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Optimal Longevity Hedging Framework for Insurance Companies Considering Basis and Mispricing Risks

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Abstract

This article studies the optimal hedging strategy to deal with longevity risk for the life insurer considering basis risk. We build up a longevity hedging framework that incorporates not only the internal natural hedging but also the external hedging by using the q-forwards. The optimal hedging strategy is obtained by a minimizing-variance approach that can minimize the impact of longevity risk on the insurer's profit function. To investigate the basis risk, instead of using population mortality, we adopt a unique mortality data set of annuity and life insurance policies that enable us to calibrate the multi-population mortality dynamics for different lines of insurance policies. We consider three different hedging strategies: the natural hedging strategy, the external hedging strategy, and combining both natural hedging, and external hedging strategies. The hedge effectiveness for different hedging strategies is evaluated. In addition, the mortality forecast model based on VECM and ARIMA are used to examine the impact of basis risk on hedge effectiveness. As a result, combining both internal and external hedging strategies is the most effective way to manage longevity risk. Ignoring the basis risk will decrease the hedge effectiveness.

Introduction

The recent increase in longevity has increased pressures on defined benefit (DB) pension plan providers and annuity providers. Longevity risk has become non-

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negligible and its influence is increasing gradually and globally. Hedging longevity risks has taken on an increasingly important role for life insurance companies. Finding a way to hedge longevity risk has received great attention in both academic and practices.

In general, the hedging strategy can be categorized as an internal or external method. Natural hedging is regarded as the internal hedging strategy that insurers can hedge longevity risks with their own business products between life insurance and annuity because these two types of products are sensitive in opposing ways to the changes in mortality rates. Cox and Lin (2007) find empirical evidence that annuity-writing insurers who have more balanced business in life and annuity risks tend to charge lower premiums than otherwise similar insurers. Wang et al. (2010) propose an immunization model and Tsai, Wang, and Tzeng (2010) further use a conditional value at risk to investigate the natural hedging strategy. Alternatively, the life insurer and pension provider can seek to hedge longevity risk externally using capital market solutions. Blake and Burrows (2001) first proposed that issuing survivor bonds could help a pension fund insure against longevity risk. To utilize the capital market for transferring longevity risk, more recent studies focus on the issue of securitization of longevity risk, and a variety of survivor securities and survivor derivatives have been developed in both academic and practice (e.g., Lin and Cox, 2005; Blake, Cairns, and Dowd, 2006; Cox, Lin, and Wang, 2006; Dowd et al., 2006; Denuit, Devolder, and Goderniaux, 2007; Biffis and Blake, 2009; Blake et al., 2010; Dawson et al., 2010). For example, the EIB/BNP longevity bond aimed at transfering longevity risk, though it was never ultimately issued. The world's first capital market derivative transaction, a q-forward contract between JPMorgan and the U.K. company Lucida, took place in January 2008; the first capital market longevity swap, executed in July 2008, enabled Canada Life to hedge its U.K.-based annuity policies. In December 2010, Swiss Re launched a series of 8-year longevity-based insurance-linked security notes, which it called Kortis notes. Blake et al. (2013) note that the emergence of a traded market in longevity-linked capital market instruments would act a catalyst to help facilitate the development of annuity markets.

There are some discussions regarding external and internal hedging strategies for the life insurer. Cox and Lin (2007) suggest that natural hedging is good but may be too expensive to be effective in the context of internal life insurance and annuity products. They show that insurers that exploit natural hedging by using a mortality swap can charge a lower risk premium than others. In addition, the restriction of using the natural hedging strategy for the insurers is that they must adjust the sales volume of life insurance and annuity products to remain an optimal liability proportion, which is sometime not feasible in practice. To overcome such a restriction, a natural hedging strategy has been developed in some new forms in practice. For example, in November 2012, in the United States, General Motors (GM) has offloaded a huge amount of risk by transferring $25.1 billion of future pension obligations to Prudential Financial. Since Prudential has a huge life assurance portfolio, its strategy is motivated by a desire to exploit natural hedging that builds up its annuity exposure by providing group annuities to GM. Regarding the external hedging strategy, Ngai and Sherris (2011) investigate the effectiveness of static hedging strategies for longevity risk management using longevity bonds and derivatives for annuity products. Results show that static hedging using q-forwards or longevity bonds reduces the longevity risk substantially for life annuities. Extendedly, this study intends to propose a longevity hedging framework that considers not only the natural hedging strategy but also the external hedging strategy by using longevity-linked securities. For a life insurer with both annuity and life insurance business, greater longevity risk implies that the insurer can earn profits from selling life insurance policies but suffer losses from selling annuity insurers. The annuity providers usually have more annuity policies than life insurance policies in their liability; that is, the longevity risk cannot be fully natural hedged and the insurer can consider the external hedging by using the longevity-linked securities to deal with the remaining longevity risk. We examine the hedge effectiveness for three hedging strategies: the natural hedging strategy, the external hedging strategy, and combining both natural hedging and external hedging strategies. To find the optimal hedging strategy, we set the hedging objective according to the insurer's profit function that is calculated based on both the cash flows of the insurer's liabilities and the cash flows of the longevity-linked securities. The optimal hedging strategy is obtained by a minimizing-variance approach that attempts to minimize the impact of longevity risk on the insurer's profit function. In addition, longevity risk exposure may be different in different lines of business and population groups. The great concern in hedging longevity risk is longevity basis risk. Coughlan, Epstein, Ong, et al. (2007) show that a simple demonstration of the natural hedging phenomenon can be performed by a historical simulation of the prices of annuity and life assurance policies based on a time series of historical mortality rates. However, they point out that a meaningful degree of risk reduction can be achieved between a life book and an annuity book, assuming the two populations are fairly similar demographically. Coughlan et al. (2011) further point out the importance of basis risk in longevity hedging because the mortality experience may differ from that of life insurance and annuity portfolio, the hedge will be imperfect and leave a residual amount of risk, known as basis risk. The use of population-based mortality indices for managing the longevity risk inherent in specific blocks of annuitant liabilities may result in basis risk. However, the existing literature on longevity hedging mainly demonstrates with population mortality experience or treating life insurance and annuity business on the same mortality basis. For example, Wang et al. (2010) and Tzeng, Wang, and Tsai (2011) employ the same mortality rate measure (population mortality rates) for both life insurance and annuity products in finding the optimal product mix. It may happen the mismatch in mortality rates between life insurance and annuity products. Li and Hardy (2011) examine basis risk in index longevity hedges by considering the dependence between the population underlying the hedging instrument and the population being hedged. Using data from the female populations of Canada and the United States, they discover that the augmented common factor model is preferred in terms of both goodness of fit and *ex post* forecasting performance. This model is then used to quantify the basis risk. Ngai and Sherris (2011) study the external hedging strategy based on the Australian population data, as Australian annuitant data are not available. They find q-forwards are effective in hedging the longevity risk, but they contain a significant amount of additional basis risk compared to longevity bonds. Although Ngai and Sherris consider the basis risk, they point out that an analysis of available Australian annuitant mortality based on industry experience in comparison to population mortality would be required to confirm the analysis in this article, which is based on U.K. data. Zhu and Bauer (2014) consider parametric and nonparametric mortality forecasting model to examine the performance of natural hedging. These models are calibrated to female U.S. population mortality data. Therefore, to hedge longevity risk, it is important to consider basis risk and use the industry mortality experience rather than the population mortality experience.[1] We then examine the hedge effectiveness in the presence of the basis risk. Wang, Huang, and Hong (2013) propose a natural hedging model that can account for both the variance and mispricing effects of longevity risk at the same time. We also investigate the mispricing risk in this research.[2]

