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Non-productive consumption loans and threshold effects in the inflation-growth relationship

By Fu-Sheng Hung

Department of Economics, National Taipei University, Taipei 104, Taiwan, ROC; e-mail: fshung@mail.ntpu.edu.tw

Recent empirical evidence indicates that two inflation thresholds exist in the inflation-growth relationship. Pre-existing theoretical models, however, fail to generate such a pattern. By adding consumption loans (which are non-productive) into a standard model of imperfect information, this paper finds that an increase in the inflation rate may increase, decrease, or have no significant effect on economic growth for inflation rates below a threshold level; however, for inflation rates higher than this threshold level, an increase in the inflation rate significantly reduces economic growth. Moreover, the marginal impact of an increase in the inflation rate in terms of reducing economic growth increases with the rise in the inflation rate, until the inflation rates reach the second threshold level, from which such a marginal effect significantly decreases. These results accord well with recent empirical evidence.

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1. Introduction

Ever since the seminal work of Tobin (1965), the effect of inflation on capital investment and economic growth has long been one of the important topics in macroeconomics. Theoretically, depending on how money is introduced into the model, the early literature has established that an increase in the inflation rate may lead to an increase (as in Tobin, 1965, where money is a substitute for capital in the portfolio), a decrease (as in Stockman, 1981, where capital investment is subject to the cash-in-advance constraint), or have no effect (as in Sidrauski, 1967, where money enters into the utility function) on capital investment and economic growth. On the other hand, early empirical studies have reported a mixed correlation between inflation and economic growth, until recently numerous studies have found non-linear correlations between inflation and economic growth.

Fischer (1993) first points out the possibility that the effect of an increase in the inflation rate on economic growth may differ at low levels and high levels of inflation. Specifically, by choosing 15% and 40% inflation rates as the break points, Fischer (1993) finds that an increase in the inflation rate leads to an increase in economic growth for inflation rates below 15%; however, there is a reduction in economic growth for inflation rates above 15%. Moreover, the marginal (and negative) impact of an increase in the inflation rate on economic growth is substantially lower for inflation rates above 40% than for those between 15% and 40%. Consequently, there are two inflation thresholds (a lower threshold at 15% inflation and a higher threshold at 40%) at which the effect of an increase in the inflation rate on economic growth changes.

By adopting the break points proposed by Fischer (1993), Barro (1997) finds a negative correlation between inflation and economic growth for all ranges of inflation; however, the coefficients of the inflation rates in the growth regressions are equal to −0.023 for inflation rates below 15%, −0.055 for inflation rates between 15% and 40%, and −0.029 for inflation rates above 40%. In a sense, Barro's (1997) finding is very consistent with that of Fischer (1993), as there are two inflation thresholds in the inflation-growth relationship.[1] By applying newly developed econometric techniques, Ghosh and Phillips (1998) find a positive correlation between inflation and economic growth for very low levels of inflation rates. For other levels of inflation rates, however, there is a negative correlation between inflation and economic growth, corroborating the existence of the lower inflation threshold. Moreover, the negative correlation is convex, namely, the marginal impact of an increase in the inflation rate in reducing economic growth is higher for inflation rates between 10% and 20% than for those between 40% and 50%, implying that a higher threshold level does exist. Khan and Senhadji (2001) and Burdekin *et al.* (2004) also confirm Ghosh and Phillips's (1998) findings.[2]

Though recent studies reach an agreement on the existence of two inflation thresholds, they disagree on the inflation-growth correlation for inflation rates below the lower threshold. Such a disagreement, in fact, can also be found in other empirical studies. For example, Bruno and Easterly (1998) find that the negative correlation between inflation and economic growth is only observed with high levels of inflation; for low levels of inflation, there is no cross-country correlation between inflation and economic growth. As a result, recent empirical studies have found two inflation thresholds in the inflation-growth relationship with the following scenario. Below the lower inflation threshold, the effect of an increase in the inflation rate on economic growth is uncertain. Between the two thresholds, an increase in the inflation rate powerfully reduces economic growth. Above the higher threshold, there continues to be a negative effect of inflation on economic growth, but it is a small one. While recent empirical studies have

[1] Indeed, according to Barro's (1997) result, the marginal effect of an increase in the inflation rate in reducing economic growth is small for inflation rates below the lower threshold level as well as for inflation rates above the higher threshold level. If inflation rates are located in between the lower and higher thresholds, then inflation has a relatively large impact on economic growth.

[2] In particular, Fig. 2 in Khan and Senhadji (2001) replicates the inflation-growth relationship that we just outlined.

confirmed the existence of two inflation thresholds, recent theoretical models fail to generate such a pattern for the inflation-growth relationship.[3] The purpose of this paper is to develop a model that is able to yield such a pattern for the inflation-growth relationship and thereby provide a possible theoretical explanation.

Although recent theoretical studies fail to generate two inflation thresholds, attempts have been made to capture some aspects of this non-linear relationship. Bose (2002), for example, develops a model to shed light on the empirical facts that the overall output effect of inflation is negative and that there is an inflation threshold at which the magnitude of the negative output effect of inflation significantly changes. Bose's analysis is based on a simple endogenous growth model in which households (lenders) can convert output into capital by means of a home production technology or by lending to capital-producing firms (entrepreneur borrowers) that are endowed with a linear technology for capital production. Compared with firms' capital technology, the home production yields a lower return so that loans are mutually desirable to lenders and borrowers. Competition among lenders then implies that lenders' return from lending to borrowers must be equal to that from the home production. Bose (2002) further assumes that goods need to be stored in the form of money before they can be traded between lenders and borrowers, indicating that money is needed for loan transactions but not for the home technology. As a result, an increase in the inflation rate that reduces lenders' returns from lending must be associated with an increase in the nominal interest rate on loans. As is indicated by Bose (2002), this feature is very similar to Stockman's (1981) cash-in-advance constraint on capital investment so that there is an overall negative output effect of inflation.

The key ingredient of Bose's (2002) model that generates the inflation threshold is the presence of asymmetric information, which arises under the assumptions that there are two types of borrowers and that borrowers' types are private information. As is standard in the literature, when faced with such asymmetric information lenders can induce separation of borrowers by rationing credit to a fraction of borrowers (a lending regime of rationing). Besides this standard means of separation, Bose (2002) proposes a possibility that lenders can also separate borrowers through the costly screening of a fraction of borrowers (a screening regime). He then shows that an increase in the inflation rate exacerbates the problem of asymmetric information and hence leads to an increase in the incidence of rationing or screening, which is detrimental to economic growth under each lending regime. Moreover, the rate of economic growth for a given inflation rate is greater in the screening regime compared with that in the rationing regime and there exists a critical inflation rate below which lenders choose a screening regime and above

[3] Since the inflation rate does not have any effect on how money is introduced into the theoretical models, early theoretical models are not able to explain this pattern. As will be discussed in Section 4 below, recent theoretical studies are not able to fully capture the inflation-growth relationship reported by recent empirical studies.

which lenders choose a rationing regime. These results indicate that there is a sharp fall in the growth rate as inflation increases from low levels and exceeds this critical level, thus confirming the existence of an inflation threshold.

Bose's (2002) analysis is quite insightful in explaining the existence of the lower inflation threshold reported by recent empirical studies; however, his model cannot generate the higher inflation threshold as well as the possibility that an increase in the inflation rate may facilitate capital investment and economic growth for inflation rates below the lower inflation threshold.[4] To fully capture the non-linear correlations between inflation and economic growth, we add a group of borrowers who intend to borrow for consumption (i.e. consumer borrowers) into a framework that is very similar to Bose (2002). This idea is motivated by Jappelli and Pagano (1994) who focus on credit to consumers (i.e. consumption loans), rather than on credit to capital-producing firms (i.e. investment loans). In a model where credit rationing of consumption loans is exogenously given, Jappelli and Pagano (1994) show that an exogenous increase in the incidence of credit rationing to consumers, which reduces the fraction of banking resources allocated to consumers, can force the economy to save more resources for capital investment.[5] Under models of endogenous growth, this can promote economic growth; hence, an increase in the incidence of credit rationing of consumption loans is beneficial to economic growth. If we follow Bose (2002) to endogenously obtain credit rationing of consumption loans, then an increase in the inflation rate, which exacerbates the problem of asymmetric information as in Bose (2002) and hence leads to an increase in the incidence of credit rationing of consumption loans, can lead to a higher aggregate saving rate and, as is shown by Jappelli and Pagano (1994), can facilitate capital investment and economic growth.

By adding consumer borrowers into a framework based on Bose's (2002) rationing regime and endogenously deriving credit rationing of consumption loans along with that of investment loans, this paper can generate two opposite effects of inflation on growth (a positive output effect of inflation as in Jappelli and Pagano and a negative output effect of inflation as in Bose), which can potentially explain two inflation thresholds in recent empirical studies. In our model, there are three kinds of agents (i.e. lenders, borrowers, and output-producing firms) and borrowers are further classified into two groups: consumers and entrepreneurs (i.e. capital-producing firms). Consumers intend to borrow old-age income for middle-age consumption while entrepreneurs need external funding for implementing their capital projects. Similar to Bose (2002), there are two types of borrowers in each group—those with a high risk of default and those with a low risk of default—and borrowers' types are private information. Loans are intermediated by banks (established by lenders with free entry) which are subject to reserve

[4] Bose's (2002) model well captures the lower inflation threshold reported by Barro (1997), who has found that an increase in the inflation rate always reduces economic growth and that such an effect is more pronounced for inflation rates between 15% and 40% compared with inflation rates below 15%.

