On Option Pricing with Ornstein-Uhlenbeck Position Process

doi:10.30003/JRM.200707.0004

風險管理學報, 9(2), 2007

作者/Author： Mi-Hsiu Chiang;Wei-Kuang Chen;Chi-Hung Cheng

頁數/Page： 173-191

出版日期/Publication Date：2007/07

風險管理學報 第九卷 第二期 2007 年 7 月
Journal of Risk Management Vol.9 No.2 July 2007
pp173-191

On Option Pricing with Ornstein-Uhlenbeck Position Process

Mi-Hsiu Chiang* Wei-Kuang Chen Chi-Hung Cheng

Abstract

This research employs the Ornstein Uhlenbeck position process as an alternative underlying stochastic process for stock prices in markets where frictional elements are present. We derive a analytical formula for call option prices together with the hedging parameters in closed-form. We conduct sensitivity analysis to explore how this pricing model differs from the traditional Black-Scholes. Our numerical results suggest that, the impact of the frictional elements in the long term would actually be less significant. Our numerical results also show that when the underlying asset stock is highly volatile, the presence of frictional elements in the market would in fact amplifies the deviation in option prices between our model and that of the traditional Black-Scholes model.

Keywords: option pricing, Ornstein Uhlenbeck position process
JFL classification:

* Department of Money and Banking, National Chengchi University.

1. Introduction

The option pricing model of Black-Scholes (1973) makes the assumptions of a frictionless market with no taxes, transaction costs, or limits on borrowing, lending and short selling. In addition, the risk-free rate and variance of the return on the stock are assumed to be constant. While Geometric Brownian motion has long been used to characterize the behaviour of stock price movements, in the real world stock price movements can subject to frictional elements, in particular the existence of taxes, transaction costs, and price-limit constraints directly calls for alternative stochastic processes that are adequate for capturing such impacts. In this article, we propose using the Ornstein-Uhlenbeck position process as an alternative stochastic process to characterize the behaviour of stock prices subject to frictions. Based on the martingale pricing method, we derive a closed-from formula for European call prices under such assumption, and provide in-depth analysis for the theoretical of this process. As an application, we consider the pricing of options in the Taiwan Security Exchange (TSE) market where stock prices are subject to price-limit constraints. The stock prices generated by Ornstein-Uhlenbeck position process have some properties consistent with the characteristics of the prices with the imposition of price limits.

Existing literature on the impacts of frictional elements on option pricing is substantial. Cox & Ross (1976) relax the assumption of a geometric Brownian motion for the underlying stock price movements, and consider the CEV (constant elasticity of variance) process as an alternative. While both arithmetic and geometric Brownian motion can be shown to be special cases of the CEV diffusion process, Cox and Ross was motivated by Black's (1976) observation that volatility appears to be inversely related to underlying asset price movements.

Cox and Rubinstein (1985) allow for an instantaneous conditional volatility of stock returns to be a deterministic function of the stock price levels. The stochastic volatility models of Hull and White (1987), and Scott (1987) consider more general patterns of conditional volatility in similar spirits to the development of time series models such as GARCH and ARCH. The observation that stock returns are leptokurtic leads Merton (1976) to consider the underlying asset price movements are a jump diffusion process.

In dealing with transaction costs, Leland (1985) tackles this issue and derives a closed form solution resembling the Black-Scholes formula in the presence of proportional transaction costs. Merton (1990) formulates the problem in a discrete-time one-period setting while Boyle and Vorst (1992) further extends Merton's framework to multi-periods.

Goldenberg (1986) makes the observation that there exists non-zero correlation among future prices in markets where friction elements, such as transaction cost and price limits are present, and suggests using Ornstein-Uhlenbeck position process to model the future prices. Using the geometric Brownian motion as the model for underlying asset prices means that asset returns are independent and can vary with infinite velocity within an infinitesimal time interval, while future prices in markets with price constraints can only vary in a bounded range, and the employment of Ornstein-Uhlenbeck position process closely captures such feature.

In this article, we assume the Ornstein Uhlenbeck position process as the stock price driving process in markets where price limits are present. We derive a theoretical option pricing model subject to this assumption. We investigate the behavior of the stock prices in markets with the imposition of price limits and find that the employment of Ornstein Uhlenbeck position process is more consistent with the characteristics of stock price movements than that were otherwise assumed to follow a geometric Brownian motion. Close-form formulae are derived for the European calls, and the hedging parameters to allow for actual hedging practices.

The structure of this paper is organized as follows: Section 2 introduces the Ornstein Uhlenbeck position process and discuss its distributional features; In section 3, we derived a closed form formula of European call options when the stock prices are governed by the Ornstein Uhlenbeck position process, together with the relevant hedging parameters; Section 4 draws direct comparison between this option pricing model and the traditional Black-Scholes model; Section 5 considers the empirical application of our theoretical model to the case of Taiwan Securities Exchange market where price limit constraints are presents. Section 6 concludes this paper.

2. Sample path properties of the Ornstein-Uhlenbeck position process

Following Doob (1942) who applies modern probability theory to the analysis of

Ornstein Uhlenbeck position process. We identify the distributional properties of Ornstein Uhlenbeck position process through the following theorem:

Theorem: Let $u(t)$ $(-\infty < t < +\infty)$ be a one-parameter family of chance variables, determining a stochastic process with the following properties.

