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Using the hybrid Phillips curve with memory to forecast US inflation

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Abstract:
This paper adopts the Caputo fractional derivative to re-specify the hybrid Phillips curve as a dynamic process of inflation with memory. The Caputo fractional derivative contains a non-integer differencing order, providing the same insight for persistence as emphasized in the Autoregressive Fractionally Integrated Moving Average (ARFIMA) time series models. We utilize the hybrid Phillips curve with memory to forecast US inflation during 1967–2014. The results indicate that our model performs well against a traditional hybrid Phillips curve, an integrated moving average model and a naive random walk model in quasi-in-sample forecasts. In out-of-sample forecasts based on Consumer Price Index (CPI) and Personal Consumption Expenditure (PCE) data, we find that the forecasting performance of Phillips curve models depends on the sample period. Our model with CPI data can outperform others in out-of-sample forecasts during and after the most recent financial crisis (2006–2014).

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1 Introduction

Inflation forecasting is a widely explored issue in macroeconomics. Inflation forecasts are important for households to monitor the cost of living, for firms to determine production capacities and pricing strategies, and for policymakers to stabilize prices and the economy. One of the popular inflation forecasting models is the Phillips curve (Atkeson and Ohanian 2001; Stock and Watson 2007; Brissimis and Magginas 2008; Stock and Watson 2008; Riggi and Venditti 2015; Chan, Koop, and Potter 2016). An advantage of adopting the Phillips curve is that it embodies real activity measures, utilizing an economy's information for forecasting. Yet, one problem encountered when using the Phillips curve to forecast inflation is that inflation may not be a stationary process. Empirical evidences present mixed results for inflation stationarity (Culver and Papell 1997; Stock and Watson 2007; Arize and Malindretos 2012). There is no consensus whether inflation is an I(1) process, I(0) process, or neither.

With the development of advanced time series econometrics, the statistical properties of inflation have been reanalyzed. Besides being non-stationary, inflation may also have long memory. An autoregressive fractionally integrated moving average (ARFIMA) model, introduced by Granger and Joyeux (1980), is one of the approaches employed to examine the long memory property of time series. They propose that a long memory process is more consistent with the path of most macroeconomic data and provides better long-run forecasts. The long-memory property has been found in the inflation data of the United States (Baillie, Chung, and Tieslau 1996; Bos, Franses, and Ooms 2002; Bos, Koopman, and Ooms 2014; Hassler and Meller 2014; Balcilar, Gupta, and Jooste 2017) and other countries (Hassler and Wolters 1995; Franses and Ooms 1997; Reisen, Cribari-Neto, and Jensen 2003; Gadea and Mayoral 2006; Noriega, Capistran, and Ramos-Francia 2013; Belkhouja and Mootamri 2016).

In the ARFIMA models, d denotes a non-integer differencing order. According to Robinson (2003), for a series with a zero d, its impulse response to shocks diminishes exponentially and represents short memory. For a series with a d between 0 and 0.5, its impulse response to shocks decays hyperbolically and represents long memory. A series with a d between 0.5 and 1 ensures that its impulse response against transitory shocks converges to zero slowly. A series with a d equal to 1 has a unit root. Its shocks have a permanent effect. A fractionally integrated time series shows lower frequency and extends the impulse responses for longer lags. With this interpretation, d is not just a difference order. It is a memory parameter that determines the degree of persistence, that is, the medium-and long-term impact of shocks on the process.

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The purpose of this paper is to incorporate the non-stationary and memory properties of inflation into a hybrid Phillips curve to forecast inflation. A hybrid Phillips curve is expressed as a linear combination of lagged one-period inflation, expected inflation, and the output gap. Unlike previous studies that estimate the value of d to address the dynamic property of inflation rates, we adopt the Caputo (1967, 1969) fractional derivative to re-specify the Phillips curve as a dynamic process with time-dependent inflation rates and output gaps. The Caputo fractional derivative contains a non-integer order, providing the same insight for persistence as emphasized in the ARFIMA time series models. It incorporates memory effects into linear differential equations that can be solved using the Laplace transform. The Caputo fractional models have been frequently applied in mathematics, biological sciences, physics, and other research (Caputo 2001; 2003; Caputo and Cametti 2008; 2009; Constantinescu and Stoicescu 2011). In economics, Caputo extends his fractional calculus method to the Fisher equation (Caputo and Kolari 2001) and monetary policy effects (Caputo and Di Giorgio 2006). The former quantifies the persistence of stock prices against inflation shocks. The latter simulates the dynamics of output in response to negative monetary shocks.

Hereafter, the term "hybrid Phillips curve with memory" refers to the Phillips curve derived from Caputo's fractional derivative, denoted as HPCF. In the HPCF, expected inflation is calculated using current and previous inflation rates and previous output gaps. Previous inflation rates are weighted in an increasing manner with respect to time. We conduct HPCF forecasts of US inflation over 1967Q1–2014Q4 and then compare the results with a naive random walk model, an integrated moving average model, and a traditional hybrid Phillips curve model. We use two sets of inflation data, one constructed with seasonally adjusted CPI for all items and one constructed with the implicit price deflator for personal consumption expenditures (PCE), and two sets of potential output data, one constructed with the Hodrick–Prescott filter and one estimated by the Congressional Budget Office (CBO). Inflation measured with PCE data has smaller means and standard deviations than those with CPI data. CBO-estimated output gaps have larger means and standard deviations than Hodrick–Prescott filtered ones.

Our results indicate that imbedding the memory property into a hybrid Phillips curve yields better accuracy than a traditional hybrid Phillips curve, an integrated moving average (IMA) model and a naive random walk model for quasi-in-sample forecasts. In out-of-sample forecasts, our model with CPI data can outperform others during 2006Q1–2014Q4, a period in which all models generate higher root mean square errors than those in other sample periods and output gap has the greatest mean and variability among all sample periods.

Our research makes two contributions. First, we address the inflation persistence phenomenon from a mathematical perspective. To the best of our knowledge, this is the first paper to introduce memory into the hybrid Phillips curve via the Caputo fractional derivative. Our inflation forecasting methodology avoids the estimation inaccuracy of the differencing parameter and an issue that multivariate time series may be integrated of non-identical differencing orders. Second, our research adds insight to the existing debate regarding the superiority of forecasting models. Neither a naive random walk nor a Phillips curve model provides overwhelmingly accurate performances in forecasting inflation.

Relevant studies in regard to using the Phillips curve for US inflation forecasting are summarized in Table 1. Atkeson and Ohanian (2001) compare non-accelerating inflation rate of unemployment[1] (NAIRU) Phillips curve-based inflation forecasts with a naive random walk forecast for the 1984–1999 time period. Their findings suggest that none of the NAIRU models provide better predicting accuracy than a naive model. Fisher, Liu, and Zhou (2002) show that Phillips curve models perform better than others for the two-year-ahead forecast horizon. Bos, Franses, and Ooms (2002) include macroeconomic leading indicators, such as unemployment rate, short-term interest rate, and the spread between long term and short term interest rates, in a basic ARFIMA model. They conclude that univariate models provide better forecast performance than others. Stock and Watson (2008) conclude that with the inclusion of other activity variables, the Phillips curve provides good but episodic forecasts for expected inflation. Dotsey, Fujita, and Stark (2015) conclude that the Phillips curve model is generally inferior to other models in forecasting inflation. Chan, Koop, and Potter (2016) develop a time-varying Phillips curve model with bounded trend for inflation and nonaccelerating rate of unemployment. Their results indicate that a bounded bivariate model provides better forecasts than a reduced vector autoregressive model, a bivariate random walk model and others.