[5] Modigliani (1986, p.305) has a similar argument.

requirements and, following Bose (2002), money is needed for loan transactions between banks and borrowers. For a reason similar to Bose (2002), an increase in the inflation rate must be associated with an increase in the nominal interest rate on loans. We then find that an increase in the inflation rate gives rise to two effects on loans to entrepreneurs (investment loans). First, an increase in the inflation rate that erodes the purchasing power of money induces entrepreneurs to borrow more resources for financing their capital projects at a maximal scale.[6] This reduces the amount of resources left for lenders' capital production at home and hence is detrimental to economic growth. Second, it increases the nominal interest rate on loans to entrepreneurs in an asymmetric way such that the magnitude of the increase in the nominal interest rate is greater for the contract intended for high-risk entrepreneurs than that for low-risk entrepreneurs. As in Bose (2002), this gives high-risk entrepreneurs more incentives to pretend to be low-risk ones and thereby exacerbates the problem of asymmetric information. To deter this behavior, intermediaries must ration the credit of low-risk entrepreneurs more severely. Given that entrepreneurs' capital technology is better than lenders' home technology in terms of producing capital, this second effect is also detrimental to capital investment and economic growth. Under certain parameter values, the marginal effect of both first and second effects (from investment loans) increases at a decreasing rate as inflation rises.

Similar to loans to entrepreneurs, an increase in the inflation rate that raises the nominal interest rate on loans to consumers also gives rise to two effects. First, since consumer borrowers intend to borrow old-age income for middle-age consumption, an increase in the nominal interest rate, which reduces the present value of consumers' old-age income, decreases the amount each consumer wishes to borrow. Second, it also exacerbates the problem of asymmetric information and thereby leads to tighter rationing of low-risk consumers. Similar to Jappelli and Pagano (1994), both effects force the economy to save more resources for capital investment (via lenders' home production technology) and hence are beneficial to economic growth. It is found that the marginal effect of the first (respectively second) effect on growth increases (resp. declines) as inflation rises. Under some parameter values, it is further found that the marginal effect of the second (resp. first) effect on growth dominates that of the first (resp. second) effect for relatively low (resp. high) levels of inflation. As a result, the net marginal effect of both effects (from consumption loans) is decreasing (resp. increasing) in the inflation rate for relatively low (resp. high) levels of inflation.

The joint consideration of both productive investment and non-productive consumption loans can potentially explain two inflation thresholds in recent empirical studies. The positive effect of inflation on growth (from consumption loans) may dominate or be dominated by the negative one (from investment loans) for very low levels of inflation, implying that the net effect of inflation on growth

[6] As will be seen below, this maximal scale is independent of inflation, a result similar to Bose (2002).

is uncertain for very low levels of inflation. As inflation rises, the marginal effect of the negative effect from investment loans increases (at a decreasing rate) while the marginal effect of the positive effect from consumption loans declines for relatively low levels of inflation. Therefore, there must be a critical inflation rate such that the negative effect from investment loans eventually dominates the positive effect and the difference between these two effects become large after inflation rates are higher than this critical level. This case potentially explains the lower inflation threshold. Moreover, the marginal effect of the positive output effect from consumption loans turns out to be increasing in the inflation rate after inflation rates are relatively high. Due to this, there must be another critical inflation rate such that although the negative effect still dominates the positive one, the difference between these two effects becomes small as the inflation rates exceed this critical level. This gives rise to the higher inflation threshold.

The remainder of this paper proceeds as follows. Section 2 presents the basic model and Section 3 describes the equilibrium loan contracts for the purpose of investment and consumption. In Section 4 we first obtain the equilibrium growth rate and then examine how a change in the inflation rate affects the equilibrium rate of economic growth. We also compare our results with recent theoretical studies. Section 5 concludes.

2. Description of the model

The economy consists of an infinite sequence of three-period-lived overlapping generations (OG). Each generation is of identical size and composition, and contains three kinds of risk-neutral agents: lenders, borrowers and output-producing firms. Borrowers are further classified into two groups: entrepreneurs (i.e. capital-producing firms) and consumers. For simplicity, each population of entrepreneurs and output-producing firms is normalized to one while the populations of lenders and consumers are normalized to n and m, respectively.

2.1 Lenders

Lenders are endowed with a unit of labor when they are young and care only about their old-age consumption. Hence, in the first period of life a lender will sell his labor to firms to generate wage income and save this income for his old-age consumption. Each young lender has access to a home production technology that can convert one unit of time t output into $Q\varepsilon$ ($\varepsilon < 1$) units of time $t+2$ units of capital with certainty.[7] By denoting ρ_{t+2} as the rental rate of capital at time $t+2$, the rate of return on the home production technology between time t and $t+2$ is $Q\varepsilon\rho_{t+2}$. A time-t young lender can simply save his wage income by means of this home

[7] As will be seen, an entrepreneur's capital project, if successful, can convert one unit of time-t output into Q units of time-$t+2$ capital. The assumption that $\varepsilon < 1$ implies that the home technology is inferior to the entrepreneur's project in terms of producing capital.

technology for his old-age consumption. Alternatively, young lenders can extend loans to borrowers in return for time-$t+2$ output. As is the case in Azariadis and Smith (1996), it is assumed that there are financial intermediaries that attract deposits from lenders and offer loans to borrowers.

2.2 Consumer borrowers

Each consumer borrower cares about consumption in his second and third periods of life. Consumers have no endowment in the first and second periods of life; however, with a non-negative probability, each consumer will be endowed with one unit of labor in his final period of life. There are two types of consumers and consumers' types refer to the probability of getting one unit of old-age labor.[8] With probability p_i, $i=H, L$, a type-i consumer will receive one unit of labor in the final period and, with probability $1-p_i$, the consumer will be endowed with nothing. Consumers' types are private information. It is assumed that $0<p_H<p_L\leqslant 1$ so that type-L consumers have a higher probability of obtaining a unit of old-age labor than type-H consumers; hence, type-L (resp. type-H) consumers can be regarded as low-risk (resp. high-risk) consumers. A fraction λ of consumer borrowers is of type-H.

The utility function of a representative (generation-t) consumer is given as

$$U^c(c_t,c_{t+1},c_{t+2}) = c_{t+1} + \beta^c c_{t+2}, \tag{1}$$

where c_t is the consumption in time t and β^c is the discount factor. To induce borrowing, we assume that β^c is sufficiently small; hence, if possible, all consumers intend to borrow from the intermediary and consume all expected old-age income in their middle age. Following Bose (2002), consumer borrowers must apply for a loan in their first period of life, even though they are concerned about middle-age consumption. It should be noted that if no funds are forthcoming, then the expected lifetime utility of a generation-t consumer is $p_L\beta^c w_{t+2}$ for type-L consumers and $p_H\beta^c w_{t+2}$ for type-H ones, where w_{t+2} is the wage rate in time $t+2$ (i.e. the old or final period for generation-t consumers). Given that $p_L>p_H$, it is clear that type-L consumers have a lower opportunity cost of being denied credit than do type-H ones. Similar to Bose (2002), this assumption makes the separating equilibrium emerge.[9]

2.3 Entrepreneur borrowers (capital-producing firms)

Entrepreneurs care only about old-age consumption. An entrepreneur is endowed with one unit of labor as well as an investment project in his second period of life.

[8] We assume that the structure of consumers is similar to that of entrepreneurs, which will be stated below.

[9] As indicated by Bose (2002), the assumption that different types of borrowers with different opportunity costs are denied credit ensures the "single crossing properties" of the indifference curve in the contract plane.

The investment project is risky, and according to its probability of success can be classified as either high-risk (type-H) or low-risk (type-L). It should be noted that entrepreneurs are not endowed with any output; hence, external funding is necessary for an entrepreneur to implement his project. Following Bose (2002), a young entrepreneur must apply for a loan (from an intermediary) during his first period of life, even though he needs the external funding during middle age.

A middle-aged entrepreneur who obtains a loan from the intermediary can operate his investment project using his own labor to convert one unit of time-$t+1$ output into Q units of time-$t+2$ capital, with probability p_i, $i=H, L$. With probability $1-p_i$, the operation of the project fails and nothing is produced. By assumption, $0<p_H<p_L\leqslant 1$ and the types of entrepreneurs' projects are private information. Moreover, a fraction λ of entrepreneurs is assumed to have type-H projects.