1. The process is temporally homogenous

2. The process is a Markov process

3. If s, t are arbitrary distinct numbers, u(s), u(t) have a (non-singular) bi-variate Gaussian distribution.

Define m and σ_0^2 *by*

$$m = E\left[u(t)\right] \qquad \sigma^2 = E\left\{\left[u(t) - m\right]^2\right\} \tag{1}$$

Then the given process is one of the following two types.

(A) If $t_1 < t_2 < t_3 < \ldots\ldots < t_n$, *the random variables* $v_{t1}, v_{t2}, \ldots\ldots v_{tn}$ *are mutually independent Gaussian variables.*

:

(B) (O. U. process) There exist a friction coefficient β*>0. If* $t_1 < t_2 < t_3 < \ldots\ldots < t_n$, *the random variables* $v_{t1}, v_{t2}, \ldots\ldots v_{tn}$ *have an n-variate Gaussian distribution with common mean m, variance* σ_0^2 *and covariance as:*

$$E[(u_t - m)(u_s - m)] = \sigma_0^2 e^{-\beta|t-s|} \tag{2}$$

where in Case (A) the existence of white noise is considered as a generalized stochastic process; Case (B) defines the statistical features of an Ornstein Uhlenbeck process. In above definition, each time-slice of an Ornstein Uhlenbeck process is Gaussian distributed with mean m and variance σ_0^2, and correlation exists between these Gaussian distributions. Correlation is decreasing with an increasing time interval and is denoted by the coefficient β. In Case (B) we can observe a well defined velocity of asset price approaching Brownian motion and it is subject to a central elastic restoring force which represent frictional elements in the trading environment. The Ornstein Uhlenbeck position process can be show to be a solution of the Langevin Equation:

$$du_t = -\beta u_t dt + dB_t \tag{3}$$

where B_t is a nonstandard Brownian motion process with variance $2\sigma_0^2 \beta t$.
Ornstein Uhlenbeck position process $x(t)$ is defined by the integral of $u(t)$:

$$\Delta x_t = x_t - x_0 = \int_0^t u(s)ds \tag{4}$$

where the expected value, variance, and the covariance of the process are as following:

$$E[\Delta x_t] = \int_0^t E[u(s)]ds = mt \tag{5}$$

$$V[\Delta x_t] = \int_0^t \int_0^t E[u(s)u(s')]dsds' = \int_0^t \int_0^t e^{-\beta|s-s'|}dsds' = \frac{2\sigma_0^2}{\beta^2}(e^{-\beta t} - 1 + \beta t) \tag{6}$$

$$\rho\left[x_{t2} - x_{t1}, x_{s2} - x_{s1}\right] = \frac{(e^{\beta s2} - e^{\beta s1})(e^{\beta t2} - e^{\beta t1})}{2\left(e^{-\beta(s2-s1)} - 1 + \beta\left(s2 - s1\right)\right)^{1/2}\left(e^{-\beta(t2-t1)} - 1 + \beta\left(t2 - t1\right)\right)^{1/2}} \tag{7}$$

$$where \quad s1 < s2 \le t1 < t2$$

Since an Ornstein Uhlenbeck process is a solution to the Langevin Equation. By equation (3) and (4), we can express the Ornstein Uhlenbeck position process $x(t)$ directly as:

$$x(t) = x(0) + \frac{1 - e^{-\beta t}}{\beta}u(0) + \frac{1}{\beta}\int_0^t [1 - e^{-\beta(t-\tau)}]dB(\tau) \tag{8}$$

where $dB(\tau)$ is a non-standard Brownian motion with variance $2\sigma_0^2\beta t$ and mean zero.

We must note some interesting features of the variance of the Ornstein Uhlenbeck position process. As the time interval t tends to infinity, $e^{-\beta t} \to 0$. The variance is proportional to the time variable t, resembling the behaviour of the variance of an arithmetic Brownian motion. On the other hand, when the time interval t tends to zero, we can express $e^{-\beta t}$ as $1 - \beta t + \frac{1}{2}(\beta t)^2$, so that the variance approaches $\sigma_0^2 t^2$, which is less than the volatility of an arithmetic Brownian motion (c.f. Cox and Miller, 1965). This tells us that the volatility of the Ornstein-Uhlenbeck position process resembles that of an arithmetic Brownian motion in the long term, but less in magnitude in the short term.

Under the risk-neutral measure, the stock price movements can then represented as:

$$\begin{aligned} &\frac{dS}{S} = rdt + \sigma dx_t \\ &dx_t \sim N(0, v^2) \\ &where \quad v^2 = \frac{2}{\beta^2}(e^{-\beta dt} - 1 + \beta dt) \end{aligned} \tag{9}$$

where $dx_t = u_t dt$, and x_t is the integral of Ornstein-Uhlenbeck process *u(s)*. We derive the expected value, variance, and correlation of S_T in the following.

$$\begin{aligned} d\ln S_T &= rdt + \sigma dx_t - \frac{1}{2}\sigma^2(dx_t^{\ 2}) \\ &= \left(r - \frac{\sigma^2 v^2}{2dt}\right)dt + \sigma dx_t \\ &= \left[r - \frac{\sigma^2}{\beta^2 dt}\left(e^{-\beta dt} - 1 + \beta dt\right)\right]dt + \sigma dx_t \end{aligned} \tag{10}$$

where

$$\begin{aligned} &E\left[dx_t^{\ 2}\right] = \frac{2}{\beta^2}\left(e^{-\beta dt} - 1 + \beta dt\right) \\ &V\left[dx_t^{\ 2}\right] = E\left[dx_t^{\ 4}\right] - E\left[dx_t^{\ 2}\right] = 3V\left[dx_t\right]^2 - E\left[dx_t^{\ 2}\right] \to 0 \qquad \text{as dt} \to 0 \\ &so \quad that \quad (dx_t)^2 = \frac{2}{\beta^2}\left(e^{-\beta dt} - 1 + \beta dt\right) \end{aligned}$$