To fill the gap, we consider basis risk on finding the optimal hedging strategy for dealing with longevity risk. Particularly, we deal with basis risk in the following three aspects. First, instead of population mortality data, we employ unique experience mortality data from the life insurance industry that includes the mortality experience for both life insurance and life annuity for men and women separately. It enables us to capture the longevity risk for different product lines to avoid a mismatch in mortality rates. Second, we employ a multi-population mortality framework that can well capture the mortality dynamics across insurance business and assess the basis risk. A vector error correction model (VECM) has been developed to deal with the basis risk since it can capture long-term equilibrium relationship for mortality dynamics (see Li and Hardy, 2011; Salhi and Loisel, 2012; Yang and Wang, 2013). Salhi and Loisel (2012) apply the VECM for mortality rates directly. Li and Hardy (2011) and Yang and Wang (2013) both use the VECM for the mortality time trend under the Lee–Carter framework. The later approach overcomes the problem of the Lee–Carter model that does not consider the interrelationship across populations but it still poses the properties of the Lee–Carter model to forecast future mortality rates based on age and period effects. Thus, we extend Yang and Wang's (2013) model and calibrate it to investigate the basis risk. Third, we utilize the multi-population mortality model in the longevity hedging framework and calculate the profit function for different lines of life insurance business. The optimal hedging strategy is obtained by minimizing the change on the insurer's profit function. The impact of basis risk on the hedge effectiveness is examined by comparing the results based on the mortality forecasting model of VECM with ARIMA. Therefore, we can benefit from the unique experience mortality data to measure longevity risk and model mortality dynamics in the presence of basis risk. We then make the contributions of this research in threefold. As a result, combining natural and external hedging strategies provides better hedge effectiveness than using a natural or external hedging strategy individually.

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[1]Comparing with Li and Hardy (2011) and Ngai and Sherris (2011), they both investigate the external hedging based on population mortality experience. However, we consider three hedging strategies and use industry mortality experience. The similarity is that we all consider mortality dependence in analyzing basis risk but use a different mortality model. Li and Hardy (2011) also consider the VECM model but they find the augmented common factor model provides the best goodness of fit and forecast.

[2]The actuary uses a period mortality table for pricing. It happens that the annuitants live much longer than the mortality assumption based on the period mortality table.

The remainder of this article is organized as follows. In the "Modeling Mortality Dynamics for the Life Insurer Considering Basis Risk" section, we present a multi-population mortality framework to model the mortality dynamics for different product lines. To avoid basis risk, this multi-population mortality framework is calibrated to the real mortality experience from the insurance industry. The mortality data are introduced and the corresponding parameters in the mortality model are obtained. In the "Optimal Hedging Strategy for Longevity Risk" section, we build the proposed hedging framework for the insurance company. The profit function and hedging objective function are introduced. In the "Numerical Analysis" section, the optimal hedging strategy is analyzed numerically. After we provide a numerical analysis of the optimal hedging strategy, we draw some key conclusions and implications.

Modeling Mortality Dynamics for the Life Insurer Considering Basis Risk

We deal with the basis risk in the proposed longevity hedging framework that considers both natural and external hedging strategies. The mortality experiences for life annuity, insurance business, and the underlying longevity-linked securities are different. To utilize the longevity hedging strategy effectively, we consider a mortality model to project the future mortality rates for different groups of population simultaneously. The mortality modeling and calibration are described as follows.

The Multi-Population Mortality Dynamics

We adopt a multi-population model based on the Lee–Carter framework (Lee and Carter, 1992). Consider N population groups for different product lines for the life insurance company and the underlying longevity hedging instrument. The mortality rates for a person aged x in year t based on the mortality experience of the jth population group (with subscript j), denoted as $m_{x_j,t}$, is expressed as,

$$\ln m_{x_j,t} = a_{x_j} + b_{x_j}\kappa_{j,t} + e_{x_j,t},\ j = 1, \ldots, N, \tag{1}$$

where the age effect of mortality dynamics for the jth population group is captured by the coefficient a_{x_j} and the future mortality rates for the jth population changes according to an overall mortality time index $\kappa_{j,t}$, which is modulated by an age response b_{x_j}. The error term $e_{x_j,t}$ reflects a particular, age-specific historical influence that is not captured by the model. The parameters b_{x_j} and $\kappa_{j,t}$ are subject to $\Sigma_x b_{x_j} = 1$ and $\Sigma_t\ \kappa_{j,t} = 0$ to ensure the model identification.

To forecast future mortality dynamics, Lee and Carter (1992) assume that a_{x_j} and b_{x_j} remain constant over time and forecast the future dynamics of the mortality index $\kappa_{j,t}$ using a standard autoregressive moving-average time-series model (ARIMA[p,1,q]), as follows:

$$\Delta\kappa_{j,t} = c_j + \varepsilon_{j,t} + \Sigma_{i=1}^{p}\ \varphi_{j,i}\Delta k_{j,t-i} + \Sigma_{i=1}^{q}\eta_{j,i}\varepsilon_{j,t-i}. \tag{2}$$

where p and q denote the AR and MA orders, respectively; c_j, $\varphi_{j,i}$, and $\eta_{j,i}$ are the drift, AR, and MA parameters, respectively.

However, to deal with longevity exposure for a pool of insurance policies across different product lines considering basis risk, we need to forecast future mortality rates for different groups of populations simultaneously. To do so, we adopt a multivariate time-series approach to analyze the mortality time index. That is, we use a cointegration analysis to investigate whether a common stochastic trend appears in the future mortality time trends $\kappa_{j,t}$ for different product lines. If each series of $\kappa_{j,t}$ is an $I(p)$ process, that is, a nonstationary process with p unit roots, then $\kappa_{j,t}$ is cointegrated and we can model $\kappa_{j,t}$ with a VECM.[3]

In this study, we investigate the effect of basis risk by comparing the mortality modeling using a standard ARIMA time-series model with the VECM model. To present the multi-population mortality forecast under the VECM framework clearly, we express the future mortality rates for a person of age x at time t for different population groups in matrix form as:

$$\ln M_{x,t} = a_x + b_x K_t + e_{x,t}, \tag{3}$$

where $\ln M_{x,t} = \left[\ln m_{x_1,t}, \ldots, \ln m_{x_N,t}\right]'$ represents an N-by-1 vector of mortality rates; $a_x = [a_{x_1}, \ldots, a_{x_N}]'$; $b_x = \mathrm{diag}(b_{x_1}, \ldots, b_{x_N})$ represents an N-by-N diagonal matrix whose entries, starting in the upper left corner, are $b_{x_1}, \ldots, b_{x_N}$; $K_t = \left[\kappa_{1,t}, \ldots, \kappa_{N,t}\right]'$ represents an N-by-1 mortality time index vector; and $e_{x,t} = \left[e_{x_{1,t}}, \ldots, e_{x_{N,t}}\right]'$ denotes an N-by-1 vector of error terms for the aged x person at time t.