If no funds are forthcoming, the entrepreneur can then utilize his labor in the home production of goods during his second period of life.[10] Following Bose (2002), output produced by a type-i entrepreneur at time t in the home production yields $\beta_i^e w_{t-1}$ units of time t consumption goods, where w_{t-1} is the market wage rate at time $t-1$. It is assumed that $\beta_L>\beta_H=0$, implying that the opportunity cost of being rejected in regard to a loan is lower for a low-risk entrepreneur than for a high-risk one. We also follow Bose (2002) by assuming that a middle-aged entrepreneur has access to a storage technology that can convert one unit of time-$t+1$ output into one unit of time-$t+2$ output.[11] Thus, if a low-risk entrepreneur is rejected in regard to a loan, he can engage in the home production and store the proceeds for his old-age consumption.

2.4 Output-producing firms

Output-producing firms are active in their final period of life as they gain access to an output production technology when they are old. The old firm can rent capital from other old lenders and old entrepreneurs, plus hire young lenders and old consumers (who obtain labor endowment) as labor input to produce output. The output production technology in time t is given as

$$y_t = A\psi_t^{\sigma} k_t^{\alpha} L_t^{1-\alpha},\ A>0, \tag{2}$$

where ψ_t denotes the average capital stock per firm and k_t and L_t are the capital stock as well as labor employed by the firm, respectively. Capital depreciates fully after production. In the capital market equilibrium, each firm employs the same

[10] The low- (high-) risk entrepreneurs are those entrepreneurs whose projects belong to type-L (-H). Entrepreneurs' projects are not tradable.

[11] The low-risk entrepreneur can engage in home production in his second period of life if his loan application is rejected. The entrepreneur, however, cares only about old-age consumption. Hence, Bose (2002) implicitly assumes that the low-risk entrepreneur in middle age has access to a storage technology. It should be noted that this storage technology is not accessible to young borrowers, so that young borrowers must hold money. See below.

amount of capital; hence, $\psi_t = k_t$. Moreover, following Bose and Cothren (1996) and Bose (2002), it is assumed that $\sigma = 1 - \alpha$, implying that the output production technology is linear as in the *AK* model. Labor and capital are competitive so that the wage rate (w_t) and the rental rate of capital (ρ_t) at time t are given as

$$w_t = A(1-\alpha)k_t^{\sigma+\alpha}L_t^{-\alpha} = A(1-\alpha)k_tL_t^{-\alpha} \quad (3)$$

and

$$\rho_t = A\alpha\psi^{\sigma}k_t^{\alpha-1}L_t^{1-\alpha} = A\alpha L_t^{1-\alpha}. \quad (4)$$

As will become clear, the per-firm labor employment is constant over time under a separating equilibrium in the loans market.[12] Hence, the rental rate of capital is constant over time (which is denoted as ρ) as in the *AK* model.

2.5 Financial intermediation, money, and loan transactions

Loans are intermediated and each young lender can establish an intermediary without incurring any cost.[13] The assumption of free entry into the intermediary activity ensures competitive behavior among intermediaries, which will drive the intermediary's profits to zero.

It should be recalled that borrowers need external funding during their second period of life, while lenders wish to save while still young. We assume that intergenerational loans are too costly to process.[14] As a result, borrowers must contract with financial intermediaries when they are young. Once a young borrower at time t obtains a loan from an intermediary, he must exchange it for money in the same period and then use the money to buy output in the next period for capital investment or consumption. Moreover, the operation of each financial intermediary is subject to a reserve requirement policy that asks each intermediary to hold a μ $(1 > \mu > 0)$ fraction of total deposits in the form of money between time t and $t+1$.

At the beginning of time t, each young borrower applies for loans from a young intermediary.[15] Once a young intermediary reaches an agreement with a young borrower, he must offer a deposit contract to young lenders. Suppose that a young intermediary at time t agrees to offer q_t units of time-t output to a young borrower at time t. Then, in order to fulfill the borrower's need, the intermediary must also offer a deposit contract at time t to young lenders that attracts $q_t/(1-\mu)$

[12] Labor employment includes young lenders and old consumers who obtain one unit of labor endowment. Hence, $L_t = L = n + m[\lambda P_H + (1-\lambda)p_L]$.

[13] We assume that direct lending/borrowing between lenders and borrowers is more costly than indirect lending/borrowing (i.e. via financial intermediation). We also consider the limiting case where the cost of intermediation is normalized to zero.

[14] Similar to traditional OG models, money serves for intergenerational transactions while loans are for intragenerational transactions.

[15] For simplicity, the young intermediary is an intermediary operated by a young lender.

units of time-t output in the form of deposits during the same period. Once the intermediary obtains $q_t/(1-\mu)$ in deposits, he will hand over q_t ($\equiv(1-\mu)\ q_t/(1-\mu)$) units of deposits to the borrower and hold $q_t\mu/(1-\mu)$ units of deposits in the form of money between time t and $t+1$ to satisfy the reserve requirement.

Following Bose (2002), it is assumed that Q is sufficiently large so that the rate of return from lenders' home production (i.e. $Q\varepsilon\rho$) is greater than the rate of return from holding nominal money. This implies that lenders will not simply store their wage in the form of money.

The demand for money originates from the young borrowers (who want to store their loans in the form of money) as well as the young intermediary (who needs to hold money to satisfy the reserve requirement).[16] Following Bose (2002), the government accomplishes any monetary injection (denoted as M_t) by a lump-sum transfer to old lenders and borrowers. The old lenders and borrowers in turn can utilize the money (transferred from the government) to buy output for their consumption. As a result, the suppliers of money at any point in time include old lenders and borrowers (who obtain M_t from the government in the same period) plus the middle-aged intermediaries (who hold money as a reserve requirement during their young period of life) and middle-age borrowers (who utilize the money they acquire at the young period to exchange output for investment or consumption).

Denote δ_t as the fraction of the total wage incomes of time-t young lenders that are lent to young borrowers (via intermediaries). The market-clearing price (P_t) is then determined by an equation similar to Bose (2002), which can be written as $\delta_t n w_t P_t/(1-\mu) = \delta_{t-1} n P_{t-1} w_{t-1}/(1-\mu) + M_t$.[17] As stated by Bose (2002), w_t is proportional to k_t (eq. (2)), which is predetermined in time $t-1$, and P_{t-1}, δ_{t-1}, and w_{t-1} are also predetermined. Therefore, the inflation rate is determined by the change in monetary injections M_t (or withdrawals $-M_t$) as well as by the ratio of the reserve requirement μ. As both are policy variables determined by the monetary authority, we follow Bose (2002) by treating the inflation rate, denoted by $1+\tau$, as a policy variable.

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[16] In Bose (2002), young lenders who intend to lend to borrowers need to hold money. We have modified our model to be consistent with Bose (2002) in such a way that young lenders, instead of young borrowers, need to hold money. The conclusion derived below, however, does not change.

[17] Recall that the population of lenders is equal to n and each lender is endowed with one unit of labor when young. Hence, young lenders' total wage income amounts to nw_t. If young borrowers intend to borrow $\delta_t n w_t$, financial intermediaries must attract $\delta_t n w_t/(1-\mu)$ units of deposits, of which $(1-\mu)\delta_t n w_t/(1-\mu)(=\delta_t n w_t)$ is handed over to borrowers and $\mu\delta_t n w_t/(1-\mu)$ is held in the form of money. Note that borrowers will sell $(1-\mu)\delta_t n w_t/(1-\mu)$ for money and will use the money in exchange for output in the next period. Hence, the total demand for money at time t is equal to $P_t\{[\mu\delta_t n w_t + (1-\mu)\delta_t n w_t]/(1-\mu)\}$, which is equal to $\delta_t n w_t P_t/(1-\mu)$.

3. Loan markets and the equilibrium contracts

We shall now determine the equilibrium contracts for investment and consumption loans. At the beginning of each period, each time-t young intermediary must decide whether or not to finance borrowers. To finance the borrower, the young intermediary announces one set of contracts intended for entrepreneur borrowers and the other set for consumer borrowers. If a young intermediary's offer is not dominated by others, then he is approached by potential young borrowers. After the completion of the loan contracts, the intermediary offers a deposit contract to young lenders and attracts deposits to fulfill the needs of borrowers.

As in Bencivenga and Smith (1993), the equilibrium loan contracts at time t are defined such that there is no incentive for any intermediary to offer an alternative contract, taking w_t, w_{t+2}, ρ_{t+2}, $1+\tau$ (the inflation rate), and other intermediaries' offers as given. We also follow Bencivenga and Smith (1993) and Bose (2002) by focusing on the separating equilibrium such that an intermediary offers contracts that separate borrowers according to their type.

3.1 Equilibrium contracts for entrepreneurs

Similar to Bose's (2002) rationing regime, the equilibrium separating contracts intended for type-i entrepreneurs (denoted as $C^e_{t,i}$, $i=H, L$) are represented by $C^e_{t,i} \equiv (R^e_{t,i}, q^e_{t,i}, \pi^e_{t,i})$, $i=H, L$, where $R^e_{t,i}$ is the interest rate for the contract intended for a type-i entrepreneur, $q^e_{t,i}$ is the corresponding loan quantity, and $\pi^e_{t,i} \in (0,1]$ is the probability with which an intermediary offers the loan.

