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An inventory replenishment system with two inventory-based substitutable products

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ABSTRACT

In a supermarket, two mutually substitutable products with the same price are usually arranged one next to the other such as Coke and Pepsi colas, Campbell and Progresso soups, Breyer and Friendly ice creams, etc. It is evident that a large pile of products (e.g., colas, soups, baked goods, fruits, vegetables, etc.) displayed in a supermarket often induces customers to buy more because of its visibility, variety, and freshness. Hence, high inventory of one product provides consumers various choices, and makes this product preferable. In short, the demand for one product is positively influenced by its displayed stock level while negatively impacted by the displayed stock level of the other product. With the demand being stock-dependent, it may be profitable to maintain high stock level at the end of the replenishment cycle. The common inventory assumption of zero-ending inventory is extended to non-negative ending inventory. Hence, we first propose an inventory model with two inventory-based substitutable products to determine the optimal replenishment time and the ending inventory levels for both products in order to maximize the total annual profit. We then demonstrate that the total annual profit is strictly pseudo-concave with respect to the decision variables, which reduces the search for the global maximum to a local optimum. We also use simple economic interpretations to explain theoretical results. Furthermore, the theoretical results reveal that the optimal replenishment time is whenever one of two substitutable products is sold-out. Finally, numerical examples and sensitivity analyses are presented to highlight several managerial implications.

1. Introduction

The study of product substitution has recently gained considerable attention in the literature, as it contributes to the success of companies' decisions regarding material/product planning, pricing and inventory control. According to Shin et al. (2015) there are three types of substitution mechanisms. The first type is assortment-based substitution. In this type the customer voluntarily chooses a substitute, triggered by the fact that the substitute is newly added in the assortment. For example, newly baked doughnuts (or newly arrived vegetables) are more attractive to the customer than existing stale doughnuts. The second type of substitution mechanism happens if the demand for a specific product could not be satisfied, so that the demand may be fulfilled by a substitute product. This is inventory-based substitution. Likewise, the customer prefers a large pile of fresh fruits more than a small pile because of better selection and visibility. In the third substitution mechanism, the customer's behavior is driven by a change in the price of substitutable product. This type of substitution mechanism is price-based substitution. For instance, the customer buys an on-sales product from a specific store instead of a near-by store simply because of its lower price.

A variety of conventional inventory models, including periodic-review, economic order quantity (EOQ) models, and newsvendor models have been appropriately adapted to manage substitutable products. McGillivray and Silver (1978) first explored items with identical cost and a fixed substitution probability, and obtained the optimal order

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quantity by using simulation and heuristics. Parlar and Goyal (1984) studied the same problem and demonstrated that the total expected profit is concave, which implies there is a unique optimal solution. Parlar (1988) extended the single-player model to a two-player competitive market, and showed the existence of Nash equilibrium for two substitutable products with stochastic demands. Ernst and Kouvelis (1999) expanded the substitutable-demand pattern from no shortages to allow for shortages. Rajaram and Tang (2001) examined the substitution effect on order quantities as well as on expected profits. Netessine and Rudi (2003) analytically confirmed the numerical results of Rajaram and Tang (2001). Nagarajan and Rajagopalan (2008) provided the optimal policies for an inventory model with two substitutable products whose demands are negatively correlated. Hsieh and Wu (2009) modeled a supply chain in which two suppliers sell two substitutable products to a common retailer who faces random demand for those two products. Maity and Maiti (2009) studied an inventory model for deteriorating and substitutable multi-items with stock dependent demand. Gurler and Yilmaz (2010) proposed a model for two substitutable products in which the retailer is allowed to return a portion or all of the unsold products to the manufacturer for some credit. Stavrulaki (2011) established the demand stimulation and substitution effect when demand is stochastic. Krommyda et al. (2015) built an inventory problem in which demand is determined and satisfied with two mutually substitutable products. Recently Shin et al. (2015) presented a comprehensive taxonomy of the literature on the planning involved with substitutable products.

Levin et al. (1972) argued that large piles of consumer goods displayed in a supermarket will lead customers to buy more. Similarly, Silver and Peterson (1985) noted that sales at the retail level tend to be proportional to the amount of stock displayed. A variety of inventory models have thus been proposed to quantify this phenomenon in exploring the optimal inventory policies. Baker and Urban (1988) proposed specifying the demand pattern as a power function of displayed stock level. Taking a different approach, Mandal and Phaujdar (1989) specified the demand as a linear function of displayed stock level. Recently, a large number of mathematical models for stock dependent demand rate, under several assumptions, have been proposed such as Goyal and Chang (2009), Chang et al. (2010), Hsieh and Dye (2010), Yang et al. (2010), Dye and Hsieh (2011), Teng et al. (2011), and Wu et al. (2014, 2016), Chen et al. (2016), and Feng et al. (2017). Readers may refer to Urban (2005) for a review of inventory models with stock dependent demand. In summary, a brief comparison among the above mentioned models is given in Table 1.

In this paper, we study a model for two inventory-based substitutable products such as Coke and Pepsi colas, Campbell and Progresso soups, Breyer and Friendly ice creams, Tylenol and Bayer pain relief, etc., which the retailer usually arranges side by side on display next to each other. Additionally, the substitutable products are offered for the same price, and the high inventory of one product makes that particular product preferable to customers. Today's consumers become more health conscious, the demand for seasonal products has drastically increased in recent years. This change has made it very important for retailers to better manage substitutable products. It may be profitable to maintain high stock level at the end of the replenishment cycle when the demand is dependent upon on-hand inventory. Therefore, the objective is to determine the optimal cycle time and ending inventory levels for both products in order to maximize total annual profit.

2. Notation and assumptions

The following notation and assumptions are used in this paper.

2.1. Notation

For simplicity, we define the symbols for parameters, decision

Table 1
A brief comparison of related literature on substitutable products.

Authors (years)	Demand	Model formulation	Ending inventory	Decision variables
McGillivray and Silver (1978)	Constant	Cost minimization	Zero	Order-up-to-levels by a heuristic approach
Parlar and Goyal (1984)	Stochastic demand	Maximize expected profit	Zero	Order quantities for both products
Parlar (1988)	Stochastic demand	Maximize expected profit	Zero	Order quantities for two players using games theory
Ernst and Kouvelis (1999)	Stochastic demand	Maximize expected profit	Zero	Stocking levels for individual products and multiproduct packets
Rajaram and Tang (2001)	Stochastic demand	Maximize expected profit	Zero	Order quantities solved by a service rate heuristic
Netessine and Rudi (2003)	Stochastic demand	Maximize expected profit	Zero	Order quantities with centralized and competitive systems
Nagarajan and Rajagopalan (2008)	Stochastic demand	Maximize expected profit	Zero	Order quantities under stockout-based substitution
Hsieh and Wu (2009)	Stochastic demand	Maximize expected profit	Zero	Ordering and pricing decisions under revenue sharing, return policy, and combination of revenue sharing and return policy
Maity and Maiti (2009)	Stock dependent demand	Maximize profit	Zero	Production lot sizes for deteriorating multi-items
Stavrulaki (2011)	Stochastic demand	Maximize expected profit	Zero	Order quantities for both products
Krommyda et al. (2015)	Stock dependent demand	Maximize profit	Zero	Order quantities for both products
Present model	Stock dependent demand	Maximize profit	Positive or zero	Replenishment cycle time and ending inventory levels for both products

variables, and functions accordingly.

Parameters:

p_i Profit per unit for Product i ($i = 1$ or 2), in dollars.
h_i Holding cost per unit per year for Product i ($i = 1$ or 2), in dollars.
A Total ordering cost per order for both products, in dollars.
U Maximum total shelf space for both products, in units.

Decision variables:

E_i Ending inventory level for Product i ($i = 1$ or 2) in units with$E_i \geq 0$.
T Replenishment cycle time in years with $T \geq 0$.

Functions:

$D_i(t)$ Demand rate for Product i ($i = 1$ or 2) at time t, in units.
$I_i(t)$ Inventory level for Product i ($i = 1$ or 2) at time t, in units.
Q_i Order quantity for Product i ($i = 1$ or 2), in units.
Π (E_1, E_2, T) Total annual profit for both products, in dollars.

For convenience, the asterisk symbol on a variable denotes the optimal solution of the variable. For instance, $T^*$is the optimal solution of T. Next, we propose some necessary assumptions in order to build the mathematical model.

2.2. Assumptions

A large pile of products (e.g., Coke Colas, Campbell soups, vegetables, fruits, etc.) displayed in a supermarket often induces more sales and profits due to its visibility, freshness, or variety. In the literature, Levin et al. (1972) observed that "large piles of consumer goods displayed in a supermarket will lead customers to buy more." Likewise, Silver and Peterson (1985) also noticed that sales at the retail level tend to be proportional to the amount of stocks displayed. Therefore, we assume that building up stocks has a positive impact on demand.

In a supermarket, the retailer usually arranges two mutually substitutable products with the same price next to each other (e.g., Coke and Pepsi colas, Campbell and Progresso soups, Breyer and Friendly ice creams, Colombo and Yoplait yogurts, etc.). In general, the demand for both products is very steady. In addition, high inventory of one product provides consumers various choices and makes this product preferable. In addition, we assume that the percentage of stale items is negligible. Hence the model is suitable for the following two categories (*i*) products with a long shelf life (e.g., colas, soups, over-counter medicines, etc.) or (*ii*) products with a short shelf life but high demand (e.g., seasonal vegetables, fruits, daily baked goods, etc.). As a result, we assume as in Krommyda et al. (2015) that the demand rate $D_i(t)$ for Product i ($i = 1$ or 2) at time t is a function of the instantaneous stock levels $I_1(t)$ and $I_2(t)$ given as:

$$D_1(t) = a_1 + b_1 I_1(t) - b_2 I_2(t) \geq 0; \quad 0 \leq t \leq T, \quad a_1, b_1, b_2 \geq 0, \tag{1}$$

and

$$D_2(t) = a_2 - b_1 I_1(t) + b_2 I_2(t) \geq 0; \quad a_2, b_1, b_2 \geq 0, \quad 0 \leq t \leq T. \tag{2}$$

When the demand is dependent on the amount of displayed stock, it may be profitable to keep a high stock level. In contrast to the classical zero ending inventory, we assume as in Urban (1992) that the ending inventory may be zero or positive.