According to the Granger representation theorem (Engle and Granger, 1987), the VECM of order p (or VECM[p]) for k_t follows the form:

$$\Delta K_t = C + \Pi K_{t-1} + \Sigma_{d=1}^{p-1} \Gamma_d \Delta K_{t-d} + \varepsilon_t \tag{4}$$

where Δ is the first-order difference operator; C, an N-by-1 vector, contains deterministic terms (e.g., constant, trend, seasonal dummies); Π is an N-by-N long-run impact matrix; Γ_d for $d = 1, \ldots, p-1$, is an N-by-N short-run impact matrix; and $\varepsilon_t = \left[\varepsilon_{1,t}, \ldots, \varepsilon_{N,t}\right]'$ is a shock vector, where $\varepsilon_{1,t}, \varepsilon_{2,t}, \ldots, \varepsilon_{N,t}$ are mutually independent.

Calibration of the Mortality Dynamics for Annuity, Insurance Business, and Underlying Mortality Index

To deal with the basis risk in the proposed longevity hedging framework that considers both natural and external hedging strategies, the above multi-population model is calibrated to the mortality experiences for annuity, insurance business, and the underlying mortality index of the hedging instrument. For the annuity and insurance business, we employ unique experience mortality data from the life insurance industry in Taiwan. This data set covers more than 50,000,000 policies[4] issued by the life insurance companies in Taiwan from the period of 1972 to 2008 (37 years of data) and includes the actual mortality experience for both life insurance and

[3]This multi-population model has been introduced by Li and Hardy (2011) and Yang and Wang (2013).

[4]The data set may contain the same policyholder with multiple policies.

life annuity and for men and women separately. Because the insurance company in Taiwan has a special product strategy that sells life insurance with a series of survival benefits instead of life annuity product, we then regard the mortality experience such life insurance policies as the proxy for annuity mortality experience.[5] The summaries of policy numbers are shown in Table 1 that we divide all policies into two groups: with or without survival benefits. Thus, we can capture the actual mortality pattern for different lines of business rather than demonstrating with population mortality rates.

For the external hedging, we demonstrate that the insurer uses q-forward contracts as the external hedging instrument. The q-forward was first introduced as an instrument for hedging longevity risk by JP Morgan in 2007. The reference mortality index for the first q-forward contract is based on the LifeMetrics graduated mortality rate for 65-year-old males in the reference year for England and Wales national population. To investigate the basis risk on longevity hedging, we consider different underlying reference mortality indices for the q-forward contract, which are Taiwan and U.K. population mortality, respectively.[6] Thus, we calibrate the multi-population model to each line of business and the underlying mortality index. In other words, we have five population groups. The first four groups are distributed from Taiwan insurance data, which include Taiwan females for annuity policies (TW-Fa), Taiwan males for annuity policies (TW-Ma), Taiwan females for life insurance policy (TW-Fl), and Taiwan males for life insurance policy (TW-Ml). The fifth group represents the mortality experience for the underlying hedging instrument based on Taiwan (TW-T) or U.K. population (U.K.-T) respectively.[7]

Figure 1 exhibits the relationship of mortality time trend $(k_{j,t})$ among the five different groups.[8] We can observe that the mortality improvement for annuity policies for both males and females seems more faster than that for life insurance policies.[9] In general, there exists the trend of mortality improvement for different lines of insurance business and such mortality time trend moves in a similar pattern across different lines of insurance business. Based on the cointegration analysis, they exist the

[5]We actually use the insurance policy with a series of survival benefits as a proxy for life annuity. For example, policyholders may receive 5 percent of face amount every 2 years for the whole lifetime after they complete their premium payments. This type of insurance is very similar to the annuity product. The main reason for issuing such an insurance policy rather than the ordinary life annuity is because the commission is much higher. As a result, there is almost no ordinary annuity in the Taiwan insurance market.

[6]The data of populations are from the Human Mortality Database (HMD).

[7]To demonstrate the effect of basis risk resulting from the mismatch of the underlying mortality index with the hedging instrument, we also compare the hedge effectiveness with the U.K. mortality index.

[8]The parameters of the Lee–Carter model are estimated using SVD method.

[9]The possible reason is that national health insurance implemented in 1995 has improved the medical quality and significantly improved the mortality rates for the elders in Taiwan. As younger individuals prefer to purchase life insurance while more seniors buy annuities, the TW-Fa (TW-Ma) are overrepresented by senior ages while TW-Fl (TW-Ml) are over-represented by young ages.

TABLE 1
Summary of Policy Nnumbers Derived From Taiwan Insurance Data

Insurance Type	Female	Male
With periodical survival benefits	7,175,200	7,509,730
No survival benefits	18,254,681	18,072,776

cointegrated effect and it indicates that there are long-term equilibrium relationships among the mortality time trend in different groups.[10] Therefore, we believe the cointegration method that can capture the interrelated long-term equilibrium relationships of mortality improvement among groups is a more suitable method than the ARIMA process to analyze hedging strategies. The parameter estimates for the mortality models based on VECM or ARIMA are shown in the Appendix.

OPTIMAL HEDGING STRATEGY FOR LONGEVITY RISK

Profit Function of the Insurance Company

We consider a life insurance company that suffers longevity risk for their annuity policies. To study the optimal hedging strategy for longevity risk, we consider three different hedging strategies: (1) the natural hedging strategy, where the insurer issues life insurance policies to hedge longevity risk; (2) the external hedging strategy, where the insurer uses longevity-linked securities to reduce longevity risk; and (3) combining both the natural hedging and external hedging strategies, where the insurer finds the optimal hedging strategy to minimize the impact of longevity risk on its profit function. Therefore, the insurer's profit function considering longevity hedging can be defined as the cash flows of the insurer's liability and the cash flows of the hedging instrument. Let $U(t)$ denote the insurer's profit at time t, which is

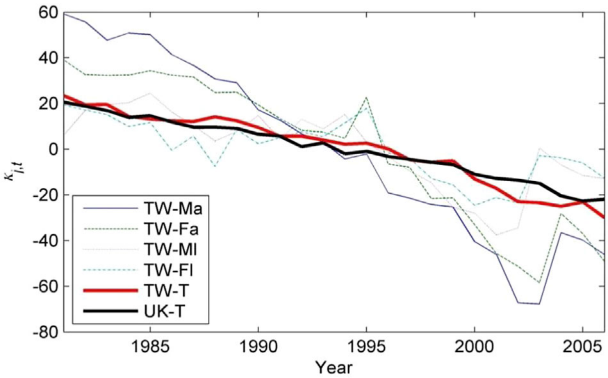
$$U(t) = [\Sigma_i M_i V_i^H(t, T) + \Sigma_h c_{A,h} V^A(x_{j,h}, t)], \tag{5}$$

where $V_i^H(t, T)$ is the value of hedging instrument resulting from the ith hedge strategy at time t with maturity date T, and M_i is the unit of the hedge instrument. $V^A(x_{j,h}, t)$ denotes the expected present value of the hth annuity policy of \$1 per year payable annually in advance for the policyholder aged x in jth population group and $c_{A,h}$ is the corresponding annual payment for the hth annuity policy.

Equation (5) is a general form of the profit function for the insurance company that allows for different longevity hedging strategies. We consider three different hedge strategies. The corresponding value of $V_i^H(t, T)$ depends on the underlying hedging instruments, where $i = L, q, q + L$. That is, we let $V_L^H(t, T)$ denote the value for internal hedging, $V_q^H(t, T)$ for external hedging, and $V_{q+L}^H(t, T)$ for combining both internal and external hedging. The values of these hedging instruments are described in the next subsection.

