International Review of Economics and Finance 51 (2017) 545–561

Contents lists available at ScienceDirect

International Review of Economics and Finance

journal homepage: www.elsevier.com/locate/iref

International R&D funding and patent collateral in an R&D-based growth model

Wei-Chi Huang [a], Ching-Chong Lai [a,b,c,d], Ping-Ho Chen [b,*]

[a] Department of Economics, National Cheng Chi University, Taiwan
[b] Institute of Economics, Academia Sinica, Taiwan
[c] Institute of Economics, National Sun Yat-Sen University, Taiwan
[d] Department of Economics, Feng Chia University, Taiwan

A R T I C L E I N F O

JEL classification:
E44
O31
O40

Keywords:
International R&D funding
Patent collateral
R&D-based growth model
Optimal patent protection

A B S T R A C T

This paper develops an R&D-based growth model featuring international R&D funding and patent collateral. Several main findings emerge from the analysis. First, with an inelastic labor supply, a rise in the fraction of patent collateral is beneficial to both innovations and economic growth. Second, when labor supply is inelastic, a rise in either the foreign interest rate or the fraction of borrowed R&D funding is harmful to innovations and economic growth. Third, our numerical results show that the above two findings are robust when labor is supplied elastically. Finally, our numerical results indicate that, regardless of whether labor supply is inelastic or elastic, the government can implement an optimal patent breadth policy to maximize the social welfare level. Our numerical results also point out that this optimal patent breadth will decrease in response to a reduction in the foreign interest rate, a rise in the fraction of the collateral, and a reduction in the fraction of borrowed R&D labor costs.

1. Introduction

In this paper, we develop an R&D-based growth model that features international R&D funding and patent collateral. We then use the model to examine how the international borrowing interest rate and the fraction of patent collateral will affect innovations and economic growth. In addition to providing a positive analysis of R&D investment and economic growth, this paper also presents a normative analysis regarding how the government will set its optimal patent protection from the viewpoint of welfare maximization.

This paper is motivated by the following three observations. Firstly, R&D entrepreneurs are subject to difficulties in obtaining finance. According to Zúñiga-Vicente, Alonso-Borrego, Forcadell, and Galán (2014), there are some reasons why R&D firms find it difficult to obtain sufficient funds from the banking system. The first reason is that R&D projects are subject to extreme uncertainty about their success. The second reason is that, to prevent bankers from revealing the information on R&D projects to industrial competitors, R&D firms are reluctant to disclose the details of their R&D projects in loan application documents. The third reason is that R&D projects are featured with the idea-based nature and the lack of tangible products. Due to the shortfall between R&D expenditure and funding in the form of loans from banks, R&D firms are forced to borrow from households and other non-bank funding sources to meet their R&D costs.

Secondly, the financing of business enterprise R&D from abroad is observed in OECD data. It is commonly believed that R&D funding

* Corresponding author. Institute of Economics, Academia Sinica, Nankang, Taipei 115, Taiwan.
E-mail address: 97258503@nccu.edu.tw (P.-H. Chen).

http://dx.doi.org/10.1016/j.iref.2017.07.009
Received 14 February 2017; Received in revised form 7 July 2017; Accepted 10 July 2017
Available online 20 July 2017
1059-0560/

is critical for the growth effect of R&D investment. In their recent paper, using empirical data, Aghion, Farhi, and Kharroubi (2012) find that, by virtue of credit and liquidity constraints, R&D is more affected by a countercyclical monetary policy than by physical investment. To reflect this fact, Chu and Cozzi (2014) set up a Schumpeterian growth model that features a cash-in-advance (CIA) constraint on R&D investment. A notable specification of their model is that R&D entrepreneurs fully fund their investment from the home country. However, based on practical data, OECD (2011, p. 92) documents the following statement. "On average, R&D funding from abroad plays quite an important role in the funding of business R&D. In the EU, it represented around 10% of total business enterprise R&D in 2008. … For Austria, Ireland, the Slovak Republic and the United Kingdom, funds from abroad represented 20% or more of total business enterprise R&D."[1] As is obvious, the Chu and Cozzi (2014) specification ignores the fact that R&D companies obtain a considerable portion of their R&D funding from abroad.

Thirdly, the financing of business enterprise R&D is observed to be subject to patent collateral. It is quite possible that R&D firms will face financial frictions when they source R&D funding from the home country and/or abroad. A significant number of empirical studies, such as Brown, Martinsson, and Petersen (2012), Hochberg, Serrano, and Ziedonis (2014), and Mann (2016), point out that R&D patents often serve as collateral when entrepreneurs issue bonds to borrow funds for R&D. Among existing studies, Mann (2016) finds that in the U.S. during the period from 1990 to 2013 there has been an increasing tendency for patenting firms to pledge their patents as collateral. In 2013, about 40% of patenting firms posted their patents as collateral to obtain innovative financing. Based on these empirical findings, it is interesting to shed light on how patent collateral provides a vehicle to affect R&D investment and economic growth.

Up till now, to the best of our knowledge, no theoretical analysis has been devoted to dealing with international R&D funding and patent collateral in an R&D-based model.[2] To address the importance of these two R&D-related factors, this paper develops an R&D-based growth model that is able to reflect the realistic situation where R&D firms can obtain R&D funding from the international market and R&D patents can serve as collateral. With this framework, we are able to analyze how the international borrowing interest rate and the fraction of patent collateral will affect R&D investment, economic growth and social welfare.[3]

The normative analysis of this paper focuses on the factors determining optimal patent protection. In this regard, our study is most closely related to the literature on the optimal patent protection level. Within the literature, Iwaisako and Futagami (2003) and Futagami and Iwaisako (2007) show that stronger patent protection generates two conflicting effects on social welfare. On the one hand, it encourages R&D investment, and hence is beneficial to the growth rate and the social welfare level. On the other hand, a stronger patent protection tends to raise the markup price of intermediate goods. This tends to lower output production and the consumption of final goods, and hence is harmful to the social welfare level. Accordingly, the government will choose its optimal patent protection policy at the level where these two conflicting effects are balanced. Moreover, Chu and Furukawa (2011) find that under a centralized economy, the optimal patent protection level increases with the size of a quality improvement but decreases with the rate of time preference. In departing from these existing studies, this paper highlights how the optimal patent protection level interacts with international R&D funding and patent collateral.

The remainder of this paper is organized as follows. In Section 2, we construct an R&D-based growth model featuring international R&D funding and patent collateral. In Section 3, by focusing on the case where labor supply is perfectly inelastic, we discuss the growth effects of R&D-related shocks, and then analyze the optimal patent breadth policy and how it reacts to international R&D funding and patent collateral. Section 4 deals with whether our results in Section 3 are robust when labor supply is elastic. Finally, in Section 5, the main findings of the analysis are summarized.

2. The model

In this section we set up an R&D-based growth model that can be treated as an extension of the pioneering work by Romer (1990). In the Romer (1990) model, R&D investment leads to the creation of new varieties of intermediate goods. We extend the expanding-variety Romer (1990) model by bringing international R&D funding and patent collateral in R&D firms into the picture. In what follows, we will briefly describe the economy's structure.

2.1. Households

Consider an economy that is populated by a large number of identical and infinitely-lived households. Each household is endowed with one unit of time that is divided between labor L and leisure $H(=1-L)$. The lifetime utility of the representative household is given by:

$$\int_0^\infty \left[lnC_t + \Omega ln(1-L_t)\right]e^{-\rho t}dt; \quad \Omega > 0, \rho > 0, \tag{1a}$$

[1] See OECD (2011, p. 92) for the real values of R&D funds from abroad in OECD countries.

[2] In their open-economy R&D-growth models, Aghion, Howitt, and Mayer-Foulkes (2005) and Chu, Cozzi, Pan, and Zhang (2016) build up a distance-to-frontier R&D-based growth model, in which R&D entrepreneurs are subject to credit constraints rather than patent collateral constraints. However, these studies stress that a backward country's innovations will make its growth rate converge to the leading country's *exogenous* growth rate. This paper instead examines how international R&D funding and the international borrowing interest rate affect the *endogenous* growth rate. In addition, Amable, Chatelain, and Ralf (2010) set up an R&D-based growth model that features patents as collateral. However, their analysis does not involve international R&D funding.

[3] Turnovsky (1997) and Lai and Chin (2010) develop an open-economy endogenous model that features international funding and an imperfect world capital market. However, in their studies the economy's growth is driven by capital accumulation. Our analysis instead focuses on the relation between international funding and economic growth in an R&D-driven endogenous growth model.

where C is the consumption of final goods and t refers to time. The parameters ρ and Ω denote, respectively, the subjective discount rate and leisure preference. It should be noted that this paper deals with two distinct situations, namely, *inelastic* and *elastic* labor supply. Under the situation where labor supply is perfectly inelastic, labor supply is associated with a fixed value (i.e., $L_t = \overline{L}$), while under the situation where labor supply is elastic, labor supply is chosen by the household.

The household's budget constraint can be expressed as:

$$\dot{K}_t + \dot{a}_t = r_t K_t + \left(r_t^A + \dot{V}_t / V_t\right) a_t + w_t L_t + r_t D_t - C_t, \tag{1b}$$

where K is the stock of physical capital, $a(= VA)$ is the value of equity shares of monopolistic firms owned by the household, A is the number of equity shares (i.e., the number of varieties of intermediate goods), V is the value of an invented variety, r is the interest rate of the home country, r^A is the rate of dividends, $\dot{V}/V$ is the rate of capital gain or loss in equity shares, w is the wage rate,[4] and D is the amount of loans lent to R&D firms.

The optimum conditions for the representative household with respect to the indicated variables are:

$$C_t : \frac{1}{C_t} = \lambda_t, \tag{2a}$$

$$L_t : \frac{\Omega}{1 - L_t} = w_t \lambda_t, \tag{2b}$$

$$K_t : \lambda_t r_t = -\dot{\lambda}_t + \rho \lambda_t, \tag{2c}$$

$$a_t : \lambda_t \left(r_t^A + \dot{V}_t / V_t\right) = -\dot{\lambda}_t + \rho \lambda_t, \tag{2d}$$

$$\lambda_t : \dot{K}_t + \dot{a}_t = r_t K_t + \left(r_t^A + \dot{V}_t / V_t\right) a_t + w_t L_t + r_t B_t - C_t, \tag{2e}$$

where λ denotes the shadow value of wealth, and wealth is defined as the sum of physical capital K and the value of equity shares a.