Table 1: A summary of related literature.

Related literature	Forecasting method	Sample period	Measures of inflation	Results

Atkeson and Ohanian (2001)	(1) Non-accelerating inflation rate of unemployment (NAIRU) Phillips curves: $\pi_{t+12}^{12} - \pi_t^{12} = \alpha + \beta(L)x_t + \gamma(L)(\pi_t - \pi_{t-1}) + \varepsilon_{t+12}x_t$ represents unemployment rate or other activity index (2) Naive forecast: $\alpha = \beta(L) = \gamma(L) = 0$	1984Q1–1999Q3	PCE and CPI	The naive random walk model outperforms NAIRU Phillips curve models
Fisher, Liu, and Zhou (2002)	(1) Atkeson and Ohanian's naive forecast model (12 months ahead): $\pi_{t+12}^{12} = \pi_t^{12} + \varepsilon_{t+12}^{12}$ (2) Atkeson and Ohanian's naive forecast model (24 months ahead): $\pi_{t+24}^{12} = \pi_t^{12} + \varepsilon_{t+24}^{12}$ (3) Generalized Phillips curve models: $\pi_{t+h}^{12} - \pi_t^{12} = \alpha + \beta(L)a_t + \gamma(L)(\pi_t - \pi_{t-1}) + \varepsilon_{t+h}$, $h = 12, 24$ a_t denotes the value of the Chicago Fed National Activity Index (CFNAI)	1977M1–2000M1	CPI, Core CPI, and PCE	The naive forecast models outperform other models for 1985–2000 Phillips curve models perform better than others when the forecast horizon is 2 years
Bos, Franses, and Ooms (2002)	$(1-L)^d\gamma(L)(\pi_t - \Omega'_t\beta - v'_{t-h}\,\Phi_h) = \Theta(L)\varepsilon_t\Omega_t$ contains the dummy variables representing structural shifts in the mean of inflation, h is the forecast horizon and Φ_h contains the macroeconomic variables. $\gamma(L) = 1 - \varphi_1 L - \ldots - \varphi_p L^p$, $\Theta(L) = 1 + \theta_1 L + \ldots + \theta_q L^q$	1984M1–1999M12	Core CPI	US postwar inflation has long memory with an order of $d = 0.3$. The univariate models outperform other models in 2-year-ahead forecasting
Stock and Watson (2008)	(1) Stock and Watson (2007) unobserved components stochastic volatility (UC-SV) model: $\pi_t = \tau_t + \eta_t$, where $\eta_t = \sigma_{\eta,t}\xi_{\eta,t}$, $\ln\sigma_{\eta,t}^2 = \ln\sigma_{\eta,t-1}^2 + v_{\eta,t}$, $\tau_t = \tau_{t-1} + \varepsilon_t$, where $\varepsilon_t = \sigma_{\varepsilon,t}\xi_{\varepsilon,t}$, $\ln\sigma_{\varepsilon,t}^2 = \ln\sigma_{\varepsilon,t-1}^2 + v_{\varepsilon,t}$ Logarithms of the variances of η_t and ε_t evolve as independent random walks. (2) Atkeson and Ohanian's naive forecast model: $\pi_{t+4}^4 = \pi_t^4 + \varepsilon_{t+4}^4$ (3) Autoregressive model: $\pi_{t+h}^h - \pi_t = \alpha^h + \gamma^h(L)(\pi_t - \pi_{t-1}) + \varepsilon_{t+h}^h$ (4) Gordon (1998) Phillips curve forecast: $\pi_{t+1} = \alpha + \beta(L)u_{t+1} + \gamma^G(L)\pi_t + \mu(L)z_t + \varepsilon_{t+1}u_t$ is the unemployment rate. z_t represents supply shocks (5) Mishkin (1990) multivariate forecast: $\pi_{t+4}^4 - \pi_t = \alpha + \beta^h(L)b_t + \gamma^h(L)(\pi_t - \pi_{t-1}) + \varepsilon_{t+4}^4 b_t$ represents the interest spread between 1-year Treasury bonds and 90-day Treasury bills	1960Q1–2007Q4	CPI, Core-CPI, PCE, Core-PCE, and GDP deflator	The performance of Phillips curve forecasts depends on the sample period

Dotsey, Fujita, and Stark (2015)	(1) Phillips curve:$\pi^h_{t+h} - \pi_t = \alpha^h + \beta^h(L)\,\hat{u}_t + \gamma^h(L)(\pi_t - \pi_{t-1}) + \varepsilon^h_{t+h}, h = 2, 4, 6, 8.\hat{u}_t$ is defined as the difference between the actual unemployment rate and the HP-filtered-trend unemployment rate. (2) Naive forecast:$E_t(\pi^h_{t+h} - \pi^4_{t-1}) = 0, \pi^h_t = (400/h)[\log(p_t) - \log(p_{t-h})]$, p_t is the price index for core personal consumption expenditures (3) Stock and Watson (2007) : First-order integrated moving average (IMA(1,1)) model$\pi_t = \tau_t + \eta_t, \tau_t = \tau_{t-1} + \varepsilon_t\tau_t$ is a stochastic trade of inflation. η_t and ε_t are serially uncorrelated error terms	1969Q1–2014Q2; 1984Q1–2014Q2	PCE	Phillips curve model is generally inferior to the other two models
Chan, Koop, and Potter (2016)	(1) Phillips curve with bounded trend:$\pi_t - \tau^\pi_t = \rho^\pi_t(\pi_{t-1} - \tau^\pi_{t-1}) + \lambda_t(u_t - \tau^u_t) + \varepsilon^\pi_t u_t - \tau^u_t = \rho^\pi_1(u_{t-1} - \tau^u_{t-1}) + \rho^\pi_2(u_{t-2} - \tau^u_{t-2}) + \varepsilon^u_t \varepsilon^\pi_t \sim N(0, e^{h_t}), \varepsilon^u_t \sim N(0, \sigma^2_u), h_t = h_{t-1} + \varepsilon^h_t, \varepsilon^h_t \sim N(0, \sigma^2_h)\tau^\pi_t$ and τ^u_t are the trend inflation and the nonaccelerating inflation rate of unemployment, respectively. (2) Vector autoregressive model of order 2 (3) Bivariate random walk model (4) Univariate unobserved components for inflation and an AR(2) for the unemployment rate. (5) Stella and Stock (2013) : Multivariate unobserved components stochastic volatility (UCSV) model	1975Q1–2013Q1	CPI	The bounded bivariate model forecasts better than other models

π_t is the rate of inflation. h denotes the number for the h-step ahead. L is the lag operator. $\gamma(L)$ and $\beta(L)$ represent the number of lagged values in inflation and other variables. All the studies above conduct out-of-sample forecasts. CPI stands for consumer price index and PCE stands for personal consumption expenditures.