A retailer sells two mutually substitutable products, say Products 1 and 2. At time 0, the retailer has E_i ($i = 1$ and 2) units from the previous cycle, and receivesQ_i ($i = 1$ and 2) units of Product i. Hence, at the beginning of the replenishment cycle, the retailer has on-hand inventory of E_i+Q_i ($i = 1$ and 2) units for Product i. During the time interval [0, T], the inventory level for both products is gradually depleted by the stock-dependent consumption rate. The replenishment cycle ends at time T when the ending inventory for Product i ($i = 1$ and 2) is E_i. At that time a new order for both products is placed and hence a new replenishment cycle is repeated again.

Most retailers have limited shelf space. Hence, we assume that the maximum shelf space for both products is U.

3. Model formulation and solution

Based on the above assumptions, the inventory levels for Products 1 and 2 at time t during the time period [0, T] are governed by the following two differential equations:

$$\frac{dI_1(t)}{dt} = I_1' = -D_1(t) = -a_1 - b_1 I_1(t) + b_2 I_2(t); \quad 0 \leq t \leq T \tag{3}$$

and

$$\frac{dI_2(t)}{dt} = I_2' = -D_2(t) = -a_2 + b_1 I_1(t) - b_2 I_2(t); \; 0 \leq t \leq T, \tag{4}$$

with the boundary conditions:

$$I_1(T) = E_1, \quad I_2(T) = E_2, \quad I_1(0) = E_1 + Q_1, \quad I_2(0) = E_2 + Q_2, \text{ and } I_1(0) + I_2(0) \leq U. \tag{5}$$

Notice that $D_1(t) + D_2(t) = a_1 + a_2$, for all $0 \leq t \leq T$. Hence, It is clear that $Q_1 + Q_2 = (a_1 + a_2)T$, and thus

$$I_1(0) + I_2(0) = E_1 + E_2 + Q_1 + Q_2 = E_1 + E_2 + (a_1 + a_2)T \leq U. \tag{6}$$

Solving the differential equations (3) and (4) with the boundary conditions $I_1(T) = E_1$ and $I_2(T) = E_2$, we obtain

$$I_1(t) = e^{(b_1+b_2)(T-t)}\left[\frac{b_1E_1 - b_2E}{b_1 + b_2} + \frac{a_1b_1 - a_2b_2}{(b_1 + b_2)^2}\right] + \frac{(a_1 + a_2)b_2}{b_1 + b_2}(T - t) + \frac{b_2(E_1 + E_2)}{b_1 + b_2} - \frac{a_1b_1 - a_2b_2}{(b_1 + b_2)^2}, \tag{7}$$

and

$$I_2(t) = e^{(b_1+b_2)(T-t)}\left[\frac{b_2E_2 - b_1E_1}{b_1 + b_2} + \frac{a_2b_2 - a_1b_1}{(b_1 + b_2)^2}\right] + \frac{(a_1 + a_2)b_1}{b_1 + b_2}(T - t) + \frac{b_1(E_1 + E_2)}{b_1 + b_2} - \frac{a_2b_2 - a_1b_1}{(b_1 + b_2)^2}. \tag{8}$$

To ensure the demand rate $D_i(t)$ for $i = 1$ or 2 is not negative over time t, we get

$$D_1(t) = e^{(b_1+b_2)(T-t)}\left[b_1E_1 - b_2E_2 + \frac{a_1b_1 - a_2b_2}{b_1 + b_2}\right] + \frac{(a_1 + a_2)b_2}{b_1 + b_2} \geq 0, \tag{9}$$

and

$$D_2(t) = e^{(b_1+b_2)(T-t)}\left[b_2E_2 - b_1E_1 + \frac{a_2b_2 - a_1b_1}{b_1 + b_2}\right] + \frac{(a_1 + a_2)b_1}{b_1 + b_2} \geq 0. \tag{10}$$

Combining (9) and (10), we obtain

$$-\frac{(a_1 + a_2)b_1}{b_1 + b_2} \leq e^{(b_1+b_2)(T-t)}\left[b_2E_2 - b_1E_1 - \frac{a_1b_1 - a_2b_2}{b_1 + b_2}\right] \leq \frac{(a_1 + a_2)b_2}{b_1 + b_2}, \quad \text{for all } 0 \leq t \leq T. \tag{11}$$

Since $0 < e^{(b_1+b_2)(T-t)} < e^{(b_1+b_2)T}$ for all $0 < t \leq T$, we can reduce (11) to

$$-\frac{(a_1+a_2)b_1}{b_1+b_2} \le e^{(b_1+b_2)T}\left[b_2E_2-b_1E_1-\frac{a_1b_1-a_2b_2}{b_1+b_2}\right] \le \frac{(a_1+a_2)b_2}{b_1+b_2}$$

Hence, to ensure the demand rate is not negative, the following condition must be satisfied:

$$-\frac{(a_1+a_2)b_1}{b_1+b_2}e^{-(b_1+b_2)T} \le \left[b_2E_2-b_1E_1-\frac{a_1b_1-a_2b_2}{b_1+b_2}\right] \le \frac{(a_1+a_2)b_2}{b_1+b_2}e^{-(b_1+b_2)T}. \tag{12}$$

Applying $I_1(0)=E_1+Q_1$ and $I_2(0)=E_2+Q_2$, we get

$$Q_1 = e^{(b_1+b_2)T}\left[\frac{b_1E_1-b_2E}{b_1+b_2}+\frac{a_1b_1-a_2b_2}{(b_1+b_2)^2}\right]+\frac{(a_1+a_2)b_2T}{b_1+b_2}+\frac{b_2(E_1+E_2)}{b_1+b_2}-\frac{a_1b_1-a_2b_2}{(b_1+b_2)^2}-E_1 = [e^{(b_1+b_2)T}-1]\left[\frac{b_1E_1-b_2E_2}{b_1+b_2}+\frac{a_1b_1-a_2b_2}{(b_1+b_2)^2}\right]+\frac{(a_1+a_2)b_2T}{b_1+b_2}, \tag{13}$$

and

$$Q_2 = e^{(b_1+b_2)T}\left[\frac{b_2E_2-b_1E_1}{b_1+b_2}+\frac{a_2b_2-a_1b_1}{(b_1+b_2)^2}\right]+\frac{(a_1+a_2)b_1T}{b_1+b_2}+\frac{b_1(E_1+E_2)}{b_1+b_2}-\frac{a_2b_2-a_1b_1}{(b_1+b_2)^2}-E_2 = [e^{(b_1+b_2)T}-1]\left[\frac{b_2E_2-b_1E_1}{b_1+b_2}+\frac{a_2b_2-a_1b_1}{(b_1+b_2)^2}\right]+\frac{(a_1+a_2)b_1T}{b_1+b_2}. \tag{14}$$

Hence, the inventory holding cost for Product 1 per replenishment cycle is

$$h_1\int_0^T I_1(t)dt$$

$$= h_1\int_0^T\left\{e^{(b_1+b_2)(T-t)}\left[\frac{b_1E_1-b_2E_2}{b_1+b_2}+\frac{a_1b_1-a_2b_2}{(b_1+b_2)^2}\right]+\frac{(a_1+a_2)b_2}{b_1+b_2}(T-t)+\frac{b_2(E_1+E_2)}{b_1+b_2}-\frac{a_1b_1-a_2b_2}{(b_1+b_2)^2}\right\}dt$$
$$= h_1\left\{\left[\frac{b_1E_1-b_2E_2}{(b_1+b_2)^2}+\frac{a_1b_1-a_2b_2}{(b_1+b_2)^3}\right][e^{(b_1+b_2)T}-1]+\frac{(a_1+a_2)b_2}{2(b_1+b_2)}T^2+\left[\frac{b_2(E_1+E_2)}{b_1+b_2}-\frac{a_1b_1-a_2b_2}{(b_1+b_2)^2}\right]T\right\} \tag{15}$$

Similarly, the inventory holding cost for Product 2 per cycle time is

$$h_2\int_0^T I_2(t)dt$$

$$= h_2\left\{\left[\frac{b_2E_2-b_1E_1}{(b_1+b_2)^2}+\frac{a_2b_2-a_1b_1}{(b_1+b_2)^3}\right][e^{(b_1+b_2)T}-1]+\frac{(a_1+a_2)b_1}{2(b_1+b_2)}T^2+\left[\frac{b_1(E_1+E_2)}{b_1+b_2}-\frac{a_2b_2-a_1b_1}{(b_1+b_2)^2}\right]T\right\}. \tag{16}$$

Consequently, the total annual profit $\Pi\,(E_1, E_2, T)$ is given as

$$\prod(E_1,E_2,T)=\frac{1}{T}\left\{p_1Q_1+p_2Q_2-A-h_1\int_0^T I_1(t)dt-h_2\int_0^T I_2(t)dt\right\}$$
$$=\frac{1}{T}\left\{p_1\left[\left[e^{(b_1+b_2)T}-1\right]\left[\frac{b_1E_1-b_2E_2}{b_1+b_2}+\frac{a_1b_1-a_2b_2}{(b_1+b_2)^2}\right]+\frac{(a_1+a_2)b_2T}{b_1+b_2}\right]\right.$$
$$+p_2\left[\left[e^{(b_1+b_2)T}-1\right]\left[\frac{b_2E_2-b_1E_1}{b_1+b_2}+\frac{a_2b_2-a_1b_1}{(b_1+b_2)^2}\right]+\frac{(a_1+a_2)b_1T}{b_1+b_2}\right]-A$$
$$-h_1\left[[e^{(b_1+b_2)T}-1]\left[\frac{b_1E_1-b_2E_2}{(b_1+b_2)^2}+\frac{a_1b_1-a_2b_2}{(b_1+b_2)^3}\right]+\frac{(a_1+a_2)b_2}{2(b_1+b_2)}T^2+\left[\frac{b_2(E_1+E_2)}{b_1+b_2}-\frac{a_1b_1-a_2b_2}{(b_1+b_2)^2}\right]T\right]$$
$$\left.-h_2\left[[e^{(b_1+b_2)T}-1]\left[\frac{b_2E_2-b_1E_1}{(b_1+b_2)^2}+\frac{a_2b_2-a_1b_1}{(b_1+b_2)^3}\right]+\frac{(a_1+a_2)b_1}{2(b_1+b_2)}T^2+\left[\frac{b_1(E_1+E_2)}{b_1+b_2}-\frac{a_2b_2-a_1b_1}{(b_1+b_2)^2}\right]T\right]\right\}. \tag{17}$$

Thus, the optimization problem here is

$$Max_{E_1,E_2,T}\,\Pi\,(E_1,E_2,T) \tag{18}$$

subject to:

$$-\frac{(a_1+a_2)b_1}{b_1+b_2}e^{-(b_1+b_2)T} \le b_2E_2-b_1E_1-\frac{a_1b_1-a_2b_2}{b_1+b_2} \le \frac{(a_1+a_2)b_2}{b_1+b_2}e^{-(b_1+b_2)T},$$

$$E_1+E_2+(a_1+a_2)T \le U,$$

$E_1 \ge 0E_2 \ge 0$, and $T \ge 0$.