Solving the stochastic differential equation (10) gives us:

$$\begin{aligned} &S_T = S_0 * e^{\left(r - \frac{\sigma^2 v^2}{2T}\right)T + \sigma \Delta x_T} \\ &v^2 = \frac{2}{\beta^2}\left(e^{-\beta T} - 1 + \beta T\right) \end{aligned} \tag{11}$$

So the expected stock price, variance, and correlation are found to be:

$$E\left[\ln S_T - \ln S_0\right] = \left[r - \frac{\sigma^2 v^2}{2T}\right]T \tag{12}$$

$$V\left[\ln S_T - \ln S_0\right] = \frac{2\sigma^2}{\beta^2}(e^{-\beta T} - 1 + \beta T) \tag{13}$$

$$\rho\left[\ln S_{t2} - \ln S_{t1}, \ln S_{s2} - \ln S_{s1}\right] = \frac{(e^{\beta s2} - e^{\beta s1})(e^{-\beta t1} - e^{-\beta t2})}{2\left(e^{-\beta(s2-s1)} - 1 + \beta(s2-s1)\right)^{1/2}\left(e^{-\beta(t2-t1)} - 1 + \beta(t2-t1)\right)^{1/2}} \tag{14}$$

$where \quad s1 < s2 \le t1 < t2$

From the above derivation, we know that the volatility of the log price differences inherits the property of the Ornstein Uhlenbeck position process, which approaches the volatility of Brownian motion as T is large and less in magnitude as T is small. Besides, the correlation of log stock price differences decreases as the distance of these two increments increases. As the correlation approaches zero, the underlying asset returns becomes independent, and approach that otherwise generated by a Brownian motion.

3. Option pricing in Ornstein Uhlenbeck position process

Goldenberg (1986) leads us to consider in above section the Ornstein Uhlenbeck position process as a dynamic description of the undelrying asset prices in a frictional market. In this section we proceed under the martingale valuation framework to derive a closed form formula for the price of European call options under a risk neutral probability measure when Ornstein Uhlenbeck position process is used.

We define the final payoff of a European call option as:

$$C_T = Max(0, S_T - K) \tag{15}$$

and the risk-neutral value of the call at initial time can be expressed as:

$$
\begin{aligned}
C &= e^{-rT} * E^{Q}\left[S_T - KI_{S_T>K}\right] \\
&= e^{-rT} * E^{Q}[S_0 e^{(r-\frac{\sigma^2 v^2}{2T})T+\sigma\Delta x_T} - KI_A] \\
&= e^{-rT} * S_0 e^{rT} E^{Q}\left[e^{-\frac{\sigma^2 v^2}{2}T+\sigma\Delta x_T} I_A\right] - Ke^{-rT}E^{Q}\left[I_A\right] \\
&= S_0 E^{Q}\left[e^{-\frac{\sigma^2 v^2}{2}T+\sigma\Delta x_T} I_A\right] - Ke^{-rT}E^{Q}\left[I_A\right]
\end{aligned}
\tag{16}
$$

$$where \quad A = S_T > K$$

$$
\begin{aligned}
E^{Q}\left[e^{-\frac{\sigma^2 v^2}{2T}T+\sigma\Delta x_T} I_A\right] &= \int e^{-\frac{\sigma^2 v^2}{2T}T+\sigma\Delta x_T} I_A \frac{1}{\sqrt{2\pi v}} e^{-\frac{(\Delta x_T - 0)^2}{2v^2}} d(\Delta x_t^Q) \\
&= \int I_A \frac{1}{\sqrt{2\pi v}} e^{-\frac{(\Delta x_T)^2 - 2v^2\sigma\Delta x_T + (\sigma v^2)^2}{2v^2}} d(\Delta x_t^Q) \\
&= \int I_A \frac{1}{\sqrt{2\pi v}} e^{-\frac{(\Delta x_T - v^2\sigma)^2}{2v^2}} d(\Delta x_t^Q) \\
&= \int I_A \frac{1}{\sqrt{2\pi v}} e^{-\frac{(\Delta x_T{}^R - 0)^2}{2v^2}} d(\Delta x_t^R)
\end{aligned}
$$

$$Let \qquad \Delta x_T{}^Q - v^2\sigma = \Delta x_T{}^R$$

$$
\begin{aligned}
E^{Q}\left[e^{-\frac{\sigma^2 v^2}{2T}T+\sigma\Delta x_T} I_A\right] &= E^{R}\left[I_A\right] = P^{R}(S_0 e^{(r-\frac{\sigma^2 v^2}{2T})T+\sigma(\Delta x_T{}^R + v^2\sigma)} > K) \\
&= N\left(\frac{\ln(\frac{S_0}{K}) + (r - \frac{\sigma^2 v^2}{2T})T + v^2\sigma^2}{\sigma v}\right) \\
&= N\left(\frac{\ln(\frac{S_0}{K}) + (r + \frac{\sigma^2 v^2}{2T})T}{\sigma v}\right)
\end{aligned}
$$

$$E\left[I_A\right] = P(S_0 e^{(r-\frac{\sigma^2 v^2{}^2}{2T})T+\sigma\Delta x_T} > K) = N\left(\frac{\ln(\frac{S_0}{K}) + (r - \frac{\sigma^2 v^2}{2T})T}{\sigma v}\right)$$