One general conclusion, which can be drawn from existing studies, is that the forecasting performance of Phillips curve models depends on the sample period. Fisher, Liu, and Zhou (2002) argue that an environment with low inflation volatility and a stable monetary policy regime favors the naive random walk forecasts. In our sample, the period of 1996Q1–2005Q4 has the lowest inflation volatility. The root mean square errors[2] (RMSEs) generated by all models during this period are relatively low compared to those in other forecast periods. As Fisher, Liu, and Zhou (2002) indicate, we do find that a naive random walk model outperforms Phillips curve models during this period. Stock and Watson (2008) propose that when the unemployment gap is larger than 1.5 in absolute value, the forecast accuracy of the Phillips curve will improve under the unobserved components stochastic volatility (UC-SV) model. Our HPCF model uses output gap instead of unemployment gap as a measure of economic activity. The output gap has the greatest mean in absolute value and standard deviation during 2006Q1–2014Q4. Our results show that when inflation is measured with CPI data, the HPCF models can outperform other models in out-of-sample forecasts during and after the great recession. In line with previous research (Atkeson and Ohanian 2001; Stock and Watson 2007), a naive random walk model tends to perform well in an environment with smaller inflation, particularly during the Great Moderation (1984–2014).

The remainder of the paper is organized as follows. Section 2 introduces the evolvement and application of the Phillips curve. Section 3 presents a hybrid Phillips curve with memory. Section 4 describes the methodology and simulation results. Section 5 concludes.

2 The Phillips curve: evolvement and application

The traditional Phillips curve can be expressed as

$$\pi_t = \lambda x_t, \tag{1}$$

where π_t is the inflation rate at time t. λ describes the relationship between the inflation rate and the output gap and is expected to be a positive number. x_t is the output gap, defined as the deviation of actual output from potential output. In the formation of (1), the output gap is a substitute for the deviation of the unemployment rate from its natural rate. When the economy is booming, greater aggregate demand drives up the price level and creates more job opportunities. Low unemployment is associated with positive output gaps. On the other hand, slack demand depresses the price level and leads to high unemployment and negative output gaps. This simple equation presents a dilemma for policymakers when attempting to achieve low inflation and low unemployment.

After taking expected inflation into account, an expectations-augmented Phillips curve is written as[3]

$$\pi_t = \lambda x_t + \beta E_t \pi_{t+1}. \tag{2}$$

Here β is the discount factor and $E_t\pi_{t+1}$ is the expected inflation rate at $t+1$ given the information at time t. Equation (2) implies that inflation follows a forward-looking process. As we solve equation (2) forward, inflation becomes a function of the present discounted value of a sequence of current and future output gaps.[4]

In the 1990s, the new Keynesian framework considered the assumptions of monopolistic competition and nominal rigidities to modify the Phillips curve as a new Keynesian Phillips curve.

$$\pi_t = \frac{(1-\varpi)(1-\beta\varpi)}{\varpi} mc_t + \beta E_t \pi_{t+1}, \tag{3}$$

$$mc_t = kx_t, \tag{4}$$

where mc_t is the real marginal cost proportional to the output gap. With Calvo (1983) staggered price setting, which assumes that for each period a fraction $1-\varpi$ of all firms can adjust their prices flexibly, ϖ becomes a measure of the degree of nominal rigidity. The format of the new Keynesian Phillips curve is similar to the old one when $\lambda = k(1-\varpi)(1-\beta\varpi)/\varpi$. Nevertheless, in the new Keynesian Phillips curve, real marginal cost becomes the driving source of the inflation process. To reconcile the empirical relationship between inflation and output gap,[5] a hybrid Phillips curve appears:

$$\pi_t = \phi\pi_{t-1} + (1-\phi)E_t\pi_{t+1} + \delta x_t, \tag{5}$$

with $0 < \phi < 1$. Current inflation is a linear combination of previous inflation, expected inflation, and the output gap. Equation (5) also indicates that a temporary shock to output gaps not only affects the inflation for one period but over the duration of the shock.

Besides being used to forecast inflation, the Phillips curve also provides two other applications. First, it can be used to measure the inflation persistence and its correlation with excess demand. The coefficients of expected inflation, lagged inflation and output gap are estimated to examine their relative importance on the determination of current inflation (Fuhrer and Moore 1995; Gali and Gertler 1999; Rudda and Whelan 2005). Second, with inflation on the vertical axis and output gap on the horizontal axis, the slope of a Phillips curve evaluates the size of real effects of nominal shocks (Hutchison and Walsh 1998; Daniels and VanHoose 2006).

3 A hybrid Phillips curve with memory

This section consists of two parts. First, we briefly lay out an ARFIMA framework and summarize its applications to inflation persistence and inflation forecasting. Second, we introduce the hybrid Phillips curve with memory constructed via Caputo's fractional derivative.

When applying a multivariate ARFIMA model to forecast inflation, Bos, Franses, and Ooms (2002) first estimate the differencing order of all the variables and implement recursive out-of-sample forecasts. It is noted that inflation rates along with all other macroeconomic variables are assumed to have the same differencing order in their application. Our methodology begins by explicitly specifying a hybrid Phillips curve that emphasizes the memory property of inflation, and then predictions are generated. We draw the insight from an ARFIMA model in terms of a fractional differencing order and allow the differencing order to vary between 0.01 and 0.99. This avoids the issues that the differencing parameter may not be precisely estimated and multivariate time series may be integrated of non-identical differencing orders.

3.1 ARFIMA model

Suppose y_t is a series such that after being differenced d times, the resulting series is white noise with finite variance. Then the series y_t is said to be integrated of order d. When d is between 0 and 1, y_t is an ARFIMA (p, d, q) series expressed as

$$(1-L)^d\gamma(L)y_t = \Theta(L)\varepsilon_t, \tag{6}$$

where $\gamma(L) = 1 - \varphi_1 L - \ldots - \varphi_p L^p$ and $\Theta(L) = 1 + \theta_1 L + \ldots + \theta_q L^q$ are the autoregressive and moving average polynomials, respectively. ε_t is the error term and is assumed to be independent and identically distributed with a zero mean and a variance of σ_t^2.

There is extensive research discussing inflation persistence in the fractionally integrated models. Hassler and Wolters (1995) test on monthly data of the United States, the United Kingdom, Germany, France and Italy and find that the inflation rates in these countries exhibit various degrees of dependence. Gadea and Mayoral (2006) argue that compared to autoregressive integrated moving average (ARIMA) models, ARFIMA is a parsimonious and flexible method to estimate the persistence property of inflation and conclude that the values of d for OECD countries are between 0.6 and 0.8.