Equipped with equations (2c) and (2d), the no-arbitrage condition between holding physical capital and equity shares is given by:

$$r_t = r_t^A + \dot{V}_t / V_t. \tag{3a}$$

From equations (2a) and (2b), the optimality condition for labor supply is:

$$1 - L_t = \Omega \frac{C_t}{w_t}. \tag{3b}$$

Equipped with equations (2a) and (2c), the usual Keynes-Ramsey rule is written as:

$$\dot{C}_t / C_t = r_t - \rho. \tag{3c}$$

To simplify the notation, in the following analysis we omit the time subscript unless it is necessary.

2.2. Final goods

The domestic final goods Y are treated as the numéraire. They are produced by competitive firms using labor and a continuum of intermediate goods in the form:

$$Y = L_Y^{1-\alpha} \int_0^A x_i^\alpha di; \quad 1 > \alpha > 0, \tag{4a}$$

where L_Y is the labor input in the production of final goods, x_i represents the intermediate goods for $i \in [0, A]$, and A is the number of varieties of intermediate goods.

Let p_i be the price of x_i. The profit function of the final good firms can then be written as:

$$\pi_Y = Y - w L_Y - \int_0^A p_i x_i di, \tag{4b}$$

Therefore, the conditional demand functions for L_Y and x_i are:

[4] We assume that workers are perfectly mobile across sectors. This implies that a unified wage rate w is present in the domestic economy.

$$L_Y = \frac{(1-\alpha)Y}{w}, \tag{4c}$$

$$x_i = L_Y \left(\frac{\alpha}{p_i}\right)^{\frac{1}{1-\alpha}}. \tag{4d}$$

2.3. Intermediate goods

There is a continuum of differentiated intermediate goods, and each intermediate good firm is owned by a monopolist. Following Romer (1990), physical capital is the factor input used to produce intermediate goods, and one unit of physical capital produces one unit of intermediate good. The production function can then be expressed as $x_i = k_i$, where k_i is the capital input used by the type-i intermediate firm. Therefore, the monopolistic profit of the type-i intermediate firm π_{x_i} is:

$$\pi_{x_i} = p_i x_i - r k_i. \tag{5a}$$

Accordingly, the profit-maximizing pricing of the type-i firm is:

$$p_i = r/\alpha. \tag{5b}$$

Here we follow Goh and Olivier (2002), Iwaisako and Futagami (2013) and Chu, Cozzi, Lai, and Liao (2015) to introduce a policy variable denoted by η which can measure the patent breadth. We assume that $\eta \in [1, 1/\alpha]$, and therefore the pricing rule of the type-i firm is:

$$p_i = \eta r. \tag{5c}$$

Equation (5c) implies that the pricing decisions of all intermediate good firms are symmetric. Thus, we can drop the notation i for the variables $\{x,\ p,\ k,\ \pi_x\}$. The profit function can then be represented as:

$$\pi_x = (\eta - 1) r x. \tag{6}$$

2.4. R&D with international funding

In the R&D sector, the value of any variety V is equal to $V = \int_t^\infty \pi_x e^{-r(\tau - t)} d\tau$. This implies that V follows the no-arbitrage condition:

$$rV = \pi_x + \dot{V}. \tag{7}$$

The return on investment in R&D will be equal to the profit from the monopolistic intermediate good firm π_x plus the capital gain $\dot{V}$. In line with Romer (1990), the R&D firm hires R&D labor L_A to produce new varieties of the knowledge-driven form:

$$\dot{A} = \varsigma A L_A; \quad \varsigma > 0, \tag{8}$$

where the parameter ς reflects the R&D productivity.[5]

In line with Christiano, Eichenbaum, and Evans (2005) and Neumeyer and Perri (2005), R&D firms have to pay a specific fraction of their production costs before cashing their output sales. This creates the need for working capital. In each period, the R&D firm needs working capital to pay for a fraction of the labor costs θ in advance, where $\theta \in [0, 1]$. The total wage payment for the R&D labor is wL_A, and hence the R&D firm needs to borrow the amount of funds $\theta w L_A$. In this economy, the R&D firm can choose to fund the shortage of working capital from both the foreign and home countries. Let ε be the proportion of the shortage of working capital borrowed from foreign countries, where $\varepsilon \in [0, 1]$. Moreover, to reflect the empirical fact that R&D funding from abroad plays quite an important role in the funding of business R&D, we assume that the foreign interest rate $\bar{r}$ is lower than the domestic interest rate r, that is, $\bar{r} < r$.[6] Therefore, the rational R&D firm tends to borrow as much as possible from foreign countries. However, it is not possible for the R&D firm to borrow without limit from foreign countries, because it should offer the market value of its patents as collateral.

The main reason for patent collateral is that creditors wish to mitigate the possible loss in default arising from risky R&D projects. Similar to the specification in Aghion and Banerjee (2005, p. 24), Kunieda and Shibata (2014), and Lai, Chin, and Chen (2017), the amount of debt that the R&D firm borrows cannot exceed a specific proportion of the value of its new patents.

[5] Our analytical results in Subsection 3.1 remain unchanged when the R&D firm uses final goods to produce new varieties in the lab-equipment form. To save space, the detailed derivations are not reported here but are available from the authors upon request.

[6] To simplify our analysis, we assume that the home country is a small open economy, and the foreign interest rate is treated as given. See, for example, Turnovsky (1996) for a similar assumption.

$$\varepsilon\theta wL_A \leq \varphi V\dot{A}; \quad 1>\varphi>0. \tag{9a}$$

The parameter φ captures the extent of the credit constraints.[7]

Due to $\bar{r}<r$, the profit-maximizing R&D firm will choose a value of ε such that the *inequality* constraint (9a) is *binding*.[8] To be more specific, the value of ε is chosen so as to satisfy the following constraint:

$$\varepsilon = \frac{\varphi V\dot{A}}{\theta wL_A}. \tag{9b}$$

Accordingly, the remaining proportion of the shortage of working capital is funded by the home households, i.e.,

$$D = (1-\varepsilon)\theta wL_A. \tag{9c}$$

Let π_A denote the profit of the R&D firm. The R&D firm's maximization problem can be written as:

$$Max \quad \pi_A = V\dot{A} - wL_A - \theta[(1-\varepsilon)r + \varepsilon\bar{r}]wL_A, \tag{10a}$$

$$s.t.\ \dot{A} = \varsigma AL_A. \tag{10b}$$

The free entry condition for R&D is given by:

$$V = \{1+\theta[(1-\varepsilon)r+\varepsilon\bar{r}]\}\frac{w}{\varsigma A}. \tag{10c}$$

Equation (10c) reveals that the free entry condition guarantees zero profit for the R&D firm, and hence the value of R&D is equal to the total cost of R&D.

Combining (9b), (10b) and (10c) together yields:

$$\varepsilon = \frac{\phi(1+\theta r)}{\theta[1+\phi(r-\bar{r})]}. \tag{11}$$

Equation (11) shows that the shortage of working capital borrowed from foreign countries (ε) is affected by the fraction of the collateral (ϕ), the fraction of borrowed R&D labor costs (θ), and the foreign interest rate ($\bar{r}$).

2.5. Market clearing and aggregation

The market-clearing condition for the labor market is:

$$L_Y + L_A = L = 1 - H. \tag{12a}$$

Equation (12a) indicates that total labor demand is equal to labor supply.

With its symmetric feature, the market-clearing condition for physical capital is expressed as:

$$\int_0^A x_i di = Ax = Ak = K, \tag{12b}$$

where Ak is the aggregate capital demand for all intermediate firms and K is the supply of capital provided by the households.

In Appendix A we show that the household's budget constraint can be alternatively written as:

$$\dot{K} = Y - \theta\varepsilon\bar{r}wL_A - C, \tag{12c}$$

where $\theta\varepsilon\bar{r}wL_A$ is the payment for the cost of international R&D borrowings, and, for ease of exposition, $Y-\theta\varepsilon\bar{r}wL_A$ can be treated as the household's disposable income. It should be noted that, since the government sector is absent from the model, the household's budget constraint stated in equation (12c) is equivalent to the economy's resource constraint.

Given that the property of transitional dynamics, the balanced economic growth rate and dynamic welfare are closely related to whether labor supply is inelastic or elastic, in the following two sections we will deal with two distinct situations. The first situation considers an inelastic labor supply, and the second situation considers an elastic labor supply.

[7] We consider that R&D firms finance the shortage of working capital by way of international borrowings. In line with Hochberg et al. (2014), R&D firms are allowed to issue venture debt. We assume that the international funding market is an asymmetric information market. Therefore, to avoid lending risk, foreign lenders will ask the home country's R&D firms to provide some collateral.

[8] Equation (9a) indicates that only the international borrowings of the R&D firm are subject to the patent collateral constraint. Our analytical results are robust when domestic borrowings are also subject to the patent collateral constraint.

3. Inelastic labor supply

The first scenario concerns the situation where labor supply is perfectly inelastic and is associated with $L = \overline{L}$. This situation can be treated as a benchmark case, and is intended for comparison with the other situation.

