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Growth, intergenerational welfare, and environmental policies in an overlapping generations economy

Hsun Chu[1] | **Chu-chuan Cheng**[2] | **Ching-chong Lai**[3,4,5]

[1]Department of Economics, Tunghai University, Taiwan

[2]Department of Accounting, Feng Chia University, Taiwan

[3]Institute of Economics, Academia Sinica, Taiwan

[4]Department of Economics, National Cheng Chi University, Taiwan

[5]Institute of Economics, National Sun Yat-Sen University, Taiwan

Correspondence
Hsun Chu, Department of Economics, Tunghai University, No. 1727, Sec. 4, Xitun Dist., Taiwan Boulevard, Taichung, 40704 Taiwan.
Email: hchu0824@gmail.com

Abstract

This paper examines the effects of the environmental tax on long-run growth and intergenerational welfare in a discrete-time overlapping generations (OLG) model. We highlight that the role regarding how the environmental tax revenues are distributed between the young or old generations has important implications for the growth and welfare effects. Our results indicate that raising the environmental tax can exert different effects on the environmental utility of the existing young and old generations, implying an intergenerational welfare conflict of the environmental policy. However, if tax revenues are distributed appropriately, our numerical simulation shows that it is possible for a higher environmental tax to improve the welfare of all generations.

1 | INTRODUCTION

Externalities lie at the heart of environmental economics. These externalities are usually not only intragenerational—polluters affect currently living humans, but also intergenerational—they impose a cost on future generations. Environmental policies, therefore, should be responsible for internalizing both types of externalities. In this study, we examine the intertemporal welfare effects of the environmental policies. We ask the following questions: What is the environmental policy impact on the welfare of different generations? Does an intergenerational welfare conflict emerge from the implementation of a tighter environmental policy? Is it possible for an environmental policy to improve the welfare of all generations?

To address these questions, we construct a two-period overlapping generations (OLG) model à la Diamond (1965) featuring endogenous growth and environmental externalities. There are multiple externalities in this model. First, in order to introduce endogenous growth, we consider the capital externality à la Romer (1986) and Lucas (1988) on the production side. Moreover, the production process emits pollution leading to a deterioration in the environmental quality, which consequently generates two types of environmental externalities. The first environmental externality affects the individual's utility. As is obvious, a worse environment reduces people's happiness. For instance it can negatively affect people's health or reduce their satisfaction with outdoor leisure. The second environmental externality influences the firm's productivity. A worse environment harms workers' health, and bad air quality can accelerate the depreciation of equipment. These facts indicate that a poor environment reduces the efficiency of the production process. Our specifications that pollution negatively affects both the individuals' utility and firms' productivity are in accordance with these observations.[1]

A notable feature of our analysis is that we highlight the role of how the environmental tax revenues are distributed between the young or old generations. This role has important implications both for the growth and welfare effects of the environmental tax. For the growth effect, we show that raising the environmental tax tends to stimulate economic growth when the tax revenues are largely transferred to the young generation. The intuition is that tax revenues transferred to the old generation are entirely consumed, while some part of the tax revenues transferred to the young generation is used to accumulate physical capital, which is beneficial to growth. Accordingly, the result of the positive growth effect of the environmental tax does not necessarily rely on the presence of an environmental externality in production.

With respect to the welfare effect of the environmental tax, our results characterize the intergenerational welfare conflicts between the elderly and young, and between existing and future generations. When the government raises the environmental tax, the environmental quality will be improved, and at the same time it has an ambiguous effect on economic growth. For the existing old generation who only lives in the current period, it does not have time to wait for the improvement of the environmental quality, neither can it have time to enjoy a higher economic growth. Therefore, its welfare is determined primarily by the rebates of tax revenues it can receive. Obviously, the existing old generation will experience a welfare gain if a large portion of tax revenues is transferred to the old generation. By contrast, for the existing young and future generations, both the environment and economic growth matter for their welfare. In the case where the environmental tax enhances growth, all generations (except the existing old generation) are able to experience a better environment and higher growth, which represents a definite welfare gain. In the case where the environmental tax depresses economic growth, the welfare effect of the environmental tax becomes uncertain, but generations born in the very distant future will definitely lose because the growth effect plays a dominant role in evaluating their welfare.

In a counterpart of this study, Bovenberg and Heijdra (1998) investigate the intergenerational welfare effect of an environmental tax in an OLG model à la Yaari (1965) and Blanchard (1985).[2] Our results differ from theirs in several important respects. First, in their model, a welfare gain from a better environment is identical among all existing generations regardless of whether they are old or young. In our model, by contrast, raising the environmental tax benefits the environmental utility of existing young generations more than of existing old generations, by virtue of the feature that existing old generations can hardly wait for the improvement of the environment. Second, in their model, the existing (very) old generations certainly lose from raising the environmental tax; as a consequence, a Pareto-improving environmental tax is not possible in their model. In our model, the existing old generation may be better off if it receives adequate rebates of tax revenues. Importantly, our result implies that a Pareto-improving environmental tax is feasible as long as the tax revenues are properly distributed. Moreover, our paper provides a numerical analysis to demonstrate the theoretical possibility of a Pareto-improving environmental tax.

This study is also related to the huge body of literature on the interplay between environmental policies and endogenous economic growth. Most of these studies confine their analysis to the model with the infinitely lived household.[3] Others deal with an OLG model either based on the Yaari–Blanchard framework (Pautrel, 2008, 2009) or on the Samuelson–Diamond framework (John & Pecchenino, 1994; John, Pecchenino, Schimmelpfennig & Schreft, 1995; Ono, 2003, 2007a,b; Wendner, 2005; Jouvet, Pestieau & Ponthiere, 2010; Mariani, Perez-Barahona & Raffin, 2010; Wang, Zhao & Bhattacharya, 2015). Within these existing OLG models, the model structure we present is closer to that of Ono (2003, 2007a,b). However, our paper departs from these studies in the following ways. First, we consider the possibility of a positive environmental externality in the production sector. Second, we abstract from private investment in environmental maintenance.[4] Finally, these studies do not focus on the conflicting intergenerational welfare effects and the possibility of a Pareto-improving environmental policy, which is our main concern in this paper.