Bos, Franses, and Ooms (2002) include macroeconomic leading indicators, such as unemployment rate, short-term interest rate, and the spread between long term and short term interest rates, in a basic ARFIMA model. Their modified model is defined as

$$(1-L)^d\gamma(L)(\pi_t - \Omega'_t\beta - v'_{t-h}\Phi_h) = \Theta(L)\varepsilon_t, \tag{7}$$

where π_t is a vector of monthly inflation rates, Ω_t contains the dummy variables representing structural shifts in the mean of inflation, h is the forecast horizon and Φ_h contains the macroeconomic variables. Their estimation results show that d is about 0.25 and is significantly different from 0 and 0.5. For the period 1960–1999, US inflation was a long memory process. The inclusion of real activity variables somewhat improves the forecasting accuracy for short-run horizons. However, the univariate models substantially outperform other models for two-year-ahead forecasting.

3.2 Our model

We use the Caputo fractional derivative to build a hybrid Phillips curve with memory. Caputo (1967, 1969) defines a fractional operator which provides initial and boundary conditions to address the memory property of a time-dependent variable. Its initial condition has a derivative of integer order whose physical interpretation can be properly addressed. Due to its tractability, the Caputo fractional derivative has been widely applied in a variety of fields. The Caputo fractional derivative of fractional order u is defined as

$$\frac{\partial^u f(t)}{\partial t^u} = \frac{1}{\Gamma(1-u)}\int_0^t \left(\frac{1}{(t-\tau)^u}\frac{df(\tau)}{d\tau}\right)d\tau, \tag{8}$$

where u is subject to the limits $0 < u < 1$. Γ is the gamma function. The weight of the first order derivative $\left(\frac{df(\tau)}{d\tau}\right)$ in the time interval $[0, t]$ increases as τ approaches t. The memory effect occurs since the dynamics of $f(t)$ involve its past values. We express equation (5) in Caputo's memory function form as

$$\begin{aligned} &\frac{d}{dt}E_t\pi_{t+1} = a\pi_t + bD^{(u)}\pi_t - \delta x_t, \\ &where\ D^{(u)}f(t) = \frac{\partial^u f(t)}{\partial t^u}. \end{aligned} \tag{9}$$

Here u is the order of the fractional derivative, subject to $0 < u < 1$. Equation (9) depicts the expected inflation dynamics. People formulate their expected change in inflation based on current inflation, the current output gap and previous inflation. Different from a traditional hybrid Phillips curve, the value of expected inflation contains a weighted average of previous inflation rates. The weights decrease as the lag number extends. We aim to forecast the level of inflation instead of the change of inflation. Consequently, by using Laplace transforms, the expected inflation is obtained in the following proposition.

Proposition 3.1 *A solution to*

$$\frac{d}{dt}E_t\pi_{t+1} = a\pi_t + bD^{(u)}\pi_t - \delta x_t,$$
$$for\ 0 < u < 1\ is \tag{10}$$
$$E_t\pi_{t+1} = E_0\pi_1 + a\int_0^t \pi(\tau)d\tau + \frac{b}{\Gamma(1-u)}\int_0^t (t-\tau)^{-u}\pi(\tau)d\tau - \frac{b\pi_0 t^{1-u}}{\Gamma(2-u)} - \delta\int_0^t x(\tau)d\tau.$$

Proof. See the Appendix for the complete derivation.

4 Methodology and simulation results

In this section, we apply the hybrid Phillips curve derived from Caputo's fractional derivative (HPCF) to forecast US inflation. Assuming that inflation and output gap are constant in the interval $(k-1, k]$ for all positive integers k, equation (10) with n lags can be written as equation (11).

$$\begin{aligned} E_t\pi_{t+1} &= \pi_{t-(n-1)} + a\sum\nolimits_{k=t-(n-1)}^{t}\pi_k \\ &+ \frac{b}{\Gamma(2-u)}\sum\nolimits_{k=t-(n-1)}^{t}((t-k+1)^{1-u} - (t-k)^{1-u})\pi_k \\ &- \frac{b\pi_{t-(n-1)}n^{1-u}}{\Gamma(2-u)} - \delta\sum\nolimits_{k=t-(n-1)}^{t} x_k,\ \text{for } 0 < u < 1. \end{aligned} \tag{11}$$

Equation (11) expresses the expected inflation for period $t+1$ as a linear equation involving the inflation rate during period $t-(n-1)$, a sum of inflation rates between periods $t-(n-1)$ and t with constant weights, a weighted sum of inflation rates between periods $t-(n-1)$ and t with increasing weights, a deduction factor in relation to the inflation rate during period $t-(n-1)$, and a sum of output gaps between periods $t-(n-1)$ and t with constant weights.

In our application of the HPCF, we only use one-quarter forecast horizons since longer horizons require predictions of the output gap. Our data set consists of US CPI for all items, implicit price deflator for PCE, and GDP values from 1957Q1 to 2014Q4. The seasonally-adjusted values of CPI, PCE, and GDP are obtained from the Federal Reserve Economic Data (FRED) database, which is maintained by the Federal Reserve Bank of St. Louis. A year-over-year quarterly inflation rate π_k is calculated using the average of three year-over-year monthly CPI changes corresponding to the quarter. Two measures of output gap x_k, the deviation of GDP from its Hodrick-Prescott-filtered trend as well as the output gap estimated by the Congressional Budget Office, are used in the simulation. The CBO estimates potential output based on determinants of labor supply and labor productivity. It is constantly updated and provides an economic rationale instead of filtering values by statistical techniques.

Our implementation of the HPCF model involves updating the parameters a, b, δ, u every quarter. Two other parameters that need to be chosen, either a priori or by data analysis, are the number of lags (n) and the number of equations (m) used in ordinary least squares estimation of a, b, and δ. One necessary condition of the general implementation is that $n+m$ cannot exceed the number of data set quarters preceding the beginning quarter of the forecast period. For example, if 1976Q1 is the beginning of the forecast period, $n+m$ must be no more than 76.[6] The general implementation of the HPCF proceeds as follows.

First, a forecast period is chosen along with a fixed number of lags n and the number of equations m used in regressions. Second, for each quarter t in the forecast period, we calculate $E_{t-1}\pi_t$ in the following manner:

a. A system of m equations similar to equation (11) is set up as

$$\begin{aligned} \pi_{t-j} &= \pi_{t-(j+n)} + a\sum\nolimits_{k=t-(j+n)}^{t-(j+1)}\pi_k \\ &+ \frac{b}{\Gamma(2-u)}\sum\nolimits_{k=t-(j+n)}^{t-(j+1)}((t-k+1)^{1-u} - (t-k)^{1-u})\pi_k \\ &- \frac{b\pi_{t-(j+n)}n^{1-u}}{\Gamma(2-u)} - \delta\sum\nolimits_{k=t-(j+n)}^{t-(j+1)} x_k, j = 1, \ldots, m. \end{aligned}$$

b. For each u from 0.01 to 0.99 in one one-hundredth increments, the method of least squares is performed on the above m equations to estimate a, b, and δ.

c. Of the ninety-nine quadruples of the form (a, b, δ, u) obtained in part b), let $(\hat{a}, \hat{b}, \hat{\delta}, \hat{u})$ denote the one that yields the least root mean square error with regard to fitting the inflation values $\pi_{t-1}, \pi_{t-2}, \ldots, \pi_{t-m}$.