[10]The test results of cointegration are available upon request from the authors.

Figure 1
The Estimated Mortality Time Trend ($k_{j,t}$) for Different Population Groups



Hedging Instruments Underlying Different Hedging Strategies

The internal hedge strategy utilizes the life insurance policies to offset the longevity risk for annuity business. The value of internal hedging, $V_L^H(t,T)$, is expressed as

$$V_L^H(t,T) = \Sigma_h c_{L,\mathrm{h}} V^L\left(x_{j,h}, t\right), \tag{6}$$

where $V^L\left(x_{j,h}, t\right)$ is the expected present value of the hth life insurance policy of \$1 death benefit payable at the end of the year of death for the policyholder aged x in jth population group, and $c_{L,h}$ is the corresponding total benefit for this insurance policy.

Regarding the external hedge strategy, we use the q-forward contract as the hedge instrument. The q-forward contract specifies a maturity date at which the realized mortality rate for a given population is exchanged in return for a fixed (mortality) rate that is agreed at the initiation of the contract. To hedge the longevity risk, the insurer could purchase the q-forward contracts in which it receives fixed mortality rates and pays realized mortality rates. At maturity, the hedge will pay out to the insurer an amount that increases as mortality rates fall to offset the correspondingly higher value of annuity liabilities. We can express the value of the net payoff amount at current time t for a series q-forward contracts with different maturity date from $t+1$ to T, $V_q^H(t,T)$, as

$$V_q^H(t,T) = \Sigma_{u=t+1}^{T} c_q (1+r)^{-(u-t)} \left(q_{x_j}^f(t,u) - q_{x_j}^{\mathrm{real}}(t,u)\right). \tag{7}$$

Thus, for using only one q-forward contract with maturity date T to hedge longevity risk, Equation (7) can be simplified as

$$V_q^H(t,T) = c_q (1+r)^{-(T-t)} \left(q_{x_j}^f(t,T) - q_{x_j}^{\mathrm{real}}(t,T)\right), \tag{8}$$

where $q_{x_j}^f(t,T)$ is the q-forward rate for aged x person at maturity T at time t and $q_{x_j}^{\mathrm{real}}(t,T)$ is the actual mortality rate for aged x person at maturity T at time t with face value c_q for the q-forward contract. In this research, $q_{x_j}^{\text{real}}(t,T)$ is captured by the VECM model and $q_{x_j}^{f}(t,T)$ is priced fairly according to the underlying mortality experience. Thus, no payment changes hands at the inception of the transaction and a net payment will be determined at maturity. The details of q-forward contracts can be found in Coughlan, Epstein, Ong, et al. (2007) and Coughlan, Epstein, Sinha, and Honig (2007).

Objective Function of Finding the Optimal Strategy

According to the profit function shown in Equation (5), we set the objective for the insurer to find an effective strategy for minimizing unexpected change on the profit caused by the longevity risk. That is to minimize the variance of the change of the insurer's profit function, which is

$$\min_{M} \operatorname{Var}[\Delta U(t)],$$

where $\Delta U(t)$ denotes the change in the insurer's profit due to longevity risk.

We investigate three different strategies based on minimizing variance approach. Our objective is to determine the optimal value of M, that is, how many units of the hedging instrument that the insurer should take to hedge longevity risk. The hedging strategy is static. That is, we did not adjust M at different times. In addition, we consider the same optimal quantity for both male and female policies.

NUMERICAL ANALYSIS

Example

For simplicity and without loss of generality, we assume the annuity and life insurance policies are issued at aged 60 male and female insured, respectively. The annual payment for the annuity policy is assumed to be \$5 and the death benefit for the life insurance policy is \$30. That is, $c_{A,h} = 5$ and $c_{L,h} = 30$. The face values of q-forwards with different maturity dates are all assumed to be \$1,000. The actuarial mortality index $\left(q_{x_j}^{\text{real}}(t,T)\right)$ for the q-forward contract is based on Taiwan mortality experience for 60-year-old persons and the fixed rate is obtained from the fair pricing of q-forward contracts. The fixed rate is 0.0174 for a 10-year maturity contract and 0.0229 for a 15-year maturity contract. The interest rate, r, is assumed to be 0.02.

In the following numerical analysis, we generate 50,000 sample paths of mortality rate and then adopt the optimization program to compute the optimal longevity hedging strategy. We attempt to explore the hedge effectiveness in the following aspects. We first analyze the hedge effectiveness for different hedge strategies under the VECM mortality forecasting model and illustrate with the policy maturity of 10 years in "The Effect of Different Hedging Strategies" section. The model risk and basis risk are investigated in "The Effect of Model Risk on Hedge Effectiveness" and "The Effect of Basis Risk on Hedge Effectiveness" sections. Thus, Taiwan mortality experience is used for our main numerical analysis. The U.K. mortality experience of q-forward contract is only used for analyzing the basis risk in "The Effect of Basis Risk on Hedge Effectiveness" section. The implementation of the Solvency II approach is shown in the "Analysis of Hedge Effectiveness According to Solvency II Approach" section. The mispricing risk is then discussed in the "Discussion of Mispricing Risk on Hedge Effectiveness" subsection.

Analysis of Hedge Effectiveness

The Effect of Different Hedging Strategies. We first examine the hedge effectiveness for different hedging strategies under the VECM model. Three hedging strategies are investigated: (1) the internal hedging strategy, (2) the external hedging strategy, and (3) combining both the internal and external hedging strategies. According the above example setting, assume the insurer attempts to deal with the longevity risk for the annuity business.[11] The longevity risk exposure of the annuity business is measured in Table 2. We present the simulated expected present values (EPVs)[12] of the annuity policies for males and females using the VECM model based on Taiwan mortality experience. In the following analysis, the insurer attempts to hedge the longevity risk on the total liability of the annuity products. In Table 2, we measure the impact of longevity risk on life annuity by calculating the statistics of "unhedged $\Delta U(t)$."

To assess the hedge effectiveness, we compare the distribution of of the change in the insurer's profit before and after hedging in Figure 2. We report the variance and value at risk of the change of the insurer's profit after hedging in Table 3. In addition, following Li and Hardy (2011), we also compare the longevity risk reduction and longevity value at risk reduction after hedging. The amount of risk reduction is calculated according the variance of the change of the insurer's profit. That is, $1 - \left[\mathrm{Var}\left(\Delta \mathrm{U}(0)'\right)/\mathrm{Var}(\Delta \mathrm{U}(0))\right]$. $\mathrm{Var}(\Delta \mathrm{U}(0))$ and $\mathrm{Var}\left(\mathrm{U}(0)'\right)$ denote the variance of the change of the insurer's profit before and after hedging, respectively. Similarly, longevity value at risk reduction is computed based on the measurement of value at risk. We consider the value at risk at a 95 percent confidence level.

According to Table 3 and Figure 2, we can see some interesting findings. The internal hedging strategy can reduce more risk than the external hedging strategy. However, combining the internal and external hedging strategy is the most effective way to reduce longevity risk in terms of both longevity risk reduction and longevity value at risk reduction. Moreover, using a series of q-forward contracts with different maturity dates enhances the hedge effectiveness more than a q-forward contract with the same maturity date, which is 10 years in the illustrative example. It reduces the risk up to 46.77 percent. Therefore, the proposed hedging framework that incorporates the internal natural hedging and the external hedging can help the insurer manage longevity risk more effectively.