3.1. Transitional dynamics

With inelastic labor supply, the household's optimality condition reported in (2b) is absent from the model. Moreover, the household's optimality condition for labor supply stated in (3b) is replaced by:

$$L = \overline{L}. \tag{13a}$$

Equation (3b) implies that the household's leisure also remains intact at a fixed value, i.e.,

$$H = \overline{H}. \tag{13b}$$

In order to derive the dynamic equation that summarizes the entire model, we define one transformed variable: $f = C/K$. After some manipulations, we can derive three differential equations in terms of the ratio between consumption and physical capital f, the ratio between physical capital and the R&D stock x,[9] and the interest rate r that summarize the dynamics of the economy. This result leads us to establish the following proposition:

Proposition 1. *With an inelastic labor supply, the dynamics of the economy is expressed by the following three differential equations*:

$$\dot{f}/f = r - \rho - \frac{\eta r}{\alpha} + f + \frac{(1-\alpha)\phi\overline{r}(1+\theta r)}{x[1-\phi(\overline{r}-r)]}\left(\frac{\alpha}{\eta r}\right)^{\frac{\alpha}{1-\alpha}}\left(1 - \overline{H} - \left(\frac{\eta r}{\alpha}\right)^{\frac{1}{1-\alpha}}x\right), \tag{14a}$$

$$\dot{x}/x = \frac{\eta r}{\alpha} - f - \left\{\frac{(1-\alpha)\phi\overline{r}(1+\theta r)}{\varsigma x[1-\phi(\overline{r}-r)]}\left(\frac{\alpha}{\eta r}\right)^{\frac{\alpha}{1-\alpha}} + 1\right\}\varsigma\left(1 - \overline{H} - \left(\frac{\eta r}{\alpha}\right)^{\frac{1}{1-\alpha}}x\right), \tag{14b}$$

$$\dot{r}/r = (1-\alpha)\frac{r}{\vartheta}\left\{\frac{\psi\varsigma x[1-\phi(\overline{r}-r)]\left(\frac{\eta r}{\alpha}\right)^{\frac{1}{1-\alpha}}}{(1+\theta r)r} - 1\right\}, \tag{14c}$$

where $\vartheta = \alpha - \frac{(1-\alpha)\theta r}{1+\theta r} + \frac{(1-\alpha)\phi r}{1-\theta(\overline{r}-r)}$ *and* $\psi = \frac{\alpha}{1-\alpha}\frac{\eta-1}{\eta}$.

Proof. See Appendix B.

3.2. Steady-state growth

At the balanced growth equilibrium, the economy is characterized by $\dot{f} = \dot{x} = \dot{r} = 0$. Due to the complexity of the dynamic system, both Appendix C and the later quantitative analysis in Subsection 3.4 show that the economy is featured by a unique balanced growth equilibrium via a numerical simulation. Let $\tilde{f}$, $\tilde{x}$ and $\tilde{r}$ denote the stationary values of f, x and r. Then, from Equations (14a), (14b) and (14c) it is straightforward to infer that the steady-state values $\tilde{f}$, $\tilde{x}$ and $\tilde{r}$ satisfy the following stationary relationships:

$$\tilde{r} - \rho - \frac{\eta\tilde{r}}{\alpha} + \tilde{f} + \frac{(1-\alpha)\phi\overline{r}(1+\theta\tilde{r})}{\tilde{x}[1-\phi(\overline{r}-\tilde{r})]}\left(\frac{\alpha}{\eta\tilde{r}}\right)^{\frac{\alpha}{1-\alpha}}\left(1 - \overline{H} - \left(\frac{\eta\tilde{r}}{\alpha}\right)^{\frac{1}{1-\alpha}}\tilde{x}\right) = 0, \tag{15a}$$

$$\frac{\eta\tilde{r}}{\alpha} - \tilde{f} - \left\{\frac{(1-\alpha)\phi\overline{r}(1+\theta\tilde{r})}{\varsigma\tilde{x}[1-\phi(\overline{r}-\tilde{r})]}\left(\frac{\alpha}{\eta\tilde{r}}\right)^{\frac{\alpha}{1-\alpha}} + 1\right\}\varsigma\left(1 - \overline{H} - \left(\frac{\eta\tilde{r}}{\alpha}\right)^{\frac{1}{1-\alpha}}\tilde{x}\right) = 0, \tag{15b}$$

$$\frac{\psi\varsigma\tilde{x}[1-\phi(\overline{r}-\tilde{r})]\left(\frac{\eta\tilde{r}}{\alpha}\right)^{\frac{1}{1-\alpha}}}{(1+\theta\tilde{r})\tilde{r}} = 1. \tag{15c}$$

By some simple manipulations to delete $\tilde{f}$ and $\tilde{x}$, we can derive the following quadratic function in terms of $\tilde{r}$:

$$(\phi\psi + \theta)\tilde{r}^2 + \left[1 - \phi\psi\left(\varsigma + \rho - \varsigma\overline{H}\right) + \psi(1-\phi\overline{r})\right]\tilde{r} - \psi(1-\phi\overline{r})\left(\varsigma + \rho - \varsigma\overline{H}\right) = 0. \tag{16a}$$

[9] It should be noted that the market-clearing condition for physical capital reported in equation (12b) requires that $Ax = K$.

We can solve for two values of $\tilde{r}$ to satisfy (16a). One is positive and the other is negative. To make the analysis meaningful, we exclude the negative interest rate. Therefore, the reasonable equilibrium value of the domestic interest rate can be expressed as:

$$\tilde{r} = \frac{-\Phi + \sqrt{\Phi^2 + 4(\phi\psi + \theta)\psi(1-\phi\bar{r})(\varsigma + \rho - \varsigma\overline{H})}}{2(\phi\psi + \theta)}, \tag{16b}$$

where $\Phi = 1 - \phi\psi(\varsigma + \rho - \varsigma\overline{H}) + \psi(1 - \phi\bar{r})$.

Given $f = C/K$, $x = K/A$, $p = \eta r$, and $\int_0^A p_i x_i di = Apx = \alpha Y$, along the balanced growth $\dot{f} = \dot{x} = \dot{r} = 0$ imply that consumption, physical capital, R&D, and output grow at a common rate $\tilde{g}$. From (3c) and (16b), we can derive the balanced growth rate as follows:

$$\tilde{g} = \frac{-\Phi + \sqrt{\Phi^2 + 4(\phi\psi + \theta)\psi(1-\phi\bar{r})(\varsigma + \rho - \varsigma\overline{H})}}{2(\phi\psi + \theta)} - \rho. \tag{17}$$

Differentiating (17) with respect to ϕ, $\bar{r}$, θ, and η yields the following results:

$$\frac{\partial\tilde{g}}{\partial\phi} = \frac{2\psi\varsigma\tilde{L}_Y(\tilde{r} - \bar{r})\theta}{(\phi\psi + \theta)\Lambda} > 0, \tag{18a}$$

$$\frac{\partial\tilde{g}}{\partial\bar{r}} = \frac{-\psi\phi\varsigma\tilde{L}_Y}{\Lambda} < 0, \tag{18b}$$

$$\frac{\partial\tilde{g}}{\partial\theta} = \frac{-\phi\psi\tilde{r}\{\varsigma(1 - \overline{H} - \tilde{L}_Y) + \rho + \theta\tilde{r}\}}{(\phi\psi + \theta)\Lambda} < 0, \tag{18c}$$

$$\frac{\partial\tilde{g}}{\partial\eta} = \frac{\phi\mu\varsigma\tilde{L}_Y(2\phi\psi + \theta)(\tilde{r} - \bar{r}) + \mu\tilde{r}\left[\frac{\theta^2\varepsilon}{\phi\psi} + \phi\varepsilon\tilde{r} + (\theta\varepsilon - \phi) + \theta(\varepsilon - \phi\tilde{r})\right]}{(\phi\psi + \theta)\Lambda} > 0, \tag{18d}$$

where $\mu = \alpha/\eta^2(1-\alpha) > 0$, $\theta(\varepsilon - \phi\tilde{r}) = \phi\theta[(1-\varepsilon)\tilde{r} + \varepsilon\bar{r}] + \phi(1-\theta\tilde{r}) > 0$, $(\theta\varepsilon - \phi) = \theta[(1-\varepsilon)\tilde{r} + \varepsilon\bar{r}] > 0$ and $\Lambda = 2(\phi\psi + \theta)\tilde{r} + \Phi > 0$.[10] The results in (18a)-(18d) lead us to establish the following proposition:

Proposition 2. *With an inelastic labor supply, a rise in either the fraction of the collateral (ϕ) or patent breadth (η) raises the balanced growth rate, while a rise in either the foreign interest rate ($\bar{r}$) or the fraction of borrowed R&D labor costs (θ) lowers the balanced growth rate.*

The economic intuition behind Proposition 1 is quite obvious. A higher fraction of the collateral (ϕ) implies that the home country's R&D firms can obtain a larger amount of cheaper funds from foreign countries. This encourages the R&D firms to hire more labor, and hence leads to more innovations and higher economic growth. A larger patent breadth (η) increases the intermediate good firms' profit, which in turn increases the value of R&D and encourages R&D firms to devote more resources to R&D investment, thereby leading to a higher economic growth rate. Similarly, in response to a higher foreign interest rate ($\bar{r}$) or a higher fraction of borrowed R&D labor costs (θ), the R&D firm is motivated to reduce its R&D labor. This in turn leads to a decline in the home country's innovations and economic growth.[11]

3.3. Dynamic welfare

This subsection turns to deal with the normative analysis, and analyzes the optimal patent breadth that maximizes the social welfare level. To provide a more complete and precise picture, in line with Maebayashi, Hori, and Futagami (2017), we consider that the welfare analysis is implemented in a manner in which the equilibrium path includes transitional dynamics.[12]

Based on the Keynes-Ramsey rule reported in equation (3c), we can derive the expression: $C_t = C_0 \exp[\int_0^t (r_s - \rho)ds]$, where C_0 is the initial consumption level. Taking logarithms for both sides of this expression yields:

$$lnC_t = lnC_0 + \int_0^t (r_s - \rho)ds. \tag{19a}$$

Substituting equation (19a) into (1a), the social welfare function (i.e., indirect lifetime utility of households) in association with $L = \bar{L}$ is given by:

[10] From equation (16b), we have $2(\phi\psi + \theta)\tilde{r} + \Phi = \sqrt{\Phi^2 + 4(\phi\psi + \theta)\psi(1-\phi\bar{r})(\varsigma + \rho - \varsigma\overline{H})}$. Given $1 > \phi > 0$, $1 > \bar{r} > 0$ and $1 > \overline{H} > 0$, we can infer that $\Phi^2 + 4(\phi\psi + \theta)\psi(1-\phi\bar{r})(\varsigma + \rho - \varsigma\overline{H}) > 0$. This implies that $\Lambda = 2(\phi\psi + \theta)\tilde{r} + \Phi > 0$ holds.