d. An ex-post forecast of $E_{t-1}\pi_t$ is performed.

$$\begin{aligned} E_{t-1}\pi_t = \pi_{t-n} + \hat{a} \sum_{k=t-n}^{t-1} \pi_k \\ + \frac{\hat{b}}{\Gamma(2-\hat{u})} \sum_{k=t-n}^{t-1} ((t-k)^{1-\hat{u}} - (t-1-k)^{1-\hat{u}})\pi_k \\ + \frac{\hat{b}\pi_{t-n} n^{1-\hat{u}}}{\Gamma(2-\hat{u})} - \hat{\delta} \sum\nolimits_{k=t-n}^{(t-1)} x_k. \end{aligned}$$

We apply the HPCF to quasi-in-sample and out-of-sample forecasts. Quasi-in-sample forecasting invokes the general implementation over many combinations of m and n for the forecast period, and chooses the series of predictions which has the least RMSE. We consider values of n between[7] 1 and 12 and all possible corresponding values of m. Since the calculations of RMSE involve the forecast period's actual inflation values, we define such a forecast as quasi-in-sample in terms of m and n.

Out-of-sample forecasting splits the data set into an in-sample period (estimation period) and an out-of-sample period (evaluation period). The estimation period conveys information used to fit the model while the evaluation period is used to compare the forecast accuracy. In our implementation, an out-of-sample HPCF forecast calculates the predicted inflation values for the evaluation period, using values of m and n determined from a quasi-in-sample forecast of an estimation period prior to the evaluation period.

We compare our results with the forecasts from three models: a traditional hybrid Phillips curve (HPC), defined as equation (5), an integrated moving average (IMA(1,1)) model from Stock and Watson (2007) ,[8] and a naive random walk model. In the estimation of the HPC, n always equals 2, and ϕ takes the place of u in part b). A quasi-in-sample HPC forecast with a fixed n is implemented over all possible values of m. A naive random walk model simply assumes that for any period t, $E_t\pi_{t+1} = \pi_t$. It can be viewed as an ARIMA(0,1,0) model and is usually considered as a benchmark for inflation prediction (Atkeson and Ohanian 2001; Chan, Koop, and Potter 2016; Dotsey, Fujita, and Stark 2015; Elliot and Timmermann 2008; Fisher, Liu, and Zhou 2002; Stella and Stock 2013).

Table 2 and Table 3 present the results with CPI data, and Table 4 and Table 5 present the results with PCE data. The numbers in bold signify a lower RMSE than the naive random walk model, the IMA(1,1) model and the corresponding traditional Phillips curve model for the same forecast period. The tables indicate that PCE data produces inflation rates that generally have lower means and standard deviations than inflation rates derived from CPI data. The HP-filtered output gap data generally have lower means and standard deviations than CBO-estimated data. Quasi-in-sample forecasts are conducted over six forecasts periods: 1967Q1–2014Q4, 1967Q1–1975Q4, 1976Q1–1985Q4, 1986Q1–1995Q4, 1996Q1–2005Q4 and 2006Q1–2014Q4. Compared to the alternative models, the quasi-in-sample Phillips curve forecasts, particularly HPCFs, are more accurate in each forecast period. Out-of-sample forecasts are conducted over four forecast periods: 1976Q1–1985Q4, 1986Q1–1995Q4, 1996Q1–2005Q4 and 2006Q1–2014Q4. Our results indicate that the HPCF model with CPI data and HP-filtered output gaps outperforms others in 2006Q1–2014Q4 out-of-sample forecasts. A naive random walk model with PCE data forecasts better than others in 2006Q1–2014Q4 out-of-sample forecasts. During 2006Q1–2014Q4, output gaps have the greatest mean and standard deviation among all sample periods. The RMSEs of all models for 2006Q1–2014Q4 are also greater than those in other sample periods. The IMA(1,1) model is mildly superior to Phillips curve models based on CPI and PCE data for 1976Q1–1985Q4. The naive random walk model outperforms Phillips curve models based on CPI and PCE data for 1996Q1–2005Q4.

Table 2: Forecasting results (CPI & HP-filtered output gap).

Forecast period	Actual inflation		Output gap (HP-filtered)		Naive model	Rolling window IMA(1,1) model	Quasi in-sample forecasts		Out-of-sample forecasts			
	Mean	Standard deviation	Mean	Standard deviation			HPCF	HPC	HPCF	HPC	HPCF	HPC

												Root mean square errors (RMSEs)
Whole sample period 1967Q1–2014Q4	4.26	2.88	−0.67	128.99	0.768	0.725	0.743 (37,2)	0.740 (38,2)	–	–	–	–
Sub-sample period (1) 1967Q1–1975Q4	5.80	2.71	−0.14	14.47	0.755	0.601	0.557 (34,2)	0.500 (14,2)	–	–	–	–
Sub-sample period (2) 1976Q1–1985Q4	7.22	3.40	2.78	50.31	0.888	0.710	0.777 (27,8)	0.813 (73,2)	0.859 (34,2)	0.895 (14,2)	0.809	0.805
Sub-sample period (3) 1986Q1–1995Q4	3.55	1.16	−4.22	52.20	0.566	0.584	**0.548** (43,2)	0.564 (25,2)	0.750 (27,8)	0.587 (73,2)	0.588	0.577
Sub-sample period (4) 1996Q1–2005Q4	2.51	0.73	−28.96	128.20	0.445	0.472	**0.439** (91,6)	0.445 (84,2)	0.477 (43,2)	0.493 (25,2)	0.481	0.48
Sub-sample period (5) 2006Q1–2014Q4	2.17	1.42	30.35	253.95	1.062	1.120	**1.030** (100,5)	1.059 (112,2)	**1.053** (91,6)	1.105 (84,2)	1.121	1.093
A-O (2001); Bos, Franses, and Ooms (2002) 1984Q1–1999Q4	3.28	1.15	3.18	49.97	0.507	0.504	**0.495** (43,2)	0.507 (38,2)	0.543 (64,4)	0.521 (60,2)	0.538	0.515
Fisher, Liu, and Zhou (2002) 1977Q1–2000Q4	4.76	3.10	7.66	61.37	0.680	0.607	0.644 (68,4)	0.658 (78,2)	0.686 (34,2)	0.668 (24,2)	0.651	0.650
Stock and Watson (2008) 1976Q1–2007Q4	4.34	2.85	10.82	117.67	0.671	0.614	0.642 (69,4)	0.651 (69,2)	0.683 (34,2)	0.711 (14,2)	0.655	0.653
Dotsey, Fujita, and Stark (2015) 1979Q1–2014Q4	3.68	2.80	0.39	148.65	0.781	0.771	**0.758** (71,5)	0.779 (71,2)	0.789 (46,2)	0.795 (43,2)	0.776	0.781
Dotsey, Fujita, and Stark (2015) 1984Q1–2014Q4	2.84	1.26	1.00	158.46	0.717	0.747	0.721 (105,2)	0.729 (100,2)	0.774 (64,4)	0.750 (60,2)	0.769	0.742
Chan, Koop, and Potter (2016) 1976Q1–2012Q4	4.04	2.83	−3.03	144.92	0.787	0.766	0.767 (71,5)	0.781 (71,2)	0.807 (34,2)	0.866 (14,2)	0.777	0.781