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[11]For the internal hedging strategy, we find the optimal unit for the insurance policy; for the external hedging strategy, we find the optimal unit for the q forward contract; for the strategy combining both the internal and external hedging, we find the optimal unit for the insurance policy and the q forward contract. We can use the existing product mix of the insurer to find the optimal unit for the external hedging instrument according the setting of our hedging strategies in Equations (5)–(8).

[12]The EPV is calculated by discounting the future net cash flows resulting from the actual and expected mortality rates.

TABLE 2
The Impact of Longevity Risk on Life Annuity, Unhedged $\Delta U(t)$

Maturity	Gender	Mean	Variance	VaR (95)
10	Male	−0.0001	0.0063	−0.1215
	Female	0.0000	0.0022	−0.0700
	Total	−0.0001	0.0113	−0.1640
15	Male	−0.0007	0.0541	−0.3534
	Female	−0.0003	0.0194	−0.2105
	Total	−0.0010	0.0996	−0.4849

Note: Total liability includes one male and one female policy. VaR (95) denotes the 95% value at risk.

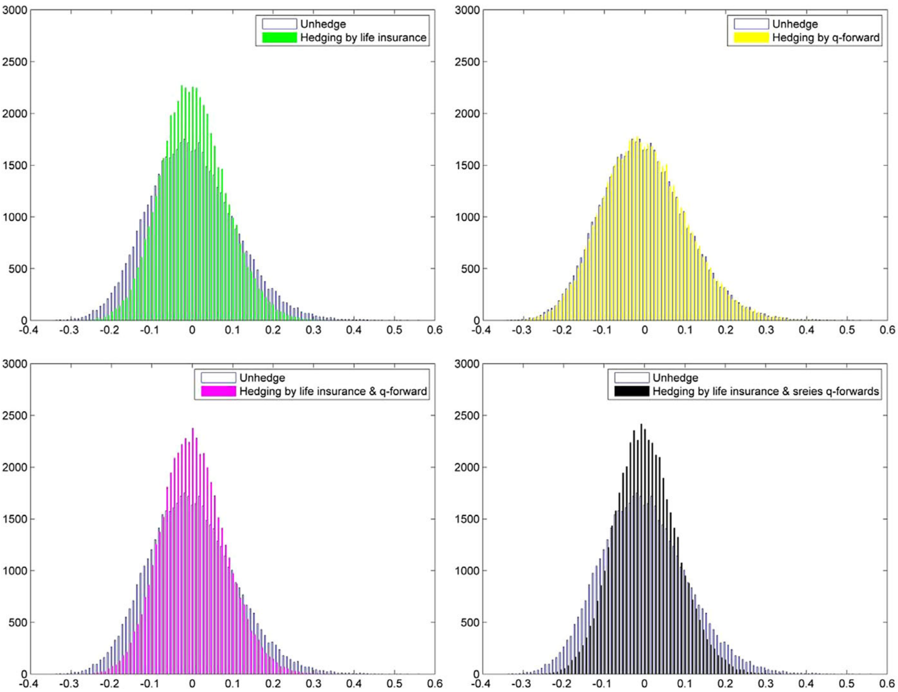
TABLE 3
The Optimal Hedging Unit and Hedge Effectiveness for Different Hedging Strategies, Hedged $\Delta U(t)$

Hedging Method	Optimal M*	Variance	VaR (95)	Longevity Risk Reduction	Longevity Value at Risk Reduction
Internal hedging	0.1284	0.0065	−0.1248	0.4245	0.0392
External hedging (single q-forward)	0.0290	0.0107	−0.1599	0.0519	0.0041
External hedging (series q-forward)	0.0076	0.0103	−0.1571	0.0843	0.0069
Internal + external hedging (single q-forward)	(0.1244, 0.0170)	0.0063	−0.1232	0.4418	0.0408
Internal + external hedging (series q-forward)	(0.1230, 0.0055)	0.0060	−0.1202	0.4677	0.0438

The Effect of Model Risk on Hedge Effectiveness. We further examine the model risk by comparing the mortality forecast for the insurance business and the underlying mortality index of the q-forward contract using the ARIMA model with the VECM model. We only show the simulated profit distributions based on the two models according the hedging strategy that combines both internal and external hedging (a single-duration q-forward) in Figure 3. We find that it can reduce 44.18 percent of the risk based on the VECM model but only 0.00 percent for the ARIMA model. In terms of the tail risk, longevity value at risk reduction is 0.0408 under the VECM model and 0.0001 under the ARIMA model. The difference is smaller for using the ARIMA model. Therefore, the use of multi-population mortality model can deal with the basis risk and increase the hedge effectiveness. Such effect also applies to the internal and external hedging strategy individually.

The Effect of Basis Risk on Hedge Effectiveness. We examine the impact of the underlying mortality index of the q-forward contract on the hedge effectiveness. To

Figure 2
The Simulated Distribution of the Change of the iInsurer's Profit Before and After Hedging



Note: Left Top: Internal hedging strategy; Right Top: External hedging strategy; Left Bottom: Combining both internal and external hedging strategies with a single-duration q-forward contract; Right Bottom: Combining both internal and external hedging strategies with a series q-forward contract.

hedge the longevity risk for the annuity polices for Taiwan population, it may exist basis risk and result in hedge ineffectiveness if the underlying morality index of the q-forward contract is different from the pooled risk. In Figure 4, we compare two underlying mortality indices for q-forward contracts, which are based on Taiwan and U.K. mortality indices for 60-year-old persons separately. We find that it can reduce 44.18 percent of the risk based on the Taiwan mortality index but only 42.83 percent for the U.K. mortality index. In terms of the tail risk, longevity value at risk reduction is 0.0408 under the Taiwan mortality index and 0.0395 under the U.K. mortality index. The hedge effectiveness is better when the underlying mortality index is much closer to the mortality experience of the annuity business. Our results are in line with the existing literature using q-forward contracts as the hedging instrument (see Coughlan, Epstein, Sinha, and Honig, 2007). The underlying reference of q-forward contract can affect the hedge effectiveness. Coughlan, Epstein, Sinha, and Honig (2007) point out that basis risk can be managed and minimized by careful design and calibration of the hedge. Therefore, the selection of the underlying

FIGURE 3
The Simulated Distribution of the Change of the Insurer's Profit Based on VECM and ARIMA Model (Internal and External Hedging Strategy)

