma_z^2 s}};\ d_4=\frac{-\mu_y(t_3-t_2)-a\,u_z s}{\sqrt{\sigma_y|t_3-t_2|+a^2\sigma_z^2 s}};\ \rho_{3,4}=corr(\frac{W_{t_2-t_1}}{\sqrt{|t_2-t_1|}},\frac{W_{t_3-t_2}}{\sqrt{|t_3-t_2|}});$$

$$d_5=\frac{\mu_y(t_3-t_1)-a\,u_z s}{\sigma_y\sqrt{|t_3-t_1|+a^2\sigma_z^2 s}};\ d_6=\frac{\mu_y(t_3-t_2)-a\,u_z s}{\sqrt{\sigma_y|t_3-t_2|+a^2\sigma_z^2 s}};\ \rho_{5,6}=corr(\frac{W_{t_3-t_1}}{\sqrt{|t_3-t_1|}},\frac{W_{t_3-t_2}}{\sqrt{|t_3-t_2|}}).$$

Proof. See Appendix C.

Furthermore, Proposition 3 must be adopted to derive the solution of the fair price

DOI:10.6814/DIS.NCCU.RMI.001.2018.F08

of the Swiss Re bond without coupons.

Proposition 3. When $Y_{Max}=Y_{t_i}$, the following equation can be derived from Proposition 2:

$$E^{Q}\left[1_{\{S_T<1\}}\middle|Y_{Max}=Y_{t_i}\right]=\Phi(\Phi^{-1}(1-F_{Y_{t_i}}(K_1))+\theta_i), \tag{3.18}$$

in which $F_{Y_{t_i}}(K_1)=P_r^P(Y_{t_i}\le K_1)=\sum_{s=0}^{\infty}\frac{e^{-\lambda_t}(\lambda_t)^s}{s!}\Phi\left(\frac{\ln\frac{K_1}{Y_{t_0}}-\mu_y(t_i-t_0)-su_za}{\sqrt{\sigma_y^2(t_i-t_0)+s\sigma_z^2a^2}}\right)$, i=1, 2, 3;

θ_i is the risk premium of Y_{t_i}; $\mu_y=\frac{1}{a_1+a_2+...+a_5}\sum_{i=1}^{5}a_i(\mu_i-\frac{1}{2}\sigma_i^2)$; $u_z=\frac{\sum_{i=1}^{5}s_ia_iu_{x_i}}{s}$;

$\sigma_z^2=\frac{\sum_{i=1}^{5}s_ia_i^2\sigma_{x_i}^2}{s}$; $\lambda_t=\sum_{i=1}^{5}(\lambda_{\tilde{I}_t^i}+\lambda_{\Gamma_{i,t}})$.

Proof. See Appendix D.

Following the previous procedure, we can derive Equation (3.19) through Proposition 3.

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$$
\begin{aligned}
& E^{Q}\left[S_T 1_{\{S_T<1\}} \middle| Y_{Max}=Y_{t_i}\right] \\
&= E^{Q}\left[S_T 1_{\{S_T<1\}} \middle| Y_{Max}=Y_{t_1}\right] P^{Q}(Y_{Max}=Y_{t_1}) \\
&\quad + E^{Q}\left[S_T 1_{\{S_T<1\}} \middle| Y_{Max}=Y_{t_2}\right] P^{Q}(Y_{Max}=Y_{t_2}) \\
&\quad + E^{Q}\left[S_T 1_{\{S_T<1\}} \middle| Y_{Max}=Y_{t_3}\right] P^{Q}(Y_{Max}=Y_{t_3}) \\
&= E^{Q}\left[\frac{Max(Y_{t_1}-K_1,0)-Max(Y_{t_1}-K_2,0)}{K_2-K_1} 1_{\{S_T<1\}} \middle| Y_{Max}=Y_{t_1}\right] P^{Q}(Y_{Max}=Y_{t_1}) \\
&\quad + E^{Q}\left[\frac{Max(Y_{t_2}-K_1,0)-Max(Y_{t_2}-K_2,0)}{K_2-K_1} 1_{\{S_T<1\}} \middle| Y_{Max}=Y_{t_2}\right] P^{Q}(Y_{Max}=Y_{t_2}) \\
&\quad + E^{Q}\left[\frac{Max(Y_{t_3}-K_1,0)-Max(Y_{t_3}-K_2,0)}{K_2-K_1} 1_{\{S_T<1\}} \middle| Y_{Max}=Y_{t_3}\right] P^{Q}(Y_{Max}=Y_{t_3}) \\
&= \left[1-\Phi(\Phi^{-1}(F_{Y_{t_i}}(K_1))+\theta_i)\right] \times \\
& \left\{
\begin{array}{l}
\Phi(\Phi^{-1}(F_{Y_{t_i}}(K_2))+\theta_i) + \\
\left[\Phi(\Phi^{-1}(F_{Y_{t_i}}(K_2))+\theta_i) - \Phi(\Phi^{-1}(F_{Y_{t_i}}(K_1))+\theta_i)\right] \times \\
\left\{\dfrac{1}{K_2-K_1}\left[\displaystyle\sum_{s=0}^{\infty}\frac{e^{-\lambda_t}(\lambda_t)^s}{s!} Y_{t_0} e^{\left(\mu_y+\frac{1}{2}\sigma_y^2\right)(t_i-t_0)+s\,(au_z+\frac{1}{2}a^2\sigma_z^2)} + \theta_i\sqrt{Var^{P}(Y_{t_i})}\right] - \dfrac{K_1}{K_2-K_1}\right\}
\end{array}
\right\}, \qquad (3.19)
\end{aligned}
$$

with $$F_{Y_{t_i}}(K_1)=P_r^P(Y_{t_i}\le K_1)=\sum_{s=0}^{\infty}\frac{e^{-\lambda_t}(\lambda_t)^s}{s!}\Phi\left(\frac{\ln\frac{K_1}{Y_{t_0}}-\mu_y(t_i-t_0)-asu_z}{\sqrt{\sigma_y^2(t_i-t_0)+s\,a^2\sigma_z^2}}\right),$$

$$F_{Y_{t_i}}(K_2)=P_r^P(Y_{t_i}\le K_2)=\sum_{s=0}^{\infty}\frac{e^{-\lambda_t}(\lambda_t)^s}{s!}\Phi\left(\frac{\ln\frac{K_2}{Y_{t_0}}-\mu_y(t_i-t_0)-asu_z}{\sqrt{\sigma_y^2(t_i-t_0)+s\,a^2\sigma_z^2}}\right), \text{ and}$$

$$
\begin{aligned}
Var^{P}(Y_{t_i}) &= \sum_{s=0}^{\infty}\frac{e^{-\lambda_t}(\lambda_t)^s}{s!} Y_{t_0}^2 e^{2\mu_y(t_i-t_0)+2sau_z+2\sigma_y^2 a^2(t_i-t_0)} - \\
&\quad \left\{\sum_{s=0}^{\infty}\frac{e^{-\lambda_t}(\lambda_t)^s}{s!} Y_{t_0} e^{2\mu_y(t_i-t_0)+2s(au_z+a^2\sigma_z^2)+\frac{1}{2}a^2\sigma_y^2(t_i-t_0)}\right\}^2,
\end{aligned}
$$

for i=1, 2, 3.

Consequently, by substituting Equations (3.17), (3.18), and (3.19) into Equation (3.16), the fair price of the Swiss Re bond without coupons can be obtained.

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3.4 Empirical Results

In this section, we first use the mortality data from the HMD to estimate the parameters $(\mu_i, \sigma_i, u_{\Lambda_i}, \sigma_{\Lambda_i}, u_{\pi_i}, \sigma_{\pi_i})$ for the United States, the United Kingdom, France, Italy, and Switzerland in the proposed infectious mortality model. The time window is 1933–2007.

3.4.1 Parameter Estimation and Goodness of Fit of the Infectious Mortality Model

A calibration approach is adopted to estimate the variables $(\mu_i, \sigma_i, u_{\Lambda_i}, \sigma_{\Lambda_i}, u_{\pi_i}, \sigma_{\pi_i})$ for the five focal countries. Calibration refers to estimating the best fitting parameters in a parametric model in comparison with a chosen observable quantity. Comparative information typically consists of the historical data of liquid instruments. Prices are fitted based on the assumption that a trader agrees that the historical data are consistent with a true process. Different jump-diffusion processes are calibrated using actual log returns of the population mortality index for each country. The detailed procedure is as follows:

(1) Collect the actual log returns of the population mortality indices of the United States, the United Kingdom, France, Italy, and Switzerland. Consider $d(\ln \hat{q}_{i,t})$, the model log returns of the population mortality indices of the five countries from Equation (3.4), and $d(\ln q_{i,t})$, the observed log returns of the population mortality index of

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each country. The difference $d(\ln q_{i,t}) - d(\ln \hat{q}_{i,t})$ is a function of the values of $\Theta = (\mu_i, \sigma_i, u_{\Lambda_i}, \sigma_{\Lambda_i}, u_{\pi_i}, \sigma_{\pi_i})$.

(2) Given the initial values of $(\mu_i, \sigma_i, u_{\Lambda_i}, \sigma_{\Lambda_i}, u_{\pi_i}, \sigma_{\pi_i})$ illustrated in Table 3.3, the initial values of μ_i and σ_i are chosen as the mean and volatility of mortality indices of the United States, the United Kingdom, France, Italy, and Switzerland, respectively. Find the parameter vector Θ to solve the nonlinear sum of squared errors as follows:

$$SSE = \underset{\Theta}{Min} \sum_{j=1}^{n} \left| \varepsilon_j[\Theta] \right|^2$$

Using the above procedure, the estimated parameters characterizing the proposed infectious mortality model for the five countries are shown in Table 3.4.

Table 3.3 Initial Values of the Calibrated Parameters for Five Countries

	US	UK	France	Italy	Switzerland
μ_i	0.006907	0.011709	0.011921	0.011997	0.013883
σ_i	0.000438	0.001319	0.001469	0.001348	0.001363
u_{π_i}	0.001	0.001	0.001	0.001	0.001
σ_{π_i}	0.002	0.002	0.002	0.002	0.002
u_{Λ_i}	0.001	0.001	0.001	0.001	0.001
σ_{Λ_i}	0.002	0.002	0.002	0.002	0.002

Note: Initial values of μ_i and σ_i are chosen as the mean and volatility of the mortality indices of the United States, the United Kingdom, France, Italy, and Switzerland, respectively.

DOI:10.6814/DIS.NCCU.RMI.001.2018.F08

Table 3.4
Parameter Estimates in the Infectious Mortality Dynamics for Five Countries

	US	UK	France	Italy	Switzerland
μ_i	-0.007635698 (0.00028)	-0.0019212853 (0.00033)	-0.0023350499 (0.00021)	-0.0022268845 (0.00035)	-0.0017903048 (0.00025)
σ_i	0.0353039713 (0.00031)	0.0303390187 (0.00033)	0.0208596021 (0.00028)	0.0346345882 (0.00021)	0.0041059500 (0.00030)
u_{π_i}	-0.4080070179 (0.00025)	-0.0685499403 (0.00028)	-0.0480721838 (0.00027)	-0.0797427948 (0.00021)	-0.0545940218 (0.00024)
σ_{π_i}	0.1727495194 (0.00032)	0.0287313469 (0.00039)	0.0201543124 (0.00037)	0.00328499675 (0.00031)	0.0227485367 (0.00041)
u_{Λ_i}	-0.1932588391 (0.00051)	-0.0413981341 (0.00059)	-0.0315769919 (0.00051)	-0.06456981451 (0.00058)	-0.0445981165 (0.00055)
σ_{Λ_i}	0.3129031480 (0.00068)	0.3398761175 (0.00061)	0.2659823115 (0.00069)	0.20038971226 (0.00058)	0.29913998715 (0.00071)

Notes: Parameter estimates in Equation (3.4) for i = the United States, the United Kingdom, France, Italy, and Switzerland. Standard errors are shown in parentheses.