The remainder of this paper is organized as follows. Section 2 describes the economy. Section 3 characterizes the equilibrium and the balanced-growth path. Section 4 analytically investigates the growth and welfare effects of an environmental tax in the absence of a positive environmental production externality. Section 5 examines the possibilities of a Pareto-improving environmental policy via numerical simulation. Section 6 concludes.

2 | THE MODEL

We consider an infinite-horizon economy comprised of finitely lived individuals, perfectly competitive firms, and the government. Production creates pollution that damages environmental quality, which is treated as a renewable resource and can possibly be beneficial to both individuals' utility and productive activities. In what follows, we in turn describe the structure of the economy.

2.1 | Individuals

Time is discrete. A new generation (called generation *t*) is born in each period *t* = 1, 2,…, and lives for two periods. There is also an initial old generation (called generation 0) that lives only in period 1. For simplicity we assume no population growth and the size of each generation is normalized to unity. All individual agents are identical except for their ages. Accordingly, the representative generation *t* has the following utility function:

$$U_t = \begin{cases} \ln c_t^y + \eta \ln E_t + \rho(\ln c_{t+1}^o + \eta \ln E_{t+1}) & \text{for } t \geq 1 \\ \ln c_{t+1}^o + \eta \ln E_{t+1} & \text{for } t = 0 \end{cases}, \tag{1}$$

where c_t^y is consumption in youth age in period t and c_{t+1}^o is consumption in old age in period $t+1$. E_t is environmental quality in period t. $\rho \in (0,1)$ is the subjective discount factor, and $\eta > 0$ denotes the weight in terms of the utility attached to environmental quality.

All individual agents live for two periods. In the first period (in youth age) each of the agents is endowed with one unit of labor inelastically, and it allocates its total income (the sum of wage income and government transfer payments) between savings and young-age consumption. In the second period (in old age), each of the agents is retired from the labor market and receives the return from savings and governments' transfer payments as its old-age consumption. Therefore, the budget constraints of generation t in youth and old age are respectively given by:

$$c_t^y + s_t = w_t + (1-\theta)g_t, \tag{2}$$

$$c_{t+1}^o = R_{t+1}s_t + \theta g_{t+1}, \tag{3}$$

where s_t is savings, w_t is labor income, R_{t+1} is the gross return on savings, and g_t denotes the government transfer payments. Equations 2 and 3 state that, in each period, the government returns environmental tax revenues to the young and the elderly as lump-sum transfer payments according to the proportions $1-\theta$ and θ, respectively.[5]

Notice that, for generation 0, there are no decisions to make in period 1. Each of the agents possesses s_0 as its initial asset and passively receives both transfer payments and the return from savings as its consumption in old age. Without loss of generality, we assume $s_0 = 1$ in the following analysis. For generation $t \geq 1$, each of the agents maximizes U_t in Equation 1 subject to Equations 2 and 3, and yields the following consumption and saving functions:

$$c_t^y = \frac{1}{1+\rho}\left[w_t + (1-\theta)g_t + \frac{\theta}{R_{t+1}}g_{t+1}\right], \tag{4}$$

$$c_{t+1}^o = \frac{\rho R_{t+1}}{1+\rho}\left[w_t + (1-\theta)g_t + \frac{\theta}{R_{t+1}}g_{t+1}\right], \tag{5}$$

$$s_t = \frac{1}{1+\rho}\left[\rho w_t + \rho(1-\theta)g_t - \frac{\theta}{R_{t+1}}g_{t+1}\right]. \tag{6}$$

2.2 | Production

There is a continuum of identical and perfectly competitive firms. The number of firms is normalized to unity. The representative firm produces a single final good Y_t using the following production function:

$$Y_t = \Lambda_t K_t{}^{\alpha} P_t{}^{\beta} L_t{}^{\nu}; 1 > \alpha,\ \beta,\ \nu > 0, \alpha + \beta + \nu = 1, \tag{7}$$

where Λ_t is the technology level that stands for the production externalities, K_t is the aggregate physical capital, L_t is the aggregate labor, and P_t is a "dirty input," which can be thought of as oil or other energy (see, e.g., Agnani, Gutiérrez & Iza, 2005; Aguiar-Conraria & Wen, 2008). Firms hire labor, capital, and dirty inputs to maximize profits taking all factor prices and the technology level as given. The representative firm's problem can be written as:

$$\underset{K_t, L_t, P_t}{\text{Max}}\ \Pi_t = Y_t - r_t K_t - w_t L_t - (1+\tau)b_t P_t,^{6} \tag{8}$$

$$s.t. Y_t = \Lambda_t K_t{}^{\alpha} P_t{}^{\beta} L_t{}^{\nu},$$

where Π_t is the profits, r_t is the capital rental rate, and $\tau \geq 0$ denotes the flat environmental tax that the government levies on dirty inputs. The private price of dirty inputs b_t is assumed to exogenously evolve with the aggregate capital, that is, $b_t = bK_t$ where $b > 0$ is a constant parameter.[7] The first-order conditions for the firm's optimizing problem, in per-worker terms, are:

$$\alpha\Lambda_t {k_t}^{\alpha-1} {p_t}^{\beta} = r_t, \tag{9}$$

$$\beta\Lambda_t {k_t}^{\alpha} {p_t}^{\beta-1} = (1+\tau)b_t, \tag{10}$$

$$\nu\Lambda_t {k_t}^{\alpha} {p_t}^{\beta} = w_t, \tag{11}$$

where $k_t = K_t/L_t$ and $p_t = P_t/L_t$. Equations 9 to 11 indicate that the firm equates the marginal product of the capital, labor, and pollution to their respective marginal cost.

We assume that there exist two kinds of positive externalities in the production sector. The first one is the "capital externality" suggested by the standard literature of endogenous growth theory such as Romer (1986) and Lucas (1988).[8] The second one is the "environmental production externality," which indicates that the output level can rise with a better environmental quality (see, e.g., Bovenberg & Smulders, 1995; Fullerton & Kim, 2008; Chu & Lai, 2014). The technology level can be specified in the following form:

$$\Lambda_t = A {K_t}^{1-\alpha} {E_t}^{\lambda}, \tag{12}$$

where $A > 0$ is a constant, and $\lambda \geq 0$ is a parameter that reflects the extent of the environmental externality.