Before determining the equilibrium contract intended for entrepreneurs, it should first be noted that the entrepreneurs' capital technology is linear (in the case of success); hence, each will intend to borrow as much as possible. As pointed out by Bencivenga and Smith (1993), a maximal scale of the project is needed to bound the size of each investment loan. Bencivenga and Smith (1993) also indicate that this maximal scale in the presence of financial intermediaries should be related to k_t.[18] As w_t is linear in k_t, we assume that the maximal scale for each generation-t entrepreneur's project is equal to w_t.

The following proposition describes the equilibrium contracts for both types of entrepreneurs:

Proposition 1 Define $(1+\bar{\tau}^e_L) \equiv [Q\rho p_L(1-\mu)+\mu-\beta^e_L(1-\mu)]/Q\varepsilon\rho$ and $(1+\bar{\tau}^e_H) \equiv [Q\rho p_H(1-\mu)+\mu]/Q\varepsilon\rho$. Supposing that $1+\tau \leq \min\{(1+\bar{\tau}^e_L)$ $(1+\bar{\tau}^e_H)\}$, then the equilibrium separating contracts are given by

$$R^e_{t,i} = \frac{Q\varepsilon\rho - \mu/(1+\tau)}{p_i(1-\mu)},\ i=H,L, \tag{5a}$$

[18] It should be noted that the assumption that entrepreneurs' capital projects have a maximal scale is commonly encountered in the literature on asymmetric information. See Bencivenga and Smith (1993) for the case of adverse selection and Ma and Smith (1996) for the case of costly state verification.

$$q^e_{t,i} = (1+\tau)w_t,\ i = H,L, \tag{5b}$$

$$\pi^e_{t,H} = 1, \tag{5c}$$

$$\pi^e_{t,L} = \frac{p_L\{Q\rho[p_H(1-\mu)-(1+\tau)\varepsilon]+\mu\}}{p_H\{Q\rho[p_L(1-\mu)-(1+\tau)\varepsilon]+\mu\}}. \tag{5d}$$

The formal proof of this proposition is available upon request. Some intuition for the results of Proposition 1 is as follows. First, competition among intermediaries indicates that each contract, $C^e_{t,L}$ and $C^e_{t,H}$, must separately yield zero expected profit to an intermediary under the separating equilibrium. Since each lender can establish an intermediary without any cost and since each intermediary must attract deposits from other young lenders, this further implies that each expected rate of return to an intermediary from $C^e_{t,L}$ and $C^e_{t,H}$ must be equal to the deposit rate the intermediary offered to young lenders. Competition among lenders further implies that the deposit rate offered by an intermediary is equal to the rate of return from lenders' home production. To lend a unit of time-t output to an entrepreneur, an intermediary must attract $1/(1-\mu)$ units of deposit from lenders in which one unit is directly handed to the entrepreneur and the remainder (i.e. $\mu/(1-\mu)$) is exchanged for nominal money to satisfy the reserve requirement. As a result, the expected rate of return to an intermediary from lending to entrepreneurs is equal to $p_i R^e_{t,i}$ plus $\mu/(1-\mu)(1+\tau)$, where $\mu/(1-\mu)(1+\tau)$ is the rate of return from holding $\mu/(1-\mu)$ units of money.[19] Because the rate of return from lenders' home production is equal to $Q\varepsilon\rho$, the expected rate of return to an intermediary from lending (i.e. $p_i R^e_{t,i} + \mu/(1-\mu)(1+\tau)$) must be equal to $Q\varepsilon\rho/(1-\mu)$, which yields the expression $R^e_{t,i}$, $i = H, L$.

Second, eq. (5a) indicates that an increase in the inflation rate $(1+\tau)$ raises the interest rate on loans to entrepreneurs. The participation constraint for type-L (resp. type-H) entrepreneurs indicates that the expected payoff to generation-t entrepreneurs from borrowing must be greater than or equal to $\beta^e_L w_t$ (resp. $\beta^e_H w_t = 0$), the return from the home production of output for type-L (resp. type-H) entrepreneurs. Since the entrepreneurs' expected payoff from borrowing is negatively correlated with the interest rate on loans, the participation constraints imply that the inflation

[19] We have assumed that the young intermediary is asked to hold the money between time t and $t+1$ and, similar to entrepreneurs, each middle-age intermediary at time $t+1$ has access to a storage technology that can convert one unit of time $t+1$ output into one unit of time-$t+2$ output. Thus, the intermediary can simply store $\mu/(1-\mu)(1+\tau)$ units of time-$t+1$ output and repay them to his depositors at time $t+2$. Note that if the intermediary is required to hold the money for two periods, then the rate of return from money is equal to $1/(1+\tau)^2$ and the interest rate is equal to $R^e_{t,i} = \{Q\varepsilon\rho - [\mu/(1+\tau)^2]\}/p_i(1-\mu)$. We have verified that our results as obtained below do not change.

rate cannot be too high; otherwise, entrepreneurs have no incentive to borrow. Specifically, to satisfy the participation constraint it is required that $(1+\tau) \leq (1+\bar{\tau}_L^e)$ for type-L entrepreneurs and $(1+\tau) \leqslant (1+\bar{\tau}_H^e)$ for type-H entrepreneurs. In other words, both types of entrepreneurs will borrow if $(1+\tau) \leqslant \min\{(1+\bar{\tau}_L^e),(1+\bar{\tau}_H^e)\}$. Third, with a linear technology in the production of capital, each entrepreneur intends to borrow as much as possible, implying that each entrepreneur intends to implement his capital project at the maximal scale w_t. Since entrepreneurs obtain loans in the first period of life and operate their capital projects in the second period of life, each entrepreneur must borrow $(1+\tau)w_t$ units of goods in the first period of life and exchange them for money in the same period. In the next period, the entrepreneur can exchange the money for w_t units of time-$t+1$ goods in order to implement his capital project at the maximal scale. Consequently, $q_{t,i}^e = (1+\tau)w_t$, $i = H, L$.

Finally, according to eq. (5a), $R_{t,H}^e > R_{t,L}^e$. This together with eq. (5b) will induce type-H entrepreneurs to apply for $C_{t,L}^e$ and enjoy a lower interest rate. The separating equilibrium must satisfy the incentive constraint that prevents type-H entrepreneurs from applying for $C_{t,L}^e$ (intended for type-L entrepreneurs). Similar to Bose's (2002) rationing regime, the incentive constraint can be satisfied by offering the type-H entrepreneurs their first best contract (which leads to eq. (5c)) and the type-L entrepreneurs a distorted contract such that type-H entrepreneurs have no incentive to apply for $C_{t,L}^e$. More specifically, the contract $C_{t,L}^e$ is distorted in such a way that any borrower who applies for this contract may be rejected with probability $1-\pi_{t,L}^e$. Note that the assumption of $1+\tau \leq (1+\bar{\tau}_L^e)$ implies that the expected payoff of type-L entrepreneurs (who apply for $C_{t,L}^e$) is increasing in $\pi_{t,L}^e$, which in turn implies that the incentive contract must be binding and the value of $\pi_{t,L}^e$ can be obtained accordingly. Note that $\pi_{t,L}^e$ should be non-negative, which holds for $(1+\tau) \leq (1+\bar{\tau}_H^e)$.

Proposition 1 leads to the following result:

Corollary 1 An increase in $(1+\tau)$ for $1+\tau \leq \min\{(1+\bar{\tau}_L^e),(1+\bar{\tau}_H^e)\}$ raises the incidence of credit rationing of type-L entrepreneurs.

This result is very intuitive. Since Since $p_L > p_H$, eq. (5a) implies that an increase in the inflation rate will cause $R_{t,H}^e$ to increase more than $R_{t,L}^e$. This gives type-H entrepreneurs more incentives to apply for $C_{t,L}^e$ (intended for type-L entrepreneurs) and thereby exacerbates the problem of asymmetric information. To deter this behavior, the incentive constraint implies that, as inflation rises, intermediaries must ration credit to type-L entrepreneurs more severely (i.e. to decrease the value of $\pi_{t,L}^e$). In other words, an increase in the inflation rate is associated with a decrease in $\pi_{t,L}^e$, which raises the incidence of credit rationing of type-L entrepreneurs.