3.4.2 Numerical Analysis

The fair price of the Swiss Re bond can be obtained according to the parameters shown in Tables 3.2 and 3.4. Using the principal of $1 as an example and for comparison purposes, the risk premium of 0.83 is assumed for the bonds following the trend reported by Cox et al. (2006). We perform a scenario analysis based on three cases. Case 1 (normal situation): according to the base parameters of $\theta = 0.83$, $\lambda_t = 0.05$, $\alpha = 0.71112$[2], $u_z = -0.001$, and $\sigma_z = 0.1$, the fair par price of the Swiss Re bond is 0.7393. Case 2 (low infection): given $\theta = 0.83$, $\lambda_t = 0.05$, $\alpha = 0.99$, $u_z = -0.001$ and $\sigma_z = 0.1$, the fair par price of the Swiss Re bond is 0.8457. Case 3 (high infection):

[2] This is calculated as the average of $\tilde{V}_t^i$ for i = the United States, the United Kingdom, France, Italy, and Switzerland.

DOI:10.6814/DIS.NCCU.RMI.001.2018.F08

given $\theta = 0.83$, $\lambda_t = 0.05$, $\alpha = 0.001$, $u_z = -0.001$, and $\sigma_z = 0.1$, the fair par price of the Swiss Re bond is 0.6992.

From the scenario analysis, the fair prices are lower than the par value of $1, and those reported by Tsai and Tzseng (2013) (0.9966). Thus, ignoring the effects of infectious mortality rates significantly overestimates the price of mortality bonds. In other words, considering the phenomenon of infectious mortality rates enables the fair price of a mortality security to be more fitted to real-world values.

Furthermore, we numerically investigate the price of the mortality bond by using the proposed infectious mortality model. Table 3.5 demonstrates the impacts of the major parameters, mean and volatility, on the magnitudes of infectious mortality, threshold values (α), and jump intensities of the par spread of the bonds. According to the literature on mortality bonds,[3] we assume that the risk premiums range from 1 to 2. Table 3.5 shows a common phenomenon: the fair price decreases as mortality increases. In Panel A, the impacts of the mean of the magnitudes of infectious mortality on bond prices are uncertain. However, bond prices decrease as the volatilities of the magnitudes of infectious mortality increase due to higher mortality rates (Panel B).

Panel C demonstrates that the relationship of threshold values with the fair prices

[3] The risk premiums presented by Cox et al. (2006), Lin and Cox (2008), Chen and Cox (2009), and Lin, Liu, and Yu (2013) were 0.83, 0.8657, 1.5, and 1.21, respectively.

DOI:10.6814/DIS.NCCU.RMI.001.2018.F08

is positive, because the higher the threshold values, the lower the infectious mortality is. Thus, the loss of principal of Vital Capital I declines, and the prices then increase. Conversely, Panel D reveals that when jump intensities increase, mortality rates generally increase, and bond prices decline. The sensitivity of the volatilities of the magnitudes of infectious mortality is the largest among the model parameters, whereas that of threshold values is the smallest.

Table 3.5 reveals a common phenomenon, in which the volatilities of the magnitudes of infectious mortality exert significant effects on the fair prices.

DOI:10.6814/DIS.NCCU.RMI.001.2018.F08

Table 3.5 Impacts of Various Parameters on Fair Prices of the Swiss Re Bond

Parameter	$\theta = 0.83$	$\theta = 0.8657$	$\theta = 1.21$	$\theta = 1.5$
u_z	Panel A: u_z changes			
-0.001	0.5163	0.5196	0.5897	0.6125
-0.003	0.4987	0.5011	0.5734	0.5813
-0.005	0.4593	0.4972	0.5539	0.5712
-0.007	0.4886	0.5313	0.5618	0.5896
-0.009	0.5098	0.5478	0.5715	0.5947
σ_z	Panel B: σ_z changes			
0.1	0.9125	0.9237	0.9358	0.9399
0.2	0.8143	0.8168	0.8915	0.9141
0.3	0.7759	0.7825	0.8598	0.8611
0.4	0.5647	0.5998	0.6315	0.7014
0.5	0.3325	0.3985	0.4918	0.5481
α	Panel C: α changes			
0.70	0.8169	0.8198	0.8245	0.8266
0.75	0.8256	0.8267	0.8309	0.8351
0.80	0.8321	0.8357	0.8401	0.8416
0.85	0.8395	0.8400	0.8415	0.8423
0.90	0.8411	0.8425	0.8438	0.8509
λ_t	Panel D: λ_t changes			
0.01	0.6458	0.6511	0.6715	0.6798
0.02	0.6135	0.6212	0.6598	0.6613
0.03	0.6123	0.6437	0.6997	0.6011
0.04	0.5978	0.6198	0.5498	0.5599
0.05	0.4569	0.4986	0.5058	0.5149

3.5 Conclusion

Transferring catastrophic losses using mortality-linked securities has become pertinent for the insurance industry. Many life insurers operate their businesses internationally. According to patterns of mortality experience, we find that catastrophic events may cause the comovement of mortality rates across countries. Although studies consider mortality rates with jumps, they explain the comovement of mortality rates by using common jumps across countries. However, the mortality trend empirically reveals

DOI:10.6814/DIS.NCCU.RMI.001.2018.F08

that mortality comovement may occur only when there is a catastrophic event that causes considerable mortality in all countries. Studies rarely model the phenomenon of infectious mortality rates. To fill this research gap, this study offers a new perspective of the effects of mortality rates on the valuation of mortality securities. We accordingly propose an infectious mortality model: using the Wang transform, a valuation formula for the mortality bond is derived through the proposed infectious mortality model.

An empirical analysis reveals that the fair price of Vital Capital I in the model is far higher than that reported by Cox et al. (2006) and is closer to the actual bond price. Therefore, considering the infectious effects of mortality rates enables mortality bond prices to fit real-world values, which is helpful for pricing mortality securities and for managing catastrophic mortality risk for reinsurers.

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Chapter 4 Valuation and Analysis of Fixed-Coupon and Floating-Coupon Mortality Bonds in the Infectious Mortality Model

This chapter examines coupon mortality bonds (fixed-coupon and floating-coupon bonds). These bonds are similar to the Swiss Re bond. The closed-form solution of a fixed-coupon mortality bond is derived, and we assume that the coupons of floating-coupon mortality bonds are linked to a stochastic interest rate, which follows the CIR interest rate model. Monte Carlo simulation is employed to evaluate the sensitivities of the fair prices of floating-coupon bonds.

4.1 Model Formulation

Let $H_{i,t} = \dfrac{V_t^{\tilde{i}}}{1 - V_t^{\tilde{i}}}$, which is the odds ratio of the i^{th} country for $i = 1, 2, 3, ..., m$ at time t. This odds ratio reflects the ratio of the previously calculated death ratio of all countries except that of the i^{th} country relative to death ratio of the i^{th} country. We assume that the natural logarithm returns of $H_{i,t}$ for $i = 1, 2, 3, ..., m$ at time t follow a geometric Brownian motion, such that

$$d(\ln H_{i,t}) = \mu_{H_i} dt + \sigma_{H_i} dW_{H,t}, \qquad (4.1)$$

where μ_{H_i} and σ_{H_i} denote the drift term and volatility, respectively, and $W_{H,t}$ is a one-dimensional standard Brownian motion under the original probability measure P. From Equation (4.1), we can confirm that $0 \le V_t^{\tilde{i}} \le 1$.

By means of the raw mortality data of five countries (i.e., the United States, the

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United Kingdom, France, Italy, and Switzerland) obtained from the Human Mortality Database (HMD) from 1933 to 2007, Figures 4.1-4.5 show actual odds ratios except for US., UK., France, Italy and Switzerland. Furthermore, we calibrate the parameters of Equation (4.1) with the HMD through the initial values of μ_{H_i} and σ_{H_i} set as the mean and volatility of the odds ratio for the United States, the United Kingdom, France, Italy, and Switzerland, respectively. As a result, the estimated parameters of Equation (4.1) are illustrated in Table 4.1. Based on Table 4.1, the estimated the odds ratio are demonstrated in Figures 4.6-4.10 for the United States, the United Kingdom, France, Italy, and Switzerland, respectively. In addition, the mean squared errors of $H_{i,t}$ in the United States, the United Kingdom, France, Italy and Switzerland are 0.00125, 0.00254, 0.00291, 0.00312 and 0.00113, respectively. In average, the mean squared errors are 0.00219 for all five countries.

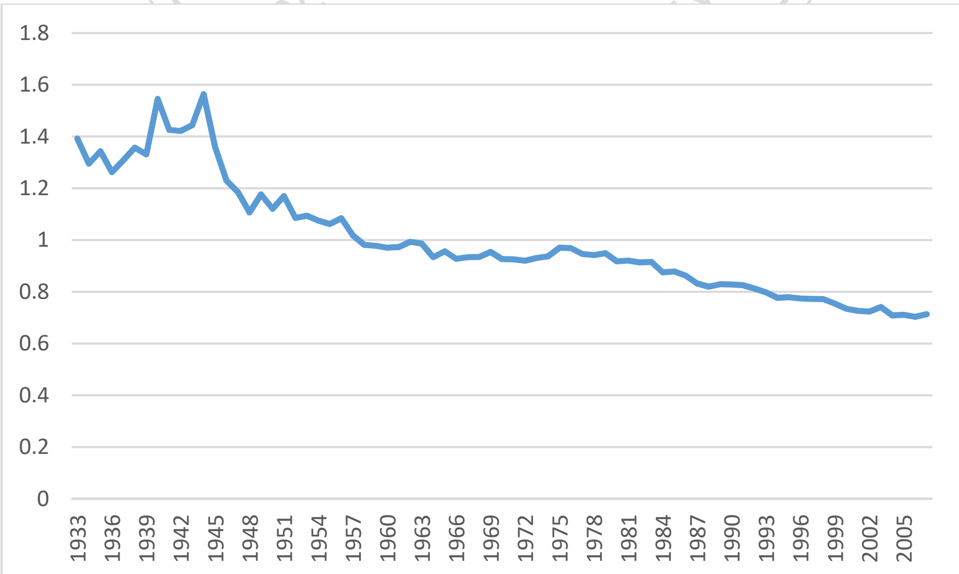

Figure 4.1 Actual Odds Ratio Except for US.