2.3 | Environmental quality

The natural environment is treated as a renewable resource, which grows and declines in the following manner:

$$E_{t+1} - E_t = \Phi(E_t) - P_t, \tag{13}$$

where $\Phi(E_t)$ is the environmental regeneration function, which relates to the current state of environmental quality. To obtain tractable results, we specify a linear form of regeneration function $\Phi(E_t) = \delta(\bar{E} - E_t)$,[9] where $\delta > 0$ is a regeneration parameter, and $\bar{E}$ denotes the maximum level of environmental quality (i.e., the environmental quality corresponding to zero pollution).[10] We impose a condition on $(\delta, \bar{E})$ to assume that they are large enough to avoid negative environmental quality ($E_t > 0 \forall t$). Equation 13 indicates that environmental quality in the next period is specified to be positively related to the regeneration capacity of the environment $\Phi(E_t)$ and negatively related to the level of dirty inputs used.

2.4 | Government

The government is subject to a balanced-budget requirement, which levies an environmental tax on pollution and transfers the revenue to individuals. Let g_t be total transfer payments. In each period t, the young (generation t) receive $(1 - \theta)g_t$ while the elderly (generation $t - 1$) receive θg_t. Hence, the government budget constraint in period t is given by:

$$\tau b_t P_t = (1-\theta)g_t + \theta g_t. \tag{14}$$

The weight parameter θ plays an important role throughout the analysis. It stands for the revenue weight that the government assigns to the young and the elderly. As we will see later, θ is also a parameter that captures the welfare conflict between different generations. It can be seen from the individual's budget constraint reported in Equations 2 and 3 that, when $\theta = 0$, the whole of the tax revenues are returned to the young. However, when $\theta = 1$, the elderly receive all of the tax revenues and we can treat this case as a kind of pay-as-you-go public pension system financed by environmental taxes. Furthermore, we refer to the case of $\theta = 0.5$ as an "equal transfer policy" that indicates that tax revenues are equally distributed to each generation.

3 | COMPETITIVE EQUILIBRIUM

This section deals with the competitive equilibrium and characterizes the balanced-growth path. We first deal with the market clearing condition for physical capital. In line with the literature on Samuelson–Diamond OLG models, we assume that capital fully depreciates in the process of production. Hence, given that labor is stationary and normalized to unity, the market clearing condition for physical capital is:

$$s_t = k_{t+1}. \tag{15}$$

This condition indicates that savings from young agents determine the stock of physical capital in the next period. Accordingly, the gross return on the individual's savings is equal to the capital rental rate, that is, $R_t = r_t$.

Definition 1. A competitive equilibrium is an infinite sequence of allocations $\{c_{t^y}, c_{t^o}, s_t, p_t, k_{t+1}, g_t\}_{t=1}^{\infty}$, prices $\{w_t, r_t, b_t, R_t\}_{t=1}^{\infty}$, and environmental tax policies $\{\tau,\theta\}$, such that, given the initial condition $s_0 > 0$, in each period:

(i) for generation $t \geq 1$, agents choose $\{c_t^y, c_{t+1}^o, s_t\}$ to maximize utility taking $\{w_t, R_{t+1}, g_t, g_{t+1}, \theta\}$ as given;
(ii) firms choose $\{k_t, p_t\}$ to maximize profit taking $\{w_t, r_t, b_t, \tau\}$ and the technology level Λ_t as given;
(iii) markets clear;
(iv) the government budget constraint is balanced, that is, $\tau b_t p_t = g_t$.

3.1 | The balanced-growth path

The balanced-growth path is characterized by a set of constant growth rates of all economic variables. Let γ_z denote the ratio z_{t+1}/z_t for all variables along the balanced-growth path.[11] In line with the environmental growth literature, we provide the following definition that describes the balanced-growth path in our economy.

Definition 2. A balanced-growth path in this model is defined as a competitive equilibrium where (i) pollution and environmental quality remain constant, that is, $\gamma_p = \gamma_E = 1$, and (ii) all other variables grow at a common endogenous growth rate, which implies that $\tilde{\gamma} = \gamma_Y = \gamma_{c^y} = \gamma_{c^o} = \gamma_k = \gamma_g$.

Our analysis focuses on steady-state solutions, that is, the solutions along the balanced-growth path. Hence, it would be useful for us to define the following transformed variables. Let a tilde denote the steady-state values. We define $\tilde{x}^{gro} \equiv x_t^{gro}/k_t$ for growing variables ($x^{gro} = c^y, c^o, w, g$), and $\tilde{x}^{non} \equiv x_t^{non}$ for nongrowing variables ($x^{non} = r, p, E$).

4 | POLICY EFFECTS WITHOUT ENVIRONMENTAL PRODUCTION EXTERNALITY

In this section, we examine the growth and welfare effects under the situation where environmental quality is not beneficial to the production process (i.e., $\lambda = 0$). We will temporarily ignore this productivity benefit of a cleaner environment for the following two reasons. First, doing so would be helpful for us to obtain analytical results. Second, doing so enables us to clarify the channels through which an environmental tax influences the welfare of different generations.

By imposing $\lambda = 0$ and substituting the transformed variables and the underlying technology $\Lambda_t = Ak_t^{1-\alpha}$ into Equations 9–11, it is easy to obtain the following steady-state values of pollution and factor prices: $\tilde{p} = (\beta A/(1+\tau)b)^{1/(1-\beta)}$, $\tilde{r} = \alpha A(\beta A/(1+\tau)b)^{\beta/(1-\beta)}$, and $\tilde{w} = \nu A(\beta A/(1+\tau)b)^{\beta/(1-\beta)}$. It follows that an increase in an environmental tax reduces pollution and the returns of both physical capital and labor inputs. The intuition is clear. A rise in the environmental tax increases the cost of the dirty input, and thereby reduces the pollution. Given less pollution in production, the marginal product of the other two factors, capital and labor, must decrease as well.

4.1 | Growth effect

To examine how the environmental tax affects the growth rate, we first derive the balanced growth rate.