For convenience, we define

$$\omega_1 = p_1 - p_2 + \frac{-h_1+h_2}{b_1+b_2} \tag{19}$$

and

$$\omega_2 = b_2h_1 + b_1h_2 > 0 \tag{20}$$

Taking the first-order partial derivatives of (17) with respect to E_1 and E_2, and rearranging terms, we get:

$$\frac{\partial\prod(E_1,E_2,T)}{\partial E_1} = \frac{b_1\omega_1}{b_1+b_2}\left[\frac{e^{(b_1+b_2)T}-1}{T}\right]-\frac{\omega_2}{b_1+b_2} \equiv K_1 \tag{21}$$

and

$$\frac{\partial\prod(E_1,E_2,T)}{\partial E_2} = -\frac{b_2\omega_1}{b_1+b_2}\left[\frac{e^{(b_1+b_2)T}-1}{T}\right]-\frac{\omega_2}{b_1+b_2} \equiv K_2 \tag{22}$$

Consequently, if $\omega_1 \ge 0$,then $K_1 \ge K_2$,and $K_2 < 0$. Otherwise, we have $K_1 \le K_2$,and $K_1 < 0$.Thus, we have the following three possible cases: (1) ($\omega_1 \ge 0$ and $K_1 \le 0$) or ($\omega_1 \le 0$ and $K_2 \le 0$) (2) $\omega_1 \ge 0$ and $K_1 > 0$ and (3) $\omega_1 \le 0$ and $K_2 > 0$. Let us discuss them separately.

It is important to understand the meanings of K_1 and K_2 before we derive theoretical results. The simple economic interpretations of K_1 and K_2 are as follows. Utilizing the Taylor series expansion of $e^{(b_1+b_2)T} \approx 1+(b_1+b_2)T$, and simplifying terms, we get

$$K_1 = \frac{b_1\omega_1}{b_1+b_2}\left[\frac{e^{(b_1+b_2)T}-1}{T}\right]-\frac{\omega_2}{b_1+b_2} \approx b_1\omega_1-\frac{\omega_2}{b_1+b_2} = b_1(p_1-p_2)-h_1,$$

and

$$K_2 = \frac{-b_2\omega_1}{b_1+b_2}\left[\frac{e^{(b_1+b_2)T}-1}{T}\right]-\frac{\omega_2}{b_1+b_2} \approx -b_2\omega_1-\frac{\omega_2}{b_1+b_2} = b_2(p_2-p_1)-h_2.$$

We know from (1) and (2) that a unit increase in inventory of Product 1 increases not only the inventory cost byh_1, but also the sales of Product 1 by $b_1$units while reducing the sales of Product 2 by $b_1$units. Hence, K_1 represents the profit received from a unit increase in inventory of Product 1. As a result, if $K_1 \le 0$, then building up inventory of Product 1 is not profitable; and vice versa. Similarly, by using the same analogous argument we know that if $K_2 \le 0$, then building up inventory of Product 2 is not profitable; and vice versa.

Case 1. ($\omega_1 \ge 0$ and $K_1 \le 0$) or ($\omega_1 \le 0$ and $K_2 \le 0$)

In this case, both K_1 and K_2 are less than or equal to zero. We can easily obtain the following results.

Theorem 1. *If* ($\omega_1 \ge 0$ *and* $K_1 \le 0$) *or* ($\omega_1 \le 0$ *and* $K_2 \le 0$), *then the optimal values for ending inventory levels,* E_1, E_2, *are either*

$$E_1^* = 0 \text{ and } E_2^* = \max\left\{0, \frac{1}{b_2(b_1+b_2)}\left[a_1b_1-a_2b_2-(a_1+a_2)b_1e^{-(b_1+b_2)T}\right]\right\}, \tag{23}$$

or

$$E_1^* = \max\left\{0, \frac{1}{b_1(b_1+b_2)}\left[a_2b_2 - a_1b_1 - (a_1+a_2)b_2e^{-(b_1+b_2)T}\right]\right\}, E_2^* = 0 \tag{24}$$

Proof. See Appendix A.

A simple economic interpretation of Theorem 1 is as follows: Since both $K_1 \le 0$ and $K_2 \le 0$, then building up inventory of Product 1 or 2 is not profitable. Hence, the ending inventory level for Product 1 or 2 should be as low as possible (i.e., the ending inventory level reaches 0 or the boundary constraint in (12)). From Theorem 1, the optimal values for E_1 and E_2 are one of the following three solutions:

(i) $E_1^* = 0$ and $E_2^* = 0$,
(ii) $E_1^* = 0$ and $E_2^* = [a_1b_1 - a_2b_2 - (a_1+a_2)b_1e^{-(b_1+b_2)T}] / [b_2(b_1+b_2)] > 0$,
or
(iii) $E_1^* = [a_2b_2 - a_1b_1 - (a_1+a_2)b_2e^{-(b_1+b_2)T}] / [b_1(b_1+b_2)] > 0$ and $E_2^* = 0$.

For simplicity, we discuss the first case only. The reader can easily obtain similar results by using the same analogous argument for the other two cases. Substituting $E_1^* = 0$ and $E_2^* = 0$ into (17), and simplifying terms, we get

$$\begin{aligned}
\prod\nolimits_1(T) = \frac{1}{T}\Bigg\{ & p_1\left[\frac{a_1b_1-a_2b_2}{(b_1+b_2)^2}[e^{(b_1+b_2)T}-1] + \frac{(a_1+a_2)b_2T}{b_1+b_2}\right] \\
& + p_2\left[\frac{a_2b_2-a_1b_1}{(b_1+b_2)^2}[e^{(b_1+b_2)T}-1] + \frac{(a_1+a_2)b_1T}{b_1+b_2}\right] - A \\
& - h_1\left[\frac{a_1b_1-a_2b_2}{(b_1+b_2)^3}[e^{(b_1+b_2)T}-1] + \frac{(a_1+a_2)b_2}{2(b_1+b_2)}T^2 - \frac{a_1b_1-a_2b_2}{(b_1+b_2)^2}T\right] \\
& - h_2\left[\frac{a_2b_2-a_1b_1}{(b_1+b_2)^3}[e^{(b_1+b_2)T}-1] + \frac{(a_1+a_2)b_1}{2(b_1+b_2)}T^2 - \frac{a_2b_2-a_1b_1}{(b_1+b_2)^2}T\right]\Bigg\} \\
= \frac{1}{T}\Bigg\{ & \omega_1\left[\frac{a_1b_1-a_2b_2}{(b_1+b_2)^2}[e^{(b_1+b_2)T}-1]\right] - \frac{\omega_2(a_1+a_2)}{2(b_1+b_2)}T^2 - A \\
& + \frac{(a_1+a_2)(b_2p_1+b_1p_2)}{b_1+b_2}T \\
& + (h_1-h_2)\frac{a_1b_1-a_2b_2}{(b_1+b_2)^2}T\Bigg\}
\end{aligned} \tag{25}$$

Theorem 2. $\prod_1(T)$ *in* (25) *is strictly pseudo-concave in* T, *and hence there exists a unique optimal solution* T^*.

Proof. See Appendix B.

The first-order condition for the optimal solution T_1 of $\prod_1(T)$ without (6) and (12) is:

$$\omega_1\frac{a_1b_1-a_2b_2}{b_1+b_2}\left[Te^{(b_1+b_2)T} - \frac{e^{(b_1+b_2)T}-1}{b_1+b_2}\right] - \frac{\omega_2(a_1+a_2)}{2(b_1+b_2)}T^2 + A = 0 \tag{26}$$

Applying Theorem 2 and the two conditions in (6) and (12), we get $T^* = T_1$ if T_1 satisfies both (6) and (12). If T_1 in (26) does not satisfy (6) or (12), then the optimal solution for T_1 is on the boundary point. Since $E_1^* = 0$ and $E_2^* = 0$ in this case, we can reduce (6) and (12) to $(a_1+a_2)T \le U$, and $-(a_1+a_2)b_1e^{-(b_1+b_2)T} \le -(a_1b_1-a_2b_2) \le (a_1+a_2)b_2e^{-(b_1+b_2)T}$, respectively. Thus, we have the following results:

If $a_1b_1 - a_2b_2 > 0$, and $T_1 \ge \min\left\{\frac{U}{a_1+a_2}, \frac{1}{b_1+b_2}\ln\left[\frac{(a_1+a_2)b_1}{a_1b_1-a_2b_2}\right]\right\} \equiv L_1$, then $T^* = L_1$.

If $a_1b_1 - a_2b_2 = 0$, and $T_1 \ge U/(a_1+a_2)$, then. $T^* = U/(a_1+a_2)$.

If $a_1b_1 - a_2b_2 < 0$, and $T_1 \ge \min\left\{\frac{U}{a_1+a_2}, \frac{1}{b_1+b_2}\ln\left[\frac{(a_1+a_2)b_2}{a_2b_2-a_1b_1}\right]\right\} \equiv L_2$, then $T^* = L_2$.

Next, we discuss the case of $\omega_1 \ge 0$ and $K_1 > 0$.

Case 2. $\omega_1 \ge 0$ and $K_1 > 0$

In this case, $K_1 > 0$ and $K_2 < 0$. We can derive the following results.

Theorem 3. *If* $\omega_1 \ge 0$ *and* $K_1 > 0$, *then the optimal*

$$E_1^* = \min\left\{U - (a_1+a_2)T, \frac{1}{b_1+b_2}\left[(a_1+a_2)e^{-(b_1+b_2)T} - \frac{a_1b_1-a_2b_2}{b_1}\right]\right\}, \text{ and } E_2^* = 0. \tag{27}$$

Proof. See Appendix C.