The closed form formula of European call options is found to be:

$$C = S_0 N\left(\frac{\ln(\frac{S_0}{K}) + (r + \frac{\sigma^2 v^2}{2T})T}{\sigma v}\right) - Ke^{-rT} N\left(\frac{\ln(\frac{S_0}{K}) + (r - \frac{\sigma^2 v^2}{2T})T}{\sigma v}\right) \qquad (17)$$

where

S_0 : stock price at the initial time

K : exercise price

r : annual int erest rate

T:number of periods (by year)

$v^2 = \frac{2}{\beta^2}(e^{-\beta T} - 1 + \beta T)$

β is correlation parameter

σ^2 is the parameter

As mentioned in the previous section, Ornstein Uhlenbeck position process approaches a geometric Brownian motion that represents log price difference when either the time-to-maturity tends to infinity or with zero correlation. Here, we analyse the behaviour of the first two moments of stock returns under respectively an Ornstein Uhlenbeck position process and a Brownian motion subject to these two conditions. We first show that when T is large:

$$E[\ln S_T - \ln S_0] = (r - \frac{\sigma^2 v^2}{2T})T \to (r - \frac{\overline{V}}{2})T$$

$$V[\ln S_T - \ln S_0] = \sigma^2 v^2 = \frac{2\sigma^2}{\beta^2}(e^{-\beta T} - 1 + \beta T) \to \overline{V}T$$

where $(r - \frac{\overline{V}}{2})T$ and $\overline{V}T$ are the mean and variance of the log stock price in geometric Brownian motion.

The above two relations will hold when:

$$\sigma^2 v^2 = \frac{2\sigma^2}{\beta^2}(e^{-\beta T} - 1 + \beta T) \to \frac{2\sigma^2}{\beta}T = \overline{V}T \qquad (18)$$

$$\sigma^2 = \frac{\beta \overline{V}}{2}$$

To facilitate direct comparisons with Brownian motion, let the parameter σ^2 be a function of $\sigma^2(\overline{V},\beta)$, where $\overline{V}$ is the instaneous variance of the log price differences represented by a geometric Brownian motion.

As the time-to-maturity approaches infinity:

$$C = S_0 N\left(\frac{\ln(\frac{S_0}{K}) + (r + \frac{\sigma^2 v^2}{2T})T}{\sigma v}\right) - Ke^{-rT} N\left(\frac{\ln(\frac{S_0}{K}) + (r - \frac{\sigma^2 v^2}{2T})T}{\sigma v}\right)$$

$$\to \quad S_0 N\left(\frac{\ln(\frac{S_0}{K}) + (r + \frac{1}{2}\overline{V})T}{\sqrt{\overline{V}T}}\right) - Ke^{-rT} N\left(\frac{\ln(\frac{S_0}{K}) + (r - \frac{1}{2}\overline{V})T}{\sqrt{\overline{V}T}}\right)$$

which confirms the fact that European call values under the Orstein Uhlenbeck position process converges to the Black-Scholes theoretical call values as T is large.

Similarly, when the correlation coefficient ρ approaches zero, β then becomes infinite, and with $\sigma^2 = \frac{\beta\overline{V}}{2}$, we then have

$$\sigma^2 v^2 = \frac{2\sigma^2}{\beta^2}(e^{-\beta T} - 1 + \beta T) = \overline{V}(\frac{e^{-\beta T} - 1}{\beta} + T) \to \overline{V}T$$

This implies that the mean and variance of the log price differences under the Ornstein Uhlenbeck position process will approach that of a Brownian motion:

$$E[\ln S_T - \ln S_0] = (r - \frac{\sigma^2 v^2}{2T})T \to (r - \frac{\overline{V}}{2})T$$

$$V[\ln S_T - \ln S_0] = \sigma^2 v^2 = \frac{2\sigma^2}{\beta^2}(e^{-\beta T} - 1 + \beta T) \to \overline{V}T$$

Therefore, as the correlation coefficient ρ approaches zero, we see that European call values under both processes coincide:

$$C = S_0 N\left(\frac{\ln(\frac{S_0}{K}) + (r + \frac{\sigma^2 v^2}{2T})T}{\sigma v}\right) - Ke^{-rT} N\left(\frac{\ln(\frac{S_0}{K}) + (r - \frac{\sigma^2 v^2}{2T})T}{\sigma v}\right)$$

$$\to \quad S_0 N\left(\frac{\ln(\frac{S_0}{K}) + (r + \frac{1}{2}\overline{V})T}{\sqrt{\overline{V}T}}\right) - Ke^{-rT} N\left(\frac{\ln(\frac{S_0}{K}) + (r - \frac{1}{2}\overline{V})T}{\sqrt{\overline{V}T}}\right)$$