[11] In our model, a higher fraction of borrowed R&D labor costs (θ) implies that it is more difficult for R&D firms to obtain funds from banks. This forces R&D firms to borrow more funds from households, and hence a higher fraction of borrowed R&D labor costs (θ) is associated with a lower fraction of bank loans. Proposition 1 shows that a lower fraction of bank loans leads to a decline in innovations, which is consistent with the empirical finding in Xin, Zhang, and Zheng (2017).

[12] By using a Romer (1990) type R&D-based growth model, Scrimgeour (2015) and Chen, Chu, Chu, and Lai (2017) numerically examine the transitional dynamics of relevant macroeconomic variables numerically following a change in the tax rate on asset income.

Table 1
Benchmark Parameterization.

Definition	Parameter	Value	Source/Target
Capital share	α	0.3	Gourio and Rudanko (2014)
Discount rate	ρ	0.05	Acemoglu and Akcigit (2012)
Foreign interest rate	$\bar{r}$	0.045	Schubert and Turnovsky (2011)
Inelastic labor supply	$\bar{L}$	1/3	Linnemann and Schabert (2003)
R&D productivity	ς	2.9691	Output growth rate = 2%
Fraction of borrowed R&D labor costs	θ	0.076	Hochberg et al. (2014)
Fraction of the collateral	ϕ	0.6396	Data
Markup (Patent breadth)	η	1.2	Jaimovich and Floetotto (2008)

$$\begin{aligned} W &= \int_0^\infty \left[lnC_t + \Omega\left(1-\bar{L}\right)\right]e^{-\rho t}dt \\ &= \int_0^\infty \left[lnC_0 + \int_0^t (r_s-\rho)ds\right]e^{-\rho t}dt + \frac{\Omega\left(1-\bar{L}\right)}{\rho}. \end{aligned} \tag{19b}$$

It should be noted that, following an adjustment in η, ϕ, $\bar{r}$, and θ, the welfare level stated in equation (19b) will change in response by way of changes in C_0 and the transitional paths of r_t.[13,14] Based on the fact that equation (19b) is too complex, it is very difficult for us to provide a clear analytical result to solve how the welfare is affected in response to a change in η, ϕ, $\bar{r}$, and θ. Accordingly, we must resort to a numerical analysis.

3.4. Calibration

This subsection offers a quantitative assessment by resorting to a numerical analysis, and then uses it to study numerically the macroeconomic effects on economic growth and social welfare. The parameters we set are adopted from commonly-used values in the existing literature or calibrated to match the U.S. data.

In line with Gourio and Rudanko (2014) and Acemoglu and Akcigit (2012), the capital share and the discount rate are set to the common values 0.3 and 0.05, respectively. Based on Schubert and Turnovsky (2011), the foreign interest rate is set to 0.045. As in Linnemann and Schabert (2003), the share of time endowment that household devotes to working is set to 1/3. According to Hochberg et al. (2014), the fraction of borrowed R&D labor costs is set to 0.076. Jaimovich and Floetotto (2008) point out that the estimated markups in the value added data range from 1.2 to 1.4, and so the benchmark markup is set to 1.2. The fraction of the collateral is given by 0.6396, which is the average loan-to-value ratio in the U.S. for the period from 2011 to 2015 according to the Statista database.[15] Finally, we calibrate the R&D productivity as 2.9691 to match the output growth rate observed in the U.S. economy, which is 2%. A summary of these benchmark parameter values is reported in Table 1.

Based on the benchmark parameter values reported in Table 1, we can infer three characteristic roots of the dynamic system: one is negative and two are positive. The numerical values for these three roots are −0.0164, 0.2537, and 1.0055. As indicated in Appendix C, this verifies the existence of the economy's balanced growth equilibrium. Moreover, we can compute the growth effect arising from changes in the fraction of the collateral, the foreign interest rate, the fraction of borrowed R&D labor costs and patent breadth. The results are summarized in Table 2.

Table 2 reveals that a higher fraction of the collateral (ϕ) and a larger patent breadth (η) will stimulate economic growth. However, economic growth is decreasing in the foreign interest rate ($\bar{r}$) and the fraction of borrowed R&D labor costs (θ). The numerical results reported in Table 2 are the same as those of the comparative statics revealed in equations (18a)-(18d). The economic intuition behind equations (18a)-(18d) is provided in Subsection 3.2, and hence we do not repeat it here.

We now turn to deal with the normative analysis, and analyze the optimal patent breadth that maximizes the social welfare level. Based on our benchmark parameter values, the solid line in Fig. 1 plots the relationship between the level of social welfare and the patent breadth in association with an inelastic labor supply, which exhibits an inverse U-shaped relationship. The optimal value of the patent breadth is $\eta^{opt} = 1.3510$.[16] The empirical estimate for the markup in the literature, e.g., Jones and Williams (2000), suggests that the value lies within a range of between 5% and 40%, and thus the optimal value $\eta^{opt} = 1.3510$ in association with an inelastic labor supply in Fig. 1 fits the evidence and usual observation.

The intuition behind the optimal patent breadth can be explained as follows. A higher patent breadth leads R&D firms to be more profitable, which in turn provides R&D firms with an incentive to conduct more R&D investment by means of hiring more R&D labor.

[13] The Keynes-Ramsey rule in equation (3c) reveals the result: $g_t^C = r_t - \rho$, where g_t^C stands for the growth rate of consumption. As a result, the welfare effect stemming from the transitional paths of r_t reflects the welfare effect arising from the transitional paths of g_t^C.

[14] Appendix D provides a Proof to show why C_0 will jump in response to an adjustment in η, ϕ, $\bar{r}$, and θ.

[15] The Statista database indicates that the U.S. loan-to-value ratios from 2011 to 2015 are 71.3%, 69.1%, 61.9%, 59.7% and 57.8%, respectively. Therefore, the value of the average loan-to-value ratio in these five years is 63.96%.

[16] It should be noted that the optimal value of the patent breadth $\eta^{opt} = 1.3510$ remains intact regardless of the value of the leisure preference Ω. However, as exhibited in Fig. 1, the value of the leisure preference Ω does affect the welfare level in association with $\eta^{opt} = 1.3510$. To make a consistent comparison between an inelastic labor supply and an elastic labor supply, in our numerical analysis concerning the inelastic labor supply in this subsection Ω is set to 1.541, which is equal to the calibrated value under the elastic labor supply. The reasoning for the calibrated value of Ω is provided in Section 4 below.

Table 2
Economic growth effect of a change in parameters (inelastic labor supply).

A change in parameters	$\Delta\tilde{g}$
$\Delta\phi$(63.96% to70%)	0.010%
$\Delta\bar{r}$(4.5%to5.5%)	−0.021%
$\Delta\theta$(7.6%to8.6%)	−0.005%
$\Delta\eta$(1.2to1.3)	2.570%

This will give rise to two conflicting effects on the level of social welfare. First, more R&D investment raises the economic growth rate, and hence, as indicated in equation (19b), generates a positive effect on social welfare. Second, a higher patent breadth leads the household to bear a higher cost for its international R&D borrowings, thereby causing a reduction in the household's disposable income, as exhibited in equation (12c). Then, the household is inclined to instantly reduce its current consumption due to the income effect. As indicated in equation (19b), a decline in current consumption generates a negative effect on social welfare. Consequently, a reduction in the patent breadth can remedy the distortion (welfare loss) stemming from the overuse of international R&D borrowings. This is the reason why the optimal patent breadth is set to a finite value $\eta^{opt} = 1.3510$.[17]

We are now in a position to discuss how the optimal patent breadth will react following a change in the foreign interest rate, the fraction of the collateral, and the fraction of borrowed R&D labor costs.

The solid line in Fig. 2 portrays the relationship between the optimal patent breadth and the foreign interest rate when labor supply is inelastic. It indicates that, following a decline in the foreign interest rate, the optimal patent breadth will decrease in response. For instance, when the foreign interest rate falls from the benchmark value of 4.5%–3.5%, the optimal patent breadth will decline from 1.3510 to 1.3490. Intuitively, a reduction in the foreign interest rate will lower the R&D cost, and R&D firms will thereby be inclined to devote more resources to R&D investment and use more R&D labor. This leads to a greater overuse of international R&D borrowings. Accordingly, the government should choose a lower optimal patent breadth to eliminate the distortion resulting from a greater overuse of international R&D borrowings.

The solid line in Fig. 3 displays the negative relationship between the optimal patent breadth and the fraction of the collateral associated with an inelastic labor supply. For instance, it reveals that, in response to a rise in the fraction of the collateral from the benchmark value of 63.96%–70%, the optimal patent breadth will decrease from 1.3510 to 1.3504 in response. Intuitively, a higher fraction of the collateral leads R&D firms to obtain cheaper foreign funds from the international market. This motivates R&D firms to engage in more R&D investment and use more R&D labor, and hence leads to a greater overuse of international R&D borrowings. To achieve social welfare maximization, the government should choose a lower optimal patent breadth to correct for the unduly high level of international R&D borrowings.

The solid line in Fig. 4 presents the positive relationship between the optimal patent breadth and the fraction of borrowed R&D labor costs. For instance, it indicates that, following a reduction in the fraction of borrowed R&D labor costs from the benchmark value of 7.6%–6%, the optimal patent breadth will fall from 1.3510 to 1.3506 in response. The rationale for this result can be explained intuitively. A reduction in the fraction of borrowed R&D labor costs reduces the interest payment on borrowing, thereby causing a decline in R&D costs. This provides R&D firms with an incentive to engage in more R&D investment and use more R&D labor, and hence leads to a greater overuse of international R&D borrowings. Accordingly, to achieve social welfare maximization, the government should adopt a lower optimal patent breadth to correct for the unduly high level of international R&D borrowings.

4. Elastic labor supply

This section deals with a more general situation where labor supply is elastic, and then analyzes how the international borrowing interest rate and the fraction of patent collateral will affect economic growth and social welfare.