A simple economic interpretation of Theorem 3 is as follows: From the facts $K_1 > 0$ and $K_2 \le 0$, we know that building up inventory of Product 1 is profitable although it is not profitable for Product 2. Consequently, the ending inventory level for Product 1 should be as high as possible (i.e., the ending inventory level reaches one of the two upper bounds of (6) and (12)). Meanwhile, the ending inventory of Product 2 should be 0.

Substituting $E_1^* = \min\left\{U - (a_1+a_2)T, \frac{1}{b_1+b_2}\left[(a_1+a_2)e^{-(b_1+b_2)T} - \frac{a_1b_1-a_2b_2}{b_1}\right]\right\}$ and $E_2^* = 0$ into (17), and rearranging terms, we have

$$\begin{aligned}
\prod\nolimits_2(T) = \frac{1}{T}\Bigg\{ & \omega_1\left[\frac{b_1E_1^*}{b_1+b_2} + \frac{a_1b_1-a_2b_2}{(b_1+b_2)^2}\right][e^{(b_1+b_2)T}-1] - \frac{\omega_2(a_1+a_2)}{2(b_1+b_2)}T^2 \\
& - A + \frac{(a_1+a_2)(b_2p_1+b_1p_2)}{b_1+b_2}T + (h_1-h_2)\frac{a_1b_1-a_2b_2}{(b_1+b_2)^2}T - \omega_2\frac{E_1^*}{b_1+b_2}T\Bigg\}
\end{aligned} \tag{28}$$

Theorem 4. $\prod_2(T)$ *in* (28) *is strictly pseudo-concave in* T, *and hence there exists a unique optimal solution* T^*.

Proof. See Appendix D.

The first-order condition for the optimal solution T_2 of $\prod_2(T)$ without (6) and (12) is:

$$\begin{aligned}
& -b_1\omega_1\frac{a_1+a_2}{b_1+b_2}[e^{(b_1+b_2)T}-1]T + \omega_1\left[b_1E_1^* + \frac{a_1b_1-a_2b_2}{b_1+b_2}\right]e^{(b_1+b_2)T}\left(T - \frac{1}{b_1+b_2}\right) \\
& + \omega_1\left[\frac{b_1E_1^*}{b_1+b_2} + \frac{a_1b_1-a_2b_2}{(b_1+b_2)^2}\right] + \omega_2\frac{a_1+a_2}{2(b_1+b_2)}T^2 + A = 0
\end{aligned} \tag{29}$$

Applying Theorem 4 and the two conditions in (6) and (12), we obtain $T^* = T_2$ if T_2 satisfies both (6) and (12). If T_2 in (29) does not satisfy (6) or (12), then the optimal solution for T_2 is on the boundary point. Consequently, we have the following results:

If $a_1b_1 - a_2b_2 > 0$, and $T_2 \ge \min\left\{\frac{U-E_1^*}{a_1+a_2}, \frac{1}{b_1+b_2}\ln\left[\frac{(a_1+a_2)b_1}{a_1b_1-a_2b_2}\right]\right\} \equiv L_3$, then $T^* = L_3$.

If $a_1b_1 - a_2b_2 = 0$, and $T_2 \ge (U - E_1^*)/(a_1+a_2)$, then $T^* = U/(a_1+a_2)$.

If $a_1b_1 - a_2b_2 < 0$, and $T_2 \ge \min\left\{\frac{U-E_1^*}{a_1+a_2}, \frac{1}{b_1+b_2}\ln\left[\frac{(a_1+a_2)b_2}{a_2b_2-a_1b_1}\right]\right\} \equiv L_4$, then $T^* = L_4$.

Finally, we discuss the case of $\omega_1 \le 0$ and $K_2 > 0$.

Case 3. $\omega_1 \le 0$ and $K_2 > 0$

In this case, $K_2 > 0$ and $K_1 < 0$. Using the same analogous argument as in Case 2, one can derive the following results.

Theorem 5. *If* $\omega_1 \le 0$ *and* $K_2 > 0$, *then the optimal*

$$E_1^* = 0, \text{ and } E_2^* = \min\left\{U - (a_1+a_2)T, \frac{1}{b_1+b_2}\left[(a_1+a_2)e^{-(b_1+b_2)T} + \frac{a_1b_1-a_2b_2}{b_2}\right]\right\}, \tag{30}$$

Proof. See Appendix E.

A simple economic interpretation of Theorem 5 is as follows: We know from $K_1 \le 0$ and $K_2 > 0$that building up inventory of Product 1 is not profitable although it is profitable for Product 2. Therefore, the ending inventory level for Product 1 should be as low as possible (i.e., 0) whereas the ending inventory level of Product 2 should be as high as possible (i.e., it reaches one of the two upper bounds of (6) and (12)).

Combining the results in Theorems 1, 3, and 5, we know that the optimal replenishment time is whenever one of two substitutable products is sold out. As a result, the optimal replenishment policy is easy to implement and understand.

Substituting $E_1^* = 0$ and $E_2^* = \min\left\{U - (a_1 + a_2)T,\ \frac{1}{b_1 + b_2}\left[(a_1 + a_2)e^{-(b_1+b_2)T} + \frac{a_1b_1 - a_2b_2}{b_2}\right]\right\}$ into (17), and rearranging terms, we have

$$\prod\nolimits_3 (T) = \frac{1}{T}\left\{\omega_1\left[\frac{-b_2E_2^*}{b_1+b_2} + \frac{a_1b_1 - a_2b_2}{(b_1+b_2)^2}\right][e^{(b_1+b_2)T} - 1] - \frac{\omega_2(a_1+a_2)}{2(b_1+b_2)}T^2 \right.$$
$$\left. - A + \frac{(a_1+a_2)(b_2p_1 + b_1p_2)}{b_1+b_2}T + (h_1 - h_2)\frac{a_1b_1 - a_2b_2}{(b_1+b_2)^2}T - \omega_2\frac{E_2^*}{b_1+b_2}T\right\} \quad (31)$$

Theorem 6. .

$\prod_3 (T)$ *in* (31) *is strictly pseudo-concave in* T*, and hence there exists a unique optimal solution* T^*.

Proof. See Appendix F.

The first-order condition for the optimal solution T_3 of $\prod_3 (T)$ without (6) and (12) is:

$$b_2\omega_1\frac{a_1+a_2}{b_1+b_2}[e^{(b_1+b_2)T} - 1]T + \omega_1\left[-b_2E_2^* + \frac{a_1b_1 - a_2b_2}{b_1+b_2}\right]e^{(b_1+b_2)T}\left(T - \frac{1}{b_1+b_2}\right)$$
$$+ \omega_1\left[\frac{-b_2E_2^*}{b_1+b_2} + \frac{a_1b_1 - a_2b_2}{(b_1+b_2)^2}\right] + \omega_2\frac{a_1+a_2}{2(b_1+b_2)}T^2 + A = 0 \quad (32)$$

Applying Theorem 6 and the two conditions in (6) and (12), we have $T^* = T_3$ if T_3 satisfies both (6) and (12). If T_3 in (32) does not satisfy (6) or (12), then the optimal solution for T_3 is on the boundary point. Hence, we have the following results:

If $a_1b_1 - a_2b_2 > 0$, and $T_3 \ge \min\left\{\frac{U - E_2^*}{a_1+a_2},\ \frac{1}{b_1+b_2}\ln\left[\frac{(a_1+a_2)b_1}{a_1b_1 - a_2b_2}\right]\right\} \equiv L_5$, then $T^* = L_5$.

If $a_1b_1 - a_2b_2 = 0$, and $T_3 \ge (U - E_2^*)/(a_1 + a_2)$, then $T^* = U/(a_1 + a_2)$.

If $a_1b_1 - a_2b_2 < 0$, and $T_3 \ge \min\left\{\frac{U - E_2^*}{a_1+a_2},\ \frac{1}{b_1+b_2}\ln\left[\frac{(a_1+a_2)b_2}{a_2b_2 - a_1b_1}\right]\right\} \equiv L_6$, then $T^* = L_6$.

In the next section, we provide a couple of numerical examples to illustrate theoretical results as well as to gain managerial insights.

4. Numerical examples

In this section two numerical examples are presented along with their sensitivity analysis from which managerial insights are drawn.

Example 1. The first example refers to products that have symmetrical profits and holding costs, i.e. $p_1 = p_2$,and $h_1 = h_2$. For this example, the following data are used: $a_1 = 200$, $a_2 = 400$,$b_1 = 3$, $b_2 = 6$, $p_1 = 25$, $p_2 = 25$, $h_1 = 5$,$h_2 = 5$, $A = 50$, and $U = 500$. One can easily obtain that $\omega_1 = 0$,. $K_1 = -5$,and $K_2 = -5$. The unique optimal solution (using Theorems 1 and 2) is:

$E_1^* = 20.73$, $E_2^* = 0.00$, $T^* = 0.12$, $Q_1^* = 18.22$, $Q_2^* = 52.81$, and $\Pi^* = 14{,}296.45$.

The demand rates and the inventory levels for Products 1 and 2 are shown in Figs. 1 and 2, respectively. The graphical representation of $\Pi(E_1, E_2, T^* = 0.12)$ is given in Fig. 3, which is a concave function in E_1 and E_2. Similarly, the graphical representation of $\Pi(E_1^* = 20.73, E_2^* = 0, T)$ is shown in Fig. 4 as a strictly pseudo-concave function in T.

Example 2. For the second example, the following data are used: $a_1 = 200$, $a_2 = 400$,$b_1 = 3$, $b_2 = 6$, $p_1 = 25$, $p_2 = 20$, $h_1 = 5$,$h_2 = 4$, $A = 50$, and $U = 500$. One can easily obtain that $\omega_1 = 4.89 > 0$, $K_1 \ge b_1(p_1 - p_2) - h_1 = 10 > 0$,and $K_2 < 0$. By using Theorems 3 and 4, the unique optimal solution is as follows:

$E_1^* = 85.80$, $E_2^* = 0.00$, $T^* = 0.14$, $Q_1^* = 71.32$, $Q_2^* = 11.89$, and $\Pi^* = 13{,}581.20$.