The numbers in bold signify a lower RMSE than the naive random walk model, the IMA model and the corresponding hybrid Phillips curve model for the same forecast period. The ordered pairs represent (m, n), where m is the number of equations per OLS calculation and n is the number of lags. HPCF represents a hybrid Phillips curve with memory. HPC represents a traditional hybrid Phillips curve. HPCFM represents a hybrid Phillips curve with memory including maximum m. HPCM represents a traditional hybrid Phillips curve including maximum m. An output gap is defined as the deviation of GDP from its Hodrick-Prescott-filtered trend. The forecast periods in Stock and Watson (2008), in Dotsey, Fujita, and Stark (2015) and in Chan, Koop, and Potter (2016) are revised to 1976Q1–2007Q4, 1979Q1–2014Q4 and 1976Q1–2012Q4, respectively. A-O (2001) stands for Atkeson and Ohanian (2001) .

Table 3: Forecasting results (CPI & CBO estimated output gap).

Forecast period	Actual inflation		Output gap (HP-filtered)		Naive model	Rolling window IMA(1,1) model	Quasi in-sample forecasts		Out-of-sample forecasts			
	Mean	Standard deviation	Mean	Standard deviation			HPCF	HPC	HPCF	HPC	HPCFM	HPCM
												Root mean square errors (RMSEs)

Whole sample period 1967Q1–2014Q4	4.26	2.88	−129.13	214.22	0.768	0.725	0.739 (37,2)	0.732 (38,2)	–	–	–	–
Sub-sample period (1) 1967Q1–1975Q4	5.80	2.71	−0.75	36.20	0.755	0.601	**0.450** (16,1)	0.466 (14,2)	–	–	–	–
Sub-sample period (2) 1976Q1–1985Q4	7.22	3.40	−70.27	84.34	0.888	0.710	0.788 (70,2)	0.771 (70,2)	0.899 (16,1)	0.897 (14,2)	0.798	0.762
Sub-sample period (3) 1986Q1–1995Q4	3.55	1.16	−103.52	72.85	0.566	0.584	0.550 (43,2)	0.548 (42,2)	0.569 (70,2)	0.570 (70,2)	0.565	0.565
Sub-sample period (4) 1996Q1–2005Q4	2.51	0.73	−66.14	146.03	0.445	0.472	0.439 (84,5)	0.439 (76,2)	0.455 (43,2)	0.451 (42,2)	0.470	0.468
Sub-sample period (5) 2006Q1–2014Q4	2.17	1.42	−421.38	312.20	1.062	1.120	**1.043** (105,5)	1.070 (109,2)	1.065 (84,5)	1.116 (76,2)	1.144	1.099
A-O (2001); Bos, Franses, and Ooms (2002) 1984Q1–1999Q4	3.28	1.15	−67.06	86.14	0.507	0.504	0.509 (43,2)	0.502 (42,2)	0.561 (25,1)	0.518 (27,2)	0.536	0.511
Fisher, Liu, and Zhou (2002) 1977Q1–2000Q4	4.76	3.10	−61.73	96.71	0.680	0.607	0.638 (68,2)	0.630 (68,2)	0.749 (16,1)	0.703 (14,2)	0.645	0.622
Stock and Watson (2008) 1976Q1–2007Q4	4.34	2.85	−72.81	107.06	0.671	0.614	0.636 (68,2)	0.627 (68,2)	0.733 (16,1)	0.700 (14,2)	0.642	0.629
Dotsey, Fujita, and Stark (2015) 1979Q1–2014Q4	3.68	2.80	−170.68	232.17	0.781	0.771	**0.765** (71,2)	0.769 (71,2)	0.926 (16,1)	0.955 (11,2)	0.790	0.767
Dotsey, Fujita, and Stark (2015) 1984Q1–2014Q4	2.84	1.26	−181.15	245.46	0.717	0.747	0.723 (105,2)	0.728 (106,2)	0.785 (25,1)	0.805 (27,2)	0.764	0.740
Chan, Koop, and Potter (2016) 1976Q1–2012Q4	4.04	2.83	−142.61	220.95	0.787	0.766	0.766 (71,2)	0.768 (71,2)	0.926 (16,1)	0.926 (14,2)	0.790	0.766

The numbers in bold signify a lower RMSE than the naive random walk model, the IMA model and the corresponding hybrid Phillips curve model for the same forecast period. The ordered pairs represent (m, n), where m is the number of equations per OLS calculation and n is the number of lags. HPCF represents a hybrid Phillips curve with memory. HPC represents a traditional hybrid Phillips curve. HPCFM represents a hybrid Phillips curve with memory including maximum m. HPCM represents a traditional hybrid Phillips curve including maximum m. An output gap is defined as the deviation of GDP from its Hodrick-Prescott-filtered trend. The forecast periods in Stock and Watson (2008), in Dotsey, Fujita, and Stark (2015) and in Chan, Koop, and Potter (2016) are revised to 1976Q1–2007Q4, 1979Q1–2014Q4 and 1976Q1–2012Q4, respectively. A-O (2001) stands for Atkeson and Ohanian (2001) .

Table 4: Forecasting results (PCE & HP-filtered output gap).

Forecast period	Actual inflation		Output gap (HP-filtered)		Naive model	Rolling window IMA(1,1) model	Quasi In-sample forecasts		Out-of-sample forecasts			
	Mean	Standard deviation	Mean	Standard deviation			HPCF	HPC	HPCF	HPC	HPCFM	HPCM
					Root mean square errors (RMSEs)							
Whole sample period 1967Q1–2014Q4	3.72	2.49	−0.67	128.99	0.568	0.530	0.524 (37,2)	0.523 (38,2)	–	–	–	–