4.1. Transitional dynamics and steady-state growth

Similar to the derivations under the situation where labor supply is inelastic, the dynamic system in association with an elastic labor supply can be expressed by three differential equations in terms of the ratio between consumption and physical capital f, the ratio between physical capital and the R&D stock x, and the interest rate r. Thus, we can establish the following proposition:

Proposition 3. *With an elastic labor supply, the dynamics of the economy is expressed by the following three differential equations*:

$$\dot{f}/f = r - \rho - \frac{\eta r}{\alpha} + f + \frac{\phi\bar{r}(1-\alpha)(1+\theta r)}{x[1-\phi(\bar{r}-r)]}\left(\frac{\alpha}{\eta r}\right)^{\frac{\alpha}{1-\alpha}}\Theta, \tag{20a}$$

[17] In his survey paper, Chu (2009) provides a similar explanation regarding the optimal level of intellectual property right protection (IPR). Chu (2009) summarizes the macroeconomic effects of IPR and points out that the optimal level of IPR should trade off the social benefits of enhanced growth against the social costs of distortions. The social costs of distortions mainly stem from the following two sources. First, the markup price distorts the relative consumption between monopolistic goods and competitive goods from the socially optimal level. Second, under an endogenous labor supply, the markup price distorts the labor supply decision from the optimal level. Therefore, by way of these distortions, even though higher IPR protection could enhance economic growth, it does not necessarily enhance social welfare.

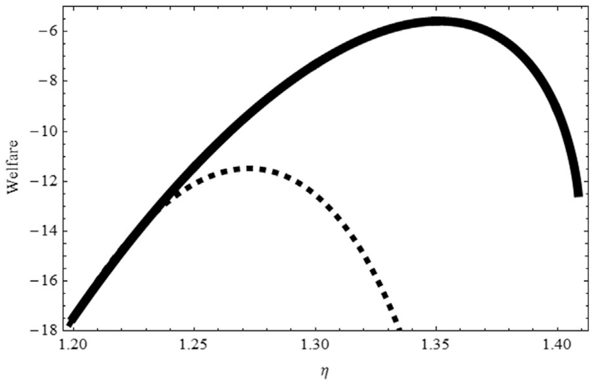

Fig. 1. The optimal patent breadth.

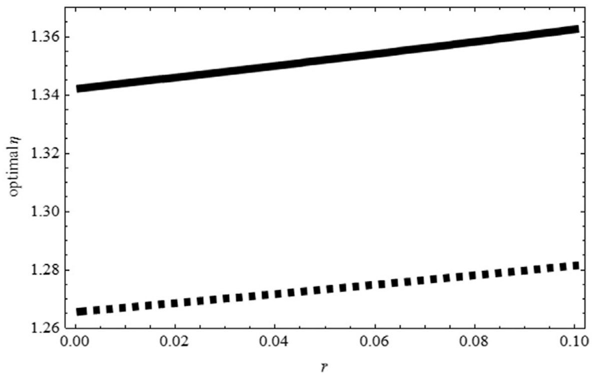

Fig. 2. The optimal patent breadth (a change in $\bar{r}$).

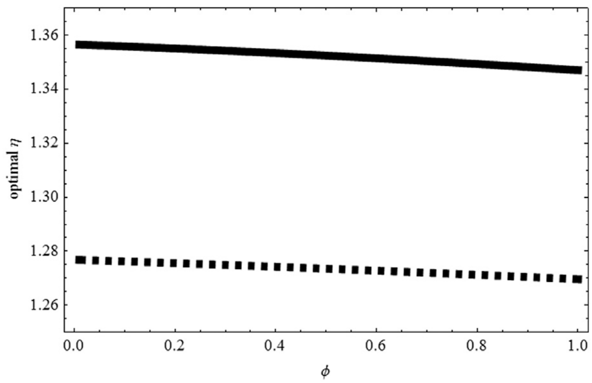

Fig. 3. The optimal patent breadth (a change in ϕ).

$$\dot{x}/x = \frac{\eta r}{\alpha} - f - \left\{ \frac{\phi \bar{r}(1-\alpha)(1+\theta r)}{x[1-\phi(\bar{r}-r)]} \left(\frac{\alpha}{\eta r}\right)^{\frac{\alpha}{1-\alpha}} + \varsigma \right\} \Theta, \tag{20b}$$

$$\dot{r}/r = (1-\alpha)\frac{r}{\vartheta} \left\{ \frac{\psi \varsigma x[1-\phi(\bar{r}-r)]\left(\frac{\eta r}{\alpha}\right)^{\frac{1}{1-\alpha}}}{(1+\theta r) r} - 1 \right\}, \tag{20c}$$

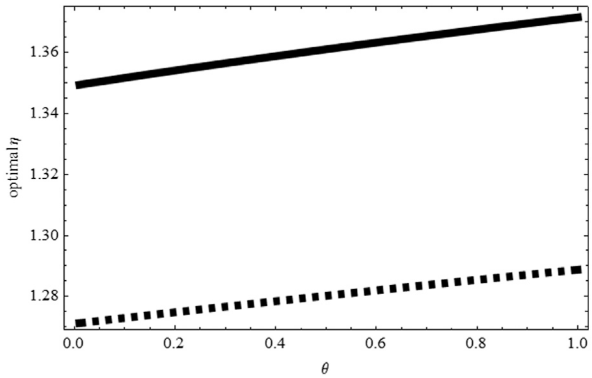

Fig. 4. The optimal patent breadth (a change in θ).

Table 3
Economic growth effect of a change in parameters (elastic labor supply).

A change in parameters	$\Delta\tilde{g}$
$\Delta\phi(63.96\%\text{to}70\%)$	0.012%
$\Delta\bar{r}(4.5\%\text{to}5.5\%)$	−0.025%
$\Delta\theta(7.6\%\text{to}8.6\%)$	−0.006%
$\Delta\eta(1.2\text{to}1.3)$	3.085%

where $\Theta = L - L_Y = 1 - \frac{\Omega fx}{1-\alpha}\left(\frac{\eta r}{\alpha}\right)^{\frac{\alpha}{1-\alpha}} - \left(\frac{\eta r}{\alpha}\right)^{\frac{1}{1-\alpha}} x$.

Proof. See Appendix E.

With similar derivations for equation (16a), we can derive the following quadratic equation in terms of the steady-state interest rate $\tilde{r}$:

$$\theta\tilde{r}^2 + \{1 - \frac{\phi\psi(1-\alpha)(\varsigma - \tilde{r} + \rho)}{(1-\alpha) - \Omega\Gamma}\}\tilde{r} - \frac{(1-\phi\bar{r})\psi(1-\alpha)(\varsigma - \tilde{r} + \rho)}{(1-\alpha) - \Omega\Gamma} = 0, \tag{21}$$

where $\Gamma = \left(\frac{\alpha}{\eta}\right)\left(\frac{\tilde{r}-\rho}{\tilde{r}}\right) + (1-\alpha)\bar{r}\phi\psi\left(\frac{\tilde{r}-\rho}{\tilde{r}}\right) > 0$. Once the steady-state interest rate $\tilde{r}$ is solved, the common growth rate in equation (17) can be expressed as:

$$\tilde{g} = \tilde{r} - \rho. \tag{22}$$

Based on the fact that equation (21) is too complex, it is very difficult for us to provide a clear analytical result to solve the interest rate $\tilde{r}$ and then explore the growth effect from equation (22). Accordingly, we must resort to a numerical analysis.

When we deal with the numerical analysis on elastic labor supply, two points should be noted. First, the parameter $\bar{L} = 1/3$ should be removed from Table 1 since in this situation labor supply is determined endogenously. Second, in addition to seven parameters ($\bar{L} = 1/3$ is excluded) used in Table 1, one additional parameter, namely, the leisure preference Ω, should be calibrated. To be more specific, in line with existing studies, the leisure preference Ω is calibrated as 1.541 so as to make hours worked be one third.

Similar to the inelastic labor supply, by using the relevant parameters for elastic labor supply, we can derive three characteristic roots of the dynamic system: one is negative and two are positive. The numerical values for these three roots are −0.0132, 0.2670, and 2.9683. Similar to the logic reported in Appendix C, the economy exists a unique balanced growth equilibrium.

Table 3 reveals that a higher fraction of the collateral (ϕ) and a larger patent breadth (η) will stimulate economic growth. However, economic growth is decreasing in the foreign interest rate ($\bar{r}$) and the fraction of borrowed R&D labor costs (θ). By comparing the results in Table 3 with those in Table 1, it is clear that the comparative results in association with an elastic labor supply are qualitatively the same as those in association with its inelastic counterpart.

We then turn to deal with the normative analysis, and analyze the optimal patent breadth that maximizes the social welfare level. When labor supply is elastic, the social welfare function (i.e., indirect lifetime utility of households) is given by:

$$\begin{aligned} W &= \int_0^\infty [lnC_t + \Omega(1-L_t)]e^{-\rho t}dt \\ &= \int_0^\infty \left[lnC_0 + \int_0^t (r_s - \rho)ds + \Omega(1-L_t)\right]e^{-\rho t}dt. \end{aligned} \tag{23}$$

Since equation (23) is too complex to solve how the welfare is affected in response to a change in η, ϕ, $\bar{r}$, and θ, we thus resort to a numerical analysis.[18]

Based on our benchmark parameter values, the dashed line in Fig. 1 plots the relationship between the level of social welfare and the patent breadth, which exhibits an inverse U-shaped relationship. The optimal value of the patent breadth is $\eta^{opt} = 1.2725$.

It is quite clear from Fig. 1 that the optimal value of the patent breadth under an elastic labor supply (1.2725) is lower than that under its inelastic counterpart (1.3510). This result can be interpreted intuitively as follows. As stated previously under the situation where the labor supply is inelastic, faced with a higher patent breadth the household needs to pay a higher cost for its international R&D borrowings, thereby causing a reduction in the household's disposable income. With the income effect, the household tends to depress both consumption and leisure since both are normal goods. The fall in leisure (coupled with the rise in labor supply) is the additional effect for the situation where the labor supply is elastic. Compared with the inelastic labor supply, as indicated in equation (23), a rise in labor supply generates an additional negative effect on social welfare when the labor supply is elastic. This is the reason why in Fig. 1, in association with a given value of η, the welfare level under an elastic labor supply is smaller than that under an inelastic labor supply. With the additional welfare loss arising from the overuse of the labor supply under an elastic labor supply, the government should choose a lower optimal patent breadth compared to the situation where the labor supply is inelastic.