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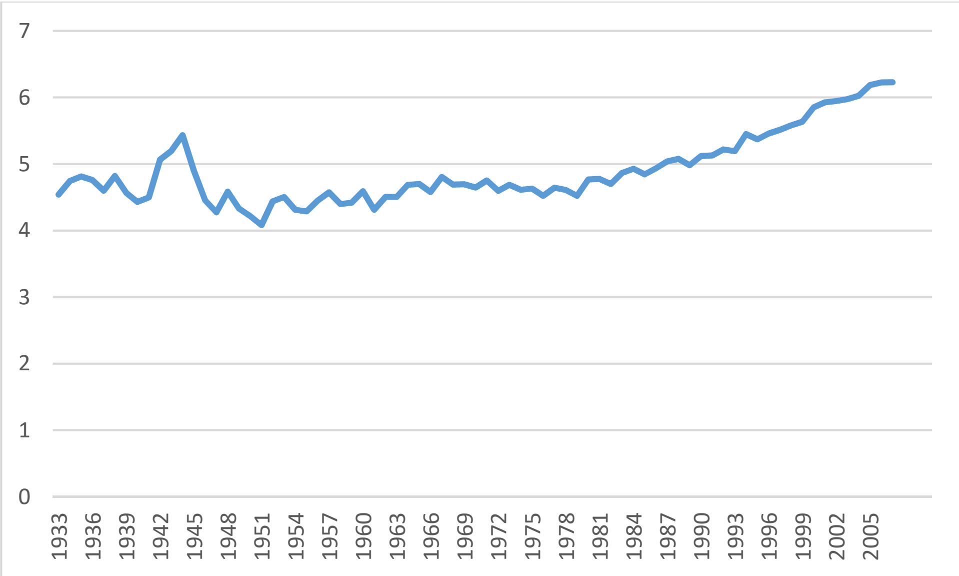

Figure 4.2 Actual Odds Ratio Except for UK.

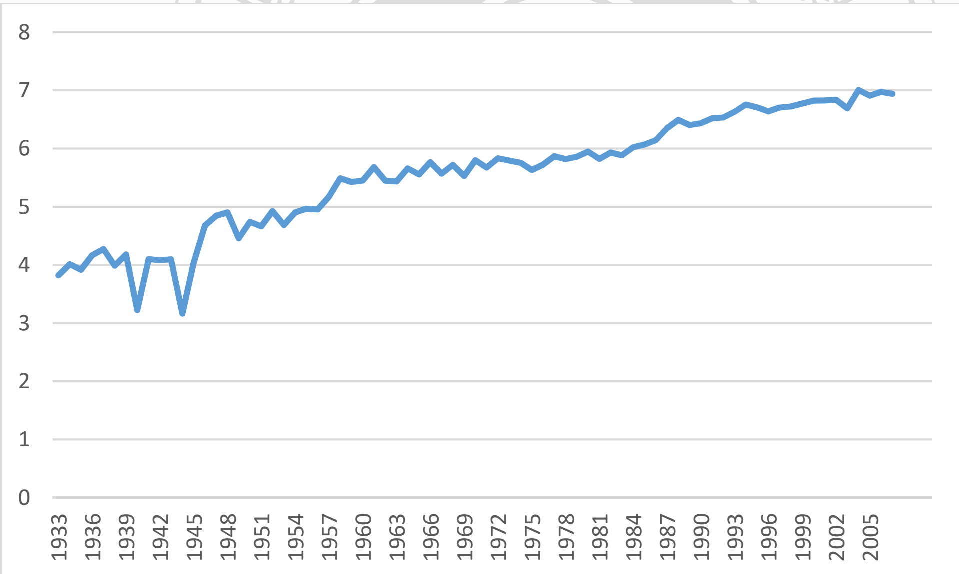

Figure 4.3 Actual Odds Ratio Except for France

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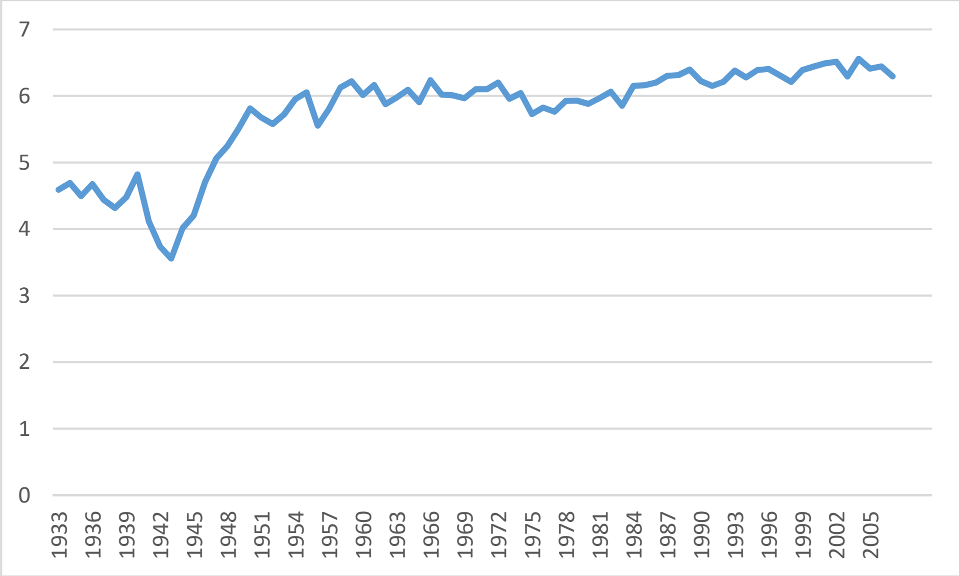

Figure 4.4 Actual Odds Ratio Except for Italy

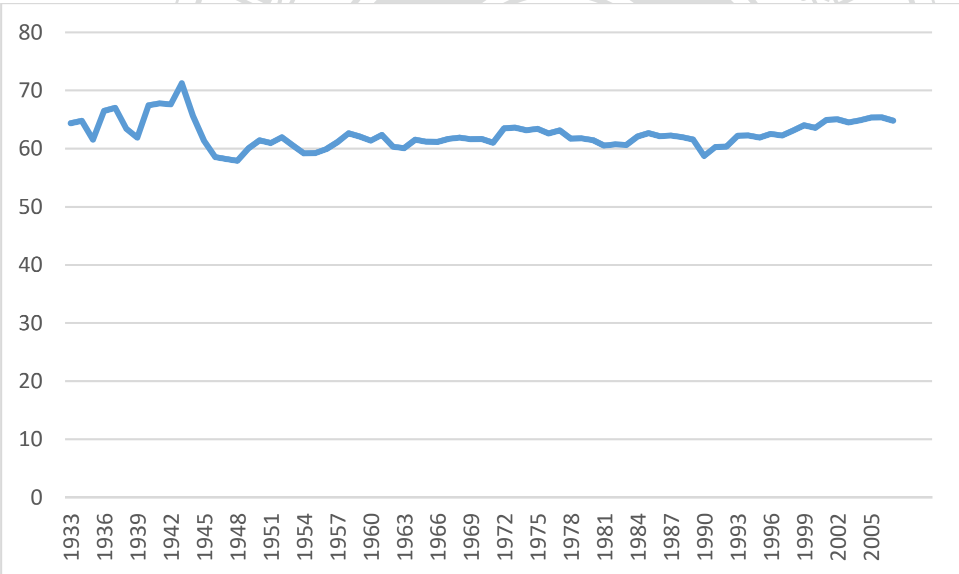

Figure 4.5 Actual Odds Ratio Except for Switzerland

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Table 4.1 Parameter Estimation of Dynamic Processes of $\ln H_{i,t}$ through Calibration

	US	UK	France	Italy	Switzerland
μ_{H_i}	-0.0043215 (0.000012)	0.0020975 (0.000011)	0.0041256 (0.000012)	0.0020159 (0.000010)	0.0001984 (0.000011)
σ_{H_i}	0.0215689 (0.000034)	0.0201452 (0.000032)	0.0331209 (0.000042)	0.0269122 (0.000035)	0.0154381 (0.000039)

Notes: The parameter estimates are derived using Equation (4.1) for $i =$ the United States, the United Kingdom, France, Italy, or Switzerland. $\theta = 0.83$, $a_1=0.7$, $a_2=0.15$, $a_3=0.075$, $a_4=0.05$, $a_5=0.025$, $\alpha_1 = 0.998$, $\alpha_2 = 4.893$, $\alpha_3 = 5.547$, $\alpha_4 = 5.744$, and $\alpha_5 = 62.487$. Standard errors are shown in parentheses. The initial values of μ_{H_i} and σ_{H_i} are the mean and volatility of the odds ratios the United States, the United Kingdom, France, Italy, and Switzerland.

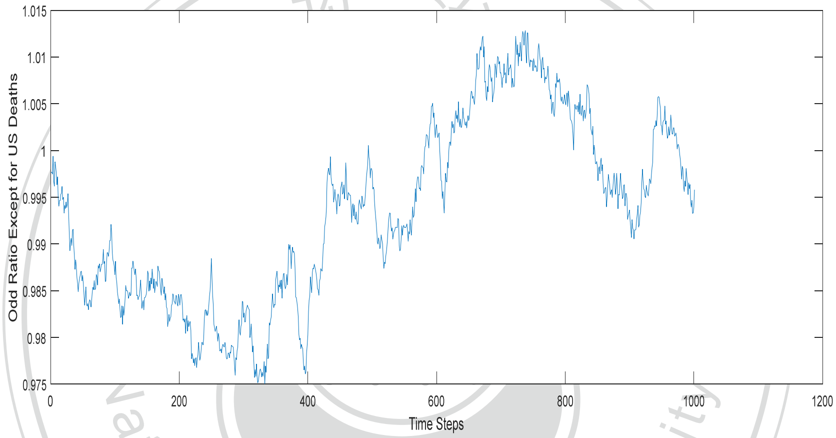

Figure 4.6 The Estimated Odds Ratio Except for US.

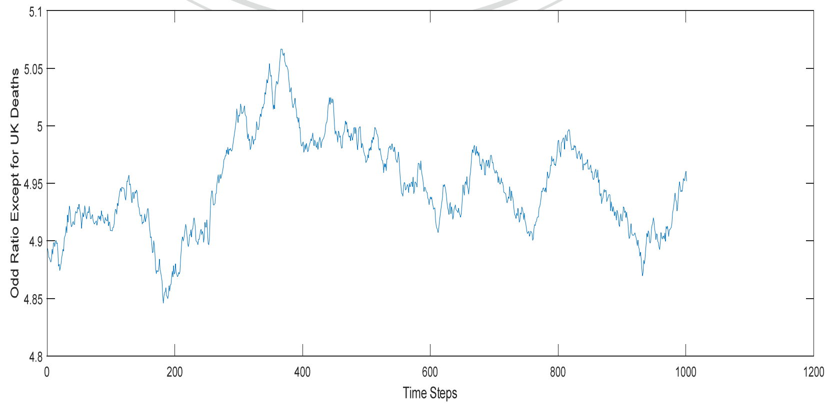

Figure 4.7 The Estimated Odds Ratio Except for UK.