Sub-sample period (1) 1967Q1–1975Q4	5.26	2.52	−0.14	14.47	0.723	0.584	0.564 (11,1)	0.538 (23,2)	–	–	–	–
Sub-sample period (2) 1976Q1–1985Q4	6.46	2.36	2.78	50.31	0.539	0.484	**0.433** (60,2)	0.440 (60,2)	0.775 (11,1)	0.484 (23,2)	0.630	0.461
Sub-sample period (3) 1986Q1–1995Q4	3.04	0.95	−4.22	52.20	0.429	0.416	**0.413** (90,9)	0.420 (31,2)	0.442 (60,2)	0.448 (60,2)	0.440	0.442
Sub-sample period (4) 1996Q1–2005Q4	1.91	0.66	−28.96	128.20	0.329	0.335	**0.310** (83,3)	0.317 (84,2)	0.357 (90,9)	0.358 (31,2)	0.345	0.341
Sub-sample period (5) 2006Q1–2014Q4	1.89	1.05	30.35	253.95	0.743	0.762	**0.705** (114,2)	0.719 (114,2)	0.755 (83,3)	0.751 (84,2)	0.775	0.746
A-O (2001); Bos, Franses, and Ooms (2002) 1984Q1–1999Q4	2.74	1.09	3.18	49.97	0.371	0.365	**0.363** (43,1)	0.374 (31,2)	0.379 (48,2)	0.389 (55,2)	0.382	0.383
Fisher, Liu, and Zhou (2002) 1977Q1–2000Q4	4.09	2.58	7.66	61.37	0.450	0.426	**0.409** (43,2)	0.416 (43,2)	0.421 (35,2)	0.418 (30,2)	**0.417**	0.423
Stock and Watson (2008) 1976Q1–2007Q4	3.73	2.40	10.82	117.67	0.452	0.432	**0.424** (69,2)	0.426 (69,2)	0.593 (11,1)	0.443 (23,2)	0.495	0.439
Dotsey, Fujita, and Stark (2015) 1979Q1–2014Q4	3.11	2.27	0.39	148.65	0.529	0.530	**0.500** (71,2)	0.508 (71,2)	**0.513** (43,2)	0.520 (43,2)	**0.511**	0.518
Dotsey, Fujita, and Stark (2015) 1984Q1–2014Q4	2.38	1.07	1.00	158.46	0.508	0.516	**0.505** (91,6)	0.511 (99,2)	0.518 (48,2)	0.523 (55,2)	0.516	0.519
Chan, Koop, and Potter (2016) 1976Q1–2012Q4	3.47	2.37	−3.03	144.92	0.537	0.528	**0.505** (71,2)	0.513 (71,2)	0.688 (11,1)	0.553 (23,2)	0.575	0.522

The numbers in bold signify a lower RMSE than the naive random walk model, the IMA model and the corresponding hybrid Phillips curve model for the same forecast period. The ordered pairs represent (m, n), where m is the number of equations per OLS calculation and n is the number of lags. HPCF represents a hybrid Phillips curve with memory. HPC represents a traditional hybrid Phillips curve. HPCFM represents a hybrid Phillips curve with memory including maximum m. HPCM represents a traditional hybrid Phillips curve including maximum m. An output gap is defined as the deviation of GDP from its Hodrick-Prescott-filtered trend. The forecast periods in Stock and Watson (2008), in Dotsey, Fujita, and Stark (2015) and in Chan, Koop, and Potter (2016) are revised to 1976Q1–2007Q4, 1979Q1–2014Q4 and 1976Q1–2012Q4, respectively. A-O (2001) stands for Atkeson and Ohanian (2001) .

Table 5: Forecasting results (PCE & CBO estimated output gap).

Forecast period	Actual inflation		Output gap (HP-filtered)		Naive model	Rolling window IMA(1,1) model	Quasi in-sample forecasts		Out-of-sample forecasts			
	Mean	Standard deviation	Mean	Standard deviation			HPCF	HPC	HPCF	HPC	HPCFM	HPCM
					Root mean square errors (RMSEs)							
Whole sample period 1967Q1–2014Q4	3.72	2.49	−129.13	214.22	0.568	0.530	0.528 (38,2)	0.524 (30,2)	–	–	–	–
Sub-sample period (1) 1967Q1–1975Q4	5.26	2.52	−0.75	36.20	0.723	0.584	**0.473** (15,1)	0.503 (21,2)	–	–	–	–

Sub-sample period (2) 1976Q1–1985Q4	6.46	2.36	−70.27	84.34	0.539	0.484	0.439 (71,2)	0.439 (68,2)	0.596 (15,1)	0.473 (21,2)	0.520	0.443
Sub-sample period (3) 1986Q1–1995Q4	3.04	0.95	−103.52	72.85	0.429	0.416	**0.414** (43,9)	0.418 (32,2)	0.437 (71,2)	0.438 (68,2)	0.431	0.430
Sub-sample period (4) 1996Q1–2005Q4	1.91	0.66	−66.14	146.03	0.329	0.335	**0.314** (83,3)	0.319 (84,2)	0.413 (43,9)	0.357 (32,2)	0.359	0.338
Sub-sample period (5) 2006Q1–2014Q4	1.89	1.05	−421.38	312.20	0.743	0.762	**0.713** (114,2)	0.740 (135,2)	0.769 (83,3)	0.766 (84,2)	0.788	0.755
A-O (2001); Bos, Franses, and Ooms (2002) 1984Q1–1999Q4	2.74	1.09	−67.06	86.14	0.371	0.365	0.375 (45,1)	0.373 (32,2)	0.385 (66,2)	0.381 (65,2)	0.378	0.376
Fisher, Liu, and Zhou (2002) 1977Q1–2000Q4	4.09	2.58	−61.73	96.71	0.450	0.426	0.412 (65,2)	0.410 (65,2)	0.533 (15,1)	0.418 (30,2)	0.451	0.410
Stock and Watson (2008) 1976Q1–2007Q4	3.73	2.40	−72.81	107.06	0.452	0.432	0.421 (69,2)	0.419 (67,2)	0.528 (15,1)	0.442 (21,2)	0.453	0.426
Dotsey, Fujita, and Stark (2015) 1979Q1–2014Q4	3.11	2.27	−170.68	232.17	0.529	0.530	**0.502** (71,2)	0.509 (71,2)	0.643 (15,1)	0.529 (43,2)	0.550	0.514
Dotsey, Fujita, and Stark (2015) 1984Q1–2014Q4	2.38	1.07	−181.15	245.46	0.508	0.516	0.508 (71,2)	0.515 (71,2)	0.519 (66,2)	0.525 (65,2)	0.520	0.520
Chan, Koop, and Potter (2016) 1976Q1–2012Q4	3.47	2.37	−142.61	220.95	0.537	0.528	**0.507** (71,2)	0.512 (71,2)	0.655 (15,1)	0.578 (21,2)	0.557	0.518

The numbers in bold signify a lower RMSE than the naive random walk model, the IMA model and the corresponding hybrid Phillips curve model for the same forecast period. The ordered pairs represent (m, n), where m is the number of equations per OLS calculation and n is the number of lags. HPCF represents a hybrid Phillips curve with memory. HPC represents a traditional hybrid Phillips curve. HPCFM represents a hybrid Phillips curve with memory including maximum m. HPCM represents a traditional hybrid Phillips curve including maximum m. An output gap is defined as the deviation of GDP from its Hodrick-Prescott-filtered trend. The forecast periods in Stock and Watson (2008), in Dotsey, Fujita, and Stark (2015) and in Chan, Koop, and Potter (2016) are revised to 1976Q1–2007Q4, 1979Q1–2014Q4 and 1976Q1–2012Q4, respectively. A-O (2001) stands for Atkeson and Ohanian (2001) .