The graphical representation of the optimal $\Pi^*$with $T^* = 0.14$, is given in Fig. 5, which reveals that the total annual profit is a concave function in E_1 and E_2. Similarly, the graphical representation of the optimal $\Pi^*$with $E_1^* = 85.80$, and $E_2^* = 0.00$ is shown in Fig. 6, which reveals that the total annual profit is a strictly pseudo-concave function in T.

Using the data in Example 2, the sensitivity analysis on the optimal solution is carried out with respect to each parameter in the appropriate unit. The computational results are given in Table 2. From these results, the following insights are gained:

1. Product 1 is the dominating product. Hence, the total annual profit $\Pi^*$is extremely sensitive to the variation of p_1, and then a_1. It is obvious that an increase in a_1, a_2, b_1, b_2, p_1, or p_2 increase Π^*. However, an increase in h_1, h_2, or A reduces Π^*.
2. An increase in p_1 causes decreases in Q_1^*, Q_2^*, and T^* while causing an increase in E_1^*. By contrast, an increase in p_2 causes increases in Q_1^*, Q_2^*, and T^* but causes a decrease in E_1^*.
3. An increase in a_1 or a_2 elevates Q_1^*, Q_2^*, and Π^* while reducing T^*. A simple economic interpretation is as follows: From (1) and (2), $a_1$is the number of customers for Product 1 who are not influenced by displayed stocks. Therefore, an increase in a_1 implies demand for Product 1 increases, which in turn increases both order quantity Q_1^*, and total annual profit Π^*while decreasing replenishment cycle time T^*.
4. A higher value of b_1 causes a higher value of Q_2^* while lowering values of Q_1^* and E_1^*. However, a higher value of b_2 causes the opposite reactions.
5. The higher the holding cost h_1 or h_2, the lower the order quantity Q_1^* or Q_2^*. In contrast, the higher the ordering cost A, the higher the order quantity Q_1^* or Q_2^*.
6. An increase in U increases Q_1^*, E_1^*, T^*, and Π^*while decreasing Q_2^*. If Product 1 is the dominating product, then an increase in shelf space U elevates order quantity Q_1^*, and ending stock level E_1^* for Product 1, and thus increases both replenishment cycle time T^* and total annual profit Π^*.

5. Conclusions

As the standard of living continues to improve, consumers become more health conscious. Hence, it is very important for retailers better to manage substitutable products. To reflect the fact that the amount of on-hand stocks stimulates the demand rate, we have proposed an inventory model with two inventory-based substitutable products in which the demand for one product is positively influenced by its own stock level while negatively influenced by the stock level of the other product. In addition to maximizing profit, we have generalized the traditional ending-inventory level from zero to non-negative. Then we have derived the optimal reorder interval and ending-stock levels for

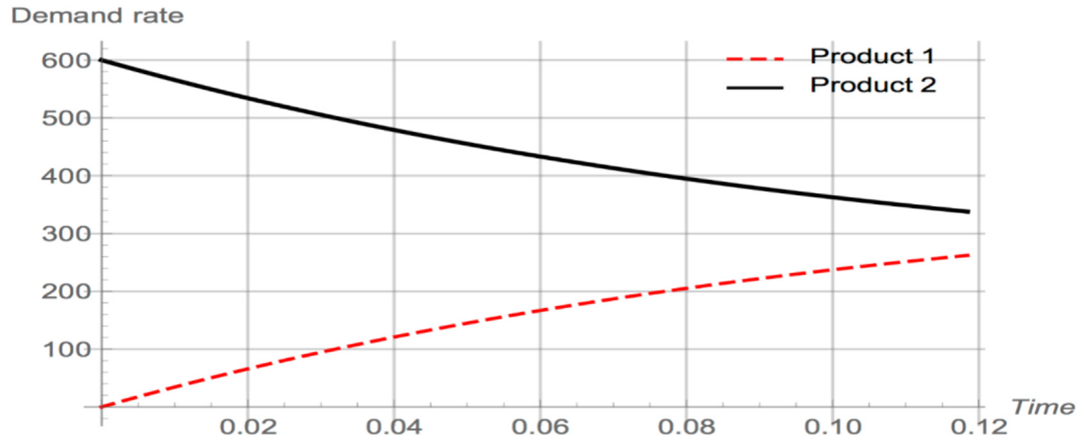

Fig. 1. Demand rates of Products 1 and 2 for Example 1.

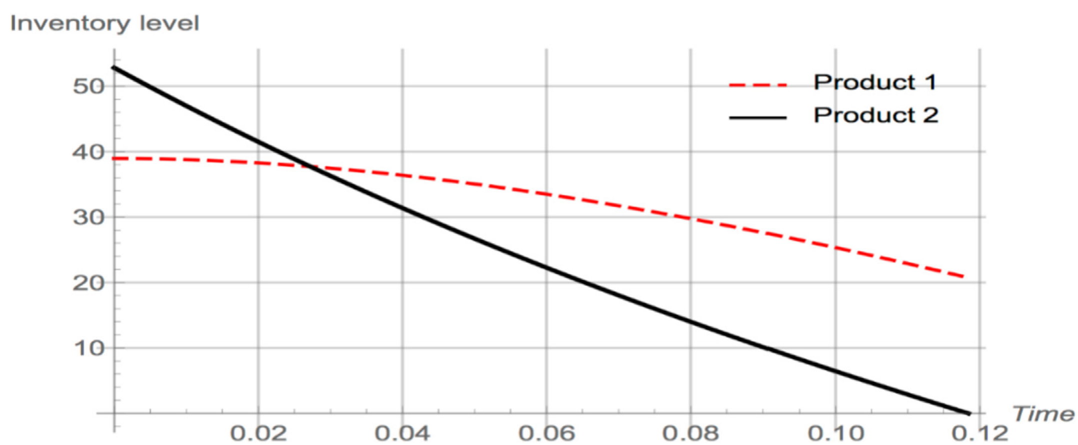

Fig. 2. Inventory levels of Products 1 and 2 for Example 1.

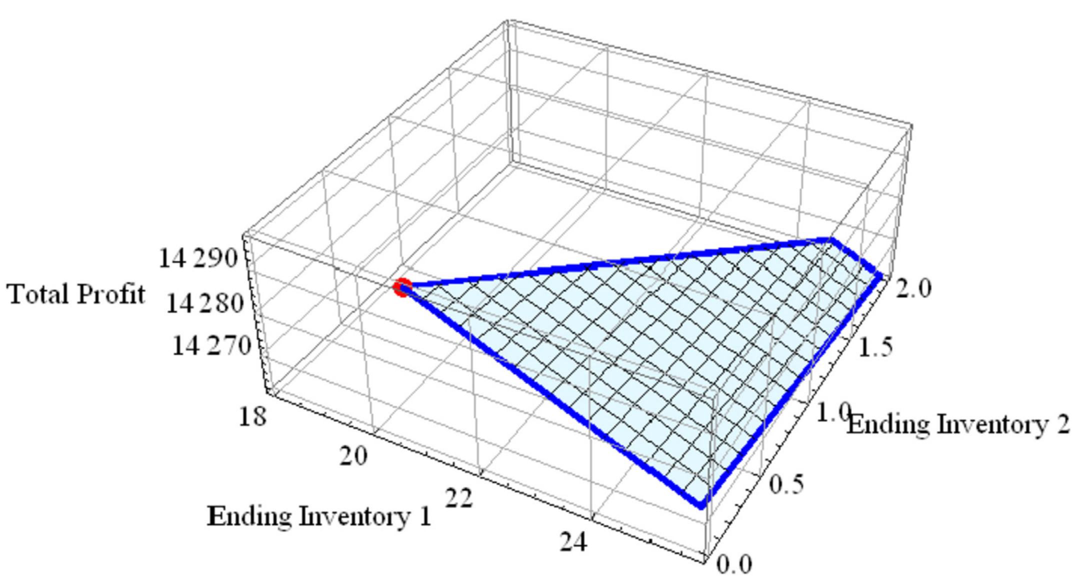

Fig. 3. Graphical representation of Π (E_1, E_2, 0.12)

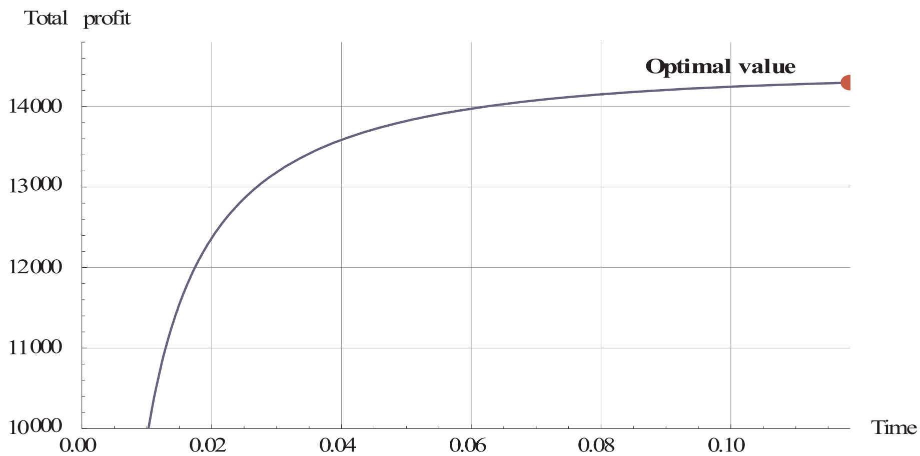

Fig. 4. Graphical representation of Π (20.73, 0, T)

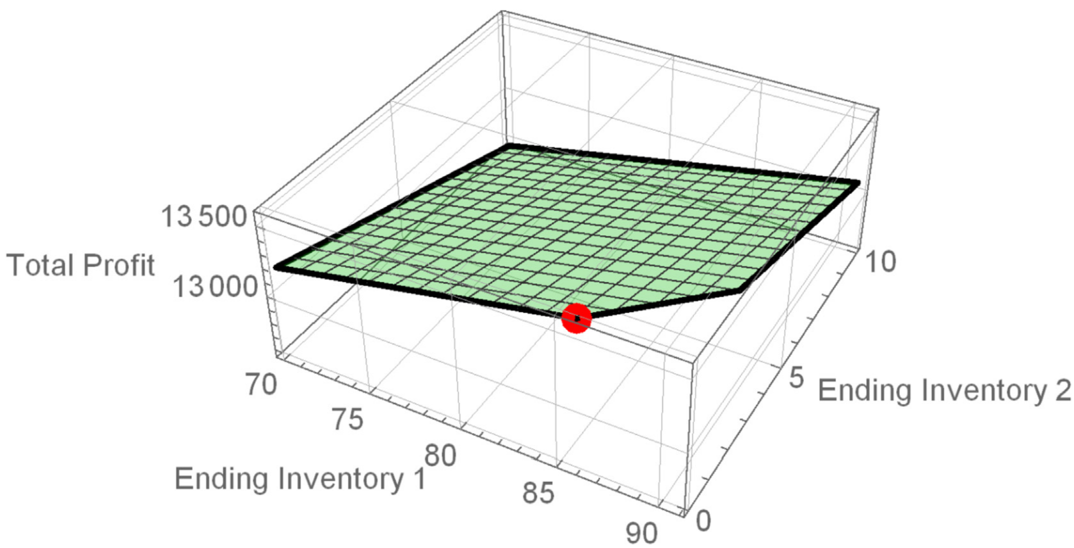

Fig. 5. Graphical representation of Π (E_1, E_2, 0.14)

both products to maximize total annual profit. We have demonstrated that the total annual profit is strictly pseudo-concave with respect to the decision variables, which simplifies the search for the global solution to a local optimum. In addition, we have shown that one of the two ending stock levels must be zero due to the substitution effect. Furthermore, we have used common sense to explain theoretical results. Finally, we have provided numerical examples and sensitivity analyses to illustrate the problem and highlight managerial implications.