3.2 The equilibrium contracts for consumers

The equilibrium contract extended to consumers shares a similar feature with that extended to entrepreneurs. Specifically, the contract offered by a lender to a type-i

consumer (denoted as $C^c_{t,i}, i=H, L$) at time t comprises a triple $\{\pi^c_{t,i}, q^c_{t,i}, R^c_{t,i}\}$, where $\pi^c_{i,i} \in [0,1]$ is the probability with which a lender offers the loan, $q^c_{t,i}$ is the quantity of loan offered, and $R^c_{t,i}$ is the interest rate that the consumer must pay back in time $t+2$ when he receives the labor endowment. The following proposition states the equilibrium contracts extended to both types of consumer:

Proposition 2 Define $(1+\bar{\tau}^c) \equiv [(1-\mu)+\beta^c\mu]/\beta^c Q\varepsilon\rho$. Suppose that $1+\tau \leq (1+\bar{\tau}^c)$ and β^c is sufficiently small. Then, the equilibrium separating contracts extended to consumers are

$$R^c_{t,i} = \frac{Q\varepsilon\rho - \mu/(1+\tau)}{p_i(1-\mu)},\ i = H,L \tag{6a}$$

$$q^c_{t,i} = \frac{w_{t+2}}{R^c_{t,i}},\ i = H,L \tag{6b}$$

$$\pi^c_{t,H} = 1 \tag{6c}$$

$$\pi^c_{t,L} = \frac{p_H(1-\mu) - \beta^c p_H[(1+\tau)Q\varepsilon\rho - \mu]}{p_L(1-\mu) - \beta^c p_H[(1+\tau)Q\varepsilon\rho - \mu]}. \tag{6d}$$

The intuition underlying the results of Proposition 2 is similar to that for Proposition 1. Specifically, similar to the equilibrium contracts extended to entrepreneurs, the interest rate on consumption loans is obtained based on the condition that the expected rate of return from lending to consumers must be equal to that from lenders' home production. This also implies that the interest rate on consumption loans is increasing with the inflation rate. Since the consumers' expected payoff from borrowing is negatively correlated with the interest rate, the participation constraint for both types of consumer borrowers requires that the inflation rate must be less than or equal to $(1+\bar{\tau}^c)$. Moreover, under the assumption that β^c is sufficiently small, consumers intend to borrow all of their expected old-age income for their middle-age consumption; hence, $q^c_{t,i} = w_{t+2}/R^c_{t,i}$.[20] The underlying mechanism for obtaining $\pi^c_{t,H}$ and $\pi^c_{t,L}$ is similar to that for $\pi^e_{t,H}$ and $\pi^e_{t,L}$. In particular, the incentive constraint must be binding under the assumption of $1+\tau \leqslant (1+\bar{\tau}^c)$. Note that the assumption of $1+\tau \leq (1+\bar{\tau}^c)$ also ensures that the value of $\pi^c_{t,L}$ is non-negative.

[20] Since consumers obtain loans in their first period of life and consume in their second period of life, the actual amount a consumer can consume in his middle age is equal to $q^c_{t,i}/(1+\tau)$.

Proposition 2 gives rise to the following result:

Corollary 2 An increase in $(1+\tau)$ for $1+\tau \leq (1+\bar{\tau}^c)$ raises the incidence of credit rationing of type-L consumers.

For a reason similar to Corollary 1, the value of $\pi^c_{t,L}$ is decreasing with the inflation rate, indicating that an increase in the inflation rate raises the incidence of credit rationing.

4. Inflation, capital formation, and economic growth

After we obtain the equilibrium contracts for consumers and entrepreneurs, we can examine the correlation between inflation and economic growth. Once we obtain this correlation, we will compare our results with those of recent theoretical studies.

4.1 The non-linear correlation between inflation and economic growth

Recall from borrowers' participation constraints that borrowers may not apply for loans if the initial levels of inflation are too high. Under such circumstances, capital is converted by means of lenders' home technology. On the other hand, for relatively low levels of inflation, capital investment is affected by the equilibrium contracts extended to entrepreneurs and consumers. To examine how the joint consideration of investment and consumption loans affects the inflation-growth relationship, we should focus on the inflation rate $1+\tau$ that is less than or equal to $(1+\bar{\tau}^c)$ and $\min\{(1+\bar{\tau}^e_L),(1+\bar{\tau}^e_H)\}$, which ensures that both types of entrepreneurs and consumers are willing to borrow.[21]

From the equilibrium contracts, we can see that the total amount used in consumption loans at time t is equal to $m[\lambda q^c_{t,H}+(1-\lambda)q^c_{t,L}\pi^c_{t,L}]/(1-\mu)$, while the total amount needed to finance entrepreneurs at time t is $(1+\tau)[\lambda+(1-\lambda)\pi^e_{t,L}]w_t/(1-\mu)$, which produces an amount of time-$t+2$ capital equal to $Q[\lambda p_H+(1-\lambda)p_L\pi^e_{t,L}]w_t$. By denoting k_{t+2} as the per firm capital at time $t+2$, we see that

$$k_{t+2}=\{nw_t-\frac{(1+\tau)}{(1-\mu)}[\lambda+(1-\lambda)\pi^e_{t,L}]w_t-\frac{m}{(1-\mu)}[\frac{\lambda}{R^c_{t,H}}+\frac{(1-\lambda)}{R^c_{t,L}}\pi^c_{t,L}]w_{t+2}\}Q\varepsilon \\ +Q[\lambda p_H+(1-\lambda)p_L\pi^e_{t,L}]w_t. \tag{7}$$

The first part of the RHS of eq. (7) is the amount of capital produced by lenders' home technology while the second part is that produced by entrepreneurs' investment projects. By substituting eq. (2) into eq. (7) and after performing some

[21] If the inflation rates are too high, borrowers' participation constraints are violated so that borrowers have no incentive to borrow. In this case, inflation has no effect on bank lending activity. Boyd *et al.* (2001) find evidence of this.

algebraic manipulations, we obtain the rate of economic growth between time t and time $t+2$ (denoted as g) which is given as[22]

$$\begin{aligned}\frac{k_{t+2}}{k_t} &= g \\ &= \frac{\{\varepsilon n + \lambda[p_H - ((1+\tau)\varepsilon/(1-\mu))] + (1-\lambda)[p_L - ((1+\tau)\varepsilon/(1-\mu))]\pi^e_{t,L}\}}{1 + \frac{mQ\varepsilon}{(1-\mu)}[\lambda p_H + (1-\lambda)p_L \pi^c_{t,L}](A(1-\alpha)L^{-\alpha}/Q\varepsilon\rho - [\mu/(1+\tau)])} \\ &\quad \times QA(1-\alpha)L^{-\alpha}.\end{aligned} \tag{8}$$

To see the effect of a change in the inflation rate on economic growth, we take the logs of both sides of eq. (8) and differentiate them with respect to $1+\tau$, which gives rise to

$$\frac{\partial \ln g}{\partial(1+\tau)} = \frac{\partial}{\partial(1+\tau)}\ln E^e - \frac{\partial}{\partial(1+\tau)}\ln E^c, \tag{9}$$

where

$$E^e = \varepsilon n + \lambda\left[p_H - \frac{(1+\tau)\varepsilon}{(1-\mu)}\right] + (1-\lambda)\left[p_L - \frac{(1+\tau)\varepsilon}{(1-\mu)}\right]\pi^e_{t,L} \tag{10}$$

and

$$E^c = 1 + \frac{mQ\varepsilon}{(1-\mu)}\left[\lambda p_H \frac{A(1-\alpha)L^{-\alpha}}{Q\varepsilon\rho - [\mu/(1+\tau)]} + (1-\lambda)p_L\pi^c_{t,L}\frac{A(1-\alpha)L^{-\alpha}}{Q\varepsilon\rho - [\mu/(1+\tau)]}\right]. \tag{11}$$

It is clear that $\partial \ln E^e/\partial(1+\tau)$ represents the effects from investment loans while $-\partial \ln E^c/\partial(1+\tau)$ denotes the effects from consumption loans.

Note that the effects of a change in the inflation rate on the value of E^e contain two parts. The first one is that an increase in $1+\tau$ induces entrepreneurs to borrow more resources (i.e. $q^e_{t,i} = (1+\tau)w_t$) for financing their capital projects at the maximal scale and this thereby reduces the amount of resources available for producing capital (via lenders' home technology). This is captured by the terms $[p_i - (1+\tau)\varepsilon/(1-\mu)]$, $i = H, L$, in eq. (10) (with the presence of a reserve requirement), which is denoted as TE for future reference. The second one is related to $\pi^e_{t,L}$ such that an increase in the inflation rate is associated with a decrease in $\pi^e_{t,L}$ Although this can save more resources for the lenders' home production, the economy's capital investment is adversely affected due to the fact that type-L entrepreneurs' capital technology is better than lenders' home technology. Since an increase in the inflation rate reduces TE and $\pi^e_{t,L}$, the value of $\partial \ln E^e/\partial(1+\tau)$ is negative.

[22] It is assumed that $p_H(1-\mu) > \varepsilon(1+\mu)$. Hence, capital produced by entrepreneurs is positive.

A change in $1+\tau$ similarly gives rise to two effects on the value of E^c. An increase in $1+\tau$ that increases the interest rate on loans to consumers reduces the present value of consumers' old-age income and thereby reduces the quantity of each consumption loan. This is represented by the term $A(1-\alpha)L^{-\alpha}/[Q\varepsilon\rho-\mu/(1+\tau)]$ in eq. (11), which is denoted as TC for future reference. The second effect is observed by the fact that an increase in $1+\tau$ reduces $\pi^c_{t,L}$, the probability of type-L consumers getting a loan. An increase in $1+\tau$ reduces both the values of TC and $\pi^c_{t,L}$ so that the sign of $\partial\ln E^c/\partial(1+\tau)$ is negative. The reduction in TC and $\pi^c_{t,L}$, however, raises the amount of resources for lenders' home production and thereby is beneficial to capital investment and economic growth. As a result, the output effect of inflation from consumption loans (represented by $-\partial\ln E^c/\partial(1+\tau)$ eq. (9)) is positive.