To compare hybrid Phillips curve models with previous literature, we also conduct out-of-sample forecasts over various sample periods: 1984Q1–1999Q4, 1977Q1–2000Q4, 1976Q1–2007Q4, 1979Q1–2014Q4, 1984Q1–2014Q4 and 1976Q1–2012Q4. Since our methodology is different from previous research, here we focus on the discussion of relative performance between Phillips curve models and their competing models. Our results are consistent with Atkeson and Ohanian (2001) and Bos, Franses, and Ooms (2002) that the 1984Q1–1999Q4 period is in favor of the naive random walk model and the IMA(1,1) model for out-of-sample forecasts. In line with Dotsey, Fujita, and Stark (2015), the naive random walk model outperforms other models during 1984Q1–2014Q4. Fisher, Liu, and Zhou (2002) conclude that Phillips curve models perform better than others for the two-year-ahead forecast horizon during 1977Q1–2000Q4. Within the same period, we find that the traditional Phillips curve model with PCE data generates lower RMSEs than other competing models. The IMA(1,1) model with CPI or PCE data has better performance than others in the periods of 1976Q1–2007Q4 and 1979Q1–2014Q4.

We also present results of augmented-HPCF and augmented-HPC out-of-sample forecasts, denoted as HPCFM and HPCM, respectively. They are estimated with the same n chosen from an estimation period and with m set to the largest possible value as of the beginning of the forecasted quarter's year, e.g. 1976Q1–1976Q4 all use the same value of m. As shown in Table 2 through Table 5, HPCFMs with CPI and PCE data usually yield lower RMSEs than the corresponding HPCFs in out-of-sample forecasts. Increasing the value of m generally improves the forecast performance of Phillips curve models. However, in most periods the augmented-Phillips curve models are outperformed by the naive random walk model and/or the IMA(1,1) model.

5 Conclusions

Predicting inflation with the Phillips curve has received long-lasting attention and mixed results in macroeconomics. Modifications of the Phillips curve, such as using various econometric techniques, different sample periods and different regressors, are made to improve its forecast accuracy. This paper embodies the memory property of inflation in a Phillips curve by using the Caputo fractional derivative to forecast inflation. This specification reconciles the empirical evidence that inflation has memory, the so called inflation persistence. After implementing our methodology, our results show that the hybrid Phillips curve with memory's quasi-in-sample forecasts generally provide better accuracy than the competing models. Similar with other studies, the performance of Phillips curve models for out-of-sample forecasts varies with the sample periods. None of these models provide overwhelmingly accurate performances in forecasting inflation. Our model with CPI data can outperform others in out-of-sample forecasts during and after the most recent financial crisis (2006–2014).

This paper provides an alternate method of formulating expected inflation. Our modified Phillips curve constructs time-dependent inflation rates via Caputo's fractional derivative and output gaps. A limitation imposed on our estimation is that inflation is predicted on a quarterly basis since only quarterly data are available for GDP. Another limitation is that our forecast horizon is one quarter since we do not predict output gap. Besides forecasting inflation, a hybrid Phillips curve with memory can serve as an inflation adjustment equation in the analysis of optimizing monetary policy. Future research can be extended to include a hybrid Phillips curve with memory in the conduct of monetary policy.

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Appendix

We want to solve the differential equation

$$\frac{d}{dt}E_t\pi_{t+1} = a\pi_t + bD^{(u)}\pi_t - \delta x_t \tag{12}$$

Here the fractional derivative of order u, $0 < u < 1$, is defined by

$$D^{(u)}f(t) = \frac{1}{\Gamma(1-u)}\int_0^t \frac{df(\tau)}{d\tau}\frac{d\tau}{(t-\tau)^u}$$

The Laplace transform of the function f is defined by $L\{f(t)\} = \int_0^\infty e^{-st}f(t)dt$. Another notation is $F(s) = \mathcal{L}\{f(t)\}$. Some basic Laplace transforms we will use are as follows:

$$L(1) = \frac{1}{s}$$

$$L(t^p) = \frac{\Gamma(p+1)}{s^{p+1}} \text{ where } p > -1, s > 0 \tag{13}$$

$$L\left\{\int_0^t f(t-\tau)g(\tau)d\tau\right\} = F(s)G(s) \tag{14}$$

$$L\{f'(t)\} = sF(s) - f(0)$$

The Laplace transform of $D^{(u)}\pi_t$ is calculated using (13) and (14).

$$\begin{aligned} L\{D^{(u)}\pi_t\} &= L\left\{\frac{1}{\Gamma(1-u)}\int_0^t \frac{d\pi(\tau)}{d\tau}\frac{d\tau}{(t-\tau)^u}\right\} \\ &= \frac{1}{\Gamma(1-u)}(sL\{\pi_t\} - \pi(0))\frac{\Gamma(1-u)}{s^{1-u}} \\ &= s^u L\{\pi_t\} - s^{u-1}\pi(0). \end{aligned}$$

Evaluating the Laplace transform of (12), we get

$$sL\{E_t\pi_{t+1}\} - E_0\pi_1 = aL\{\pi_t\} + bs^u L\{\pi_t\} - bs^{u-1}\pi(0) - \delta L\{x_t\}.$$

So

$$L\{E_t\pi_{t+1}\} = \frac{E_0\pi_1}{s} + \frac{aL\{\pi_t\}}{s} + \frac{bL\{\pi_t\}}{s^{1-u}} - \frac{b\pi(0)}{s^{2-u}} - \frac{\delta L\{\pi_t\}}{s}.$$

Taking inverse Laplace transforms, we obtain

$$E_t\pi_{t+1} = E_0\pi_1 + a\int_0^t \pi(\tau)d\tau + \frac{b}{\Gamma(1-u)}\int_0^t (t-\tau)^{-u}\pi(\tau)d\tau - \frac{b\pi_0 t^{1-u}}{\Gamma(2-u)} - \delta\int_0^t x(\tau)d\tau.$$

Notes

[1]The non-accelerating inflation rate of unemployment (NAIRU) refers to the level of unemployment that does not generate more inflation.

[2]The RMSE is defined as the square root of the average of squared differences between predicted values and actual values.

[3]Friedman (1968) proposes to include an expectation effect into the Phillips curve and uses previous inflation as a proxy for expected inflation. An expectations-augmented Phillips curve also empirically documented the inflation dynamics of the 1970s.

[4]

$$\pi_t = \lambda \sum_{i=0}^{\infty} \beta^i E_t\{x_{t+i}\}.$$

[5]As indicated in Gali and Gertler (1999), the New Keynesian Phillips curve implies that the current inflation rate should be negatively related to the lagged output gap. However, empirical evidences usually present a positive relationship between these two variables. Also, it predicts that the central bank can achieve disinflation at no substantial cost as long as the path of future output gaps is set to zero.

[6]$n + m \leq (1976–1957)*4 = 76$.

[7]Fisher, Liu, and Zhou (2002) allow lags to vary between 0 and 11 months by using the Bayes Information Criterion (BIC). Stock and Watson (2008) use BIC and Akaike information criterion (AIC) to determine the lag length over the range of 1 to 6 quarters. We estimate the model with a lag length between 1 and 12 quarters. The resulting lags fall between 1 and 9 across models.

[8]Stock and Watson's rolling window IMA model does not require estimating a time-varying unobserved components model. However, it has poorer performance relative to time-varying parameter models during periods of high volatility. We thank the reviewer for this constructive comment.

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