To conclude, when $\sigma^2 = \frac{\beta \bar{V}}{2}$, in both cases when the time-to-maturity is long enough or the correlation is zero, the Ornstein Uhlenbeck position process approaches a Brownian motion with mean $(r - \frac{\bar{V}}{2})T$ and variance $\bar{V}T$ We therefore rewrite the closed form of European call values as

$$C = S_0 N\left(\frac{\ln(\frac{S_0}{K}) + (r + \frac{\bar{V}(e^{-\beta T} - 1 + \beta T)}{2\beta T})T}{\sqrt{\frac{\bar{V}}{\beta}(e^{-\beta T} - 1 + \beta T)}}\right) - Ke^{-rT} N\left(\frac{\ln(\frac{S_0}{K}) + (r - \frac{\bar{V}(e^{-\beta T} - 1 + \beta T)}{2\beta T})T}{\sqrt{\frac{\bar{V}}{\beta}(e^{-\beta T} - 1 + \beta T)}}\right) \tag{19}$$

where:

S_0 : stock price at the initial time
K : exercise price
r : annual interest rate
T:number of periods (by year)
β is correlation parameter
$\bar{V}$ is instaneous variance of log stock price in geometric Brownian motion

There are six parameters in the option pricing model. Except the parameter β , which stands for the level of correlation, the other five parameters would affect the option value in the same directions as they were in the Black-Scholes option pricing model.

In the following, we derive the hedging Greeks of the option pricing model:

Delta:

$$Delta = N(d1)$$

$$where \quad d1 = \frac{\ln(\frac{S_0}{K}) + (r + \frac{\bar{V}(e^{-\beta T} - 1 + \beta T)}{2\beta T})T}{\sqrt{\frac{\bar{V}}{\beta}(e^{-\beta T} - 1 + \beta T)}} > 0 \tag{20}$$

Gamma:

$$Gamma = \frac{1}{S_0\sqrt{\frac{\overline{V}}{\beta}(e^{-\beta T} - 1 + \beta T)}} n(d1) > 0 \tag{21}$$

$$where \quad n(d1) = N'(d1)$$

Vega:

$$\text{Let } \overline{\text{V}} = \sigma *^2$$

$$\frac{\partial C}{\partial \sigma^*} = Sn(d1)\sqrt{\frac{(e^{-\beta T} - 1 + \beta T)}{\beta}} > 0 \tag{22}$$

Rho:

$$Rho = \frac{\partial C}{\partial r} = TKe^{-rT} N(d2) > 0 \tag{23}$$

Theta:

$$Theta = \frac{\partial C}{\partial t} = -(\frac{\partial C}{\partial \tau}) = -[S_0 n(d1)\frac{\sigma}{\beta}(1 - e^{-\beta T}) + Kre^{-rT} N(d2)] < 0 \tag{24}$$

$$Omega = \frac{\partial C}{\partial \beta} = S_0 n(d1) * \frac{1}{2}[\frac{\overline{V}}{\beta}(e^{-\beta T} - 1 + \beta T)]^{-\frac{1}{2}}[\frac{\overline{V}}{\beta}[1 - e^{-\beta T} - \beta T e^{-\beta T}]] > 0 \tag{25}$$

4. How our Model Contrasts the Black-Schole Model

As mentioned previously, the Orstein Ulenhbeck position process will reduce to a geometric Brownian motion with zero stock correlations or an infinite the time-to-maturity. In this section, we first explore how the theoretical option prices under the assumption of an Orstein Ulenhbeck position process differ from that were derived under the geometric Brownian motion assumption as in the Black-Scholes model by varying the level of asset correlations and the time to maturities. We assume throughout this section that the initial stock price and exercise price of the call option are both 50. Annual interest rate is 5%, and the annual variance is 50%.

In Table 1 and Table 2, with the time-to-maturity set to T=10/250, the correlation among daily stock returns is set to vary from 0.05 to 0.8. Table 1 shows the impact of different level of correlations on the call value. With an invariant variance of 50%, a higher correlation would cause the call option to reduce in value. This is understandable since given a fixed time-to-maturity (T=10/250) a higher level of correlation will directly reduce the volatility of the underlying asset, and hence results in a reduced call value. In Table 2, we introduce a difference ratio that is defined as the difference between the theoretical option prices under the assumption of an Orstein Ulenhbeck position process and the geometric Brownian motion divided by the Black-Scholes theoretical call value. We show how the daily correlation affects and the difference ratio, and we find that the difference ratio becomes larger as the daily correlation increases.

In Table 3 and Table 4, we examine the impact of time-to-maturities on the difference ration. Correlations between the consecutive-day stock prices are set to vary from 0.3 onwards for a total time period of twenty days. Table 3 shows that the difference ratio decreases as the time-to-maturity increases. It suggests that, in markets where frictional elements are present, the impact of the frictional elements in the long term would actually be less significant. And in Table 4, we see the difference ratio exhibits a downward decreasing pattern when both the time-to-maturity and the level of daily correlation are set to increase.