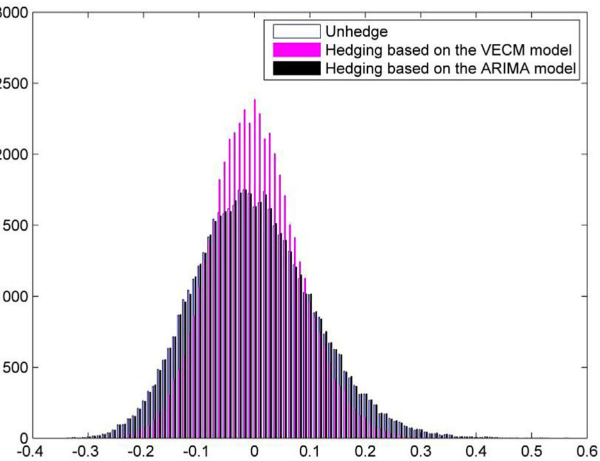
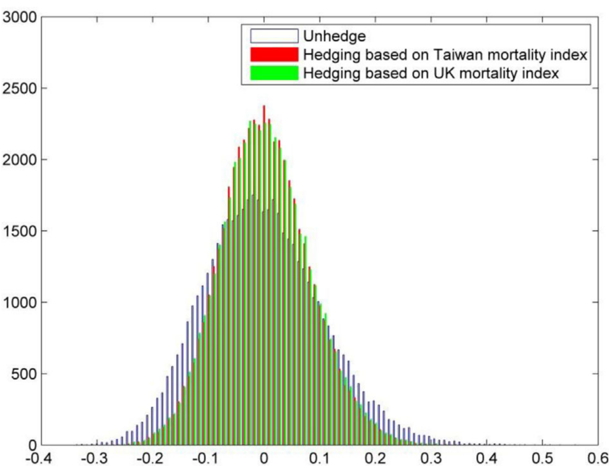


FIGURE 4
The Simulated Distribution of the Change of the Insurer's Profit Based on the Taiwan Mortality Index and the U.K. Mortality Index Underlying the q-Forward Contract (Internal and External Hedging Strategy)



mortality index for the hedging instrument can reduce the mismatch risk in longevity hedging.

Analysis of Hedge Effectiveness According to Solvency II Approach. We further use the Solvency II approach to calculate the hedge effectiveness. That is, we set up a future date (usually 1 year from now), let the randomness get realized from time 0 to

Table 4
The Optimal Hedging Unit and Hedge Effectiveness for Different Hedging Strategies (Solvency II Approach)

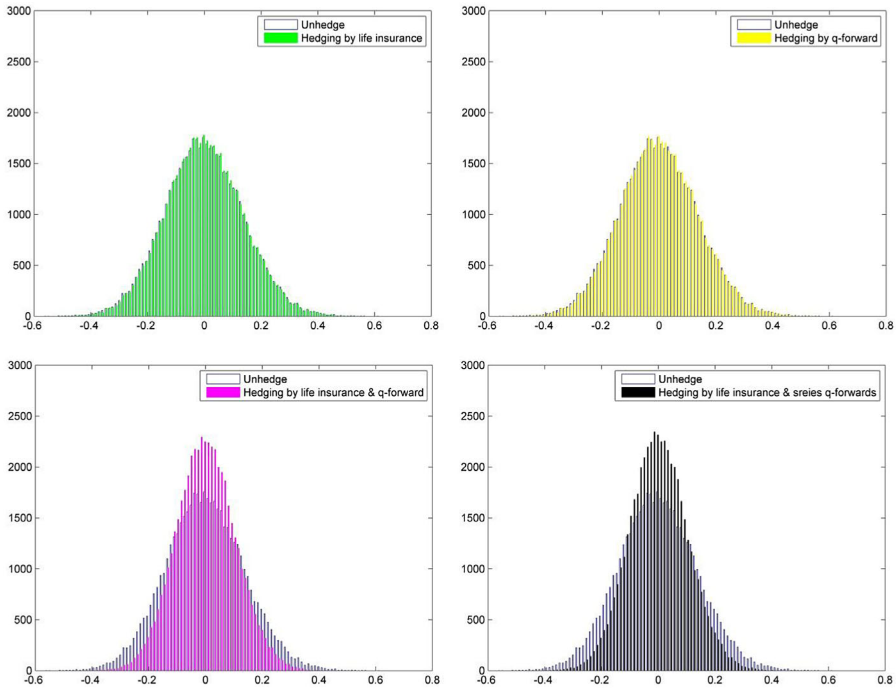
Hedging Method	Optimal M^*	Variance	VaR (95)	CTE (95)	Longevity Risk Reduction	Longevity Value at Risk Reduction
Unhedge	–	0.0192	−0.2248	−0.2804	–	–
Internal hedging	0.0018	0.0186	−0.2216	−0.2759	0.0277	0.0033
External hedging (single q-forward)	0.0006	0.0188	−0.2227	−0.2773	0.0182	0.0021
External hedging (series q-forward)	0.0001	0.0188	−0.2224	−0.2772	0.0191	0.0025
Internal + external hedging (single q-forward)	(0.1263, 0.0088)	0.0111	−0.1701	−0.2140	0.4232	0.0547
Internal + external hedging (series q-forward)	(0.1255, 0.0030)	0.0107	−0.1673	−0.2100	0.4445	0.0576
Internal hedging	0.1280	0.0066	−3.1569	−3.1860	0.4177	0.3381
External hedging (single q-forward)	0.0294	0.0107	−2.9909	−3.0257	0.0535	0.5041
External hedging (series q-forward)	0.0077	0.0104	−1.7573	−1.7912	0.0865	1.7377
Internal + external hedging (single q-forward)	(0.1239, 0.0173)	0.0064	−2.8705	−2.8995	0.4358	0.6245
Internal + external hedging (series q-forward)	(0.1225, 0.0056)	0.0061	−1.9072	−1.9353	0.4626	1.5878

time 1 (via simulation), and then calculate the risk quantities at year 1. In Table 4 and Figure 5, we find our results are consistent with the analysis in previous sections. The internal hedging strategy can reduce more risk than the external hedging strategy. However, combining the internal and external hedging strategies is the most effective way to reduce longevity risk in terms of both longevity risk reduction and longevity value at risk reduction. Moreover, using a series of q-forward contracts with different maturity dates enhances the hedge effectiveness more than a q-forward contract with the same maturity date. Therefore, the proposed hedging framework also applies to the Solvency II approach.

Discussion of Mispricing Risk on Hedge Effectiveness. In practice, the actuary may use a period mortality table for pricing life annuity. The period mortality table does not reflect the trend in mortality improvement. It implies that the annuitants live much longer than the mortality assumption based on the period mortality table; that is, the longevity risk is more significant than that using the cohort table. We call such risk the mispricing risk. Therefore, the insurer suffers not only the longevity risk but also the mispricing risk. To investigate the impact of mispricing risk on hedge effectiveness,

Figure 5

The Simulated Distribution of the Change of the Insurer's Profit Before and After Hedging (Solvency II Approach)



Note: Left Top: Internal hedging strategy; Right Top: External hedging strategy; Left Bottom: Combining both internal and external hedging strategies with a single-duration q-forward contract; Right Bottom: Combining both internal and external hedging strategies with a series q-forward contract.

Table 5 shows the impact of longevity risk on the annuity policy in the presence of mispricing risk for males and females according to the mortality experience based on Taiwan insurance data. Compared to Table 2, Table 5 discovers that the insurer suffers more significant losses for issuing annuity policy when pricing based on the

Table 5

The Impact of Longevity Risk on Life Annuity in the Presence of Mispricing Risk

Maturity	Gender	Mean	Variance	VaR (95)	CTE (95)
10	Male	−2.2083	0.0063	−2.3302	−2.3559
	Female	−1.1228	0.0022	−1.1928	−1.2072
	Total	−3.3311	0.0113	−3.4950	−3.5298
15	Male	−5.2432	0.0536	−5.5918	−5.6649
	Female	−2.8424	0.0193	−3.0516	−3.0934
	Total	−8.0857	0.0987	−8.5656	−8.6646

Note: Total liability includes one male and one female policy. VaR (95) denotes the 95% value at risk. CTE (95) is the conditional tail expectation at 95%.