We then discuss how the optimal patent breadth η^{opt} will react following a change in the foreign interest rate $\bar{r}$, the fraction of the collateral ϕ, and the fraction of borrowed R&D labor costs θ.

The dashed line in Fig. 2 depicts the relationship between the optimal patent breadth and the foreign interest rate when labor is supplied elastically. Two results displayed in Fig. 2 should be noted. First, it is revealed that, following a reduction in the foreign interest rate, the optimal patent breadth will decrease in response. For example, when the foreign interest rate declines from the benchmark value of 4.5%–3.5%, the optimal patent breadth will decrease from 1.2725 to 1.2709. The positive relationship between η^{opt} and $\bar{r}$ under an elastic labor supply is qualitatively the same as that under its inelastic counterpart. The intuition behind the positive relationship between η^{opt} and $\bar{r}$ under an elastic labor supply is similar to that under an inelastic labor supply, so to save space we do not repeat it again. Second, in association with a given value of $\bar{r}$, the optimal patent breadth under an elastic labor supply is less than that under an inelastic labor supply. The intuition underlying this result is that, as emphasized previously, an additional welfare loss arises from the overuse of labor supply under an elastic labor supply. So, to correct for this unduly high level of labor supply, the government should choose a lower optimal patent breadth compared to the situation where labor supply is inelastic.

The dashed line in Fig. 3 displays the relationship between the optimal patent breadth and the fraction of the collateral when labor supply is elastic. Two observations are found in Fig. 3. First, under an elastic labor supply the optimal patent breadth is negatively related to the fraction of the collateral. For instance, following a rise in the fraction of the collateral from the benchmark value of 63.96%–70%, the optimal patent breadth will decrease from 1.2725 to 1.2721 in response. The economic reasoning for the negative relationship between η^{opt} and ϕ under an elastic labor supply is essentially the same as that under an inelastic labor supply, and thus we do not repeat it here. Second, in association with a given value of ϕ, the optimal patent breadth under an elastic labor supply is less than that under an inelastic labor supply. The intuition underlying this result is that, as stated previously, an additional welfare loss arises from the overuse of labor supply under an elastic labor supply. Accordingly, to remedy the distortion caused by the overuse of labor supply, the government should take action to choose a lower optimal patent breadth compared to the situation where the labor supply is inelastic.

Finally, the dashed line in Fig. 4 presents the positive relationship between the optimal patent breadth and the fraction of borrowed R&D labor costs. Two observations emerge from Fig. 4. First, just as in the case of the inelastic labor supply, the optimal patent breadth η^{opt} is positively related to the fraction of borrowed R&D labor costs θ. For instance, a reduction in the fraction of borrowed R&D labor costs from the benchmark value of 7.6%–6% leads the optimal patent breadth to fall from 1.2725 to 1.2722. Second, for any given θ, the optimal patent breadth under an elastic labor supply is less than that under an inelastic labor supply. The logic behind these two observations in Fig. 4 is similar to that in Fig. 2, and hence we need not repeat it here.

5. Concluding remarks

This paper sets up an R&D-based growth model featuring international R&D funding and patent collateral, and uses it to examine how the international borrowing interest rate and the fraction of patent collateral will affect economic growth and social welfare. Several major findings emerge from the analysis. First, with an inelastic labor supply, a rise in the fraction of patent collateral is beneficial to both innovations and economic growth. Second, when labor supply is inelastic, a rise in either the foreign interest rate or the fraction of borrowed R&D funding is harmful to innovations and economic growth. Third, our numerical results show that the above two findings are robust when labor is supplied elastically. Finally, our numerical results indicate that, regardless of whether labor supply is inelastic or elastic, the government can implement an optimal patent breadth policy to maximize the social welfare level. Our numerical results also point out that this optimal patent breadth will decrease in response to a reduction in the foreign interest rate, a rise in the fraction of the collateral, and a reduction in the fraction of borrowed R&D labor costs.

Before ending this paper, one point deserves special mention here. The R&D-based growth models with expanding variety are criticized for featuring the scale effect, i.e., they predict that a rise in the size of employment (or the labor force) will lead to a higher growth rate. This result is at odds with empirical studies based on US and OECD data, for instance, Backus, Kehoe, and Kehoe (1992) and Jones (1995a; 2005). To remove this undesirable scale effect, we can follow Jones (1995b) and Eicher and Turnovsky (2000) by

[18] The Proof for the discrete jump in C_0 under an elastic labor supply is similar to that under an inelastic labor supply; the latter is shown in detail in Appendix D.

bringing the growth rate of the labor force into the model.[19] By so doing, we are able to not only show that the balanced growth rate is crucially determined by the growth rate of the labor force (rather than the level of the labor force), but also deal with whether the international borrowing interest rate and the fraction of patent collateral are powerful in affecting the "scale-adjusted" per capita output and social welfare.[20]

Acknowledgments

The authors are deeply grateful to two anonymous referees for their constructive comments, which substantially improved the paper. The authors are also indebted to Angus C. Chu, Hsun Chu and Chih-Hsing Liao for their helpful suggestions and comments regarding earlier versions of the paper. Any errors or shortcomings are the authors' responsibility.

Appendix A

This appendix provides a detailed Proof to derive an alternative form of the household's budget constraint. We first rewrite the household's budget constraint as follows:

$$\dot{K} + \dot{a} = rK + \left(r^A + \dot{V}/V\right)a + wL + rD - C. \tag{A1}$$

Differentiating $a = VA$ with respect to time yields $\dot{a} = \dot{V}A + V\dot{A}$. Then, inserting $\dot{a} = \dot{V}A + V\dot{A}$, (9c), (10a), and (12a) into (A1), we have:

$$\dot{K} + \dot{V}A = rK + \left(r^A + \dot{V}/V\right)AV + wL_Y - C - \theta\varepsilon\bar{r}wL_A. \tag{A2}$$

According to (4d), (5c), and (12b), we then have $rK = \alpha Y/\eta$. Moreover, substituting $rK = \alpha Y/\eta$ into (A2) yields:

$$\dot{K} + \dot{V}A = \frac{\alpha Y}{\eta} + \left(r^A + \dot{V}/V\right)AV + wL_Y - C - \theta\varepsilon\bar{r}wL_A. \tag{A3}$$

Inserting (3a) and (7) into (A3) gives rise to:

$$\dot{K} = \frac{\alpha Y}{\eta} + A\pi_x + wL_Y - C - \theta\varepsilon\bar{r}wL_A. \tag{A4}$$

By using (4d), (5a), (6), and $Arx = \frac{\alpha Y}{\eta}$, we have $A\pi_x = \frac{(\eta-1)\alpha Y}{\eta}$. Finally, substituting (4c) and $A\pi_x = \frac{(\eta-1)\alpha Y}{\eta}$ into (A4), the household's budget constraint reported in (1b) can be alternatively expressed as:

$$\dot{K} = Y - \theta\varepsilon\bar{r}wL_A - C. \tag{A5}$$

Appendix B

This appendix solves the dynamic system of the model under an inelastic labor supply. Equipped with (4d) and (5c), we have:

$$r = \frac{\alpha}{\eta}\left(\frac{L_Y}{x}\right)^{1-\alpha}. \tag{B1}$$

From $f = C/K$, $\dot{f}/f = \dot{C}/C - \dot{K}/K$, $x = K/A$, $Y/K = (L_Y/x)^{1-\alpha}$, (3c), (8), (9b), and (12c), we can derive:

$$\dot{f}/f = r - \rho - \left(\frac{L_Y}{x}\right)^{1-\alpha} + f + \frac{\phi V\bar{r}}{x}\varsigma L_A. \tag{B2}$$

By using $x = K/A$, $\dot{x}/x = \dot{K}/K - \dot{A}/A$, $f = C/K$, $Y/K = (L_Y/x)^{1-\alpha}$, (8), (9b), and (12c), we have:

$$\dot{x}/x = \left(\frac{L_Y}{x}\right)^{1-\alpha} - f - \left(\frac{\phi V\bar{r} + x}{x}\right)\varsigma L_A. \tag{B3}$$

Based on (4a), (4c), (10c), (11), and (B1), we can derive the value of R&D as:

$$V = \frac{(1-\alpha)(1+\theta r)}{\varsigma(1-\phi(\bar{r}-r))}\left(\frac{\alpha}{\eta r}\right)^{\frac{\alpha}{1-\alpha}}. \tag{B4}$$

Substituting (B1), (B4), and (12a) into (B2) and (B3), respectively, we obtain:

[19] Peretto (1996; 1998) and Howitt (1999) propose an alternative way to escape from the scale effect, in which their model is embodied with the feature that a rise in the scale of the aggregate economy is perfectly fragmented by the proliferation of endogenous product varieties.

