Analysis of Risk Management Strategies for Contingent Convertible Bonds

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Abstract

The contingent convertible bond (CoCo) is a structured instrument that emerged at the end of 2009. This paper explores the CoCo risk management strategy from the standpoint of investors. Taking the Equity Derivation Law as its framework, this study analyzes the hedging performance based on the static hedging of options and then introduces jumps risk to allow sudden bank defaults, observing the changes in hedging performance. By scenario analysis, this study finds that CoCo can control its investment risks via equity derivatives and that static hedging can effectively reduce the standard deviation and value-at-risk (VaR).

Key words: Contingent convertible bonds, static hedge, hedging performance, value at risk

I. Introduction

As a result of the 2007 subprime mortgage crisis, risks caused by the U.S. real estate market spread contagiously from lending banks to various financial institutions, among which insurance companies and investment banks were influenced the most. The financial crisis spread from the U.S. to the whole world, where the global financial market was deep in panic, and investors dumped or undersold stocks and bonds one after another, leading to the stock market crash, credit default swaps (CDS), and a soaring volatility index (VIX). Depositors hastily withdrew cash from the tumbledown banks, causing bank runs. Governments proposed various bailout and rescue measures to restore public confidence.

In the credit crisis between 2007 and 2008 as well as the subsequent government bailouts, we see that, first, the banks in crisis, such as Lehman Brothers, all had abundant capital before the crisis, which means that the

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original banking supervision system was evidently flawed, and additional capital injection from the government was required; second, government aid for banks did nothing but transferred taxpayers' money into these banks, which means that taxpayers had to assume the risk of the banks' business failure, while risk-seeking corporate shareholders and creditors were the only beneficiaries; third, financial institutions became "too big to fail." When large institutions suffer from a financial crisis, the government has to lend them a helping hand to stop the crisis from spreading, which implies two ethical risks: on the one hand, the repeated bailouts from the government encourage operators' speculation incentives; on the other hand, creditors and other market participants believe that financial institutions will always be rescued by governments during crises and thus fail to supervise operators carefully, reducing the market's discipline and leading to other ethical risks. Finally, the issue of "debt overhang" derives from the last two problems: When corporations experience financial deterioration, shareholders are unwilling to use their own money to take the risks but prefer debt financing. Restricted by the credit crunch and the banks' debt ceiling, however, shareholders cannot keep borrowing when the debt exceeds their loan repayment ability, and hence corporations are short of capital for the investment project which has positive net present values (NPV).

To solve these problems, an intuitive proposal is to limit loans and banking operations and raise the statutory capital requirement. This, however, would have a number of negative results. First, too many restrictions would prevent enterprises with real hedging needs from trading, and, since the equity fund would be too costly, the financial industry would lose its competitiveness. Second, raising capital requirements would cause banks to bear more losses. Most banks list their capital according to their internal risk models, but most risk models are flawed and cannot capture global and systematic risks, such as financial tsunamis. Furthermore, indexes such as the capital adequacy ratio are calculated based on accounting numbers, and hence may not reflect real market conditions and may be manipulated by operators. For example, some of the banks that received the American government's bailout, including Lehman Brothers, were all categorized as being well capitalized prior to the crisis. At that time, the concept of the contingent convertible bond (CoCo bond, or CoCo) emerged. The abovementioned problems could have been easily solved if there had been a kind of debt capital that could have automatically transformed or converted into shareholder equity to withstand losses in case of financial deterioration.

The CoCo is a bond commodity used when issuing corporations (or financial institutions) are in good financial conditions. It pays interest to CoCo investors regularly, and the principal is repaid at maturity; meanwhile, the issuing corporations enjoy interest tax shields. However, the CoCo will "automatically" and "compulsively" convert into common stocks in case of financial deterioration. With the CoCo, unlike for traditional convertible bonds, neither investors nor corporations have rights of conversion, determined by whether the predetermined conversion trigger in the CoCo issuing items has been activated. Therefore, the CoCo can withstand losses, similar to stockholders' equity, which is why it is also called "contingent capital."