Table 6
The Optimal Hedging Unit and Hedge Effectiveness for Different Hedging Strategies in the Presence of Mispricing Risk

Hedging Method	Optimal M*	Variance	VaR (95)	CTE (95)	Longevity Risk Reduction	Longevity Value at Risk Reduction
Internal hedging	0.1280	0.0066	−3.1569	−3.1860	0.4177	0.3381
External hedging (single q-forward)	0.0294	0.0107	−2.9909	−3.0257	0.0535	0.5041
External hedging (series q-forward)	0.0077	0.0104	−1.7573	−1.7912	0.0865	1.7377
Internal + external hedging (single q-forward)	(0.1239, 0.0173)	0.0064	−2.8705	−2.8995	0.4358	0.6245
Internal + external hedging (series q-forward)	(0.1225, 0.0056)	0.0061	−1.9072	−1.9353	0.4626	1.5878

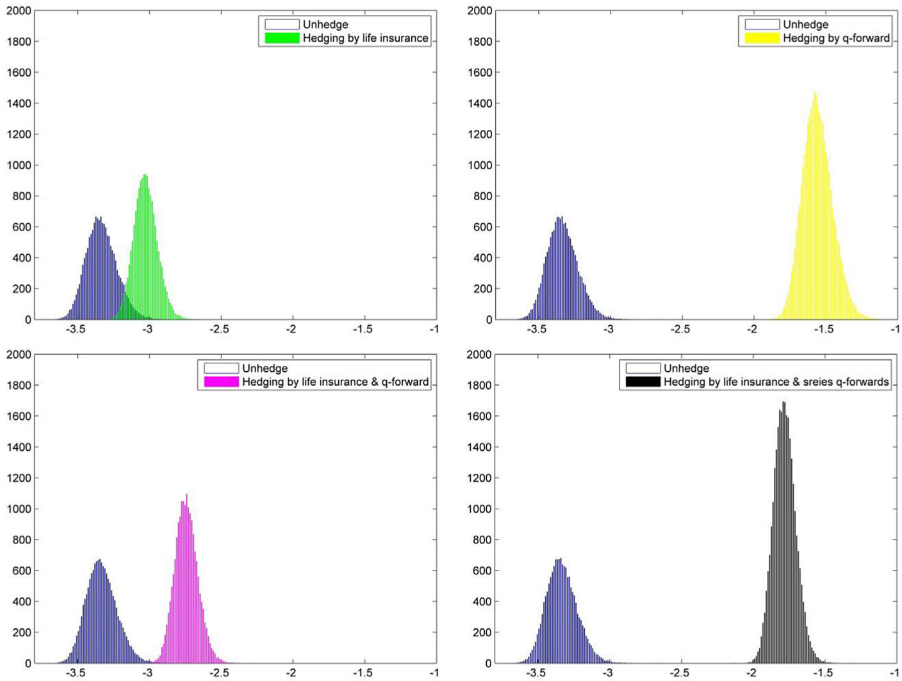
period table. The total liability of annuity business increases from 0.0001 to 3.3311 with 10-year maturity and 0.001 to 8.0857 with 15-year maturity in the presence of mispricing risk (Table 5).

We further examine the mispricing risk on the longevity hedge. That is, the insurer prices the insurance contracts based on the period table instead of the cohort table. The impacts of mispricing risk on the simulated profit distribution before and after hedging are compared in Table 6 and Figure 6. Under the three investigated hedging strategies, the hedge strategies seem to be able to reduce the longevity risk. However, it is clear to see that the longevity risk cannot be fully eliminated and the insurer still has losses after hedging. This is because the mispricing risk causes the longevity risk of annuity business too significant to be hedged. In addition, the natural hedging strategy can help to reduce the losses a bit because the mispricing risk causes the life insurance contracts have more mortality gain. In contrast, using the q-forward contracts would not help at all. The distribution of profit after external hedging does not shift. Indeed, the hedging instrument can help to reduce the risk but not to make profit.

Conclusions

This article analyzes the optimal hedging strategy to hedge longevity risk. Different from the existing literature, we utilize both internal and external hedging to deal with longevity risk. Thus, the insurance company can overcome the restriction of using the natural hedging strategy for the insurers that they must adjust the sales volume of life insurance and annuity products to remain an optimal liability proportion, which is sometimes not feasible in practice. The existing literature on hedging longevity risk only focuses on either natural hedging or external hedging, but we incorporate both into our hedging framework in this article.

FIGURE 6
The Simulated Distribution of the Change of the Insurer's Profit Before and After Hedging in the Presence of Mispricing Risk



Note: Left Top: Internal hedging strategy; Right Top: External hedging strategy; Left Bottom: Combining both internal and external hedging strategies with a single-duration q-forward contract; Left Bottom: Combining both internal and external hedging strategies with a series q-forward contract.

The evaluation of the hedge effectiveness for internal hedging, external hedging, and combining both internal and external hedging strategies is examined. We use a minimizing-variance approach to find the optimal hedging unit according to the insurer's profit function for each hedging strategy. We also take into account basis risk in the hedging framework by employing a multi-population mortality model based on the VECM to capture the mortality dynamics for annuity, life insurance business, and underlying mortality index of the hedging instrument. We adopt a unique data set of annuity and life insurance policies that enable us to calibrate the multi-population mortality dynamics for different lines of insurance policies and calculate their liabilities in the profit function. With the experienced mortality rates from life insurance companies, it makes our model able to deal with basis risk and become more realistic to apply in practice. The impact of basis risk on the hedge effectiveness is also investigated. As shown with the demonstrating example, we find the proposed hedging strategy that combines the internal and external hedging to be the most effective way to reduce longevity risk. In addition, a multi-population mortality model can help to deal with the basis risk and increase the hedge effectiveness for longevity hedging. Therefore, this study proposes a hedging framework that integrates both internal and external hedging strategies and considers basis risk, which is more suitable for the life insurer to deal with longevity risk.

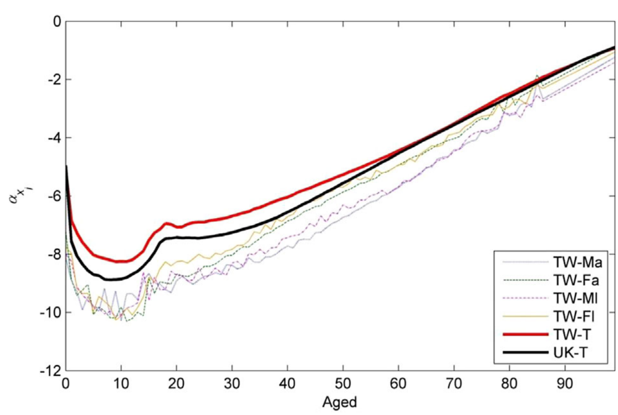
In addition, we also discuss the issue of mispricing risk on hedge longevity risk. In practice, the insurance company may price the annuity product based on the period mortality table that ignores the trend of future mortality improvement. Such a pricing assumption may increase the longevity risk, and the longevity hedging cannot fully eliminate the longevity risk. The actuary cannot ignore the mortality trend in setting the mortality assumption for pricing insurance contracts.