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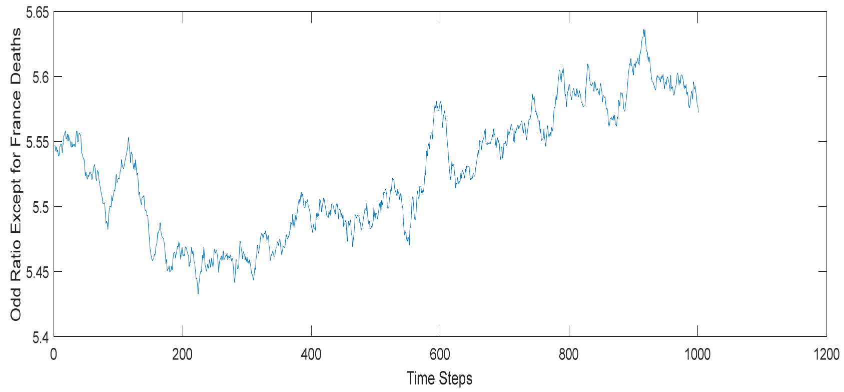

Figure 4.8 The Estimated Odds Ratio Except for France

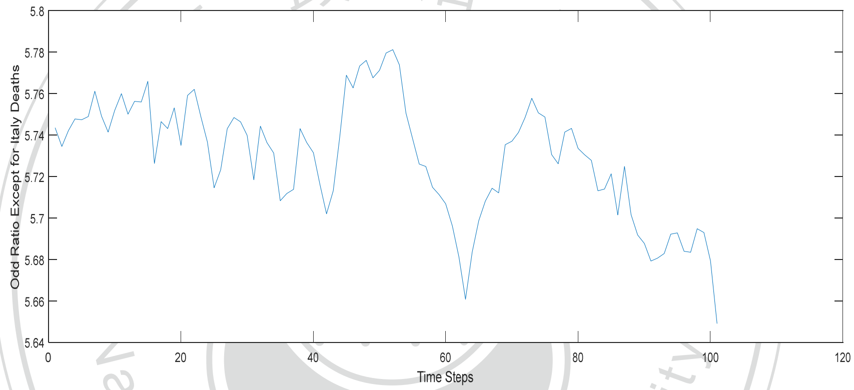

Figure 4.9 The Estimated Odds Ratio Except for Italy

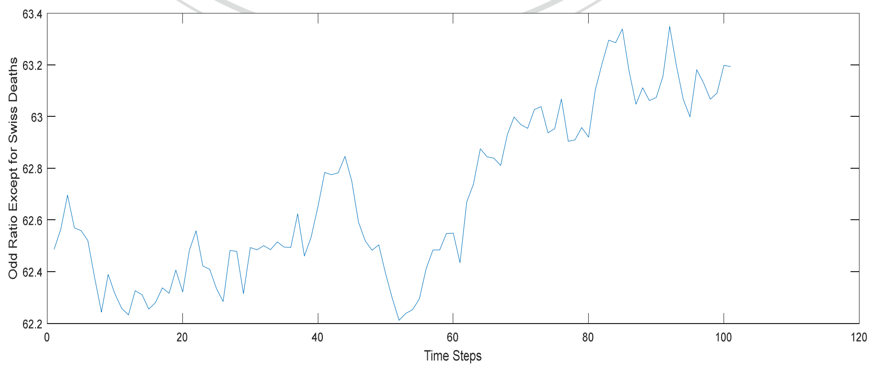

Figure 4.10 The Estimated Odds Ratio Except for Switzerland

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Next, to capture the comovement phenomenon when a catastrophic event occurs, we set a threshold for $H_{i,t}$. If $H_{i,t}$ is higher than this threshold, say α_i, then the mortality rate of the i^{th} country can be affected by the mortality rate in another country. Thus, the mortality rate of the i^{th} country at time t is modeled using two types of jumps: an infectious jump and a specific country jump. Let $I_t^{\tilde{i}}$ denote the jump frequency of the i^{th} country at time t that is infected by other countries when a catastrophic event (e.g., infectious disease) occurs; $\Gamma_{i,t}$ is the jump frequency resulting from the mortality shock in the i^{th} country at time t. We assume that both $I_t^{\tilde{i}}$ and $\Gamma_{i,t}$ follow Poisson distributions with the intensities of $\lambda_{I_t^{\tilde{i}}}$ and $\lambda_{\Gamma_{i,t}}$, respectively. Accordingly, $I_t^{\tilde{i}}$ can be expressed as

$$I_t^{\tilde{i}} = \int_0^t D_u^{\tilde{i}} du \sim \text{Poisson}(\lambda_{I_t^{\tilde{i}}}), \tag{4.2}$$

where $D_u^{\tilde{i}} = \begin{bmatrix} 1, & \text{if } H_{i,t} \geq \alpha_i \\ 0, & \text{o.w} \end{bmatrix}$.

Furthermore, we present $\lambda_{I_t^{\tilde{i}}}$ as the expectation of infectious jump frequency, which can be calculated as

$$\begin{aligned} \lambda_{I_t^{\tilde{i}}} &= E\left[I_t^{\tilde{i}} \right] \\ &= \int_0^t \Phi \left(\frac{\ln\left(\frac{H_{i,0}}{\alpha_i}\right) + (\mu_{H_i} - \frac{1}{2}\sigma_{H_i}^2)\delta}{\sigma_{H_i}\sqrt{\delta}} \right) d\delta. \end{aligned} \tag{4.3}$$

Let $q_{i,t}$ represent the mortality rate of the i^{th} country at time t. The multicountry mortality dynamics for m countries can be modeled as

DOI:10.6814/DIS.NCCU.RMI.001.2018.F08

$$\frac{dq_{1,t}}{q_{1,t}} = \mu_1 dt + \sigma_1 dW_{1,t} + (\Lambda_1 - 1) d\Gamma_{1,t} + (\pi_1 - 1) dI_t^{\tilde{1}} \ , \tag{4.4}$$

$$\frac{dq_{2,t}}{q_{2,t}} = \mu_2 dt + \sigma_2 dW_{2,t} + (\Lambda_2 - 1) d\Gamma_{2,t} + (\pi_2 - 1) dI_t^{\tilde{2}} \ ,$$

....

$$\frac{dq_{m,t}}{q_{m,t}} = \mu_m dt + \sigma_m dW_{m,t} + (\Lambda_m - 1) d\Gamma_{m,t} + (\pi_m - 1) dI_t^{\tilde{m}} \ ,$$

where μ_i and σ_i are constants, and $W_{i,t}$ is a one-dimensional standard Brownian motion under the original probability measure P. The correlation coefficient between $W_{i,t}$ and $W_{v,t}$ is $corr(dW_{v,t}, dW_{i,t}) = \rho_{v,i}$. Both $\Gamma_{i,t}$ and $I_t^{\tilde{i}}$ are independent Poisson-jump processes with the intensities of $\lambda_{\Gamma_{i,t}}$ and $\lambda_{I_t^{\tilde{i}}}$, respectively, and are driven by different risks at time t. Furthermore, $dI_t^{\tilde{i}}$ is independent of $d\Gamma_{i,t}$, and $\pi_i - 1$ is the random variable percentage in the mortality index of the i^{th} country that results from common jumps of deaths in other countries. We assume that the natural logarithm of π_i, the jump amplitude driven by deaths in other countries, follows a normal distribution with a mean of u_{π_i} and a variance of $\sigma^2_{\pi_i}$, which also can be denoted as $\ln \pi_i \sim N(u_{\pi_i}, \sigma^2_{\pi_i})$, $\pi_i > 0$, and $i = 1, 2, 3, ..., m$. By contrast, $\Lambda_i - 1$ refers to the percentage in the mortality index of the i^{th} country resulting from specific jumps in deaths of the i^{th} country, and the specific jump size distributes a normality, namely $\ln \Lambda_i \sim N(u_{\Lambda_i}, \sigma^2_{\Lambda_i})$, $\Lambda_i > 0$, and $i = 1, 2, 3, ..., m$. Finally, π_i is independent of Λ_i.

From Equation (4), $\ln \pi_i$ can denote the impact magnitude of infectious mortality of the i^{th} country driven by deaths in other countries. When the threshold (α_i) is infinite, mortality rates do not exert any infectious effects. Thus, $I_t^{\tilde{i}}$ equals 0 if α_i is infinite in Equation (4.2). This model can be reduced to the morality model introduced by Lin

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and Cox (2008).

4.2 Structure of a Mortality-Linked Bond with Coupons

This section examines the effect of infectious mortality risk on two types of mortality bonds: a floating-coupon mortality bond, which is similar to the Swiss Re mortality bond,[4] and a fixed-coupon mortality bond. For this comparison, we assume that both fixed-coupon and floating-coupon mortality bonds were issued in 2003 and matured on January 1, 2007, with the same principle; that is, both are 3-year bonds. Fixed-coupon bonds pay a fixed annual coupon, denoted as C, and floating-coupon mortality bonds pay an annual coupon linked to a stochastic spot interest rate (r_t) plus a constant proposition (β). The principal (F) underlying these two types of mortality-linked bonds is exposed to mortality risk, which is linked to the mortality index. Similarly, the Swiss Re bond is based on the average annual population mortality rates in the United States, the United Kingdom, France, Italy, and Switzerland. If this index exceeds 130% of the actual 2002 level, investors have a reduced principal payment at maturity. Let B_T denote the principal payment at maturity time T, expressed as

$$B_T = \mathrm{Max}(1-\mathrm{Loss},\ 0), \tag{4.5}$$

with $\mathrm{Loss} = \dfrac{\mathrm{Max}(Y_{Max} - 1.3\,Y_{t_0^-},\ 0) - \mathrm{Max}(Y_{Max} - 1.5\,Y_{t_0^-},\ 0)}{0.2Y_{t_0^-}}$, $Y_{Max} = \mathrm{Max}(Y_{t_1}, Y_{t_2}, Y_{t_3})$

and $Y_{t_i} = (q_{1,t_i}^{a_1} q_{2,t_i}^{a_2} ......q_{5,t_i}^{a_5})^{\frac{1}{a_1+a_2+...+a_5}}$;

where $Y_{t_0^-}$, Y_{t_0}, Y_{t_1}, Y_{t_2}, and Y_{t_3} represent the geometric average population mortality rates of the focal countries in 2002, 2003, 2004, 2005, and 2006, respectively. Furthermore, $q_{1,t}$, $q_{2,t}$, …., and $q_{5,t}$ represent the mortality indices of the United States, the United Kingdom, France, Italy, and Switzerland, respectively, and

---

[4] the coupon rate is LIBOR+135 bps.

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$a_1, a_2, ..., a_4$ and a_5 indicate the weights of their population mortality indices, respectively.