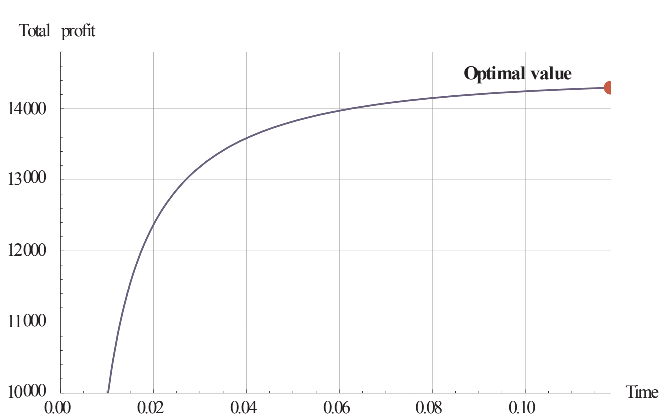

Fig. 6. Graphical representation of Π (85.80, 0, T)

Opportunities for future research are plentiful. For example, price is a major factor that impacts demand according to traditional marketing and economic theory. Hence, the proposed model can be expanded by considering pricing strategies for both products. Since most firms in the UK and US offer their products on various short-term interest-free loans (i.e., trade credit or credit term), we may generalize the present model by taking trade credit into consideration such as in Skouri et al. (2011), and Mahata (2012). Finally, we could incorporate coordination policies such as advertising, trade credit, etc., among members of the supply chain (e.g., the supplier, the retailer, and the customers) into the current model as well.

Table 2
Sensitivity analysis for Example 2.

Parameter	ω_1	K_1	K_2	Q_1^*	Q_2^*	E_1^*	E_2^*	T^*	Π^*
$a_1 = 100$	4.89	26.96	−67.93	65.00	11.71	91.74	0.00	0.15	11,149.48
$a_1 = 200$	4.89	24.52	−63.04	71.32	11.89	85.80	0.00	0.14	13,581.20
$a_1 = 300$	4.89	22.81	−59.62	77.16	12.04	80.26	0.00	0.13	16,018.48
$a_2 = 300$	4.89	26.96	−67.93	65.00	11.71	58.41	0.00	0.15	11,316.15
$a_2 = 400$	4.89	24.52	−63.04	71.32	11.89	85.80	0.00	0.14	13,581.20
$a_2 = 500$	4.89	22.81	−59.62	77.16	12.04	113.59	0.00	0.13	15,851.82
$b_1 = 2$	4.88	15.08	−64.25	85.81	10.52	145.76	0.00	0.16	13,397.90
$b_1 = 3$	4.89	24.52	−63.04	71.32	11.89	85.80	0.00	0.14	13,581.20
$b_1 = 4$	4.90	34.11	−62.67	61.50	12.63	57.44	0.00	0.12	13,619.73
$b_2 = 4$	4.86	19.81	−37.08	69.33	12.47	80.63	0.00	0.14	13,576.38
$b_2 = 6$	4.89	24.52	−63.04	71.32	11.89	85.80	0.00	0.14	13,581.20
$b_2 = 8$	4.91	30.65	−99.07	73.43	11.40	90.31	0.00	0.14	13,586.74
$p_1 = 25$	4.89	24.52	−63.04	71.32	11.89	85.80	0.00	0.14	13,581.20
$p_1 = 30$	9.89	42.71	−99.42	51.74	6.46	94.51	0.00	0.08	16,204.66
$p_1 = 35$	14.89	59.55	−133.10	41.57	4.22	100.21	0.00	0.08	18,902.13
$p_2 = 10$	14.89	59.55	−133.10	41.57	4.22	100.21	0.00	0.08	12,902.13
$p_2 = 15$	9.89	42.71	−99.42	51.74	6.46	94.51	0.00	0.10	13,204.66
$p_2 = 20$	4.89	24.52	−63.04	71.32	11.89	85.80	0.00	0.14	13,581.20
$h_1 = 3$	5.11	27.61	−65.23	72.50	12.26	85.36	0.00	0.14	13,821.01
$h_1 = 5$	4.89	24.52	−63.04	71.32	11.89	85.80	0.00	0.14	13,581.20
$h_1 = 7$	4.67	21.54	−61.09	70.35	11.59	86.17	0.00	0.14	13,341.58
$h_2 = 2$	4.67	24.72	−61.45	73.88	12.70	84.86	0.00	0.14	13,598.79
$h_2 = 4$	4.89	24.52	−63.04	71.32	11.89	85.80	0.00	0.14	13,581.20
$h_2 = 6$	5.11	24.04	−64.80	69.11	11.21	86.65	0.00	0.13	13,566.53
$A = 30$	4.89	19.76	−53.51	55.63	7.43	92.56	0.00	0.11	13,745.55
$A = 50$	4.89	24.52	−63.04	71.32	11.89	85.80	0.00	0.14	13,581.20
$A = 70$	4.89	29.49	−72.97	84.19	16.16	81.47	0.00	0.17	13,450.32
$U = 100$	4.89	15.41	−44.83	24.81	15.10	60.09	0.00	0.07	12,720.50
$U = 300$	4.89	24.52	−63.04	71.32	11.89	85.80	0.00	0.14	13,581.20
$U = 500$	4.89	24.52	−63.04	71.32	11.89	85.80	0.00	0.14	13,581.20

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Appendix A. Proof of Theorem 1

If $\omega_1 \geq 0$ and $K_1 \leq 0$, then $K_2 \leq K_1 \leq 0$. We know from (21) and (22) that $\Pi(E_1, E_2, T)$is a non-increasing function in both E_1 and E_2, which implies the optimal solution lies on the boundary conditions.

If $E_1^* = 0$, then we know from (12) that

$$-\frac{(a_1 + a_2)b_1}{b_1 + b_2}e^{-(b_1+b_2)T} \leq b_2 E_2 - \frac{a_1 b_1 - a_2 b_2}{b_1 + b_2}. \tag{A1}$$

Rearranging and simplifying terms, we have

$$E_2 \geq \frac{1}{b_2(b_1 + b_2)}\left[a_1 b_1 - a_2 b_2 - (a_1 + a_2)b_1 e^{-(b_1+b_2)T}\right] \tag{A2}$$

Hence, if $E_1^* = 0$, then

$$E_2^* = \max\left\{0, \frac{1}{b_2(b_1 + b_2)}\left[a_1 b_1 - a_2 b_2 - (a_1 + a_2)b_1 e^{-(b_1+b_2)T}\right]\right\} \tag{A3}$$

Likewise, if $E_2^* = 0$, from (12) we get

$$-b_1 E_1 - \frac{a_1 b_1 - a_2 b_2}{b_1 + b_2} \leq \frac{(a_1 + a_2)b_2}{b_1 + b_2}e^{-(b_1+b_2)T}. \tag{A4}$$

Multiplying by negative one on both sides, and simplifying terms, we obtain

$$E_1 \geq \frac{1}{b_1(b_1 + b_2)}\left[a_2 b_2 - a_1 b_1 - (a_1 + a_2)b_2 e^{-(b_1+b_2)T}\right] \tag{A5}$$

Similarly, if $E_2^* = 0$, then

$$E_1^* = \max\left\{0, \frac{1}{b_1(b_1+b_2)}\left[a_2b_2 - a_1b_1 - (a_1+a_2)b_2e^{-(b_1+b_2)T}\right]\right\} \tag{A6}$$

This completes the Proof of Theorem 1.

Appendix B. Proof of Theorem 2

Applying (25), we define

$$f_1(T) = \omega_1\left\{\frac{a_1b_1 - a_2b_2}{(b_1+b_2)^2}\left[e^{(b_1+b_2)T} - 1\right]\right\} - \frac{\omega_2(a_1+a_2)}{2(b_1+b_2)}T^2 - A + \frac{(a_1+a_2)(b_2p_1+b_1p_2)}{b_1+b_2}T + (h_1-h_2)\frac{a_1b_1-a_2b_2}{(b_1+b_2)^2}T \tag{B1}$$

and

$$g_1(T) = T. \tag{B2}$$

Taking the first-order and second-order derivatives of $f_1(T)$ with respect to T, and rearranging terms, we derive

$$f_1'(T) = \omega_1\left[\frac{a_1b_1-a_2b_2}{b_1+b_2}e^{(b_1+b_2)T}\right] - \frac{\omega_2(a_1+a_2)}{b_1+b_2}T + \frac{(a_1+a_2)(b_2p_1+b_1p_2)}{b_1+b_2} + (h_1-h_2)\frac{a_1b_1-a_2b_2}{(b_1+b_2)^2} \tag{B3}$$

and

$$f_1''(T) = \omega_1(a_1b_1 - a_2b_2)e^{(b_1+b_2)T} - \frac{\omega_2(a_1+a_2)}{b_1+b_2}. \tag{B4}$$

By using (12), we get

$$(a_1+a_2)b_1e^{-(b_1+b_2)T} \geq a_1b_1 - a_2b_2. \tag{B5}$$

Substituting (B5) into (B4), applying (20), and simplifying terms, we obtain

$$f_1''(T) \leq (a_1+a_2)\left(b_1\omega_1 - \frac{\omega_2}{b_1+b_2}\right) < 0 \tag{B6}$$

By applying the fraction concave function (e.g., see Cambini and Martein (2009, p.245), we know that $\prod_1(T) = f_1(T)/g_1(T)$ is strictly pseudo-concave in T, which completes the Proof.