[20] See, e.g., Eicher and Turnovsky (2000) for a relevant analysis.

$$\dot{f}/f = r - \rho - \frac{\eta r}{\alpha} + f + \frac{(1-\alpha)\phi\bar{r}(1+\theta r)}{x[1-\phi(\bar{r}-r)]}\left(\frac{\alpha}{\eta r}\right)^{\frac{\alpha}{1-\alpha}}\left[1-\overline{H}-\left(\frac{\eta r}{\alpha}\right)^{\frac{1}{1-\alpha}}x\right], \tag{B5}$$

$$\dot{x}/x = \frac{\eta r}{\alpha} - f - \left\{\frac{(1-\alpha)\phi\bar{r}(1+\theta r)}{\varsigma x[1-\phi(\bar{r}-r)]}\left(\frac{\alpha}{\eta r}\right)^{\frac{\alpha}{1-\alpha}} + 1\right\}\varsigma\left[1-\overline{H}-\left(\frac{\eta r}{\alpha}\right)^{\frac{1}{1-\alpha}}x\right]. \tag{B6}$$

Taking logarithms of both sides for (B4) and differentiating the resulting equation with respect to time yields:

$$\frac{\dot{V}}{V} = \frac{\theta r}{(1+\theta r)}\frac{\dot{r}}{r} - \frac{\phi r}{(1-\phi(\bar{r}-r))}\frac{\dot{r}}{r} - \frac{\alpha}{1-\alpha}\frac{\dot{r}}{r}. \tag{B7}$$

By using (7), we have:

$$r = \frac{A\pi_x}{AV} + \frac{\dot{V}}{V}. \tag{B8}$$

By substituting $A\pi_x = (\eta - 1)\alpha Y/\eta$, (4a), (B4), and (B7) into (B8) to delete $\dot{V}/V$, we can infer the expression:

$$\dot{r}/r = (1-\alpha)\frac{r}{\vartheta}\left\{\frac{\psi\varsigma x[1-\phi(\bar{r}-r)]\left(\frac{\eta r}{\alpha}\right)^{\frac{1}{1-\alpha}}}{(1+\theta r)r} - 1\right\}, \tag{B9}$$

where $\vartheta = \alpha - \frac{(1-\alpha)\theta r}{1+\theta r} + \frac{(1-\alpha)\phi r}{1-\phi(\bar{r}-r)}$ and $\psi = \frac{\alpha}{1-\alpha}\frac{\eta-1}{\eta}$.

Accordingly, as exhibited in (B5), (B6), and (B9), the dynamic system can be expressed in terms of three differential equations for f, x, and r.

Appendix C

This appendix briefly discusses the existence of the economy's unique balanced growth equilibrium. Linearizing (B5), (B6), and (B9) around the steady-state equilibrium yields:

$$\begin{pmatrix}\dot{f}\\ \dot{x}\\ \dot{r}\end{pmatrix} = \begin{pmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{pmatrix}\begin{pmatrix}f-\tilde{f}\\ x-\tilde{x}\\ r-\tilde{r}\end{pmatrix} + \begin{pmatrix}a_{14}\\ a_{24}\\ a_{34}\end{pmatrix}dz, \tag{C1}$$

where $a_{11} = \partial\dot{f}/\partial f$, $a_{12} = \partial\dot{f}/\partial x$, $a_{13} = \partial\dot{f}/\partial r$, $a_{14} = \partial\dot{f}/\partial z$, $a_{21} = \partial\dot{x}/\partial f$, $a_{22} = \partial\dot{x}/\partial x$, $a_{23} = \partial\dot{x}/\partial r$, $a_{24} = \partial\dot{x}/\partial z$, $a_{31} = \partial\dot{r}/\partial f$, $a_{32} = \partial\dot{r}/\partial x$, $a_{33} = \partial\dot{r}/\partial r$, $a_{34} = \partial\dot{r}/\partial z$, $z \in \{\eta, \bar{r}, \theta, \phi\}$. Due to the complicated calculations, we do not list the analytical results for a_{ij}, where $i \in \{1,2,3\}$ and $j \in \{1,2,3,4\}$.

Let δ_1, δ_2 and δ_3 be the three characteristic roots of the dynamic system. Due to the complexity involved in the calculations of the three characteristic roots, we cannot solve these roots analytically. In a later discussion (Subsection 3.4), we will instead show via a numerical simulation that in the dynamic system there exist two positive and one negative characteristic roots. The literature on dynamic rational expectations models, such as Burmeister (1980) and Turnovsky (2000), claims that, if the number of positive (unstable) roots equals the number of jump variables, there exists a unique perfect foresight equilibrium solution. The dynamic system in equation (C1) has two jump variables, f and r, and one predetermined variable, x,[21] and hence our numerical results reveal that the economy features a unique steady-state equilibrium.

Appendix D

This appendix provides a Proof to explain why C_0 in equation (19b) will jump in response to an adjustment in z, where $z \in \{\eta, \bar{r}, \theta, \phi\}$. Given that the discrete jump in C_0 is closely related to the discrete adjustment in f, we thus first need to show how f will react at the moment of policy implementation.

For expository convenience, let δ_1 be the negative root and δ_2 as well as δ_3 be the positive roots. Therefore, the general solution for f, x, and r is given by:

[21] Both K and A are predetermined variables, and hence $x(= K/A)$ should also be treated as a predetermined variable.

$$\begin{pmatrix} f \\ x \\ r \end{pmatrix} = \begin{pmatrix} \tilde{f}(z_v) \\ \tilde{x}(z_v) \\ \tilde{r}(z_v) \end{pmatrix} + \begin{pmatrix} 1 & 1 & 1 \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix} \begin{pmatrix} B_1 e^{\delta_1 t} \\ B_2 e^{\delta_2 t} \\ B_3 e^{\delta_3 t} \end{pmatrix}, \tag{D1a}$$

where B_1, B_2 and B_3 are undetermined coefficients and $\tilde{f}(z_v)$, $\tilde{x}(z_v)$, and $\tilde{r}(z_v)$ respectively denote the stationary values of f, x, and r in association with a specific level of the exogenous variable z, namely, z_v. Moreover, h_{21}, h_{22}, …, and h_{33} are defined as:

$$h_{2j} = \begin{vmatrix} \delta_j - a_{11} & a_{13} \\ -a_{21} & a_{23} \end{vmatrix} \Big/ \begin{vmatrix} a_{12} & a_{13} \\ a_{22} - \delta_j & a_{23} \end{vmatrix}; j \in \{1,2,3\}, \tag{D1b}$$

$$h_{3j} = \begin{vmatrix} a_{12} & \delta_j - a_{11} \\ a_{22} - \delta_j & -a_{21} \end{vmatrix} \Big/ \begin{vmatrix} a_{12} & a_{13} \\ a_{22} - \delta_j & a_{23} \end{vmatrix}; j \in \{1,2,3\}. \tag{D1c}$$

To simplify the exposition, the policy implementation that this paper deals with is expressed as the exogenous variable z, which changes from its benchmark value z_0 to a new level z_1, where $z \in \{\eta, \bar{r}, \theta, \phi\}$. By using (D1a)-(D1c), we introduce the following equations to describe the dynamic adjustment of f_t, x_t, and r_t:

$$f_t = \begin{cases} \tilde{f}(z_0); & t = 0^-, \\ \tilde{f}(z_1) + B_1 e^{\delta_1 t} + B_2 e^{\delta_2 t} + B_3 e^{\delta_3 t}; & t \geq 0^+, \end{cases} \tag{D2a}$$

$$x_t = \begin{cases} \tilde{x}(z_0); & t = 0^-, \\ \tilde{x}(z_1) + h_{21} B_1 e^{\delta_1 t} + h_{22} B_2 e^{\delta_2 t} + h_{23} B_3 e^{\delta_3 t}; & t \geq 0^+, \end{cases} \tag{D2b}$$

$$r_t = \begin{cases} \tilde{r}(z_0); & t = 0^-, \\ \tilde{r}(z_1) + h_{31} B_1 e^{\delta_1 t} + h_{32} B_2 e^{\delta_2 t} + h_{33} B_3 e^{\delta_3 t}; & t \geq 0^+, \end{cases} \tag{D2c}$$

where 0^- and 0^+ denote the instant before and after the exogenous variable change, respectively.

The three undetermined parameters B_1, B_2 and B_3 are determined by the following three conditions:

$$x_{0^-} = x_{0^+}, \tag{D3a}$$

$$B_2 = 0, \tag{D3b}$$

$$B_3 = 0. \tag{D3c}$$

Equation (D3a) indicates that $x(=K/A)$ remains intact at the instant of policy implementation since both K and A are predetermined variables. Equations (D3b) and (D3c) are the stability conditions which ensures that f_t, x_t, and r_t converge to their new steady-state values. By using equations (D2b), (D3a) and (D3b) and (D3c), we obtain:

$$B_1 = [\tilde{x}(z_0) - \tilde{x}(z_1)]/h_{21}, \tag{D4}$$

Substituting (D3b), (D3c) and (D4) into (D2a)-(D2c) yields:

$$f_t = \begin{cases} \tilde{f}(z_0); & t = 0^-, \\ \tilde{f}(z_1) + [\tilde{x}(z_0) - \tilde{x}(z_1)]/h_{21} e^{\delta_1 t}; & t \geq 0^+, \end{cases} \tag{D5a}$$

$$x_t = \begin{cases} \tilde{x}(z_0); & t = 0^-, \\ \tilde{x}(z_1) + [\tilde{x}(z_0) - \tilde{x}(z_1)] e^{\delta_1 t}; & t \geq 0^+, \end{cases} \tag{D5b}$$

$$r_t = \begin{cases} \tilde{r}(z_0); & t = 0^-, \\ \tilde{r}(z_1) + h_{31}[\tilde{x}(z_0) - \tilde{x}(z_1)]/h_{21} e^{\delta_1 t}; & t \geq 0^+. \end{cases} \tag{D5c}$$

From (D5a) the jump between f_{0^+} and f_{0^-} is given by:

$$f_{0^+} - f_{0^-} = \tilde{f}(z_1) - \tilde{f}(z_0) + [\tilde{x}(z_0) - \tilde{x}(z_1)]/h_{21}. \tag{D6}$$

According to $f = C/K$ and $x = K/A$, we can infer that $C_{0^-} = f_{0^-} x_{0^-} A_{0^-}$ and $C_{0^+} = f_{0^+} x_{0^+} A_{0^+}$. Given that both K and A are predetermined variables, $x(=K/A)$ should also be treated as a predetermined variable. This implies that $A_{0^-} = A_{0^+}$ and $x_{0^-} = x_{0^+}$, and hence it is quite clear that the jump between C_{0^+} and C_{0^-} stems from the jump between f_{0^+} and f_{0^-}.