At time 0, the expected cash flow of fixed-coupon mortality bonds for investors is

$$\begin{aligned} B_0 &= F \times e^{-r(t_3-t_0)} E^Q\left[B_T\right] + C\left[e^{-r(t_1-t_0)} + e^{-r(t_2-t_0)} + e^{-r(t_3-t_0)}\right] \\ &= F \times e^{-r(t_3-t_0)} E^Q\left[\text{Max}\left(1 - \frac{\text{Max}(Y_{Max} - K_1, 0) - \text{Max}(Y_{Max} - K_2, 0)}{K_2 - K_1}, 0\right)\right] \\ &\quad + C\left[e^{-r(t_1-t_0)} + e^{-r(t_2-t_0)} + e^{-r(t_3-t_0)}\right], \end{aligned} \tag{4.6}$$

where $E^Q(.)$ denotes the expectation value under the risk-neutral probability measure Q at time t_0, r is the constant risk-free rate, and $K_1 = 1.3Y_{t_0^-}$ and $K_2 = 1.5Y_{t_0^-}$, with $K_2 > K_1$. We provide a general valuation formula for a mortality bond with K_1 and K_2, which can be structured to reflect different payoffs for the mortality bond. However, investors must pay the face value if the mortality bonds are issued at par. Hence, $B_0 = F$, and we can obtain the fair spread (C), which also can be denoted as

$$C = \frac{F - F \times e^{-r(t_3-t_0)} E^Q\left[\text{Max}\left(1 - \frac{\text{Max}(Y_{Max} - K_1, 0) - \text{Max}(Y_{Max} - K_2, 0)}{K_2 - K_1}, 0\right)\right]}{e^{-r(t_1-t_0)} + e^{-r(t_2-t_0)} + e^{-r(t_3-t_0)}}. \tag{4.7}$$

Alternatively, at time 0, the expected income of floating-coupon mortality bonds linked to a stochastic interest rate under a forward, risk-neutral probability measure, PT, is as follows:

DOI:10.6814/DIS.NCCU.RMI.001.2018.F08

$$
\begin{aligned}
SW_{t_0,t_1}(t_0) = {} & P(t_0,t_1)E_0^{PT}\left(r_{t_1}+\beta\right) \\
& +P(t_0,t_1)E_0^{PT}P(t_1,t_2)\left(r_{t_2}+\beta\right) \\
& +P(t_0,t_1)E_0^{PT}P(t_1,t_2)P(t_2,t_3)\left(r_{t_3}+\beta\right) \\
& +P(t_0,t_1)E_0^{PT}P(t_1,t_2)P(t_2,t_3)\cdots P(t_{n-1},t_n)\left(B_{t_n}+r_{t_n}+\beta\right) \qquad (4.8)\\
= {} & P(t_0,t_1)E_0^{PT}\left(r_{t_1}+\beta\right)+P(t_0,t_1)E_0^{PT}\frac{P(t_0,t_2)}{P(t_0,t_1)}\left(r_{t_2}+\beta\right) \\
& +P(t_0,t_1)E_0^{PT}\frac{P(t_0,t_3)}{P(t_0,t_1)}\left(r_{t_3}+\beta\right) \\
& +P(t_0,t_1)E_0^{PT}\left[\frac{P(t_0,t_3)}{P(t_0,t_1)}\left(B_{t_3}+r_{t_3}+\beta\right)\right].
\end{aligned}
$$

When the bonds are issued at par (i.e., $SW_{t_0,t_1}(t_0)=F$), we obtain the fair spread (β).

Thus, the fair spread of floating-coupon bonds is

$$
\beta = \frac{1-P(t_0,t_1)E_0^{PT}r_{t_1}-P(t_0,t_2)E_0^{PT}r_{t_2}-P(t_0,t_3)E_0^{PT}(B_{t_3}+r_{t_3})}{P(t_0,t_1)+P(t_0,t_2)+P(t_0,t_3)}. \qquad (4.9)
$$

4.3 Valuation Formula for a Mortality-Linked Bond with Coupons

For pricing a fixed-coupon mortality bond, we apply the Wang transform to solve Equation (4.7). We denote the total risks at time t in the ith country as $X_{i,t} \triangleq \Gamma_{i,t}+I_t^{\tilde{i}}$, which follows a Poisson-jump process with the intensity of $\lambda_{X_{i,t}}$. Assume that x_i-1 is the percentage of the mortality index of the i^{th} country resulting from total risks, and x_i follows normal distributions with a mean of u_{x_i} and a variance of $\sigma_{x_i}^2$. Additionally,

$$
(x_i-1)dX_{i,t} \overset{d}{=} (\Lambda_i-1)\,d\Gamma_{i,t}+(\pi_i-1)\,dI_t^{\tilde{i}}. \qquad (4.10)
$$

Thus, we obtain

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$$E\left[(x_i-1)dX_{i,t}\right]=E\left[(\Lambda_i-1)\,d\Gamma_{i,t}+(\pi_i-1)\,d\tilde{I}_t^i\right], \qquad (4.11)$$

and

$$\mathrm{Var}\left[(x_i-1)dX_{i,t}\right]=\mathrm{Var}\left[(\Lambda_i-1)\,d\Gamma_{i,t}+(\pi_i-1)\,d\tilde{I}_t^i\right]. \qquad (4.12)$$

Using Equations (4.11) and (4.12), we can then obtain

$$E\left[x_i-1\right]=\frac{(e^{u_{\pi_i}+\frac{1}{2}\sigma_{\pi_i}^2}-1)\lambda_{\tilde{I}_t^i}+(e^{u_{\Lambda_i}+\frac{1}{2}\sigma_{\Lambda_i}^2}-1)\lambda_{\Gamma_i}}{\lambda_{\Gamma_i}+\lambda_{\tilde{I}_t^i}},$$

$$\Rightarrow u_{x_i}=\frac{(e^{u_{\pi_i}+\frac{1}{2}\sigma_{\pi_i}^2}-1)\lambda_{\tilde{I}_t^i}+(e^{u_{\Lambda_i}+\frac{1}{2}\sigma_{\Lambda_i}^2}-1)\lambda_{\Gamma_i}}{\lambda_{\Gamma_i}+\lambda_{\tilde{I}_t^i}}+1, \qquad (4.13)$$

$$\sigma_{x_i}^2=\frac{A+\left[(e^{u_{\pi_i}+\frac{1}{2}\sigma_{\pi_i}^2}-1)\lambda_{\tilde{I}_t^i}+(e^{u_{\Lambda_i}+\frac{1}{2}\sigma_{\Lambda_i}^2}-1)\lambda_{\Gamma_i}\right]^2}{(\lambda_{\Gamma_i}+\lambda_{\tilde{I}_t^i}+1)(\lambda_{\Gamma_i}+\lambda_{\tilde{I}_t^i})}-\left[\frac{(e^{u_{\pi_i}+\frac{1}{2}\sigma_{\pi_i}^2}-1)\lambda_{\tilde{I}_t^i}+(e^{u_{\Lambda_i}+\frac{1}{2}\sigma_{\Lambda_i}^2}-1)\lambda_{\Gamma_i}}{\lambda_{\Gamma_i}+\lambda_{\tilde{I}_t^i}}\right]^2, \qquad (4.14)$$

with

$$A=\left[e^{2u_{\Lambda_i}+\sigma_{\Lambda_i}^2}(e^{\sigma_{\Lambda_i}}-2e^{-u_{\Lambda_i}-\frac{1}{2}\sigma_{\Lambda_i}^2}+2)+1\right](\lambda_{\Gamma_i}+\lambda_{\Gamma_i}^2)-\lambda_{\Gamma_i}^2\left(e^{u_{\Lambda_i}+\frac{1}{2}\sigma_{\Lambda_i}^2}-1\right)^2$$
$$+\left[e^{u_{\pi_i}+\sigma_{\pi_i}^2}(e^{\sigma_{\pi_i}}+1)+\left(e^{u_{\pi_i}+\frac{1}{2}\sigma_{\pi_i}^2}-1\right)^2\right]\left[\lambda_{\tilde{I}_t^i}+\lambda_{\tilde{I}_t^i}{}^2\right]-\left[(e^{u_{\pi_i}+\frac{1}{2}\sigma_{\pi_i}^2}-1)\lambda_{\tilde{I}_t^i}\right]^2.$$

Under the original probability measure P, using Ito's lemma, Equation (4.2) can be rewritten as $q_{i,T}=q_{i,t_0}\,e^{(\mu_i-\frac{1}{2}\sigma_i^2)(T-t_0)+\sigma_i W_{i,T-t_0}}\prod_{l=1}^{X_{i,T}}x_{i,l}$, i=1,2,.....,5. The numbers for i indicate the United States, the United Kingdom, France, Italy, and Switzerland, respectively. Moreover,

$$\ln q_{i,T}=\ln q_{i,t_0}+(\mu_i-\frac{1}{2}\sigma_i^2)(T-t_0)+\sigma_i W_{i,T-t_0}+\sum_{l=1}^{X_{i,T}}\ln x_{i,l}. \qquad (4.15)$$

Next, let X_t represent the sum of the total risks for the United States, the United

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Kingdom, France, Italy, and Switzerland, namely $X_t \triangleq X_{1,t} + X_{2,t} + ... + X_{5,t}$, which follows a Poisson distribution with the intensity of λ_t and $\lambda_t = \sum_{i=1}^{5} (\lambda_{I_t^i} + \lambda_{\Gamma_{i,t}})$. To derive the closed-form solution of the fair price of the mortality bond, we rewrite Equation (4.7) as

$$C = \frac{F - F \times e^{-r(t_3 - t_0)} \left\{ \begin{array}{l} E^Q\left[(1-S_T)1_{\{S_T<1\}} \middle| Y_{Max} = Y_{t_1}\right] P_r^Q(Y_{Max} = Y_{t_1}) + E^Q\left[(1-S_T)1_{\{S_T<1\}} \middle| Y_{Max} = Y_{t_2}\right] P_r^Q(Y_{Max} = Y_{t_2}) \\ + E^Q\left[(1-S_T)1_{\{S_T<1\}} \middle| Y_{Max} = Y_{t_3}\right] P_r^Q(Y_{Max} = Y_{t_3}) \end{array} \right\}}{e^{-r(t_1 - t_0)} + e^{-r(t_2 - t_0)} + e^{-r(t_3 - t_0)}}. \tag{4.16}$$

The Wang transform is used to obtain the closed-form solution. Using Equation (4.16), we must solve the probability that $Y_{Max} = Y_{t_i}$ under the risk-neutral probability measure. Therefore, Propositions 1, 2, and 3 are necessary.