Table 1 Theoretical option values subject to different levels of correlation

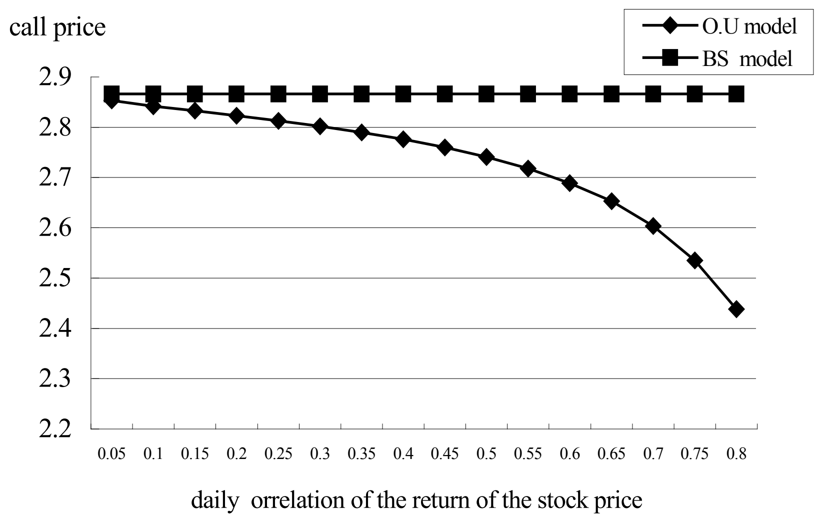


Table 2 Difference ratio subject to different levels of correlation

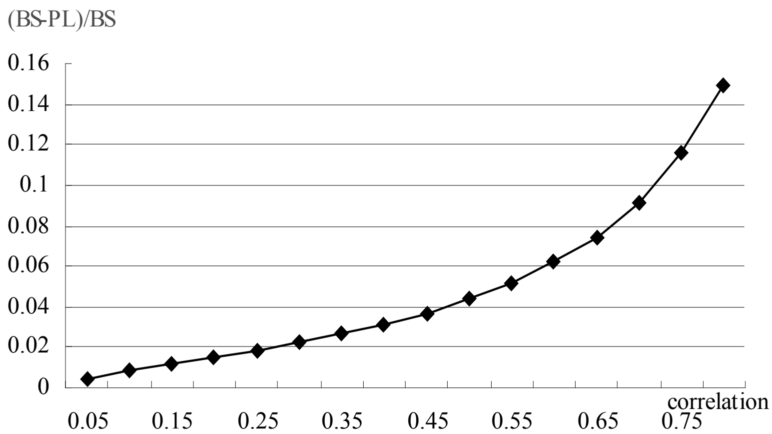
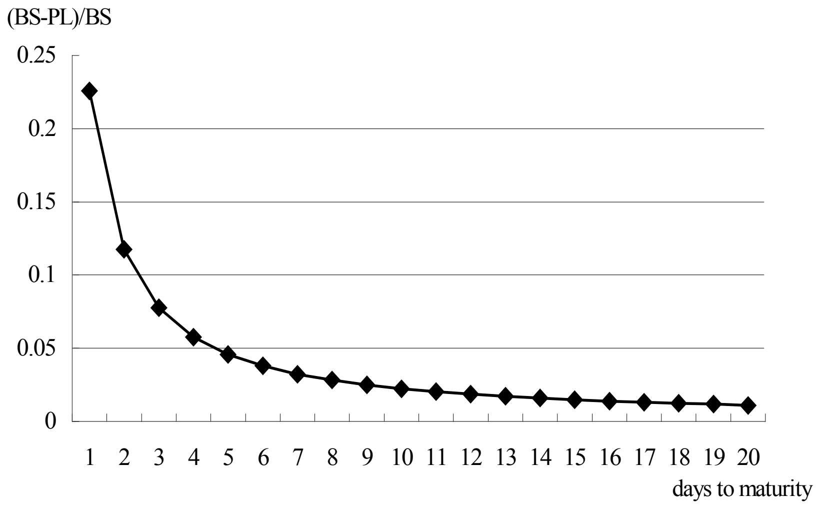


Table 3 Difference ratio subject to different time-to-maturities



The effects of moneyness, interest rates, and volatilities on the differences ratio are demonstrated in Table 5, Table 6, and Table 7. In Table 5, we assume the moneyness to range from 0.4 to 1.6, and we find that when the option is deeper in the money, the difference ratio becomes greater, and vice versa. In Table 6, we range the interest rate from 1% to 10%, and we find that as the interest rate gets higher, the difference ratio would reduce. Table 7 varies the variance from 20% to 80%, and shows that when the variance increases, the difference ratio becomes larger. It implies that if the underlying asset stock is highly volatile, the presence of frictional elements in the market would in fact amplify the deviation in option prices between our model and that of the traditional Black-Scholes model.

Table 4 Difference ration subject to joint effect to varying time-to-maturities and asset correlations