In this article, we employ the multi-population mortality model under the Lee–Carter framework. Our analysis suggests that the use of a proper multi-population mortality model is critical in longevity hedging and deserves to be studied further. We believe that insurers can make use of their mortality experience to discover an appropriate mortality model in dealing with longevity risk. Regarding the numerical analysis, we give an example for simplicity and without loss of generality. The hedging framework can be applied to a more realistic product mix according to the real case. For example, allowing different optimal quantities in male and female policies may improve the hedge effectiveness. In addition, to measure basis risk, the proposed hedging framework is based on static hedging. We believe the dynamic hedging is a direction worthy of doing in the next stage.

Appendix: Parameter Estimates of the Lee–Carter and the VECM Model

Figures A1 and A2 are shown the estimated mortality age effect (a_{x_j}) and age response (b_{x_j}) for different population groups in Lee-Carter model, respectively.

Figure A1
The Estimated Mortality Age Effect (a_{x_j}) for Different Population Groups

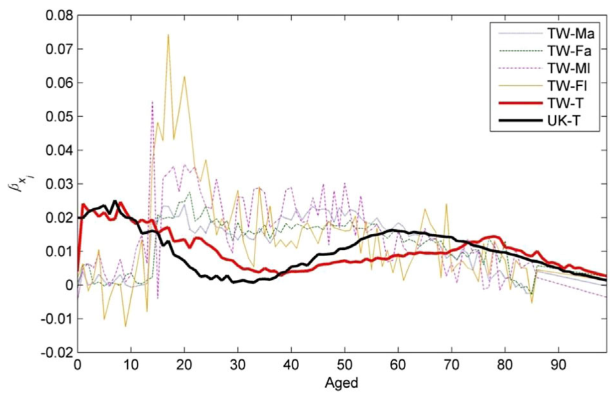


As shown in Equation (4), the VECM for K_t can be expressed in the following form:

$$\Delta K_t = C + \Pi K_{t-1} + \Sigma_{d=1}^{p-1} \Gamma_d \Delta K_{t-d} + \varepsilon_t.$$

To express to parameter estimates for each group more clearly, we use the subscription of 1, 2, 3, 4, 5, and 6 to denote the group of TW-Ma, TW-Fa, TW-Ml, TW-Fl, TW-T, and U.K.-T, respectively. The parameter estimates of K_t for the six population groups are presented as follows:

FIGURE A2
The Estimated Mortality Age Response (b_{xj}) for Different Population Groups



$$\begin{pmatrix} \Delta\kappa_{1,t} \\ \Delta\kappa_{2,t} \\ \Delta\kappa_{3,t} \\ \Delta\kappa_{4,t} \\ \Delta\kappa_{5,t} \\ \Delta\kappa_{6,t} \end{pmatrix} = \begin{pmatrix} -4.5618 \\ -5.1062 \\ -1.2338 \\ -4.2401 \\ -2.4884 \\ 2.5686 \end{pmatrix}$$

$$+ \begin{pmatrix} -0.1842 & -0.2576 & 0.3128 & 0.0155 & -0.4048 & 1.4006 \\ -0.3309 & -0.4627 & 0.5618 & 0.0278 & -0.7270 & 2.5154 \\ -0.0431 & -0.0603 & 0.0733 & 0.0036 & -0.0948 & 0.3280 \\ -0.0827 & -0.1157 & 0.1405 & 0.0069 & -0.1818 & 0.6291 \\ -0.0677 & -0.0947 & 0.1150 & 0.0057 & -0.1488 & 0.5149 \\ 0.0478 & 0.0668 & -0.0811 & 0.0040 & 0.1049 & -0.3631 \end{pmatrix} \begin{pmatrix} k_{1,t-1} \\ k_{2,t-1} \\ k_{3,t-1} \\ k_{4,t-1} \\ k_{5,t-1} \\ k_{6,t-1} \end{pmatrix}$$

$$+\begin{pmatrix} 0.1505 & -0.0761 & 0.3395 & 08412 & 0.8412 & -1.3110 \\ 0.3612 & -0.4857 & -0.0901 & 0.1687 & 1.4252 & -2.4636 \\ -0.3644 & 0.2717 & -0.1518 & 0.0836 & -1.1054 & 1.5718 \\ -0.3255 & 0.1975 & 0.1111 & -0.4422 & -0.3854 & -0.6276 \\ 0.0478 & 0.0790 & -0.1104 & 0.0040 & -0.0880 & -0.3409 \\ -0.0577 & -0.0026 & 0.0853 & -0.0300 & 0.0288 & -0.4022 \end{pmatrix}\begin{pmatrix} \Delta k_{1,t-1} \\ \Delta k_{2,t-1} \\ \Delta k_{3,t-1} \\ \Delta k_{4,t-1} \\ \Delta k_{5,t-1} \\ \Delta k_{6,t-1} \end{pmatrix}+\begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \\ \varepsilon_{4,t} \\ \varepsilon_{5,t} \\ \varepsilon_{6,t} \end{pmatrix}$$

and

$$\Sigma = \mathrm{Cov}\begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \\ \varepsilon_{4,t} \\ \varepsilon_{5,t} \\ \varepsilon_{6,t} \end{pmatrix} = \begin{pmatrix} 27.2485 & 5.7055 & 1.7159 & 8.7577 & 2.0361 & -1.0940 \\ 5.7055 & 18.5950 & 3.0447 & 20.1295 & 2.3198 & 0.7022 \\ 1.7159 & 3.0447 & 67.2412 & 37.1517 & -0.2743 & 1.1976 \\ 8.7577 & 20.1295 & 37.1517 & 48.8734 & 3.0409 & 0.0586 \\ 2.0361 & 2.3198 & -0.2743 & 3.0409 & 4.6020 & 1.3112 \\ -1.0940 & 0.7022 & 1.1976 & 0.0586 & 1.3112 & 1.6911 \end{pmatrix}.$$

TABLE A1
The Estimated Parameters of the ARIMA Model

	TW-Ma	TW-Fa	TW-Ml	TW-Fl	TW-T	U.K.-T
c	−3.5285	−4.2092	−8.3817	−0.7597	−2.1324	−1.7139
φ_1	–	–	−1.6686	–	–	–
φ_2	–	–	−0.6759	–	–	–
η_1	–	–	–	–	–	−0.6511
η_2	–	–	–	–	–	–

The general ARIMA model is described as follows.

$$\Delta\kappa_{j,t} = c_j + \varepsilon_{j,t} + \sum\nolimits_{i=1}^{p} \varphi_{j,i}\Delta\kappa_{j,t-i} + \sum\nolimits_{i=1}^{q} \eta_{j,i}\varepsilon_{j,t-i}.$$

We use AIC and BIC to be the criteria for choosing the ARIMA model. The estimated parameters of the ARIMA are shown in Table A1.

$$\Sigma_{11} = \begin{pmatrix} 9.1519 & 0 & 0 & 0 \\ 0 & 10.5419 & 0 & 0 \\ 0 & 0 & 9.6999 & 0 \\ 0 & 0 & 0 & 6.2437 \end{pmatrix}, \text{ and } \Sigma_{22} = \begin{cases} 2.5835, & \text{when } H = \mathrm{TW} - T \\ 1.4835, & \text{when } H = \mathrm{UK} - T. \end{cases}$$

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