The issuing of the CoCo has the following characteristics. (i) The CoCo provides banks with a capital buffer which avoids immediate close-downs in case of a financial tsunami and reduces the probability of default; moreover, as no additional capital injection from the government is needed, it partially prevents banks from being "too big to fail." (ii) Once the CoCo is converted compulsively, the stockholder equity will be diluted, which means that stockholders will lose control of their businesses if they operate imprudently. Thus, the CoCo can effectively restrain the speculation incentives of financial institutions, reduce agency problems, and provide market discipline. (iii) Assets can be transferred effectively. Banks that used to issue only subordinated bonds have to transfer their assets through bankruptcy recombination, which is a very costly process. However, the CoCo allows creditors to obtain equity through the conversion mechanism, allowing bank assets to be passed to more efficient operators (creditors) more efficiently, enabling the assets to be fully utilized. (iv) The CoCo can take care of "debt overhang." The key to resolving debt overhang is to reduce default risks. As long as stockholders believe that the probability of bond defaults is low, they will not worry about the effect of value transfer and will therefore be willing to increase capital for lucrative investments.

Based on the numerous advantages of the CoCo and considering the much stricter banking supervision expected in the future (some countries are already applying the CoCo to financial supervision), the CoCo is likely to eventually be listed as a bank asset. However, most studies focus on the design and evaluation of the CoCo; scholars discuss CoCo mostly from the perspective of banks or supervisors, but rarely from the standpoint of investors. In fact, investors with CoCos will want to know what the expected return and risks are and what risk management strategy to take to maximize the expected return.

This study explores CoCo risk management strategy from the perspective of investors. Taking the existing models as its framework, this paper observes the changes in the risks and losses of the CoCo under different parametric hypotheses using a Monte Carlo simulation and analyzes the static hedging of derivatives. Then, it introduces jumps to allow sudden bankruptcies so as to observe the changes in hedging performance. This study expects that risk management can eliminate the uncertainty and expected losses of investing in the CoCo and seeks to encourage investors to participate in the CoCo market by regulating and controlling investment risks with derivatives. It is further hoped that banks can improve themselves and that supervisory institutions can solve the "too big to fail" problem so as to create a triple-win situation.

The rest of this paper is organized as follows. Section II introduces the characteristics and current issuing situation, reviews the literature related to the CoCo, including research on evaluation, term design, and hedging, and discusses the research methods used and conclusions made by previous scholars. Section

III introduces the research method of this study, in which the models are divided into hedging methods with defaults considered and unconsidered and the steps of the Monte Carlo simulation. Section IV explores hedging efficiency as well as risk regulation and control methods under different parametric hypotheses. Finally, Section V provides the conclusion and suggestions for future studies.

II. A Brief Introduction to CoCo and Literature Review

A. A Brief Introduction to the CoCo

This section compiles research on the conversion mechanism of the CoCo, describes its current issuing situation in the market, and discusses the link between CoCo and bank supervision in order to provide readers with a deeper understanding of the CoCo.

The CoCo's conversion mechanism has several characteristics. First, its conversion depends on whether the capital adequacy ratio drops below the statutory limit. Before the capital adequacy ratio drops, the CoCo is the same as a common corporation bond. Second, once the conversion trigger is activated, all CoCo bonds are converted into common stocks. The number of converted stocks is determined by the stock price at that time instead of the pre-determined price. The conversion trigger, conversion form, and conversion price can all follow several designs, as explained below.

A.1. Conversion Trigger

In principle, the conversion trigger should be specific, easy to observe, and objective. Here, "specific" refers to the ability to determine banks' debt-paying ability; CoCos are not converted until banks are insolvent. The CoCo loses its effect if it is converted too early; if it is converted too late, the bank's financial conditions will likely be so poor that even the CoCo conversion will not help.

According to the categories used by Spiegeleer and Schoutens (2012), conversion triggers can be roughly divided into market triggers, accounting triggers, and regulatory triggers.[1] A market trigger is the price that can be observed on the market, such as the stock prices of banks or CDS. The advantage of the market trigger is that it is transparent and easy to observe. Moreover, the market is usually forward-looking; thus, when the price is very low and CDS is very high, this may indicate that the operational conditions of the banks have deteriorated, and the market has been affected. Another advantage is that it is very easy to evaluate a CoCo when the market trigger is applied. However, the

1 Most scholars adopt the market trigger, including Flannery (2005), McDonald (2013), and Sundaresan and Wang (2015), while the accounting trigger is adopted by Berg and Kaserer (2011), Himmelberg and Tsyplakov (2012), and Hilscher and Raviv (2014). Please refer to the literature review in Section II for details.

market trigger also has some disadvantages. First, it is not objective enough. It is hard to define a trigger that determines the actual financial condition. Second, the market price may be manipulated by institutional investors, or the price may not reflect the real value of assets due to the market disruption and irrational expectations of investors. Nevertheless, most scholars still recommend the market trigger.