By using $C_{0^-} = f_{0^-} x_{0^-} A_{0^-}$, $C_{0^+} = f_{0^+} x_{0^+} A_{0^+}$, $A_{0^-} = A_{0^+}$, $x_{0^-} = x_{0^+}$, and (D6), we can infer the discrete jump in consumption at the instant of policy implementation as the exogenous variable z that changes from its benchmark value z_0 to a new level z_1:

$$C_{0^+} = C_{0^-} + \left\{\tilde{f}(z_1) - \tilde{f}(z_0) + [\tilde{x}(z_0) - \tilde{x}(z_1)]/h_{21}\right\}x_{0^-}A_{0^-}. \tag{D7}$$

Appendix E

This appendix briefly derives the dynamic system of the model under an elastic labor supply. From (2a) and (2b), we have:

$$\frac{\Omega}{1-L} = \frac{w}{C}. \tag{E1}$$

Inserting (4a) and (4c) into (E1), we can obtain:

$$\frac{\Omega}{1-L} = \frac{(1-\alpha)(L_Y)^{-\alpha}Ax^{\alpha}/K}{C/K}. \tag{E2}$$

By using $f = C/K$ and $x = K/A$, (B1), and (E2), labor supply can be expressed as:

$$L = 1 - \frac{\Omega fx}{1-\alpha}\left(\frac{\eta r}{\alpha}\right)^{\frac{\alpha}{1-\alpha}}. \tag{E3}$$

Similar to the derivations under an inelastic labor supply stated in Appendix B, based on (B1), (B2), (B3), (B4), (B9), (E3), and the market-clearing condition for the labor market $L_Y + L_A = L$ reported in (12a), we can derive the dynamic system under an elastic labor supply as:

$$\dot{f}/f = r - \rho - \frac{\eta r}{\alpha} + f + \frac{\phi\bar{r}(1-\alpha)(1+\theta r)}{x[1-\phi(\bar{r}-r)]}\left(\frac{\alpha}{\eta r}\right)^{\frac{\alpha}{1-\alpha}}\Theta, \tag{E4a}$$

$$\dot{x}/x = \frac{\eta r}{\alpha} - f - \left\{\frac{\phi\bar{r}(1-\alpha)(1+\theta r)}{x[1-\phi(\bar{r}-r)]}\left(\frac{\alpha}{\eta r}\right)^{\frac{\alpha}{1-\alpha}} + \varsigma\right\}\Theta, \tag{E4b}$$

$$\dot{r}/r = (1-\alpha)\frac{r}{\vartheta}\left\{\frac{\psi\varsigma x[1-\phi(\bar{r}-r)]\left(\frac{\eta r}{\alpha}\right)^{\frac{1}{1-\alpha}}}{(1+\theta r)r} - 1\right\}, \tag{E4c}$$

where $\Theta = L - L_Y = 1 - \frac{\Omega fx}{1-\alpha}\left(\frac{\eta r}{\alpha}\right)^{\frac{\alpha}{1-\alpha}} - \left(\frac{\eta r}{\alpha}\right)^{\frac{1}{1-\alpha}}x$ and $\psi = \frac{\alpha}{1-\alpha}\frac{\eta-1}{\eta}$.

The Proof regarding the transitional paths of f_t, x_t, and r_t under an elastic labor supply is similar to that under an inelastic labor supply (see Appendix D), and so to save space we do not repeat it here.

References

Acemoglu, D., & Akcigit, U. (2012). Intellectual property rights policy, competition and innovation. *Journal of the European Economic Association, 10*, 1–42.
Aghion, P., & Banerjee, A. (2005). *Volatility and growth*. New York: Oxford University Press.
Aghion, P., Farhi, E., & Kharroubi, E. (2012). *Monetary policy, liquidity, and growth*. NBER Working Paper No. 18072.
Aghion, P., Howitt, P., & Mayer-Foulkes, D. (2005). The effect of financial development on convergence: Theory and evidence. *Quarterly Journal of Economics, 120*, 173–222.
Amable, B., Chatelain, J. B., & Ralf, K. (2010). Patents as collateral. *Journal of Economic Dynamics and Control, 34*, 1092–1104.
Backus, D., Kehoe, P. J., & Kehoe, T. J. (1992). In search of scale effects in trade and growth. *Journal of Economic Theory, 58*, 337–409.
Brown, J. R., Martinsson, G., & Petersen, B. C. (2012). Do financing constraints matter for R&D? *European Economic Review, 56*, 1512–1529.
Burmeister, E. (1980). On some conceptual issues in rational expectations modeling. *Journal of Money, Credit, and Banking, 12*, 800–816.
Chen, P. H., Chu, A. C., Chu, H., & Lai, C. C. (2017). Short-run and long-run effects of capital taxation on innovation and economic growth. *Journal of Macroeconomics, 53*, 207–221.
Christiano, L. J., Eichenbaum, M., & Evans, C. L. (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy, 113*, 1–45.
Chu, A. C. (2009). Macroeconomic effects of intellectual property rights: A survey. *Academia Economic Papers, 37*, 283–303.
Chu, A. C., & Cozzi, G. (2014). R&D and economic growth in a cash-in-advance economy. *International Economic Review, 55*, 507–524.
Chu, A. C., Cozzi, G., Lai, C. C., & Liao, C. H. (2015). Inflation, R&D and growth in an open economy. *Journal of International Economics, 96*, 360–374.
Chu, A. C., Cozzi, G., Pan, S., & Zhang, M. (2016). *Do stronger patents stimulate or stifle innovation? The crucial role of financial development*. MPRA Paper No. 73630.
Chu, A. C., & Furukawa, Y. (2011). On the optimal mix of patent instruments. *Journal of Economic Dynamics and Control, 35*, 1964–1975.
Eicher, T. S., & Turnovsky, S. J. (2000). Scale, congestion and growth. *Economica, 67*, 325–346.
Futagami, K., & Iwaisako, T. (2007). Dynamic analysis of patent policy in an endogenous growth model. *Journal of Economic Theory, 132*, 306–334.
Goh, A. T., & Olivier, J. (2002). Optimal patent protection in a two-sector economy. *International Economic Review, 43*, 1191–1214.
Gourio, F., & Rudanko, L. (2014). Customer capital. *Review of Economic Studies, 81*, 1102–1136.
Hochberg, Y. V., Serrano, C. J., & Ziedonis, R. Z. (2014). *Patent collateral, investor commitment, and the market for venture lending*. NBER Working Paper No. 20587.
Howitt, P. (1999). Steady endogenous growth with population and R&D inputs growing. *Journal of Political Economy, 107*, 715–730.
Iwaisako, T., & Futagami, K. (2003). Patent policy in an endogenous growth model. *Journal of Economics, 78*, 239–258.
Iwaisako, T., & Futagami, K. (2013). Patent protection, capital accumulation, and economic growth. *Economic Theory, 52*, 631–668.
Jaimovich, N., & Floetotto, M. (2008). Firm dynamics, markup variations, and the business cycle. *Journal of Monetary Economics, 55*, 1238–1252.
Jones, C. I. (1995a). Time series tests of endogenous growth models. *Quarterly Journal of Economics, 110*, 495–525.

Jones, C. I. (1995b). R&D-based models of economic growth. *Journal of Political Economy, 103*, 759–784.
Jones, C. I. (2005). In P. Aghion, & S. Durlauf (Eds.), *Growth and ideas, in handbook of economic growth* (pp. 1063–1112). Amsterdam: Elsevier.
Jones, C. I., & Williams, J. C. (2000). Too much of a good thing? The economics of investment in R&D. *Journal of Economic Growth, 5*, 65–85.
Kunieda, T., & Shibata, A. (2014). Credit market imperfections and macroeconomic instability. *Pacific Economic Review, 19*, 592–611.
Lai, C. C., & Chin, C. T. (2010). (In)determinacy, increasing returns, and the optimality of the Friedman rule in an endogenously growing open economy. *Economic Theory, 44*, 69–100.
Lai, C. C., Chin, C. T., & Chen, K. J. (2017). Oil dependence, international borrowings, and economic instability in a small open economy. *Macroeconomic Dynamics, 21*, 889–917.
Linnemann, L., & Schabert, A. (2003). Fiscal policy in the new neoclassical synthesis. *Journal of Money, Credit, and Banking, 35*, 911–929.
Maebayashi, N., Hori, T., & Futagami, K. (2017). Dynamic analysis of reductions in public debt in an endogenous growth model with public capital. *Macroeconomic Dynamics* (in press).
Mann, W. (2016). *Creditor rights and innovation: Evidence from patent collateral*. Working Paper. Anderson School of Management, UCLA.
Neumeyer, P. A., & Perri, F. (2005). Business cycles in emerging economies: The role of interest rates. *Journal of Monetary Economics, 52*, 345–380.
OECD. (2011). Innovation and growth in knowledge economies. In *OECD science, technology and industry scoreboard 2011*. Paris: OECD Publishing.
Peretto, P. F. (1996). Sunk costs, market structure, and growth. *International Economic Review, 37*, 895–923.
Peretto, P. F. (1998). Technological change, market rivalry, and the evolution of the capitalist engine of growth. *Journal of Economic Growth, 3*, 53–80.
Romer, P. M. (1990). Endogenous technological change. *Journal of Political Economy, 98*, S71–S102.
Schubert, S. F., & Turnovsky, S. J. (2011). The impact of oil prices on an oil-importing developing economy. *Journal of Development Economics, 94*, 18–29.
Scrimgeour, D. (2015). Dynamic scoring in a Romer-style economy. *Southern Economic Journal, 81*, 697–723.
Turnovsky, S. J. (1996). Fiscal policy, growth, and macroeconomic performance in a small open economy. *Journal of International Economics, 40*, 41–66.
Turnovsky, S. J. (1997). Equilibrium growth in a small economy facing an imperfect world capital market. *Review of Development Economics, 1*, 1–22.
Turnovsky, S. J. (2000). *Methods of macroeconomic dynamics* (2nd ed.). Cambridge, MA: MIT Press.
Xin, F., Zhang, J., & Zheng, W. (2017). Does the credit market impede innovation? Based on the banking structure analysis. *International Review of Economics and Finance* (in press).
Zúñiga-Vicente, J.Á., Alonso-Borrego, C., Forcadell, F. J., & Galán, J. I. (2014). Assessing the effect of public subsidies on firm R&D investment: A survey. *Journal of Economic Surveys, 28*, 36–67.