Applying Propositions 1, 2, and 3, we determine that the fair spread of fixed-coupon bonds is

$$C = \frac{F - F \times e^{-r(t_3 - t_0)} \left\{ \begin{array}{l} E^Q\left[(1-S_T)1_{\{S_T<1\}} \middle| Y_{Max} = Y_{t_1}\right] P_r^Q(Y_{Max} = Y_{t_1}) \\ + E^Q\left[(1-S_T)1_{\{S_T<1\}} \middle| Y_{Max} = Y_{t_2}\right] P_r^Q(Y_{Max} = Y_{t_2}) \\ + E^Q\left[(1-S_T)1_{\{S_T<1\}} \middle| Y_{Max} = Y_{t_3}\right] P_r^Q(Y_{Max} = Y_{t_3}) \end{array} \right\}}{e^{-r(t_1 - t_0)} + e^{-r(t_2 - t_0)} + e^{-r(t_3 - t_0)}}, \tag{4.17}$$

in which $E^Q\left[1_{\{S_T<1\}} \middle| Y_{Max} = Y_{t_i}\right] = \Phi(\Phi^{-1}(1 - F_{Y_{t_i}}(K_1)) + \theta_i)$;

$$\begin{aligned} & E^Q\left[S_T 1_{\{S_T<1\}} \middle| Y_{Max} = Y_{t_i}\right] \\ & = \left[1 - \Phi(\Phi^{-1}(F_{Y_{t_i}}(K_1)) + \theta_i)\right] \times \\ & \left\{ \begin{array}{l} \Phi(\Phi^{-1}(F_{Y_{t_i}}(K_2)) + \theta_i) + \\ \left[\Phi(\Phi^{-1}(F_{Y_{t_i}}(K_2)) + \theta_i) - \Phi(\Phi^{-1}(F_{Y_{t_i}}(K_1)) + \theta_i)\right] \times \\ \left\{ \dfrac{1}{K_2 - K_1} \left[\sum_{s=0}^{\infty} \dfrac{e^{-\lambda_t}(\lambda_t)^s}{s!} Y_{t_0} e^{\left(\mu_y + \frac{1}{2}\sigma_y^2\right)(t_i - t_0) + s\,(au_z + \frac{1}{2}a^2\sigma_z^2)} + \theta_i \sqrt{Var^P(Y_{t_i})} \right] - \dfrac{K_1}{K_2 - K_1} \right\} \end{array} \right\}, \end{aligned}$$

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$$F_{Y_{t_i}}(K_1)=P_r^P(Y_{t_i}\le K_1)=\sum_{s=0}^{\infty}\frac{e^{-\lambda_t}(\lambda_t)^s}{s!}\Phi\left(\frac{\ln\frac{K_1}{Y_{t_0}}-\mu_y(t_i-t_0)-asu_z}{\sqrt{\sigma_y^2(t_i-t_0)+s\,a^2\sigma_z^2}}\right),$$

$$F_{Y_{t_i}}(K_2)=P_r^P(Y_{t_i}\le K_2)=\sum_{s=0}^{\infty}\frac{e^{-\lambda_t}(\lambda_t)^s}{s!}\Phi\left(\frac{\ln\frac{K_2}{Y_{t_0}}-\mu_y(t_i-t_0)-asu_z}{\sqrt{\sigma_y^2(t_i-t_0)+s\,a^2\sigma_z^2}}\right), \text{ and}$$

$$\begin{aligned}Var^P(Y_{t_i})=&\sum_{s=0}^{\infty}\frac{e^{-\lambda_t}(\lambda_t)^s}{s!}Y_{t_0}^2e^{2\mu_y(t_i-t_0)+2s\,au_z+2\,\sigma_y^2\,a^2(t_i-t_0)}\\&-\left\{\sum_{s=0}^{\infty}\frac{e^{-\lambda_t}(\lambda_t)^s}{s!}Y_{t_0}e^{2\mu_y(t_i-t_0)+2s(au_z+a^2\sigma_z^2)+\frac{1}{2}a^2\sigma_y^2(t_i-t_0)}\right\}^2, \text{ i=1, 2, 3.}\end{aligned}$$

Furthermore, θ_i refers to the risk premiums of Y_{t_i};

$$\mu_y=\frac{1}{a_1+a_2+...+a_5}\sum_{i=1}^{5}a_i(\mu_i-\frac{1}{2}\sigma_i^2);\ u_z=\frac{\sum_{i=1}^{5}s_ia_iu_{x_i}}{s};\ \sigma_z^2=\frac{\sum_{i=1}^{5}s_ia_i^2\sigma_{x_i}^2}{s};$$

$$\lambda_t=\sum_{i=1}^{5}(\lambda_{I_t^i}+\lambda_{\Gamma_{i,t}});\ a=\frac{1}{a_1+a_2+...+a_5};\text{ and}$$

$$\sigma_y=\frac{1}{a_1+a_2+...+a_5}\sqrt{\begin{bmatrix}a_1\sigma_1 & a_2\sigma_2 & a_3\sigma_3 & a_4\sigma_4 & a_5\sigma_5\end{bmatrix}\begin{pmatrix}1 & \cdots & \rho_{15}\\ \vdots & \ddots & \vdots\\ \rho_{51} & \cdots & 1\end{pmatrix}\begin{bmatrix}a_1\sigma_1 & a_2\sigma_2 & a_3\sigma_3 & a_4\sigma_4 & a_5\sigma_5\end{bmatrix}'}.$$

4.4 Valuation for Floating-Coupon Mortality-Linked Bonds

Assume that the coupons of floating-coupon mortality bonds are linked to a stochastic interest rate, and the dynamic process of the stochastic interest rate follows the CIR model under an original probability measure, as follows:

$$dr_t=k(g-r_t)\,dt+\sigma_r\,dW_t, \tag{4.18}$$

where k, g, and σ_r are constants, and W_t is a wiener process with a mean of 0 and a variance of t under an original probability measure. Thus, at time t of a zero-coupon bond with maturity, T is

$$P(t,\ T)=A(t,T)\,e^{-B(t,T)\,r_t}, \tag{4.19}$$

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where $A(t, T)=\left[\dfrac{2h\ e^{[(k+h)\ (T-t)]/2}}{2h+(k+h)\left(e^{\frac{T-t}{h}}-1\right)}\right]^{2k\theta/\sigma^2}$, $B(t, T)=\dfrac{2\left(e^{[(k+h)\ (T-t)]/2}-1\right)}{2h+(k+h)\left(e^{\frac{T-t}{h}}-1\right)}$, and

$h=\sqrt{k^2+2\sigma^2}$.

From Equations (4.18) and (4.19), using Girsanov's theorem, we can obtain the dynamic process of the stochastic interest rate under a forward risk-neutral probability measure, PT, as follows:

$$dr_t=\left[kg-\left(k+B(t, T)\sigma_r^2\right)r_t\right]dt+\sigma_r\sqrt{r_t}\ dW_t^{PT}. \qquad (4.20)$$

For simplicity, we employ a Monte Carlo simulation approach to calculate the fair spread of floating-coupon mortality bonds by using Equations (4.9) and (4.20). Before performing the simulation, Equation (4.20) must be transformed into Equation (4.21) in a discrete time model. Thus,

$$\Delta r_t=\left[kg-\left(k+B(t, T)\sigma_r^2\right)r_t\right]\Delta t+\sigma_r\sqrt{r_t}\ \sqrt{\Delta t}\ \varepsilon_t, \qquad (4.21)$$

where ε_t follows a normal distribution with a mean of 0 and a volatility of 1. Accordingly, we can simulate the fair spread of floating-coupon mortality bonds by using Equations (4.9) and (4.21) over 10,000 simulation runs.

4.5 Empirical Results

In this section, we first use the mortality data from the HMD to estimate the parameters $(\mu_i, \sigma_i, u_{\Lambda_i}, \sigma_{\Lambda_i}, u_{\pi_i}, \sigma_{\pi_i})$ for the United States, the United Kingdom, France, Italy, and Switzerland in the proposed infectious mortality model. The time window is 1933–2007. With the parameter estimates, the fair price spreads of the two types of mortality linked bonds can be obtained using Equations (4.2) and (4.3). We also provide comparative statistics.

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4.5.1 Parameter Estimation and Goodness of Fit of the Infectious Mortality Model

A calibration approach is adopted to estimate the variables (μ_i, σ_i, u_{Λ_i}, σ_{Λ_i}, u_{π_i}, σ_{π_i}) for the five focal countries. Given the initial values in Table 4.2, we can obtain estimated parameters that characterize the proposed infectious mortality model for the United States, the United Kingdom, France, Italy, and Switzerland, as disclosed in Tables 4.3.

To illustrate the calibration of the parameters (k, g, σ_r) in the CIR interest rate model, we can use the interest rate of the 3-month London Interbank Offered Rate (LIBOR). This time window spans from January 1, 2013, to October, 31, 2014. The initial values are $k_0 = 0.01987$, $g_0 = 0.01523$, and $\sigma_{r_0} = 0.005$. Thus, the estimated parameters (k, g, σ_r) for the three-month LIBOR interest rate are 2.5806, 0.0023, and 0.0048, respectively.

Table 4.2 Initial Values of the Calibrated Parameters for Five Countries

	US	UK	France	Italy	Switzerland
μ_i	0.006907	0.011709	0.011921	0.011997	0.013883
σ_i	0.000438	0.001319	0.001469	0.001348	0.001363
u_{π_i}	0.001	0.001	0.001	0.001	0.001
σ_{π_i}	0.002	0.002	0.002	0.002	0.002
u_{Λ_i}	0.001	0.001	0.001	0.001	0.001
σ_{Λ_i}	0.002	0.002	0.002	0.002	0.002

Notes: The initial values of μ_i and σ_i are the mean and volatility of the mortality index for the United States, the United Kingdom, France, Italy, and Switzerland.

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Table 4.3 Parameter Estimates in Infectious Mortality Dynamics for Five Countries through Calibration

	US	UK	France	Italy	Switzerland
μ_i	-0.00654126 (0.00028)	-0.00172561 (0.00033)	-0.00217485 (0.00021)	-0.00198764 (0.00035)	-0.00134219 (0.00025)
σ_i	0.03412596 (0.00031)	0.03245149 (0.00033)	0.02469197 (0.00028)	0.03614527 (0.00021)	0.02179819 (0.00030)
u_{π_i}	-0.08798271 (0.00025)	-0.07461521 (0.00028)	-0.05165785 (0.00027)	-0.07949118 (0.00021)	-0.06143999 (0.00024)
σ_{π_i}	0.09413695 (0.00032)	0.03541679 (0.00039)	0.02914249 (0.00037)	0.01572431 (0.00031)	0.03145611 (0.00041)
u_{Λ_i}	-0.09198133 (0.00051)	-0.07984129 (0.00059)	-0.06971451 (0.00051)	-0.08191139 (0.00058)	-0.08379651 (0.00055)
σ_{Λ_i}	0.32319048 (0.00068)	0.35128811 (0.00061)	0.30149231 (0.00069)	0.25679133 (0.00058)	0.32811947 (0.00071)

Notes: The parameter estimates are derived using Equation (4.4) for $i =$ the United States, the United Kingdom, France, Italy, or Switzerland. $\theta = 0.83$, a_1=0.7, a_2=0.15, a_3=0.075, a_4=0.05, a_5=0.025, $\alpha_1 = 0.998$, $\alpha_2 = 4.893$, $\alpha_3 = 5.547$, $\alpha_4 = 5.744$, and $\alpha_5 = 62.487$. Standard errors are shown in parentheses.

4.5.2 Numerical Analysis

We first analyze the fair par spread for fixed-coupon mortality bonds according to the parameters in Table 4.3 and the valuation formula derived in Equation (4.17). Using a principal of \$1 as an example, we assume a risk premium of 0.83 for fixed-coupon bonds (Cox et al., 2006), and the base parameters are $\theta = 0.83$ and $\lambda_t = 0.05$. We then design three scenarios to discuss the impacts of the threshold value (α) on the par spreads of fixed-coupon and floating-coupon bonds. Case 1 (normal situation): given $\alpha = 16$[4], the fair par spread of fixed-coupon mortality bonds is approximately 0.7833%, whereas the fair spread of floating-coupon mortality bonds is 0.6012%, according to previous assumptions and estimated parameters (k, g, σ_r). Case 2 (low-infection situation): assuming that $\alpha = 70$, the fair par spread of fixed-coupon mortality bonds is approximately 0.9872%, whereas the fair spread of floating-coupon mortality bonds is 0.7151%. Case 3 (high-infection situation): assuming that $\alpha = 1$, the fair par spread

[4] The average threshold value is calculated as $\frac{\alpha_1 + \alpha_2 + ... + \alpha_5}{5}$, and the value is approximately 16.