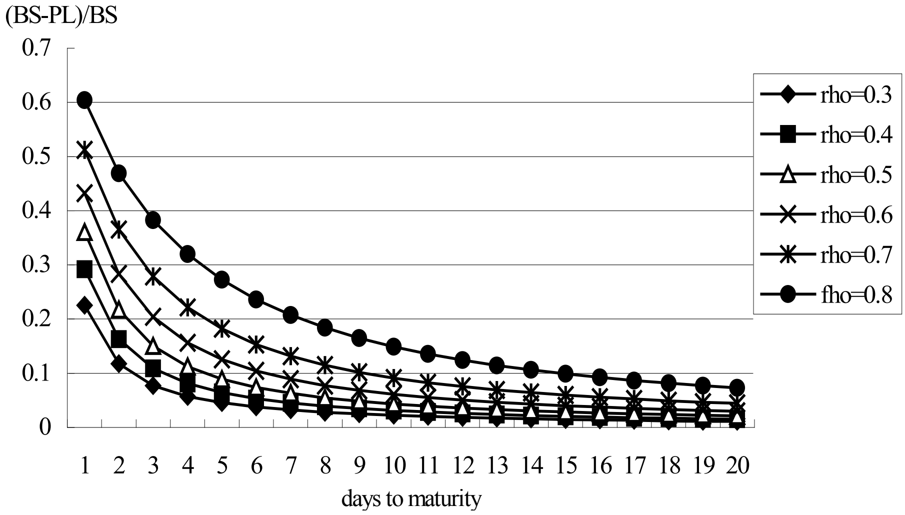


Table 5 Difference ratio subject to different levels of moneyness

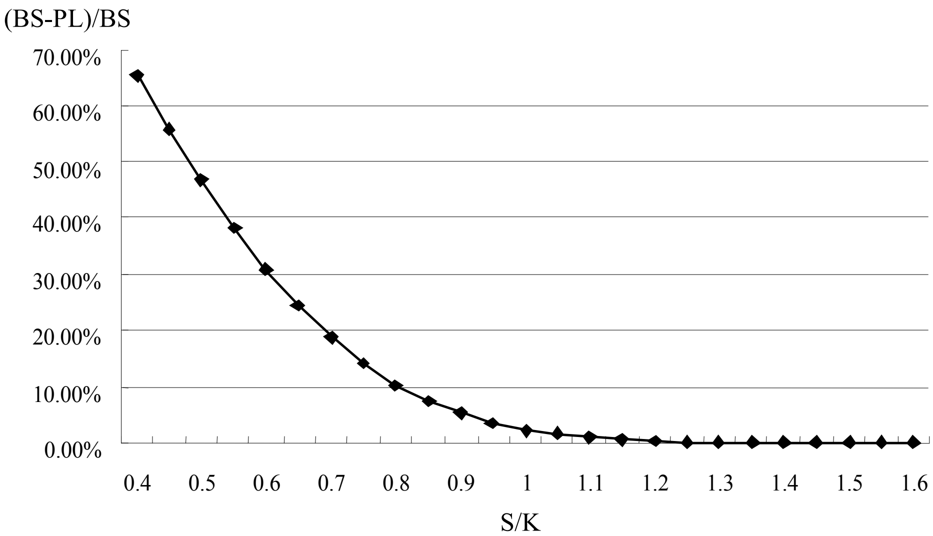


Table 6 Difference ratio subject to different levels of interest rate

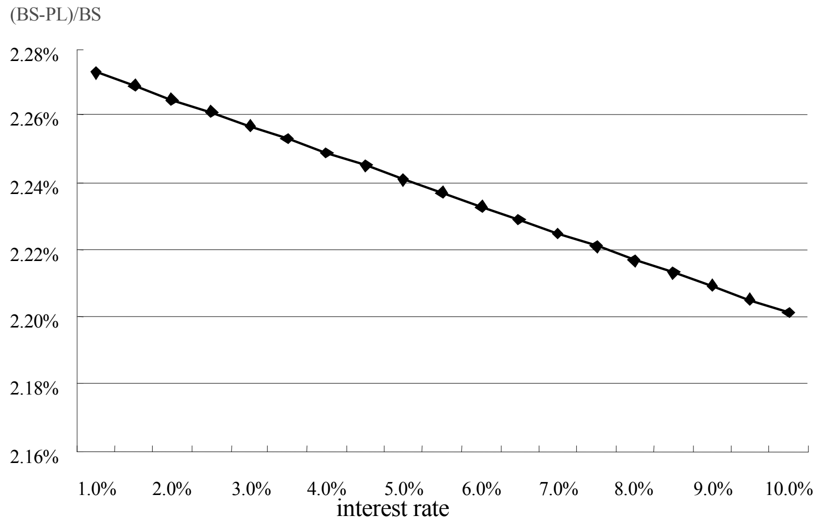
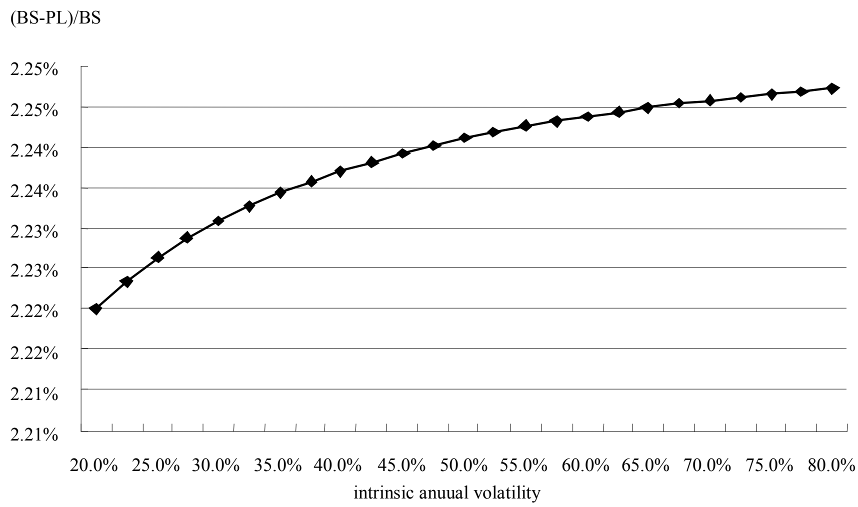


Table 7 Difference ratio subject to different levels of variance



5. Application: The TSE Market with Price Limit Constraints

In a stock market where daily price limit constraints are present, the higher or lower price trades will be suspended to the next day. as soon as the price hits the bounds.