Lemma 1. *All growing factors along the balanced-growth path grow at a common endogenous rate, given by:*

$$\tilde{\gamma}(\tau, \theta) = \frac{\rho\tilde{w} + \rho(1-\theta)\tau b\tilde{p}}{1 + \rho + \theta\tau b\tilde{p}/\tilde{r}}, \tag{16}$$

Proof. See the Appendix.

Lemma 1 indicates that the endogenous growth rate in our economy is governed by two important policy instruments, namely, the environmental tax rate τ and the distribution of tax revenues θ. Intuitively, a higher environmental tax decreases the returns of production factors. This then reduces the incentive to save and is harmful to growth. In contrast, with a higher environmental tax rate, the government collects more tax revenues that can be transferred to the young generations. This boosts their income and savings, which is beneficial to growth. The overall growth effect of the environmental tax is thus determined by these two conflicting forces, potentially yielding an inverted U-shaped relationship. Equipped with Lemma 1, the following proposition characterizes the relationship between environmental policies and the growth rate.

Proposition 1. *When tax revenues are returned to the young generation ($\theta = 0$), an environmental tax enhances (reduces) the balanced growth rate if and only if the initial tax rate* *is smaller (greater) than (1 − β − ν)/(β + ν). When tax revenues are returned to the elderly generation (θ = 0), an environmental tax unambiguously reduces the balanced growth rate.*

Proof. See the Appendix.

Proposition 1 shows that the growth effect of an environmental tax depends on how the tax revenues are split among the young and elderly. The intuition can be explained as follows. In this OLG economy, the growth rate depends upon the consumption-saving decision of young agents. The growth rate will be higher if agents save more. Hence, it is useful to look into Equation 6 to interpret how the saving decision is affected by the environmental tax. There are two conflicting forces at work. The first force is the "factor returns effect." As discussed earlier, a higher environmental tax reduces the returns on both physical capital and labor inputs. When the return on holding capital falls, agents tend to reduce savings and increase consumption; when the return on labor falls, agents' wage income declines and thus they tend to reduce both savings and consumption. The decreases in both factor returns lead to less savings, so that the factor returns effect implies a negative growth effect of the environmental tax. The second force is the "transfer effect," which means that young agents tend to save more with a higher environmental tax because they can receive more transfer income. As a consequence, the transfer effect supports a positive growth effect of the environmental tax.

With the above two conflicting effects in mind, we can proceed to discuss the intuition behind Proposition 1 in detail. First, we see that raising the environmental tax may stimulate growth in the case where θ = 0, while it always reduces growth in the case where θ = 1. This implies that a lower θ makes the environmental tax favorable in terms of enhancing growth. The intuition is obvious. The positive transfer effect is greater when young agents receive a larger portion of the tax revenues, that is, with a lower value of θ. In the extreme case where all tax revenues are distributed to the elderly generation, that is, θ = 1, the transfer effect vanishes because the young agents who make the savings decision receive no transfer income. In this case, only the negative factor returns effect is present, implying that a higher environmental tax always leads to a deterioration in the growth rate.

Moreover, we see that in the case where θ = 0, the environmental tax may stimulate growth in the case where the tax rate is initially small. The intuition is that when the initial environmental tax is small, as it goes up, tax revenues will rise significantly, which leads to a stronger transfer effect. If it outweighs the negative factor returns effect, the growth rate will be enhanced. We also note that the threshold value of the environmental tax (i.e., (1 − β − ν)/(β + ν)) is decreasing in both β and ν. In other words, a higher β and/or ν makes it less likely for an environmental tax to enhance growth. The intuition lies in the fact that the negative factor returns effect is stronger with a higher β and ν, which can be demonstrated by indicating that $\partial\tilde{w}/\partial\tau$ is decreasing in β and ν. Therefore, when these two parameters are higher, it is more difficult for the transfer effect to overturn the strong factor returns effect.

4.2 | Welfare effect

Now we turn to investigate the effect of the environmental tax on the welfare of different generations. We first deal with the welfare of the initial old generation. Note that in our model all variables are jump variables except for the capital stock and the environmental quality. Supposing that the government raises the environmental tax rate in period *t*, the levels of consumption, savings, and pollution will change instantaneously, while the capital stock and environmental quality will adjust over time. Given the utility of the initial old generation $U_0 = \ln c_1^o + \eta \ln E_1$, and since the environmental quality is a predetermined variable (E_1 is given in period 1), we can infer that for the initial old generation, an environmental tax increases (decreases) its welfare if and only if the tax increases (decreases) their present consumption. This result is quite straightforward; nonetheless it captures the important idea that the recovery of environmental quality needs to take time, while the initial old individuals have no time to wait for it. More specifically, since there is "no next period" for the initial old generation to enjoy a better environment, all their welfare concerns come from the consumption in their present period.

Turning now to the environmental tax effect on generation $t \geq 1$, we have the following lemma:

Lemma 2. *The steady-state welfare effect of raising an environmental tax rate for generation t ≥ 1 can be described by:*

$$\frac{dU_t}{d\tau} = \frac{(t-1)+\rho t}{\tilde{\gamma}}\frac{d\tilde{\gamma}}{d\tau} + \frac{1}{\tilde{c}^y}\frac{d\tilde{c}^y}{d\tau} + \frac{\rho}{\tilde{c}^o}\frac{d\tilde{c}^o}{d\tau} + \frac{\eta(1+\rho)}{\tilde{E}}\frac{d\tilde{E}}{d\tau} \mathit{for} t > 1, \tag{17}$$

$$\frac{dU_t}{d\tau} = \frac{\rho}{\tilde{\gamma}}\frac{d\tilde{\gamma}}{d\tau} + \frac{1}{\tilde{c}^y}\frac{d\tilde{c}^y}{d\tau} + \frac{\rho}{\tilde{c}^o}\frac{d\tilde{c}^o}{d\tau} + \frac{\eta\rho}{\tilde{E}}\frac{d\tilde{E}}{d\tau} \mathit{for} t = 1, \tag{18}$$

where $\tilde{E} = \bar{E} - \tilde{p}/\delta$.

Proof. See the Appendix.