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of fixed-coupon mortality bonds is approximately 0.5127%, whereas the fair spread of floating-coupon mortality bonds is 0.5819%.

The three scenarios reveal a common phenomenon that the fair spreads of these two types of bonds are higher than the 0.45%indicated by Cox et al. (2006), but lower than the 11.4% demonstrated by Tsai and Tzeng (2013). The fair spreads in our model are closer to the actual par spread of 1.35% for the Swiss Re bond. Ignoring the effects of infectious mortality rates thus significantly underestimates the par spread for mortality bonds; by contrast, considering this phenomenon provides a par spread of the mortality security that is closer to real-world values.

Assuming $\alpha = 16$, $u_z = -0.001$, and $\sigma_z = 0.1$, we numerically investigate the price of fixed-coupon mortality bonds by using the proposed infectious mortality model. The impacts of the major parameters, mean and volatility, on the magnitudes of infectious mortality, average threshold values (α), and jump intensities of the par spread of the bonds are detailed in Table 4.5. According to the literature on mortality bonds, the risk premiums are assumed to range from 1 to 2. Table 4.4 shows a common phenomenon: the fair spread of fixed-coupon mortality bonds decreases as mortality increases. In Panel A, the impacts of the mean of the magnitudes of infectious mortality on the par spread of the bonds are uncertain. However, the par spreads of the bonds decrease as the volatilities of the magnitudes of infectious mortality increase due to high mortality rates (Panel B).

Panel C illustrates the positive relationship between threshold values and the par spreads of fixed-coupon mortality bonds; the higher the threshold values, the lower the infectious mortality is. Conversely, Panel D indicates that when jump intensities increase, mortality rates increase, and that the par spread of the bonds declines. The sensitivities of the means and volatilities of the magnitudes of infectious mortality are

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greater than α and λ_t, whereas that of threshold values is the smallest.

Table 4.5 presents the sensitivity of the model parameters to floating-coupon mortality bonds. We find that the fair spreads increase as k, σ_r, or σ_z decreases; they also grow as α increases. The relationships of the fair spreads with g, u_z, or λ_t remain uncertain. Tables 4.4 and 4.5 also reveal a common phenomenon: the volatilities of the magnitudes of infectious mortality exert significant effects on the fair spreads of fixed-coupon bonds and floating-coupon bonds.

Table 4.4 Impacts of Model Parameters on Fair Spreads of Fixed-Coupon Bonds (%)

Parameter	$\theta = 0.83$	$\theta = 0.8657$	$\theta = 1.21$	$\theta = 1.5$
u_z	Panel A: u_z changes			
-0.001	0.7928	0.7919	0.7759	0.7599
-0.003	0.7816	0.7714	0.7511	0.7496
-0.005	0.7433	0.7533	0.7499	0.7411
-0.007	0.7955	0.7799	0.7633	0.7533
-0.009	0.8081	0.7854	0.7711	0.7600
σ_z	Panel B: σ_z changes			
0.1	0.7928	0.7919	0.7759	0.7599
0.2	0.7863	0.7860	0.7698	0.7580
0.3	0.7631	0.7619	0.7588	0.7499
0.4	0.7598	0.7498	0.7455	0.7396
0.5	0.7499	0.7377	0.7300	0.7277
α	Panel C: α changes			
16.0	0.7928	0.7919	0.7759	0.7599
16. 5	0.8054	0.8033	0.7865	0.7613
17.0	0.8133	0.8100	0.7900	0.7689
17.5	0.8196	0.8122	0.7936	0.7700
18.0	0.8201	0.8199	0.7999	0.7714
λ_t	Panel D: λ_t changes			
0.01	0.8254	0.8295	0.8056	0.7767
0.02	0.8190	0.8204	0.7915	0.7700
0.03	0.8144	0.8166	0.7866	0.7696
0.04	0.8016	0.8036	0.7804	0.7614
0.05	0.7928	0.7919	0.7759	0.7596

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Table 4.5 Impacts of Parameters on Fair Spreads of Floating-Coupon Bonds (%)

Parameter	Fair Spread (β)
k	Panel A: k changes
1.0	0.7964
1.5	0.7850
2.0	0.7811
2.5	0.7800
3.0	0.7746
g	Panel B: g changes
0.01	0.8011
0.02	0.8098
0.03	0.8100
0.04	0.8055
0.05	0.8016
σ_r	Panel C: σ_r changes
0.01	0.8416
0.02	0.8400
0.03	0.8376
0.04	0.8356
0.05	0.8311
u_z	Panel D: u_z changes
-0.001	0.8697
-0.03	0.8701
-0.05	0.8699
-0.07	0.8653
-0.09	0.8700
σ_z	Panel E: σ_z changes
0.1	0.8637
0.2	0.8619
0.3	0.8599
0.4	0.8536
0.5	0.8519
λ_t	Panel F: λ_t changes
0.01	0.8659
0.02	0.8675
0.03	0.8710
0.04	0.8696
0.05	0.8715
α	Panel G: α changes
16.0	0.8700
16. 5	0.8719
17.0	0.8730
17.5	0.8746
18.0	0.8766

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4.6 Conclusion

Transferring catastrophic losses using mortality-linked securities is critical to the insurance industry. Many life insurers operate their businesses internationally. According to patterns of mortality experience, we find that catastrophic events may cause the comovement of mortality rates across countries. Although researchers consider mortality rates with jumps, they explain the comovement of mortality rates by using common jumps across countries. However, mortality trends offer empirical evidence that mortality comovement may occur only if a catastrophic event causes considerable mortality in all countries. Studies rarely model the phenomenon of infectious mortality rates. To fill this gap, this study offers a new perspective of the infectious effects of mortality rates on the valuation of mortality securities. Accordingly, we propose an infectious mortality model: using the Wang transform, we derive a valuation formula for the fixed-coupon mortality bond based on our proposed infectious mortality model.

The empirical analysis reveals that the fair par spreads of fixed-coupon and floating-coupon mortality bonds in the model are far higher than those reported by Cox et al. (2006), but they are closer to the actual par spread of the Swiss Re bond. Therefore, considering the infectious effects of mortality rates enables the par spread of mortality bonds to fit real-world values, which is helpful for pricing mortality securities and for managing catastrophic mortality risk for reinsurers.

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Chapter 5 Comparison of Two Infectious Mortality Models

The infectious mortality model of the first part is named as infectious mortality model 1, and the infectious mortality model of the second part is infectious mortality model 2. The two infectious mortality models can describe the properties when multi-country mortality comovement increases significantly after mortality shocks. Table 5.1 illustrates that the mean squared errors of infectious mortality 1 in the United States, the United Kingdom, France, Italy and Switzerland are 0.00085, 0.00071, 0.00075, 0.00091 and 0.00058, respectively. In average, the mean squared errors are 0.00076 for all five countries. The mean squared errors of infectious mortality 2 in the United States, the United Kingdom, France, Italy and Switzerland are 0.00125, 0.00254, 0.00291, 0.00312 and 0.00113, respectively. In average, the mean squared errors are 0.00219 for all five countries. Obviously, the mean squared errors of the first infectious mortality model are fewer than the second infectious mortality model, whereas the second infectious mortality model can confirm the ratio of the total deaths in all countries except for those in the i^{th} country relative to the total deaths in all countries to be between 0 and 1 when all countries are only two countries.

Therefore, the two infectious mortality models have their advantages and weakness. When the payoffs of mortality-linked bonds are related to only two countries'

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mortalities, the infectious mortality model 2 is more suitable than the infectious mortality model 1. However, the means square errors of the infectious mortality model 1 is fewer than the infectious mortality model 2 as the payoffs are linked to the mortalities more than two countries.

Table 5.1 Mean Squared Errors of Two Infectious Mortality Models

Panel A: Infectious Mortality Model 1					
	US	UK	France	Italy	Switzerland
MSE	0.00085	0.00071	0.00075	0.00091	0.00058
$\overline{\text{MSE}}$	0.00076				
Panel B: Infectious Mortality Model 2					
	US	UK	France	Italy	Switzerland
MSE	0.00125	0.00254	0.00291	0.00312	0.00113
$\overline{\text{MSE}}$	0.00219				

Notes: MSE stands for mean squared errors. $\overline{\text{MSE}}$ expresses the average values of mean squared errors of US., UK., France, Italy and Switzerland.

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Chapter 6 Conclusion

In this dissertation, two infectious mortality models are developed to show that jumps occur only when there is a catastrophic event that causes considerable mortality, such as the 1918 flu pandemic. Furthermore, the models are applied to price mortality-linked securities such as the Swiss Re bond without coupons, fixed-coupon and floating-coupon mortality bonds. These models can be reduced to that introduced by Lin and Cox (2008) as model parameters are specially set. We find that the mean squared errors of the first infectious mortality model are fewer than the second infectious mortality model, whereas the second infectious mortality model can confirm the ratio of the total deaths in all countries except for those in the i^{th} country relative to the total deaths in all countries to be between 0 and 1 when all countries are only two countries. Additionally, closed-form solutions are derived for the fair prices of the Swiss Re bond without coupons and fixed-coupon mortality bonds.

The empirical results show that the fair spreads of these two types of bonds are higher than the 0.45% indicated by Cox et al. (2006) and closer to the actual par spread of 1.35% for the Swiss Re bond in infection situations. We find that ignoring the effects of infectious mortality rates significantly underestimates the par spread for mortality bonds, whereas considering this phenomenon provides a par spread of the mortality security that is closer to real-world values. This may enable insurance institutions to

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hedge infectious mortality risk.

Appendix A

Because $\Gamma_{i,t}$ and $\tilde{I}_t^i$ are independent, we have for $k \geq 0$:

$$\begin{aligned}
P(\Gamma_{i,t} + \tilde{I}_t^i = k) &= \sum_{j=0}^{k} P(\Gamma_{i,t} + \tilde{I}_t^i = k,\ \Gamma_{i,t} = j) \\
&= \sum_{j=0}^{k} P(\tilde{I}_t^i = k - j,\ \Gamma_{i,t} = j) \\
&= \sum_{j=0}^{k} P(\tilde{I}_t^i = k - j) P(\Gamma_{i,t} = j) \\
&= \sum_{j=0}^{k} e^{-\lambda_{\tilde{I}_t^i}} \frac{\left(\lambda_{\tilde{I}_t^i}\right)^{k-j}}{(k-j)!} e^{-\lambda_{\Gamma_{i,t}}} \frac{\left(\lambda_{\Gamma_{i,t}}\right)^{j}}{j!} \\
&= e^{-(\lambda_{\tilde{I}_t^i} + \lambda_{\Gamma_{i,t}})} \frac{1}{k!} \sum_{j=0}^{k} \binom{k}{j} \left(\lambda_{\tilde{I}_t^i}\right)^{k-j} \left(\lambda_{\Gamma_{i,t}}\right)^{j} \\
&= e^{-(\lambda_{\tilde{I}_t^i} + \lambda_{\Gamma_{i,t}})} \times \frac{(\lambda_{\tilde{I}_t^i} + \lambda_{\Gamma_{i,t}})^k}{k!}
\end{aligned}$$

Hence, $\Gamma_{i,t} + \tilde{I}_t^i \sim \text{Poisson}(\lambda_{\tilde{I}_t^i} + \lambda_{\Gamma_{i,t}})$, that is $X_{i,t} \sim \text{Poisson}(\lambda_{\tilde{I}_t^i} + \lambda_{\Gamma_{i,t}})$.