The net effect of inflation on output growth is determined by the sign of $\partial\ln g/\partial(1+\tau)$, which is further determined by the relative magnitudes of $\partial\ln E^e/\partial(1+\tau)$ and $\partial\ln E^c/\partial(1+\tau)$ in the absolute values. Specifically, an increase in the inflation rate will lead to a decrease (resp. an increase) in the growth rate if the absolute value of $\partial\ln E^e/\partial(1+\tau)$ (denoted as $|\partial\ln E^e/\partial(1+\tau|)$) is greater (resp. less) than that of $\partial\ln E^c/\partial(1+\tau)$ (denoted as $|\partial\ln E^c/\partial(1+\tau)|$). Note that the level of inflation plays a role in influencing the relative magnitudes of $|\partial\ln E^e/\partial(1+\tau)$ and $|\partial\ln E^c/\partial(1+\tau)|$. Hence, to examine the sign of $\partial\ln g/\partial(1+\tau)$, we depict the loci of $|\partial\ln E^e/\partial(1+\tau)|$ and $|\partial\ln E^c/\partial(1+\tau)|$ as functions of $(1+\tau)$. The following lemma characterizes the properties of $|\partial\ln E^e/\partial(1+\tau)|$:

Lemma 1 (i) If τ is equal to zero, then the value of $|\partial\ln E^e/\partial(1+\tau)|$ is a positive constant. (ii) There exists a $\bar{\tau}^e_1$, $\bar{\tau}^e_1 > \max\{\bar{\tau}^e_L,\bar{\tau}^e_H\}$, such that if $\tau=\bar{\tau}^e_1$, then $|\partial\ln E^e/\partial(1+\tau)|=0$. (iii) If the parameters are such that the sign of $\partial|\partial\ln E^e/\partial(1+\tau)|/\partial(1+\tau)$ is positive when $\tau=0$, then there exists a τ^*_e such that $\partial|\partial\ln E^e/\partial(1+\tau)|/\partial(1+\tau)>(<)0$ when $\tau<(>)\tau^*_e$.

Lemma 1 $|\partial\ln E^e/\partial(1+\tau)|$ has a configuration as depicted in Fig. 1, which is the case under the assumption that $\bar{\tau}^e_H > \bar{\tau}^e_L$. Note that the second and third results of Lemma 1 imply that $|\partial\ln E^e/\partial(1+\tau)|$ is a concave function of $(1+\tau)$ for $\tau<\tau^*_e$. For both types of entrepreneurs to borrow, it must be that $\tau\le\min\{\bar{\tau}^e_L,\bar{\tau}^e_H\}=\bar{\tau}^e_L$ so that the locus of $|\partial\ln E^e/\partial(1+\tau)|$ is depicted as a dotted line after $\tau>\bar{\tau}^e_L$. To understand the third result of Lemma 3, note that the locus of $|\partial\ln E^e/\partial(1+\tau)|$ is primarily determined by how an increase in $(1+\tau)$ affects the magnitudes of the aforementioned two effects (i.e. the magnitude of $\partial TE/\partial(1+\tau)$ and $\partial\pi^e_{t,L}/\partial(1+\tau)$) on E^e). Specifically, eq. (10) implies that

$$\left|\frac{\partial\ln E^e}{\partial(1+\tau)}\right| = -\frac{\ln E^e}{\partial(1+\tau)}$$
$$= \frac{-[\lambda+(1-\lambda)\pi^e_{t,L}]\frac{\partial TE}{\partial(1+\tau)}-(1-\lambda)[p_L-(\varepsilon(1+\tau)/(1-\mu))]\frac{\partial\pi^e_{t,L}}{\partial(1+\tau)}}{E^e}$$

The slope of $|\partial\ln E^e/\partial(1+\tau)|$ is primarily determined by differentiating this equation with respect to $(1+\tau)$. Since an increase in $(1+\tau)$ reduces E^e, the slope

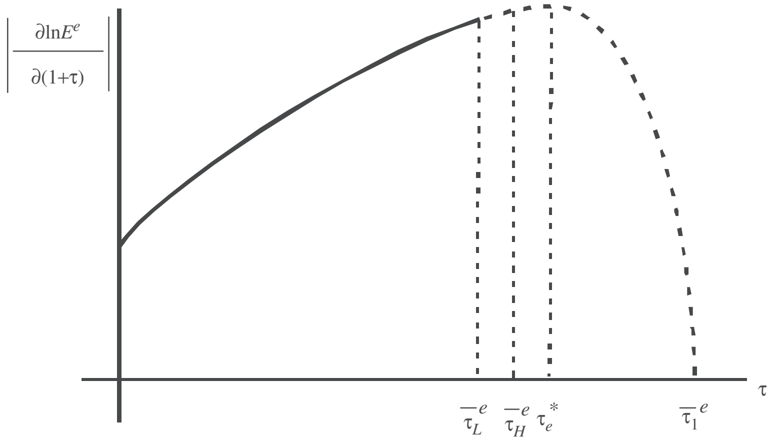

Fig. 1. The relationship between τ and $|\partial \ln E^e/\partial(1+\tau)|$

of $|\partial \ln E^e/\partial(1+\tau)|$ in turn is primarily determined by the effect of an increase in $(1+\tau)$ on the value of the numerator of this equation, which is further determined by the magnitude of the aforementioned two effects from investment loans. Note that an increase in $(1+\tau)$ decreases the value of the first term in the numerator (i.e. $-[\lambda+(1-\lambda)\pi^e_{t,L}]\partial TE/\partial(1+\tau)$) but increases the value of the second term (i.e. $-(1-\lambda)[p_L-(\varepsilon(1+\tau)/(1-\mu))][\partial\pi^e_{t,L}/\partial(1+\tau)]$). This leads to three possibilities in determining the slope of $|\partial \ln E^e/\partial(1+\tau)|$. If the effect of the second term dominates that of the first term, an increase in $(1+\tau)$ leads to an increase in the value of the numerator, which further implies that the slope of $|\partial \ln E^e/\partial(1+\tau)|$ is positive. This is the first case. On the other hand, if the effect of the first term dominates that of the second term, then an increase in $(1+\tau)$ leads to a reduction in the value of the numerator. In this situation, there are two other possibilities. If the negative effect of inflation on the value of the numerator is not large enough, then the slope of $|\partial \ln E^e/\partial(1+\tau)|$ is still positive. This is the second case. Finally, if the negative effect of inflation on the value of the numerator is large enough, then the slope of $|\partial \ln E^e/\partial(1+\tau)|$ is negative. Lemma 1 implies that the first and second cases hold for $\tau<\tau_e^*$, while the final case holds for $\tau>\tau_e^*$.

It is instructive to examine the correlation between inflation and economic growth without considering non-productive consumption loans. By not considering consumption loans, the effect of τ on economic growth can be directly derived by examining how an increase in τ affects E^e. Note that $\tau \le \max\{\bar{\tau}^e_L, \bar{\tau}^e_H\}$ and, as depicted in Fig. 1, we assume that $\bar{\tau}^e_L < \bar{\tau}^e_H < \bar{\tau}^*_e$. Given this assumption, Fig. 1 together with eq. (10) implies that an increase in τ always decreases economic growth and that the marginal impact of this negative effect is decreasing in τ for $\tau \le \bar{\tau}^e_L$. Obviously, this result is not consistent with recent empirical evidence.

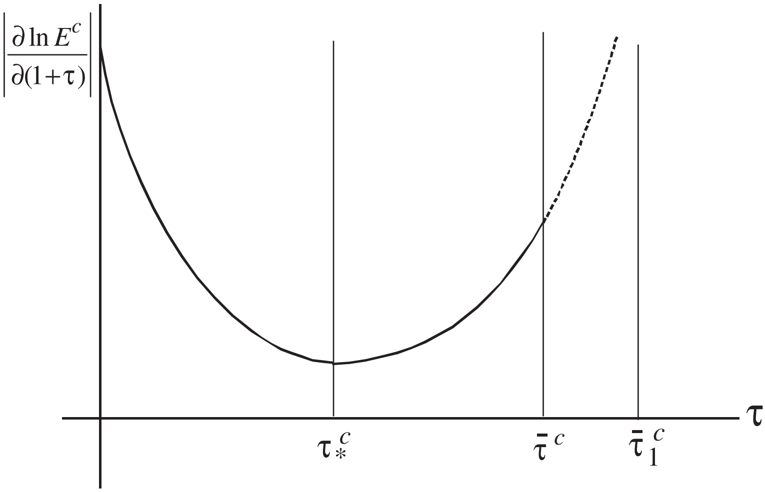

Fig. 2. The relationship between τ and $|\partial \ln E^c/\partial(1+\tau)|$

We now consider the presence of non-productive consumption loans. The following lemma characterizes the properties of $|\partial \ln E^c/\partial(1+\tau)|$ as a function of τ:

Lemma 2 (i) If iτ is equal to zero, then the value of $|\partial \ln E^c/\partial(1+\tau)|$ is a positive constant. (ii) There exists a $\bar{\tau}_1^c$, $\bar{\tau}_1^c > \bar{\tau}^c$, such that if $\tau = \bar{\tau}_1^c$, then $|\partial \ln E^c/\partial(1+\tau)| = \infty$. (iii) If the parameters are such that the sign of $|\partial \ln E^c/\partial(1+\tau)|/\partial(1+\tau)$ is negative when $\tau = 0$, then there exists a τ_c^* such that $|\partial \ln E^c/\partial(1+\tau)|/\partial(1+\tau) < (>)0$ when $\tau < (>)\tau_c^*$.