Lemma 2 decomposes the effects of an environmental tax on the welfare of generations other than the initial old generation. For the existing young and future generations, the environmental tax influences their utility via the channels of affecting the growth rate, young-age consumption, old-age consumption, and the environmental quality. It is obvious that the growth channel is stronger for further generations, as shown by the first terms on the right-hand side of Equations 17 and 18. Moreover, we see that the last terms of Equations 17 and 18 are different. This specifically means that the environmental-quality channels via which the environmental tax influences the welfare level are not analogous between the existing young and future generations. The intuition is as follows. When the environmental tax goes up, the environmental quality will improve in the next period. Thus, all future generations are ready to enjoy a life-time (two-period) environmental gain. However, given that the current state of the environment is predetermined, the existing young generation can only enjoy a next-period (one-period) environmental gain.

We do not plan to analyze the welfare effect of each generation one by one. Instead, to provide some useful hints concerning how to compare the relative extent between different channels, we turn our attention toward the change in the welfare level in association with the generation born in the very far future (i.e., $t = \infty$). To this end, based on Lemma 2, the conditions regarding how an environmental tax affects the initial old generation and the generations born in the very far future can be summarized by the following proposition:

Proposition 2. *The intergenerational welfare effects of raising the environmental tax rate have the following properties:*

(i) The initial old generation has a welfare gain (loss) if τ *is smaller (greater) than* $(\theta - \theta\beta - \alpha)/(\alpha + \theta\alpha)$*;*

(ii) generations born in the very far future have a welfare gain (loss) if environmental taxes enhance (reduce) the balanced growth rate.

Proof. See the Appendix.

Proposition 2(i) describes the condition under which an environmental tax improves the welfare level of the initial old generation. The threshold factor implies that a welfare gain is more likely to be the case when the initial τ is smaller, when θ is larger, and when α and β are smaller. The intuition can be explained as follows. First, starting from a smaller initial τ, raising the environmental tax results in more tax revenues. This, along with a larger θ, means that more revenues are transferred to the elderly, who are thus able to enjoy higher consumption and welfare. Moreover, we have mentioned earlier that an increase in the environmental tax reduces the returns of all factors. For the initial old generation, the return from savings will be reduced, which in turn worsens its welfare. This adverse effect is stronger with higher α and β (since $\partial\tilde{r}/\partial\tau$ is decreasing in both parameters). In other words, when α and β are smaller, the adverse welfare effect stemming from the decrease in the savings return is weaker, and is more easily dominated by the transfer effect. Therefore, the environmental tax is inclined to improve the initial old generation's welfare with small values of α and β. In addition, in the extreme case where the elderly receive nothing ($\theta = 0$), $\tau > (\theta - \theta\beta - \alpha)/(\alpha + \theta\beta) = 1$ is true, indicating that an environmental tax always lowers the welfare (consumption) level of the initial old generation by reducing its savings income.

We now explain the intuition underlying Proposition 2(ii). Provided that an environmental tax boosts the balanced growth rate, all generations (except for the initial old) are certainly better off by enjoying both a better environmental quality and more consumption. However, if an environmental tax depresses the balanced growth rate, the generations born in the future will suffer from a loss in nonenvironmental utility (since they consume less with a lower growth rate) and thus the overall welfare effect is uncertain. The further away the future they are born in, the larger the loss in nonenvironmental utility will be. In the endless future, the loss must eventually exceed the environmental gains. As a consequence, such welfare changes of the generations born in the very far future are governed by the growth effect (i.e., $\mathrm{sgn}[dU_\infty] = \mathrm{sgn}[d\tilde{\gamma}]$.)

5 | ENVIRONMENTAL PRODUCTION EXTERNALITY AND PARETO-IMPROVING POLICIES

In this section, we deal with the growth and welfare effects in the presence of the positive environmental externality in production (i.e., $\lambda > 0$), with the primary focus on the possibility of a Pareto-improving environmental policy. By substituting $\Lambda_t = Ak_t^{1-\alpha}E_t^{\lambda}$ into Equations 9 to 11 and implementing some calculations, the economy along the balanced-growth path can then be described by the following set of nonlinear equations:

$$\tilde{c}^y = \frac{1}{1+\rho}\left[\tilde{w} + (1-\theta)\tilde{g} + \frac{\theta}{\tilde{r}}\tilde{g}\,\tilde{\gamma}\right], \tag{19}$$

$$\tilde{c}^o = \frac{\rho(1+\tilde{r})}{1+\rho}\left[\tilde{w} + (1-\theta)\tilde{g} + \frac{\theta}{\tilde{r}}\tilde{g}\,\tilde{\gamma}\right], \tag{20}$$

$$\tilde{\gamma} = \frac{1}{1+\rho}\left[\rho\tilde{w} + \rho(1-\theta)\tilde{g} - \frac{\theta}{\tilde{r}}\tilde{g}\,\tilde{\gamma}\right], \tag{21}$$

$$\alpha A\tilde{p}^{\beta}(\bar{E} - \tilde{p}/\delta)^{\lambda} = \tilde{r}, \tag{22}$$

$$\beta A\tilde{p}^{\beta-1}(\bar{E} - \tilde{p}/\delta)^{\lambda} = (1+\tau)b, \tag{23}$$

$$\nu A\tilde{p}^{\beta}(\bar{E} - \tilde{p}/\delta)^{\lambda} = \tilde{w}, \tag{24}$$

$$\tilde{g} = \tau b\tilde{p}. \tag{25}$$

The nonlinear system expressed in Equations 19 to 25 determines seven unknowns, that is, $\tilde{c}^y$,$\tilde{c}^o$,$\tilde{\gamma}$,$\tilde{w}$,$\tilde{r}$,$\tilde{p}$, and $\tilde{g}$.