Appendix B

From Equation (3.13) and $Y_{t_i} = (q_{1,t_i}^{a_1} q_{2,t_i}^{a_2} \ldots\ldots q_{5,t_i}^{a_5})^{\frac{1}{a_1+a_2+\ldots+a_5}}$, we know

$$\begin{aligned}
\ln Y_T = \ln Y_{t_0} &+ \frac{1}{a_1+a_2+\ldots+a_5}\left[\sum_{i=1}^{5} a_i(\mu_i - \frac{1}{2}\sigma_i^2)\right] + \frac{1}{a_1+a_2+\ldots+a_5}\left[\sum_{i=1}^{5} a_i \sigma_i W_{i,T-t_0}\right] \\
&+ \frac{1}{a_1+a_2+\ldots+a_5}\left[a_1 \sum_{l=1}^{X_{1,T}} \ln x_{1,l} + a_2 \sum_{l=1}^{X_{2,T}} \ln x_{2,l} + \ldots + a_5 \sum_{l=1}^{X_{5,T}} \ln x_{5,l}\right]
\end{aligned}, \quad \text{(B1)}$$

i=1, 2,…5. Then let $\mu_y \triangleq \frac{1}{a_1+a_2+\ldots+a_5} \sum_{i=1}^{5} a_i(\mu_i - \frac{1}{2}\sigma_i^2)$;

$$\sigma_y \triangleq \frac{1}{a_1+a_2+\ldots+a_5} \sqrt{\left[a_1\sigma_1\ \ a_2\sigma_2\ \ a_3\sigma_3\ \ a_4\sigma_4\ \ a_5\sigma_5\right] \begin{pmatrix} 1 & \cdots & \rho_{15} \\ \vdots & \ddots & \vdots \\ \rho_{51} & \cdots & 1 \end{pmatrix} \left[a_1\sigma_1\ \ a_2\sigma_2\ \ a_3\sigma_3\ \ a_4\sigma_4\ \ a_5\sigma_5\right]'};$$

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and $\sum_{l=1}^{X_T} \ln Z_l \triangleq a_1 \sum_{l=1}^{X_{1,T}} \ln x_{1,l} + a_2 \sum_{l=1}^{X_{2,T}} \ln x_{2,l} + \ldots + a_5 \sum_{l=1}^{X_{5,T}} \ln x_{5,l}$, $\ln Z \sim N(u_z, \sigma_z^2)$.

Thus, Proposition 1 is completed.

Appendix C

From the Wang (2000) transform, we know the relationship between the risk-neutral probability and original probability in Equation (B1).

$$F^*(x) = \Phi(\Phi^{-1}(F(x)) + \theta). \tag{C1}$$

where $F^*(x)$ and $F(x)$ are cumulative density functions under the risk-adjusted and original probability measures, respectively; θ is a constant risk premium; and $\Phi(.)$ denotes the cumulative standard normal probability. Suppose $Y_{Max} = Y_{t_1}$. From Equation (B1), the probability of $Y_{Max} = Y_{t_1}$ under the risk-neutral measure Q can be derived as

$$P_r^Q(Y_{Max} = Y_{t_1}) = \Phi(\Phi^{-1}(P_r^p(Y_{t_1} > Y_{t_2}, Y_{t_1} > Y_{t_3})) + \theta_1), \tag{C2}$$

in which θ_1 is the risk premium of Y_{t_1}, and $P_r^p(.)$ denotes the original probability measure. Through Proposition 1, Equation (B3) also can be obtained:

$$Y_T = Y_{t_0} e^{\mu_y (T - t_0) + \sigma_y W_{T-t_0} + \frac{1}{a_1 + a_2 + \ldots + a_5} \sum_{l=1}^{X_T} \ln Z_l}. \tag{C3}$$

Substituting Equation (B3) into (B2), we find:

$$\begin{aligned} P_r^Q(Y_{Max} = Y_{t_1}) &= \Phi(\Phi^{-1}(P_r^p(Y_{t_1} > Y_{t_2}, Y_{t_1} > Y_{t_3})) + \theta_1) \\ &= \Phi(\Phi^{-1}(\Phi(d_1, d_2, \rho_{1,2})) + \theta_1), \end{aligned} \tag{C4}$$

with

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$$d_1=\frac{\mu_y(t_2-t_1)-a\,u_z s}{\sqrt{\sigma_y|t_2-t_1|+a^2\sigma_z^2 s}};\ d_2=\frac{\mu_y(t_3-t_1)-a\,u_z s}{\sqrt{\sigma_y|t_3-t_1|+a^2\sigma_z^2 s}};\rho_{1,2}=\mathrm{corr}(\frac{W_{t_2-t_1}}{\sqrt{|t_2-t_1|}},\frac{W_{t_3-t_1}}{\sqrt{|t_3-t_1|}}).$$

In the same vein, we can obtain:

$$P_r^Q(Y_{max}=Y_{t_i})=\Phi(\Phi^{-1}(P_r^p(Y_{max}=Y_{t_i}))+\theta_i)=\Phi(\Phi^{-1}(\Phi(d_{2i-1},d_{2i},\rho_{2i-1,2i}))+\theta_i), \quad \text{(C5)}$$

$i=1,2,3$. Here, θ_i is the risk premium of Y_{t_i}, and

$$d_1=\frac{\mu_y(t_2-t_1)-a\,u_z s}{\sqrt{\sigma_y|t_2-t_1|+a^2\sigma_z^2 s}};\ d_2=\frac{\mu_y(t_3-t_1)-a\,u_z s}{\sqrt{\sigma_y|t_3-t_1|+a^2\sigma_z^2 s}};\rho_{1,2}=\mathrm{corr}(\frac{W_{t_2-t_1}}{\sqrt{|t_2-t_1|}},\frac{W_{t_3-t_1}}{\sqrt{|t_3-t_1|}});$$

$$d_3=\frac{\mu_y(t_2-t_1)-a\,u_z s}{\sqrt{\sigma_y|t_1-t_2|+a^2\sigma_z^2 s}};\ d_4=\frac{-\mu_y(t_3-t_2)-a\,u_z s}{\sqrt{\sigma_y|t_3-t_2|+a^2\sigma_z^2 s}};\rho_{3,4}=\mathrm{corr}(\frac{W_{t_2-t_1}}{\sqrt{|t_2-t_1|}},\frac{W_{t_3-t_2}}{\sqrt{|t_3-t_2|}});$$

$$d_5=\frac{\mu_y(t_3-t_1)-a\,u_z s}{\sigma_y\sqrt{|t_3-t_1|+a^2\sigma_z^2 s}};\ d_6=\frac{\mu_y(t_3-t_2)-a\,u_z s}{\sqrt{\sigma_y|t_3-t_2|+a^2\sigma_z^2 s}};\rho_{5,6}=\mathrm{corr}(\frac{W_{t_3-t_1}}{\sqrt{|t_3-t_1|}},\frac{W_{t_3-t_2}}{\sqrt{|t_3-t_2|}}).$$

Appendix D

Suppose $Y_{Max}=Y_{t_1}$. We can obtain Equation (D1):

$$E^Q\left[1_{\{S_T<1\}}\middle|Y_{Max}=Y_{t_1}\right]=P_r^Q(\frac{(Y_{t_1}-K_1)1_{\{Y_{t_1}>K_1\}}-(Y_{t_1}-K_2)1_{\{Y_{t_1}>K_2\}}}{K_2-K_1}<1) \quad \text{(D1)}$$

Then let $H=(Y_{t_1}-K_1)1_{\{Y_{t_1}>K_1\}}-(Y_{t_1}-K_2)1_{\{Y_{t_1}>K_2\}}$, such that we rewrite Equation (D1)

as

$$\begin{aligned}E^Q\left[1_{\{S_T<1\}}\middle|Y_{Max}=Y_{t_1}\right]=&P_r^Q(H<K_2-K_1,\ Y_{t_1}<K_1)+P_r^Q(H<K_2-K_1,\ K_1\le Y_{t_1}\le K_2)\\&+P_r^Q(H<K_2-K_1,\ Y_{t_1}>K_2).\end{aligned} \quad \text{(D2)}$$

From Propositions 1 and 2, we next derive:

$$E^Q\left[1_{\{S_T<1\}}\middle|Y_{Max}=Y_{t_1}\right]=1-\Phi(\Phi^{-1}(F_{Y_{t_1}}(K_1))+\theta_1), \quad \text{(D3)}$$

where $F_{Y_{t_1}}(K_1)=P_r^P(Y_{t_1}\le K_1)=\sum_{s=0}^{\infty}\frac{e^{-\lambda_t}(\lambda_t)^s}{s!}\Phi(\frac{\ln\frac{K_1}{Y_{t_0}}-\mu_y(t_1-t_0)-sau_z}{\sqrt{\sigma_y^2(t_1-t_0)+sa^2\sigma_z^2}});\theta_1$ is the risk

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premiums of Y_{t_1} ; $\mu_y = \frac{1}{a_1+a_2+...+a_5}\sum_{i=1}^{5} a_i(\mu_i - \frac{1}{2}\sigma_i^2)$; $u_z = \frac{\sum_{i=1}^{5} s_i a_i u_{x_i}}{s}$; $\sigma_z^2 = \frac{\sum_{i=1}^{5} s_i a_i^2 \sigma_{x_i}^2}{s}$;

$\lambda_t = \sum_{i=1}^{5}(\lambda_{\tilde{I}_t^i} + \lambda_{\Gamma_{i,t}})$;

$$\sigma_y = \frac{1}{a_1+a_2+...+a_5}\sqrt{\left[a_1\sigma_1 \;\; a_2\sigma_2 \;\; a_3\sigma_3 \;\; a_4\sigma_4 \;\; a_5\sigma_5\right]\begin{pmatrix} 1 & \cdots & \rho_{15} \\ \vdots & \ddots & \vdots \\ \rho_{51} & \cdots & 1 \end{pmatrix}\left[a_1\sigma_1 \;\; a_2\sigma_2 \;\; a_3\sigma_3 \;\; a_4\sigma_4 \;\; a_5\sigma_5\right]'}.$$

For the same reason, if $Y_{Max} = Y_{t_i}$, then

$$E^Q\left[1_{\{S_T<1\}}\middle| Y_{Max} = Y_{t_i}\right] = \Phi(\Phi^{-1}(1 - F_{Y_{t_i}}(K_1)) + \theta_i), \tag{D4}$$

with $F_{Y_{t_i}}(K_1) = P_r^P(Y_{t_i} \le K_1) = \sum_{s=0}^{\infty}\frac{e^{-\lambda_t}(\lambda_t)^s}{s!}\Phi\left(\frac{\ln\frac{K_1}{Y_{t_0}} - \mu_y(t_i - t_0) - su_z a}{\sqrt{\sigma_y^2(t_i - t_0) + s\sigma_z^2 a^2}}\right)$, i=1, 2, 3; θ_i

as the risk premium of Y_{t_i} and the other variables as we detailed them previously.

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