Appendix C. Proof of Theorem 3

If $\omega_1 \geq 0$, then $K_2 < 0$. From (22), we know that $\Pi(E_1, E_2, T)$is a decreasing function in E_2, which implies the optimal $E_2^* = 0$. However, $K_1 > 0$, which implies that $\Pi(E_1, E_2, T)$is an increasing function in E_1. Consequently, from (6) and (12), the optimal solution E_1^* lies on the boundary point which satisfies the following two constraints:

$$E_1 + (a_1+a_2)T \leq U, \tag{C1}$$

and

$$b_1E_1 + \frac{a_1b_1 - a_2b_2}{b_1+b_2} \leq \frac{(a_1+a_2)b_1}{b_1+b_2}e^{-(b_1+b_2)T}. \tag{C2}$$

Combining (C1) and (C2), and simplifying terms, we derive

$$E_1^* = \min\left\{U - (a_1+a_2)T,\ \frac{1}{b_1+b_2}\left[(a_1+a_2)e^{-(b_1+b_2)T} - \frac{a_1b_1-a_2b_2}{b_1}\right]\right\} \tag{C3}$$

This completes the Proof.

Appendix D. Proof of Theorem 4

Let us assume $E_1^* = U - (a_1+a_2)T$ first, and then. $E_1^* = \frac{1}{b_1+b_2}\left[(a_1+a_2)e^{-(b_1+b_2)T} - \frac{a_1b_1-a_2b_2}{b_1}\right]$.

If $E_1^* = U - (a_1+a_2)T$, then applying (28), we define

$$f_{21}(T) = \omega_1\left[\frac{b_1[U-(a_1+a_2)T]}{b_1+b_2} + \frac{a_1b_1-a_2b_2}{(b_1+b_2)^2}\right]\left[e^{(b_1+b_2)T} - 1\right] - \frac{\omega_2(a_1+a_2)}{2(b_1+b_2)}T^2 - A + \frac{(a_1+a_2)(b_2p_1+b_1p_2)}{b_1+b_2}T + (h_1-h_2)\frac{a_1b_1-a_2b_2}{(b_1+b_2)^2}T - \omega_2\frac{U-(a_1+a_2)T}{b_1+b_2}T \tag{D1}$$

and

$$g_{21}(T) = T. \tag{D2}$$

We have $\prod_2(T) = f_{21}(T)/g_{21}(T)$. Taking the first-order and second-order derivatives of $f_{21}(T)$ with respect to T, and rearranging terms, we derive

$$f'_{21}(T) = -b_1\omega_1\tfrac{a_1+a_2}{b_1+b_2}[e^{(b_1+b_2)T}-1] + \omega_1\left[b_1[U-(a_1+a_2)T] + \tfrac{a_1b_1-a_2b_2}{b_1+b_2}\right] e^{(b_1+b_2)T} - \tfrac{\omega_2(a_1+a_2)}{b_1+b_2}T + \tfrac{(a_1+a_2)(b_2p_1+b_1p_2)}{b_1+b_2} + (h_1-h_2)\tfrac{a_1b_1-a_2b_2}{(b_1+b_2)^2} - \tfrac{\omega_2}{b_1+b_2}[U-2(a_1+a_2)T] \tag{D3}$$

and

$$f''_{21}(T) = -2(a_1+a_2)b_1\omega_1e^{(b_1+b_2)T} + (b_1+b_2)\omega_1\left[b_1[U-(a_1+a_2)T] + \tfrac{a_1b_1-a_2b_2}{b_1+b_2}\right]e^{(b_1+b_2)T} + \tfrac{\omega_2(a_1+a_2)}{b_1+b_2} \tag{D4}$$

By using (12), we get

$$\frac{(a_1+a_2)b_1}{b_1+b_2}e^{-(b_1+b_2)T} \geq b_1[U-(a_1+a_2)T] + \frac{a_1b_1-a_2b_2}{b_1+b_2}. \tag{D5}$$

$$f''_{21}(T) \leq -2(a_1+a_2)b_1\omega_1e^{(b_1+b_2)T} + (a_1+a_2)b_1\omega_1 + \tfrac{\omega_2(a_1+a_2)}{b_1+b_2} = -(a_1+a_2)\left\{b_1\omega_1[2e^{(b_1+b_2)T}-1] - \tfrac{\omega_2}{b_1+b_2}\right\}. \tag{D6}$$

Utilizing the Taylor series expansion that $2e^{(b_1+b_2)T}-1 = 1+2(b_1+b_2)T+(b_1+b_2)^2T^2+\ldots$, we get:

$$\frac{e^{(b_1+b_2)T}-1}{(b_1+b_2)T} = 1+\frac{1}{2}(b_1+b_2)T+\frac{1}{6}(b_1+b_2)^2T^2+\frac{1}{24}(b_1+b_2)^3T^3+\ldots \tag{D7}$$

It is clear from (D6) and (D7) that

$$2e^{(b_1+b_2)T}-1 \geq \frac{e^{(b_1+b_2)T}-1}{(b_1+b_2)T} \tag{D8}$$

and

$$f''_{21}(T) \leq -(a_1+a_2)\left\{b_1\omega_1[2e^{(b_1+b_2)T}-1] - \tfrac{\omega_2}{b_1+b_2}\right\} \leq -(a_1+a_2)\left\{\tfrac{b_1\omega_1}{b_1+b_2}\left[\tfrac{e^{(b_1+b_2)T}-1}{T}\right] - \tfrac{\omega_2}{b_1+b_2}\right\} = -(a_1+a_2)K_1 < 0 \tag{D9}$$

Consequently, by applying the fraction concave function (e.g., see Cambini and Martein (2009, p.245), we prove that $\prod_2(T) = f_{21}(T)/g_{21}(T)$ is strictly pseudo-concave in T.

Next, we discuss the other case in which $E_1^* = \frac{1}{b_1+b_2}\left[(a_1+a_2)e^{-(b_1+b_2)T} - \frac{a_1b_1-a_2b_2}{b_1}\right]$.

Similarly, let us define

$$f_{22}(T) = \tfrac{(a_1+a_2)b_1\omega_1}{(b_1+b_2)^2}[1-e^{-(b_1+b_2)T}] - \tfrac{(a_1+a_2)\omega_2}{2(b_1+b_2)}T^2 - A + \tfrac{(a_1+a_2)(b_2p_1+b_1p_2)}{b_1+b_2}T + (h_1-h_2)\tfrac{a_1b_1-a_2b_2}{(b_1+b_2)^2}T - \tfrac{\omega_2T}{(b_1+b_2)^2}\left[(a_1+a_2)e^{-(b_1+b_2)T} - \tfrac{a_1b_1-a_2b_2}{b_1}\right] \tag{D10}$$

and

$$g_{22}(T) = T. \tag{D11}$$

Then we get $\prod_2(T) = f_{22}(T)/g_{22}(T)$. Taking the first-order and second-order derivatives of $f_{22}(T)$ with respect to T, and rearranging terms, we obtain

$$f'_{22}(T) = \tfrac{(a_1+a_2)b_1\omega_1}{b_1+b_2}e^{-(b_1+b_2)T} - \tfrac{(a_1+a_2)\omega_2}{b_1+b_2}T + \tfrac{(a_1+a_2)(b_2p_1+b_1p_2)}{b_1+b_2} + (h_1-h_2)\tfrac{a_1b_1-a_2b_2}{(b_1+b_2)^2} - \tfrac{\omega_2}{(b_1+b_2)^2}\left[(a_1+a_2)e^{-(b_1+b_2)T} - \tfrac{a_1b_1-a_2b_2}{b_1}\right] + \tfrac{\omega_2T}{b_1+b_2}(a_1+a_2)e^{-(b_1+b_2)T} \tag{D12}$$

and

$$\begin{aligned} f''_{22}(T) &= -(a_1+a_2)b_1\omega_1e^{-(b_1+b_2)T} - \tfrac{(a_1+a_2)\omega_2}{b_1+b_2} + \tfrac{2(a_1+a_2)\omega_2}{b_1+b_2}e^{-(b_1+b_2)T} - (a_1+a_2)\omega_2Te^{-(b_1+b_2)T} \\ &= -(a_1+a_2)e^{-(b_1+b_2)T}\left(b_1\omega_1 - \tfrac{\omega_2}{b_1+b_2}\right) - \tfrac{(a_1+a_2)\omega_2}{b_1+b_2}e^{-(b_1+b_2)T}[e^{(b_1+b_2)T}-1] \\ &\approx -(a_1+a_2)e^{-(b_1+b_2)T}K_1 - \tfrac{(a_1+a_2)\omega_2}{b_1+b_2}e^{-(b_1+b_2)T}[e^{(b_1+b_2)T}-1] - (a_1+a_2)\omega_2Te^{-(b_1+b_2)T} < 0. \end{aligned} \tag{D13}$$

Likewise, applying the fraction concave function (e.g., see Cambini and Martein (2009, p.245), we demonstrate that $\prod_2(T) = f_{22}(T)/g_{22}(T)$ is strictly pseudo-concave in T. This completes the Proof.

Appendix EProof of Theorem 5

If $\omega_1 \leq 0$, then $K_1 < 0$. We know from (21) that $\Pi(E_1, E_2, T)$is a decreasing function in E_1, and hence the optimal $E_1^* = 0$. However, $K_2 > 0$. From (22) we know that $\Pi(E_1, E_2, T)$is an increasing function in E_2. Therefore, from (6) and (12), the optimal solution E_2^* lies on the boundary point which satisfies the following two constraints:

$$E_2 + (a_1+a_2)T \leq U, \tag{E1}$$

and

$$b_2E_2 - \frac{a_1b_1 - a_2b_2}{b_1 + b_2} \le \frac{(a_1 + a_2)b_2}{b_1 + b_2}e^{-(b_1+b_2)T}. \tag{E2}$$

Combining (E1) and (E2), and simplifying terms, we derive

$$E_2^* = \min\left\{U - (a_1 + a_2)T,\ \frac{1}{b_1 + b_2}\left[(a_1 + a_2)e^{-(b_1+b_2)T} + \frac{a_1b_1 - a_2b_2}{b_2}\right]\right\}. \tag{E3}$$

The proof is complete.