The Taiwn Stock Exchange market is clearly one of such cases where the imposition of daily price limits directly prevents stock prices form unbounded variation.

In this section, we examine the feasibility of modeling asset price dynamics based on the Orstein Uhlenbeck process. The stocks in the Taiwan Stock Exchange Corporation are constrained by daily price limits while their ADRs are traded without limits in Nasdaq. In order see how they deviates from each other, we define the difference ratios as the differences between the volatilities of stocks and their respective ADRs devided by the volatilities of the ADRs. In the following we present our numerical results:

Table 8　The volatilities of the return of the stock price in Taiwan Security Market and Nasdaq in different measure period

	Volatility	Daily	Weekly	Monthly
TSM	Stock in TSEC	0.12%	0.57%	2.78%
	ADR	0.21%	0.84%	3.91%
	difference ratio	43.10%	31.54%	28.99%
	Volatility	Daily	Weekly	Monthly
UMC	Stock in TSEC	0.12%	0.62%	2.53%
	ADR	0.25%	0.93%	4.04%
	difference ratio	53.08%	33.26%	37.31%
	Volatility	Daily	Weekly	Monthly
ASX	Stock in TSEC	0.14%	0.79%	2.92%
	ADR	0.23%	0.99%	2.76%
	difference ratio	38.69%	20.60%	-5.77%

Our results above show that the difference ratio declines as the time interval increases, and the difference between both volatility levels of the two markets is further enlarged when the time interval increases. Our results suggest that Geometric Brownian Motion could vary with an infinite velocity in an infinitesimal time interval, and its increments are mutually independent, and these two properties clearly differ from the behavior of stock prices that we can observe when price limits are present. In addition, short-term volatilities of the stock returns when price limits are present are less than that otherwise without price limit constraints. In a stock market with price limits, delayed price discovery effect directly implies that daily stock prices could in fact be correlated, and as the observed time interval increases, the delayed price discovery effect becomes lower. The Ornstein-Uhlenbeck position process closely captures such effect. When the upper/lower price limits are frequently hit, the delayed price discovery effect further prevails, this would reflect in higher correlation effects among daily stock prices. On the other hand, when the upper/lower limits are rarely hit, the impact of the delayed price discovery effect is in fact insignificant, which implies that correlation effects among stock prices are likely to be negligible, and the market is efficient. When the correlation coefficient approaches to zero, stock prices that were generated by the Ornstein-Uhlenbeck position process would coincide with that generated by Brownian motion.

6. Conclusion

Ornstein Uhlenbeck position process is the integral of Ornstein Uhlenbeck process, and it has some interesting properties. This process incorporates a description for the correlation among underlying assets. And it approaches a Brownian motion in the long run. In this paper we consider this process as the dynamic description for asset price movements subject to frictions. We derive a closed form formula for the European call value under the risk neutral measure, together with the hedging Greeks. We conduct sensitivity analysis to explore how this pricing model differs from the traditional Black-Scholes. We introduce a difference ratio that is defined as the difference between the theoretical option prices under the assumption of an Orstein Ulenhbeck position process and the geometric Brownian motion divided by the Black-Scholes theoretical call value. We find that the difference ratio becomes larger as the daily correlation increases, and the difference ratio decreases as the time-to-maturity increases. It suggests that, in markets where frictional elements are present, the impact of the frictional elements in the long term would actually be less significant. Our numerical results also show that when the underlying asset stock is highly volatile, the presence of frictional elements in the market would in fact amplify the deviation in option prices between our model and that of the traditional Black-Scholes model.

References

1. Ban, Choi, and Ku, 2000, Valuation of European options in the market with daily price limits, Applied Mathematical Finance, 7, 61-74.
2. Cox, John C. and Stephen A. Ross, 1976, "The Valuation of Options for Alternative Stochastic Processes," *Journal of Financial Economics*, 3, 145-166.
3. Cox, John C. & Rubinstein, Mark, 1985, Options Markets. Englewood Cliffs, New Jersey: Prentice-Hall.
4. Doob J.L, 1942, The Brownian movement and stochastic equations, *The annals of Mathematics*, second serious, 43, issue 2, 351-369.
5. Goldenberg D.H, 1986, Sample Path Properties of Futures Prices, *The Journal of Futures Markets*, Vol. 6, No.1, 127-140.
6. Ghiang R. & Wei, 1995, Price limits and estimation of expected return and risk, Technical Report, Working Paper, University of Miami.
7. Hull, John and Alan White, 1987, "The Pricing of Options on Assets with Stochastic Volatility," *Journal of Finance*, 42, 281-300.
8. Kim K.A & Rhee G., 1997, Price limit performance: evidence from the Tokyo Stock Exchange, *The Journal of Finance*, LⅡ, No.2, June, 885-901.
9. Merton, Robert C., 1973, "Theory of Rational Option Pricing," *Bell Journal of Economics and Management Science*, 4, 141-183.
10. Merton, Robert C., 1976, "Option Pricing When Underlying Stock Returns are Discontinuous," *Journal of Financial Economics*, 3, 125-144.
11. Scott, Louis O., 1987, "Option Pricing When the Variance Changes Randomly: Theory, Estimation, and an Application," *Journal of Financial and Quantitative Analysis*, 22, 419-438.
12. Shen C.H. & Wang L.R., 1998, Daily serial correlation, trading volume and price limits: evidence from the Taiwan stock market, *Pacific-Basin Finance Journal*, 6, 251-273.