Introducing a positive value of λ complicates the model so that closed-form solutions are no longer attainable. Therefore, we present our results via numerical simulations. Our model has ten parameters $\{\alpha, \beta, \nu, \rho, A, b, \lambda, \eta, \bar{E}, \delta\}$. We use the following basic parameter values. First, we choose $\alpha = 0.3$ and $\rho = 0.98^{30} = 0.55$, which are standard values in the literature. For the environmental parameters, we largely borrow the values from a closely related quantitative analysis by Fullerton and Kim (2008). These values include $\beta = 0.17$ (implying $\nu = 0.53$), $\lambda = 0.3$, and $\eta = 0.7$. It is worth noting that in Fullerton and Kim (2008) the extent of the environmental externality λ is chosen to be 0.77, but they vary the level to test the sensitivity within the range of [0.3, 1.2]. In our model, we choose the lowest value $\lambda = 0.3$ exercising caution not to overstate the positive externality of environmental quality. Finally, we normalize $b = 1$ and $\delta = 1$, and then $A = 5.5$ and $\bar{E} = 1.92$ are jointly calibrated such that the balanced growth rate is around 2 percent in the absence of environmental taxes.

5.1 | Growth effect

Figure 1 depicts the growth effect of an environmental tax with or without the positive environmental externality in production. Raising the environmental tax may stimulate economic growth, especially when θ is small or when the environmental externality in production is present. When θ is small, the young will save more in response to an increase in the environmental tax, which benefits the growth. When the productive externality is present, a higher environmental tax improves environmental quality, thereby causing an increase in the marginal product of capital. This also increases the incentives for the young to save; thus in this case the environmental tax is more likely to stimulate growth.

5.2 | Welfare effect and Pareto-improving policies

This subsection makes an effort to illustrate the possibilities of Pareto-improving policies. By definition, an environmental tax is Pareto-improving if it improves the welfare of at least one generation without worsening the others. One implication exhibited in Figure 1 is that, in association with a larger environmental production externality, an environmental policy with a higher probability is Pareto-improving. To see this, let us consider the case of an "equal transfer policy" (i.e., $\theta = 0.5$). In Figure 1(a) we can observe that the growth rate declines with environmental taxes in the absence of the environmental production externality ($\lambda = 0$), while in Figure 1(b) the growth rate may increase as long as environmental taxes are not too high in the presence of the

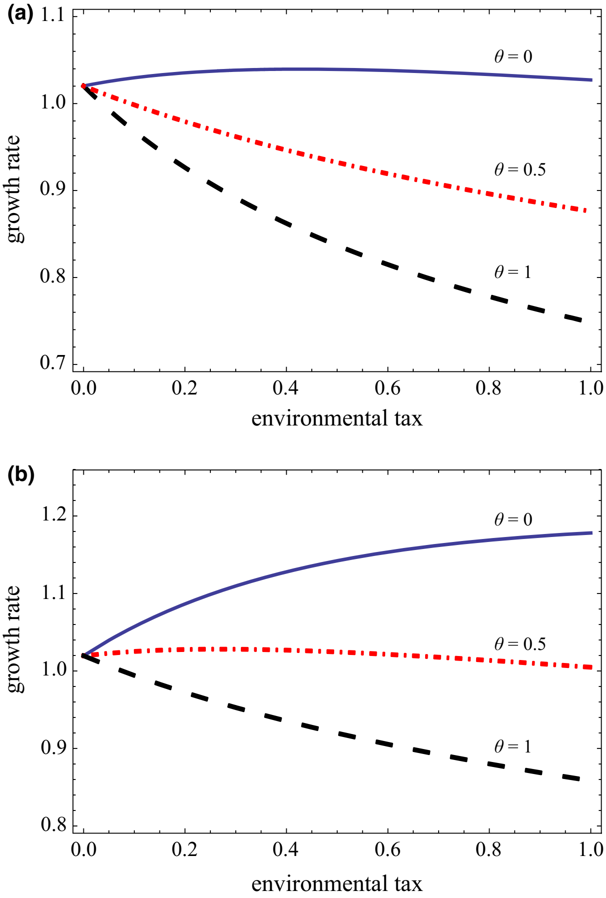

FIGURE 1 (a) Growth effect without environmental externality in production ($\lambda = 0$). (b) Growth effect with environmental externality in production ($\lambda = 0.3$) [Colour figure can be viewed at wileyonlinelibrary.com]

environmental production externality ($\lambda = 0.3$). That is to say, in association with $\lambda = 0$, if the government implements an equal transfer policy, then any rate of environmental tax can never be Pareto-improving since it will certainly worsen the generations in the very far future by reducing growth. However, in the case of $\lambda = 0.3$, an equal transfer policy is not necessarily growth-impeding. Therefore, a Pareto-improving environmental policy may possibly be achieved under such a circumstance.

To examine the intergenerational welfare effects of the environmental tax, we raise τ from 0 to 0.3 and see how the welfare levels of generations 0 to 5 respond. Table 1 reports the results under different values of (λ,θ). Some findings deserve more comments. First, with a higher proportion distributed to the existing old (a higher θ), the generation 0 is better off while all other generations are worse off from raising the environmental tax, and generations born in the more distant future lose more than generations born earlier. This is because, under a higher θ, the environmental tax tends to reduce growth, and therefore is harmful to more future generations. Second, by comparing the case $\lambda = 0.3$ with the case $\lambda = 0$, we see that the presence of a positive environmental externality increases the welfare effect of an environmental tax on all generations. Third, in the cases of $(\lambda,\theta) = (0.3,0)$ and $(\lambda,\theta) = (0.3,0.5)$, we see that the environmental tax improves the welfare of all generations 0 to 5, which implies the possibility of a Pareto-improving environmental policy.[12]

TABLE 1 The welfare changes of raising τ from 0 to 0.3

		$\theta = 0$	$\theta = 0.5$	$\theta = 1$
$\lambda = 0$	dU_0	–11.03	1.97	14.20
	dU_1	11.03	–3.90	–18.78
	dU_2	12.97	–11.19	–35.26
	dU_3	14.82	–18.12	–50.92
	dU_4	16.58	–24.71	–65.83
	dU_5	18.26	–31.00	–80.05
$\lambda = 0.3$	dU_0	2.72	15.74	27.99
	dU_1	32.09	16.95	1.85
	dU_2	41.98	17.58	–6.74
	dU_3	51.40	18.17	–14.92
	dU_4	60.37	18.74	–22.71
	dU_5	68.94	19.29	–30.16

6 | CONCLUDING REMARKS

In this study we examine the impact of the environmental tax on long-run growth and intergenerational welfare using the discrete-time OLG model à la Diamond (1965). Our analysis could provide different insights regarding the intergenerational welfare effects of an environmental tax from previous studies adopting the continuous-time Yaari–Blanchard type OLG models.