Lemma 2 implies that the locus of $|\partial \ln E^c/\partial(1+\tau)|$ is a U-shaped curve, as is depicted in Fig. 2. In particular, the second and third results of Lemma 2 indicate that $|\partial \ln E^c/\partial(1+\tau)|$ is a convex function of τ for $\tau > \tau_c^*$. For both types of consumers to borrow, it must be that $\tau \le \bar{\tau}^c$; hence, the locus of $|\partial \ln E^c/\partial(1+\tau)|$ is depicted as a dotted line after $\tau > \bar{\tau}^c$. To see the third result of Lemma 2, note that the value of $|\partial \ln E^c/\partial(1+\tau)|$ $(=-\ln E^c/\partial(1+\tau))$ is mainly determined by the magnitudes of the aforementioned two effects from consumption loans (i.e. the magnitude of $\partial TC/\partial(1+\tau)$ and $\partial \pi_{t,L}^c/\partial(1+\tau)$ in affecting E^c). Similar to the effect from investment loans, the interactions among these two effects indicate that there are two opposite effects in affecting the slope of $|\partial \ln E^c/\partial(1+\tau)|$ along with the increase in τ. In particular, one can find that $\partial^2 TC/\partial^2(1+\tau) > 0$ and $\partial^2 \pi_{t,L}^c/\partial^2(1+\tau) < 0$. The former result indicates that the slope of $|\partial \ln E^c/\partial(1+\tau)|$ is positive while the latter result implies that the slope of $|\partial \ln E^c/\partial(1+\tau)|$ is negative. Lemma 2 implies that $\partial^2 \pi_{t,L}^c/\partial^2(1+\tau)$ dominates $\partial^2 TC/\partial^2(1+\tau)$ for low levels of inflation and $\partial^2 TC/\partial^2(1+\tau)$ denominates $\partial^2 \pi_{t,L}^c/\partial^2(1+\tau)$ for high levels of inflation, leading to a U-shaped curve of $|\partial \ln E^c/\partial(1+\tau)|$.

Note that if we consider the non-productive consumption loans alone, then the effect of τ on economic growth can be obtained by examining how a change in

τ affects $1/E^c$. Without investment loans, Lemma 2 together with eq. (11) implies that an increase in $1+\tau$ always increases economic growth and that the marginal impact of this positive effect is first decreasing and then increasing. This result is also not consistent with recent empirical evidence, as recent evidence has established that an increase in τ decreases economic growth for high levels of inflation.

When we integrate productive investment loans with non-productive consumption loans, then an increase in the inflation rate can lead to an increase (a decrease) in economic growth, depending on whether i$|\partial \ln E^c/\partial(1+\tau)|$ is greater (less) than $|\partial \ln E^e/\partial(1+\tau)|$. Moreover, the marginal impact of an increase in the inflation rate on economic growth depends on the difference between $|\partial \ln E^e/\partial(1+\tau)|$ and $|\partial \ln E^c/\partial(1+\tau)|$. Note that the initial inflation rate plays an important role in determining the relative magnitudes between $|\partial \ln E^c/\partial(1+\tau)|$ and$|\partial \ln E^e/\partial(1+\tau)|$. This can be derived from Lemmas 1 and 2, as stated below.

Corollary 3 Suppose that the conditions in Lemmas 1 and 2 are satisfied and that $\tau_c^* < \min\{\bar{\tau}_L^e, \bar{\tau}_H^e\} < \tau_e^* < \min\{\bar{\tau}^c, \bar{\tau}_1^c, \bar{\tau}_1^e\}$. Then, there are two inflation thresholds—a lower threshold and a higher threshold—in the inflation-growth relationship for $\tau \leq \min\{\bar{\tau}_L^e, \bar{\tau}_H^e\}$. An increase in the inflation rate leads to an increase or a decrease in the growth rate for inflation rates below the lower threshold. For inflation rates between these two thresholds, an increase in the inflation rate reduces the growth rate and the marginal impact (the significance) of this negative effect on growth is increasing along with the rise in the inflation rate. For inflation rates higher than the higher threshold, there continues to be a negative effect of inflation on growth but the marginal impact (the significance) of this negative effect is small.

The intuition for this corollary is straightforward. Since $|\partial \ln E^e/\partial(1+\tau)|$ (the effects of investment loans) is increasing in $1+\tau$ for $\tau \leq \min\{\bar{\tau}_L^e, \bar{\tau}_H^e\}$ and $|\partial \ln E^c/\partial(1+\tau)|$ (the effects of consumption loans) is first decreasing in $1+\tau$ for $\tau < \tau_c^* \leq \min\{\bar{\tau}_L^e, \bar{\tau}_H^e\}$, it is quite possible that the effect of consumption loans dominates (is dominated by) the effect of investment loans for low (high) levels of inflation rates, leading to the lower threshold level. Moreover, the fact that $|\partial \ln E^c/\partial(1+\tau)|$ is increasing in $1+\tau$ for $\tau_c^* < \tau \leq \min\{\bar{\tau}_L^e, \bar{\tau}_H^e\}$ together with the result that $|\partial \ln E^e/\partial(1+\tau)|$ is a concave function of $1+\tau$ for $\tau \leq \min\{\bar{\tau}_L^e, \bar{\tau}_H^e\}$ implies that, although the effect from investment loans still dominates the effect from consumption loans after inflation rates are greater than the lower threshold, the difference between these two effects is decreasing in $1+\tau$ for $\tau_c^* < \tau \leq \min\{\bar{\tau}_L^e, \bar{\tau}_H^e\}$, implying that the negative output effect of inflation is decreasing in $1+\tau$. This indicates that there is a higher inflation threshold (which is greater than τ_c^*) such that the marginal (negative) impact of an increase in the inflation rate on economic growth decreases significantly after inflation rates are greater than this threshold.

It is also possible that the value of $|\partial \ln E^e/\partial(1+\tau)|$ is greater than that of $|\partial \ln E^c/\partial(1+\tau)|$ starting from $\tau = 0$. In this case, we observe a negative correlation between inflation and economic growth for all ranges of inflation. However, one can still find two inflation thresholds in this case. Specifically, the difference between $|\partial \ln E^e/\partial(1+\tau)|$ and $|\partial \ln E^c/\partial(1+\tau)|$ is small for low levels of inflation rates and hence the coefficient (negative value) of inflation is relatively low and may be insignificant. Nevertheless, the fact that $|\partial \ln E^e/\partial(1+\tau)|$ is increasing in the inflation rate (for $\tau \leq \min\{\bar{\tau}_L^e, \bar{\tau}_H^e\}$) and that $|\partial \ln E^c/\partial(1+\tau)|$ is decreasing in the inflation rate (for $\tau < \tau_c^* \leq \min\{\bar{\tau}_L^e, \bar{\tau}_H^e\}$) implies that this difference becomes large (significant) after the inflation rate is higher than a threshold. This leads to the lower inflation threshold. The higher inflation threshold is obtained when the value of $|\partial \ln E^c/\partial(1+\tau)|$ is increasing in $1+\tau$, implying that the marginal impact of inflation on economic growth decreases substantially.

To better illustrate our above analyses, we resort to numerical simulations. To do so, first note that capital's income share (α) is roughly 0.33. The reserve ratio varies quite a lot across countries, ranging from 0.05% to 25%.[23] There is no empirical evidence for choosing other parameters, however. Since our purpose is to illustrate the existence of two inflation thresholds, we intend to choose other parameters that can produce a case consistent with recent empirical evidence. Specifically, we choose other parameters to reproduce the empirical evidence obtained from recent studies such that the lower threshold level of inflation is about 10% while the higher one is located in between 40% and 50%. Moreover, the chosen parameters should yield reasonable rates of economic growth. We then consider the following example.[24]

Consider an economy with $\lambda = 0.45$, $n = 4$, $m = 0.5$, $p_L = 0.7$, $\mu = 0.23$, $p_H = 0.53$, $\varepsilon = 0.21$, $\alpha = 0.33$, $Q = 1.5$, $A = 2$, $\beta^c = 0.67$, and $\beta_L^e = 1$. In this economy, $(1+\bar{\tau}_L^e) = 1.74$, $(1+\bar{\tau}_H^e) = 1.94$, and $(1+\bar{\tau}^c) = 2.10$.[25] To allow for the presence of both investment and consumption loans, we should consider the inflation rate τ with $0 < \tau < 0.74$. The loci of $|\partial \ln E^e/\partial(1+\tau)|$ and $|\partial \ln E^c/\partial(1+\tau)|$ are depicted in Fig. 3. As is shown in Fig. 3, $|\partial \ln E^e/\partial(1+\tau)|$ is increasing in τ at a decreasing marginal effect while $|\partial \ln E^c/\partial(1+\tau)|$ is decreasing (increasing) in τ for $\tau \leq (>) 0.4 (= \tau_c^*)$.