Appendix F. Proof of Theorem 6

From Theorem 5, the optimal E_2^* has two possible solutions:

$$E_2^* = U - (a_1 + a_2)T,$$

or

$$E_2^* = \frac{1}{b_1 + b_2}\left[(a_1 + a_2)e^{-(b_1+b_2)T} + \frac{a_1b_1 - a_2b_2}{b_2}\right]$$

We prove the first case only because the Proof of the other case is similar to that in Appendix D. From (31), we define

$$\begin{aligned} f_3(T) &= \omega_1\left[\frac{-b_2[U-(a_1+a_2)T]}{b_1+b_2} + \frac{a_1b_1-a_2b_2}{(b_1+b_2)^2}\right][e^{(b_1+b_2)T} - 1] - \frac{\omega_2(a_1+a_2)}{2(b_1+b_2)}T^2 \\ &- A + \frac{(a_1+a_2)(b_2p_1+b_1p_2)}{b_1+b_2}T + \left(h_1 - h_2\right)\frac{a_1b_1-a_2b_2}{(b_1+b_2)^2}T - \omega_2\frac{U-(a_1+a_2)T}{b_1+b_2}T \end{aligned} \tag{F1}$$

and

$$g_3(T) = T. \tag{F2}$$

Taking the first-order and second-order derivatives of $f_3(T)$ with respect to T, and rearranging terms, we have

$$\begin{aligned} f_3'(T) &= b_2\omega_1\frac{a_1+a_2}{b_1+b_2}[e^{(b_1+b_2)T} - 1] + \omega_1\left[-b_2[U - (a_1 + a_2)T] + \frac{a_1b_1-a_2b_2}{b_1+b_2}\right]e^{(b_1+b_2)T} - \frac{\omega_2(a_1+a_2)}{b_1+b_2}T \\ &+ \frac{(a_1+a_2)(b_2p_1+b_1p_2)}{b_1+b_2} + (h_1 - h_2)\frac{a_1b_1-a_2b_2}{(b_1+b_2)^2} - \frac{\omega_2}{b_1+b_2}[U - 2(a_1 + a_2)T] \end{aligned} \tag{F3}$$

and

$$\begin{aligned} f_3''(T) &= 2(a_1 + a_2)b_2\omega_1e^{(b_1+b_2)T} + (b_1 + b_2)\omega_1\left[-b_2[U - (a_1 + a_2)T] + \frac{a_1b_1-a_2b_2}{b_1+b_2}\right]e^{(b_1+b_2)T} \\ &+ \frac{\omega_2(a_1+a_2)}{b_1+b_2} \end{aligned} \tag{F4}$$

By using (12), we get

$$-b_2[U - (a_1 + a_2)T] + \frac{a_1b_1 - a_2b_2}{b_1 + b_2} \ge -\frac{(a_1 + a_2)b_2}{b_1 + b_2}e^{-(b_1+b_2)T}. \tag{F5}$$

Substituting (F5) into (F4), utilizing $\omega_1 \le 0$, and simplifying terms, we obtain

$$\begin{aligned} f_3''(T) &\le 2(a_1 + a_2)b_2\omega_1e^{(b_1+b_2)T} - (a_1 + a_2)b_2\omega_1 + \frac{\omega_2(a_1+a_2)}{b_1+b_2} \\ &= (a_1 + a_2)\left\{b_2\omega_1[2e^{(b_1+b_2)T} - 1] + \frac{\omega_2}{b_1+b_2}\right\} \end{aligned} \tag{F6}$$

Applying the result from (D8) into (F6), we have

$$\begin{aligned} f_3''(T) &\le (a_1 + a_2)\left\{b_2\omega_1[2e^{(b_1+b_2)T} - 1] + \frac{\omega_2}{b_1+b_2}\right\} \le (a_1 + a_2)\left\{\frac{b_2\omega_1}{b_1+b_2}\left[\frac{e^{(b_1+b_2)T}-1}{T}\right] + \frac{\omega_2}{b_1+b_2}\right\} \\ &= -(a_1 + a_2)K_2 < 0 \end{aligned} \tag{F7}$$

Consequently, by applying the fraction concave function (e.g., see Cambini and Martein (2009, p.245), we prove that $\prod_3(T) = f_3(T)/g_3(T)$ is strictly pseudo-concave in T. This completes the Proof of Theorem 6.

References

Baker, R.C., Urban, T.L., 1988. A deterministic inventory system with an inventory level dependent demand rate. J. Oper. Res. Soc. 39, 823–831.
Cambini, A., Martein, L., 2009. Generalized Convexity and Optimization: Theory and Application. Springer-Verlag Berlin Heidelberg, USA.
Chen, S.C., Min, J., Teng, J.T., Li, F., 2016. Inventory and shelf-space management for fresh produce with freshness-and-stock dependent demand and expiration date. J. Oper. Res. Soc. 67 (6), 884–896.
Chang, C.T., Teng, J.T., Goyal, S.K., 2010. Optimal replenishment policies for non-instantaneous deteriorating items with stock-dependent demand. Int. J. Prod. Econ. 123, 62–68.
Dye, C.Y., Hsieh, T.P., 2011. Deterministic ordering policy with price- and stock-dependent demand under fluctuating cost and limited capacity. Expert Syst. Appl. 38, 14976–14983.
Ernst, R., Kouvelis, P., 1999. The effects of selling packaged goods on inventory decisions. Manag. Sci. 45, 1142–1155.
Feng, L., Chan, Y.-L., Cárdenas-Barrón, L.E., 2017. Pricing and lot-sizing policies for perishable goods when the demand depends on selling price, displayed stocks, and

expiration date. Int. J. Prod. Econ. 185, 11–20.
Goyal, S.K., Chang, C.T., 2009. Optimal ordering and transfer policy for an inventory with stock dependent demand. Eur. J. Oper. Res. 196, 177–185.
Gurler, U., Yilmaz, A., 2010. Inventory and coordination issues with two substitutable products. Appl. Math. Model. 34, 539–551.
Hsieh, C.C., Wu, C.H., 2009. Coordinated decisions for substitutable products in a common retailer supply chain. Eur. J. Oper. Res. 196, 273–288.
Hsieh, T.P., Dye, C.Y., 2010. Optimal replenishment policy for perishable items with stock-dependent selling rate and capacity constraint. Comput. Ind. Eng. 59, 251–258.
Krommyda, I.P., Skouri, K., Konstantaras, I., 2015. Optimal ordering quantities for substitutable products with stock-dependent demand. Appl. Math. Model. 39, 147–164.
Levin, R.I., McLaughlin, C.P., Lamone, R.P., Kottas, J.F., 1972. Productions/Operations Management: Contemporary Policy for Managing Operating Systems. McGraw- Hill, New York, pp. 373.
Mahata, G.C., 2012. An EPQ-based inventory model for exponentially deteriorating items under retailer partial trade credit policy in supply chain. Expert Syst. Appl. 39, 3537–3550.
Mandal, B.N., Phaujdar, S., 1989. An inventory model for deteriorating items and stock-dependent consumption rate. J. Oper. Res. Soc. 40, 483–488.
Maity, K., Maiti, M., 2009. Optimal inventory policies for deteriorating complementary and substitute items. Int. J. Syst. Sci. 40, 267–276.
McGillivray, A.R., Silver, E.A., 1978. Some concepts for inventory control under substitutable demand. Information Systems and Operational Research 16, 47–63.
Nagarajan, M., Rajagopalan, S., 2008. Inventory models for substitutable products: optimal policies and heuristics. Manag. Sci. 54, 1453–1466.
Netessine, S., Rudi, N., 2003. Centralized and competitive inventory models with demand substitution. Oper. Res. 51, 329–335.
Parlar, M., 1988. Game theoretic analysis of the substitutable product inventory problem with random demands. Nav. Res. Logist. 35, 397–409.
Parlar, M., Goyal, S.K., 1984. Optimal ordering decisions for two substitutable products with stochastic demands. Opsearch 21, 1–15.
Rajaram, K., Tang, C., 2001. The impact of product substitution on retail merchandising. Eur. J. Oper. Res. 135, 582–601.
Shin, H., Park, S., Lee, E., Benton, W.C., 2015. A classification of the literature on the planning of substitutable products. Eur. J. Oper. Res. 246, 686–699.
Silver, E.A., Peterson, R., 1985. Decision Systems for Inventory Management and Production Planning, second ed. Wiley, New York.
Skouri, K., Konstantaras, I., Papachristos, S., Teng, J.T., 2011. Supply chain models for deteriorating products with ramp type demand rate under permissible delay in payments. Expert Syst. Appl. 38, 14861–14869.
Stavrulaki, E., 2011. Inventory decisions for substitutable products with stock-dependent demand. Int. J. Prod. Econ. 129, 65–78.
Teng, J.T., Krommyda, I.P., Skouri, K., Lou, K.R., 2011. A comprehensive extension of optimal ordering policy for stock-dependent demand under progressive payment scheme. Eur. J. Oper. Res. 215, 97–104.
Urban, T.L., 1992. An inventory model with an inventory-level-dependent demand rate and relaxed terminal conditions. J. Oper. Res. Soc. 43, 721–724.
Urban, T.L., 2005. Inventory models with inventory-level-dependent demand: a comprehensive review and unifying theory. Eur. J. Oper. Res. 162, 792–804.
Wu, J., Chang, C.T., Cheng, M.C., Teng, J.T., Al-khateeb, F.B., 2016. Inventory management for fresh produce when the time-varying demand depends on product freshness, stock level and expiration date. Int. J. Syst. Sci.: Operations & Logistics 3 (3), 138–147.
Wu, J., Skouri, K., Teng, J.T., Ouyang, L.Y., 2014. A note on “optimal replenishment policies for non-instantaneous deteriorating items with price and stock sensitive demand under permissible delay in payment”. Int. J. Prod. Econ. 155, 324–329.
Yang, H.L., Teng, J.T., Chern, M.S., 2010. An inventory model under inflation for deteriorating items with stock-dependent consumption rate and partial backlogging shortages. Int. J. Prod. Econ. 123, 8–19.