For the growth effects, we show that how environmental tax revenues are transferred to different generations plays an important role in determining the effect of the environmental tax on long-run growth. Even in the absence of the positive environmental externalities in the production sector, an environmental tax still may be growth-improving. For the welfare effects, our model is capable of capturing the fact that an environmental policy has diverse environmental utility effects on the different existing generations. By conducting a numerical simulation, we show that a Pareto-improving environmental policy may be achievable.

For future extensions, our model assumes that tax revenues are transferred to the households; accordingly, an interesting extension would be to consider the case where the revenues of environmental taxation are used to finance public abatement or environmental maintenance. Furthermore, for a normative analysis, one could think of setting up and solving the maximization problem of a forward-looking social planner who takes into consideration the utility of all generations. Fruitful results might be obtained if studies were extended to include these issues.

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ENDNOTES

[1] In the environmental economics literature, the environmental externality in utility is a central feature and has been adopted in most of the literature. For studies assuming an environmental externality in production see, for example,

Bovenberg and Smulders (1995), Smulders and Gradus (1996), Fullerton and Kim (2008), and Chu and Lai (2014).

[2] The OLG models can be categorized into two branches. The first branch is the continuous-time OLG model based on the works of Yaari (1965) and Blanchard (1985). This model is closer to the Ramsey model except that agents can live for many but not infinite periods. The second branch is the discrete-time OLG model proposed by Samuelson (1958) and Diamond (1965), in which agents are generally assumed to live for two or three periods.

[3] For studies in this literature see, for example, Bovenberg and Smulders (1995), Mohtadi (1996), Smulders and Gradus (1996), Bovenberg and De Mooij (1997), Grimaud (1999), Nakada (2004), Itaya (2008), Fullerton and Kim (2008), Barman and Gupta (2010), Ayong Le Kama, Pommeret, and Prieur (2012), Chu and Lai (2014), Chu, Lai, and Liao (2016), among others.

[4] In Ono (2003), the environmental externalities are mitigated since young agents can invest in environmental maintenance in order to enjoy a better environmental quality when they are old. The intergenerational welfare conflict is also mitigated since investment in environmental capital (maintenance) serves as a bequest to future generations. However, given the fact that each individual is insignificantly small in the world, our paper assumes that no individual takes into consideration the influence that his/her decision has on the environment, and hence will not invest in any environmental maintenance activities.

[5] A more detailed discussion of θ will be provided in Subsection 2.4.

[6] It should be noted that the final good serves as the numeraire in this paper.

[7] As we have mentioned, the dirty input P_t can be thought of as petroleum, and thus b_t is the price of this energy. In the environmental and endogenous growth literature, it is often assumed that the price of dirty inputs evolves with another growing factor. This price could be the private cost (Ono, 2007a,b) or an environmental tax (Nielsen, Pedersen & Sørensen, 1995; Fullerton & Kim, 2008; Chu & Lai, 2014).

[8] Another justification of the capital externality is that the endogenous technological change could be driven by disembodied learning from net investment. See, for example, Groth and Wendner (2014).

[9] As in John et al. (1995) and Ono (2003), we consider a linear evolving function of environmental quality for the purpose of analytical tractability. In contrast, Tahvonen and Kuuluvainen (1991) and Bovenberg and Smulders (1995) consider a more complicated nonlinear form of evolving function.

[10] It can be easily seen that when $\tilde{P} = 0$, we have $\tilde{E} = \bar{E}$.

[11] It is worth noting that $\tilde{\gamma}_z$ is the growth factor of z, and $\tilde{\gamma}_z - 1$ is what we all understand as the growth rate.

[12] We cannot conclude whether a Pareto-improving environmental tax is attainable in the case of $\lambda = 0$ owing to the lack of a mathematical proof. However, by running a number of simulations and varying the parameters within a reasonable range, we find it is extremely hard, if not impossible, to implement a Pareto-improving environmental tax in the absence of environmental production externalities.

ORCID

Hsun Chu http://orcid.org/0000-0002-6076-6134

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APPENDIX A1

A1 | Proof of Lemma 1

Combining Equations 6 and 15 and dividing both sides by k_t, we have:

$$\frac{k_{t+1}}{k_t} = \frac{1}{1+\rho}\left[\rho\frac{w_t}{k_t} + \rho(1-\theta)\frac{g_t}{k_t} - \frac{\theta}{R_{t+1}}\frac{g_{t+1}}{k_{t+1}}\frac{k_{t+1}}{k_t}\right]. \tag{A1}$$

Using Definition 2, $R_{t+1} = r_{t+1}$, and substituting the transformed variables into Equation A1, on the balanced-growth path we then have:

$$\tilde{\gamma} = \gamma_k = \frac{k_{t+1}}{k_t} = \frac{1}{1+\rho}\left[\rho\tilde{w} + \rho(1-\theta)\tilde{g} - \frac{\theta}{\tilde{r}}\tilde{g}\tilde{\gamma}\right]. \tag{A2}$$

Rearranging Equation A2 yields Equation 16 in the main text.

A2 | Proof of Proposition 1

We first prove the case of $\theta = 0$. Substituting $\theta = 0$ into Equation 16 yields $\tilde{\gamma}(\tau, 0) = \rho(\tilde{w} + \tau b\tilde{p})/(1+\rho)$. Inserting $\tilde{p}$ and $\tilde{w}$ derived in the main text and differentiating $\tilde{\gamma}(\tau, 0)$ with respect to τ, we obtain:

$$\begin{aligned}
\frac{\partial\tilde{\gamma}(\tau,0)}{\partial\tau} &= \frac{\rho}{1+\rho}\left\{\frac{\beta}{1-\beta}\nu A\Omega^{\frac{2\beta-1}{1-\beta}}\frac{\beta A}{b}\frac{(-1)}{(1+\tau)^2} + b\Omega^{\frac{1}{1-\beta}} + \frac{1}{1-\beta}\tau b\Omega^{\frac{\beta}{1-\beta}}\frac{\beta A}{b}\frac{(-1)}{(1+\tau)^2}\right\} \\
&= \frac{\rho}{1+\rho}\Omega^{\frac{1}{1-\beta}}\left\{\frac{\beta}{1-\beta}\nu A\Omega^{-2}\frac{\beta A}{b}\frac{(-1)}{(1+\tau)^2} + b + \frac{1}{1-\beta}\tau b\Omega^{-1}\frac{\beta A}{b}\frac{(-1)}{(1+\tau)^2}\right\} \\
&= \frac{\rho}{1+\rho}\Omega^{\frac{1}{1-\beta}}\left\{-\frac{\nu b}{1-\beta} + b - \frac{\tau b}{(1-\beta)(1+\tau)}\right\} \\
&= \frac{b\rho}{1+\rho}\Omega^{\frac{1}{1-\beta}}\left\{\frac{-(1+\tau)\nu + (1-\beta)(1+\tau) - \tau}{(1-\beta)(1+\tau)}\right\},
\end{aligned} \tag{A3}$$

where $\Omega \equiv \beta A/(1+\tau)b > 0$. By rearranging Equation A3, we can infer the following result:

$$\tau \underset{>}{<} \left(\frac{1}{\beta+v}-1\right) \Leftrightarrow \frac{\partial\tilde{\gamma}(\tau,0)}{\partial\tau} \underset{<}{>} 0. \tag{A4}$$

We now turn to prove the case of $\theta = 1$. We first substitute $\theta = 1$ into Equation 15 to obtain

$$\tilde{\gamma}(\tau,1) = \rho\nu A\Omega^{\beta/(1-\beta)}/\Delta, \tag{A5}$$

where $\Delta \equiv 1+\rho+\tau\beta/(1+\tau)\alpha > 0$. Then, differentiating $\tilde{\gamma}(\tau,1)$ with respect to τ yields:

$$\frac{\partial\tilde{\gamma}(\tau,1)}{\partial\tau} = \frac{1}{\Delta^2}\left[\rho\nu A\frac{\beta}{1-\beta}\Omega^{\frac{2\beta-1}{1-\beta}}\frac{-\beta A}{b(1+\tau)^2}\Delta - \rho\nu A\Omega^{\frac{\beta}{1-\beta}}\frac{\beta}{\alpha(1+\tau)^2}\right], \tag{A6}$$

which can be further simplified as

$$\frac{\partial\tilde{\gamma}(\tau,1)}{\partial\tau} = \frac{1}{\Delta^2}\Omega^{\frac{1}{1-\beta}}\rho\nu b\left[-\frac{\Delta}{1-\beta} - \frac{1}{\alpha(1+\tau)}\right] < 0. \tag{A7}$$

A3 | Proof of Lemma 2

Using the transformed variables and evaluating at the balanced-growth path, we can rewrite the utility function of generation $t > 1$ as follows:

$$U_t = \ln(k_1\tilde{\gamma}^{t-1}\tilde{c}^y) + \rho\ln(k_1\tilde{\gamma}^t\tilde{c}^o) + \eta(1+\rho)\ln\tilde{E}. \tag{A8}$$

Differentiating U_t with respect to τ yields:

$$\begin{aligned}\frac{dU_t}{d\tau} &= \frac{1}{k_1\tilde{\gamma}^{t-1}\tilde{c}^y}\left[k_1(t-1)\tilde{\gamma}^{t-2}\frac{d\tilde{\gamma}}{d\tau}\tilde{c}^y + k_1\tilde{\gamma}^{t-1}\frac{d\tilde{c}^y}{d\tau}\right] \\ &+ \frac{\rho}{k_1\tilde{\gamma}^t\tilde{c}^o}\left[k_1 t\tilde{\gamma}^{t-1}\frac{d\tilde{\gamma}}{d\tau}\tilde{c}^o + k_1\tilde{\gamma}^t\frac{d\tilde{c}^o}{d\tau}\right] + \eta(1+\rho)\frac{1}{\tilde{E}}\frac{d\tilde{E}}{d\tau},\end{aligned} \tag{A9}$$

which reduces to Equation 17 in the main text. As for generation 1, the utility function can be rewritten as:

$$U_1 = \ln(k_1\tilde{c}^y) + \eta E_1 + \rho\ln(k_1\tilde{\gamma}\tilde{c}^o) + \eta\rho\ln\tilde{E}. \tag{A10}$$

Then, by differentiating U_1 with respect to τ, we can derive the expressions in Lemma 2.

A4 | Proof of Proposition 2

The initial old generation only lives in period 1 and receives transfer payments and the return from savings as their consumption in old age. This can be expressed by:

$$c_1^o = \tilde{r}s_0 + \theta\tilde{g}. \tag{A11}$$

Given that we assume $s_0 = 1$, the tax effect on the consumption of the initial old is:

$$\begin{aligned}\frac{dc_1^o}{d\tau} &= \frac{-\beta}{1-\beta}\alpha A\Omega^{\frac{2\beta}{1-\beta}}\frac{\beta A}{b(1+\tau)^2} + \theta b\left(\Omega^{\frac{1}{1-\beta}} - \frac{\tau}{1-\beta}\Omega^{\frac{\beta}{1-\beta}}\frac{\beta A}{b(1+\tau)^2}\right) \\ &= b\Omega^{\frac{1}{1-\beta}}\left[\frac{(\theta-\beta\theta-\alpha)-(\alpha+\theta\beta)\tau}{(1-\beta)(1+\tau)}\right].\end{aligned} \tag{A12}$$

Because for the initial old generation the welfare effect of the environmental tax depends solely on its effect on their current consumption, Proposition 2(i) is proved.

The proof of Proposition 2(ii) is straightforward from Equation 17 when evaluated at $t \to \infty$. Since $d\tilde{\gamma}$, $d\tilde{c}^y$, $d\tilde{c}^o$, and $d\tilde{E}$ are finite, as $t \to \infty$ the first term on the right-hand side of Equation 17 must exceed other terms. In other words, we have the result $\text{sgn}[dU_\infty] = \text{sgn}[d\tilde{\gamma}]$ provided that $d\tilde{\gamma}$ is not equal to zero.