In this example, if τ is greater than 11.5%, then the effects from investment loans dominate those from consumption loans (i.e. $|\partial \ln E^e/\partial(1+\tau)| > |\partial \ln E^c/\partial(1+\tau)|$), leading to a negative correlation between inflation and economic growth. Consequently, there is a threshold level of inflation (about 11.5%) under which the correlation between inflation and economic growth changes. Fig. 4 depicts the correlation between inflation and economic growth.

The marginal impact of an increase in $1+\tau$ on economic growth is determined by the difference between $|\partial \ln E^e/\partial(1+\tau)|$ and $|\partial \ln E^c/\partial(1+\tau)|$. As shown in Fig. 3, after the lower threshold level (i.e. $\tau > 0.115$), the difference between $|\partial \ln E^e/\partial(1+\tau)|$ and $|\partial \ln E^c/\partial(1+\tau)|$ becomes large as the inflation rate increases. This implies that the marginal impact of an increase in $1+\tau$ on economic growth

[23] These figures are reserve coefficients reported by Giorgio (1999).

[24] Note that changing the parameter values within a reasonable range of the chosen values does not alter the shapes of $|\partial \ln E^e/\partial(1+\tau)|$ and $|\partial \ln E^c/\partial(1+\tau)|$.

[25] $\bar{\tau}_1^c$ and $\bar{\tau}_1^e$ are greater than min $\{\bar{\tau}_H^e, \bar{\tau}_L^e\}$ in this example.

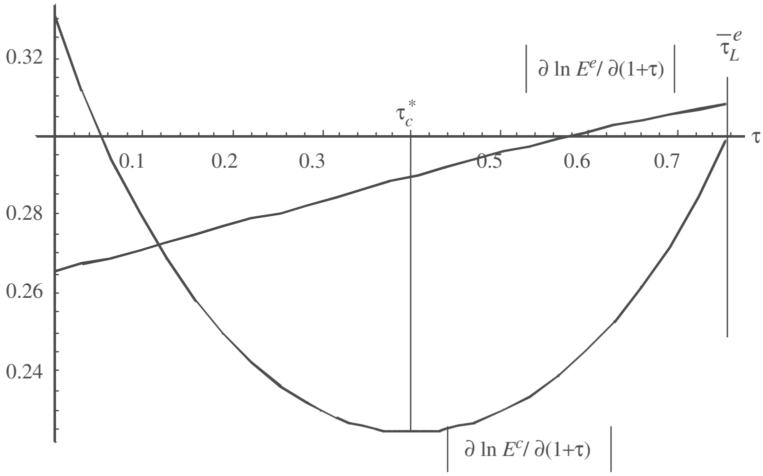

Fig. 3. The loci of $|\partial \ln E^e/\partial(1+\tau)|$ and $|\partial \ln E^c/\partial(1+\tau)|$ under the example

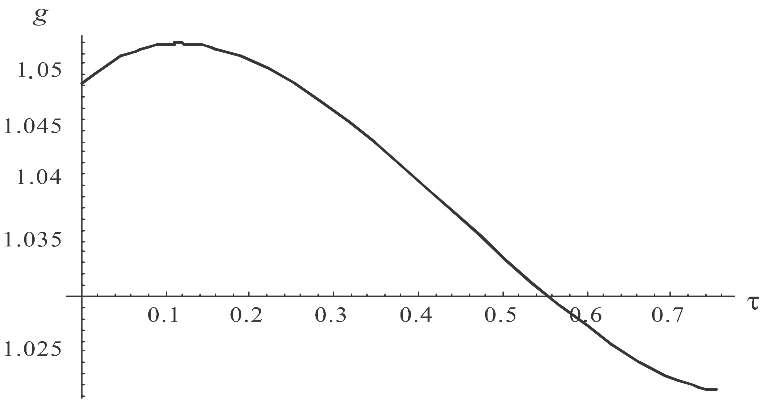

Fig. 4. Inflation and economic growth under the example

becomes more significant along with the increase in $1+\tau$. Moreover, after $\tau > 0.4$, the difference between $|\partial \ln E^e/\partial(1+\tau)|$ and $|\partial \ln E^c/\partial(1+\tau)|$ is decreasing in terms of the inflation rate, implying that the marginal impact of an increase in $1+\tau$ on economic growth decreases substantially along with an increase in $1+\tau$ after $\tau > 0.4$. This indicates that there exists a higher threshold level.

4.2 Discussion

We have shown that adding non-productive consumption into a standard model of asymmetric information can yield two inflation thresholds in the inflation-growth relationship that is consistent with recent empirical studies. Note that recent theoretical studies by Azariadis and Smith (1996), Boyd and Smith (1998), and Huybens and Smith (1999) successfully capture some aspects (but not all) of this pattern of non-linearity in the inflation-growth relationship.

In a neoclassical growth model, Azariadis and Smith (1996) consider an OG model incorporated with private information. It is shown that credit rationing arises in investment loans for high levels of inflation rates, and an increase in the inflation rate in this case exacerbates informational problems. As a result, inflation tends to increase the incidence of credit rationing on investment loans for high levels of inflation and is thereby detrimental to capital investment.[26] For low levels of inflation, however, credit is not rationed and in this case an increase in the inflation rate increases the total amount of investment loans, which facilitates capital investment. Hence, their model is able to explain the existence of the lower threshold level of inflation in the relationship between inflation and the steady-state capital stock. Nevertheless, the possibility that inflation has a negative or insignificant effect on economic growth below the first threshold level does not appear in their model. Similarly, the higher threshold level does not exist.[27]

Boyd and Smith (1998) develop a neoclassical growth model in an OG model with the presence of asymmetric information. They find two steady states in a monetary economy: one with a low capital stock and output while the other has a high capital stock and output. An increase in the money growth rate increases the steady state capital stock under the low-capital-stock steady state. However, such an increase in the high-capital-stock steady state reduces the steady state capital stock. Consequently, their model implies that the relationship between the money growth rate and the steady state capital stock depends on the initial capital stock. This is obviously not consistent with recent empirical work, which reports that the correlation between inflation and capital investment depends on the initial inflation rate.[28] Huybens and Smith (1999) examine a model with a costly-state-verification problem. They show that an increase in the inflation rate always leads to a reduction in real activity and, in particular, this negative correlation appears more pronounced at higher rates of inflation. While their study is able to capture the fact that the marginal impact of an increase in the inflation rate on economic growth increases along with an increase in the inflation rate, their models are not able to yield the higher threshold level under which the marginal effect

[26] Espinosa-Vega and Yip (1999) also develop a theoretical model whereby the inflation-growth correlation depends on the agents' degree of risk aversion. Since the degree of risk aversion is not correlated with the inflation rate in the model, their model may not capture the empirical evidence as does Bullard and Keating (1995).

[27] In fact, Azariadis and Smith (1996) do find a higher threshold level of inflation such that if inflation is greater than this threshold level, the dynamics of the economy becomes indeterminate. As the two inflation thresholds are obtained by recent empirical studies that estimate the long-run relationship between inflation and growth, the second threshold level found by Azariadis and Smith (1996) may not be related to recent empirical evidence.

[28] Boyd and Smith (1998) also find that if the inflation rate is too high, the dynamics of the economy could display limiting cycles, implying that there is no equilibrium path approaching the high-capital-stock steady state.

decreases substantially. Moreover, the possibility that an increase in the inflation rate may lead to an increase in economic growth for low levels of inflation rates also disappears.

5. Conclusion

This paper extends the work of Bencivenga and Smith (1993) and Bose (2002) by adding non-productive consumption loans into a standard model of informational imperfection in order to examine the threshold effects in the inflation-growth relationship. Without considering consumption loans, an increase in the inflation rate always leads to a reduction in the rate of economic growth, as obtained by Huybens and Smith (1999) and Bose (2002). However, the inclusion of consumption loans gives rise to an opposite effect of inflation on economic growth.

We find that the effect arising from consumption loans may dominate (be dominated by) that arising from investment loans for inflation rates below a lower threshold level of inflation, implying that the inflation-growth relationship is uncertain for inflation rates below this lower threshold. For inflation rates above this lower threshold level of inflation, we find that the negative output effect of inflation (from investment loans) always dominates the positive one (from consumption loans). Moreover, the difference between these two effects is increasing (resp. decreasing) in the inflation rate for inflation rates below (resp. above) another higher inflation threshold, implying that this negative inflation-growth correlation is convex. These observations accord well with recent empirical evidence on the inflation-growth relationship.

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