The most common accounting trigger is the capital adequacy ratio, or Tier-1 core capital ratio. These cannot be observed directly and are dependent on the information provided regularly by banks; thus, they have obvious disadvantages. The accounting information lags behind the real situation: Banks' financial conditions may worsen without general investors becoming aware until earnings are released. Moreover, accounting statements are easy for operators to manipulate, and different countries follow different accounting principles, making it difficult for the accounting trigger to follow a unified standard. The regulatory trigger has no unified standard. The conversion of CoCos is determined according to whether supervisory agencies believe that banks have debt-paying ability; CoCos can be converted if they believe that the banks are not performing well. Thus, the regulatory trigger is the most non-transparent, non-observable, and non-objective trigger, but it gives supervisors a relatively high degree of management flexibility and is most helpful in solving the "too big to fail" problem.

A.2. Conversion Form

There have been three conversion forms for all CoCos issued up to 2012. One is "conversion at par," in which the face values of all CoCos will be converted into common stocks when the trigger is activated, so that investors can recover all face values. Another is "conversion at write-down," in which only a certain percentage (e.g., 90%) is converted, while other losses are borne by investors. Thus, CoCo investors can recover only 90% of the face value. Finally, the last one, "write-down only," occurs without any conversion.[2]

A.3. Conversion Price and Conversion Ratio

The conversion price and the conversion ratio are two sides of the same coin. As long as one of them is determined, the other can be determined as well. Assume that C_P is the conversion price. Then, C_r is the conversion ratio, F is the face value of the CoCo, and α is the write-down ratio. Then, the relationship among them can be described by the following equation:

$$C_P = \alpha F / C_r, \tag{1}$$

where α=1 stands for conversion at par and 0<α<1 represents conversion at write-down. Here, C_r is implied in Equation (1) as long as C_P is determined. In practice, the conversion price is usually determined first, and then the number of

2 See Table I (CoCos Issued until 2012) for details.

converted stocks is determined according to the face value. The most common settings for C_P include the stock price during conversion (S_{trig}), the stock price at the beginning of the period (S_0), or the average stock price at a period of time after activation. Matching different values of α produces the following combinations:

(a) Fixed shares par conversion (FSP): α=1, C_P determined at the beginning.

(b) Variable shares par conversion (VSP or fixed dollar par conversion): α=1, $C_P = S_{trig}$, or the average stock price.

(c) Fixed shares write-down conversion (FSW or fixed share "premium" conversion): α<1, C_P determined at the beginning.

(d) Variable shares write-down conversion (VSW): α<1, $C_P = S_{trig}$, or the average stock price.

(e) Variable shares at par subject to maximum (VSPM): α=1, C_P = max (S_{trig}, Q), Q is a predetermined constant.

Different conversion combinations provide different levels of protection for CoCo investors and stockholders and restrain speculation incentives and market manipulation to different degrees.

A.4. Current Issuing Situation of CoCos

The first batch of CoCos was issued in the U.K. by Lloyds Bank Group (LBG) at the end of 2009 with a total face value of £7 billion. LBG named this commodity "enhanced capital notes" (ECN). The successful issuing of ECN was significant because it was generally believed that CoCos are issued in favorable economic situations, as investors may worry about the motive for issuing CoCos during economic downturns. The success of the LBG proved that this concern was unfounded; it may have encouraged other banks that were inclined to issue CoCos. In fact, many large banks, such as Swiss Bank Corporation and Credit Suisse, started issuing CoCos in 2010. Table I shows some of the important CoCos and related terms as of 2012.

B. Review of the Literature on CoCos

Flannery (2005) points out that the "market discipline" mechanism has been established in the three pillars of the Basel Accord but that bankruptcies of financial institutions usually cause several negative effects. First, bankruptcy can lead to inefficient asset utilization. In an assumed perfect market, a bankrupt reorganization just transfers assets from stockholders to others who are better at business. In reality, however, enterprise reorganization is very costly and time-consuming. The cost of bankruptcy for financial institutions is far higher. As their assets' values fluctuate widely, there may be very few asset values left after liquidation. Furthermore, the perfect operation of financial institutions depends on their good credit quality, but their credit level can never completely recover after a bankruptcy reorganization. Second, the close-down of financial

Table I
CoCos Issued until 2012

Source: Buergi (2012).

Name	Issue Date	Maturity	Nominal Amount	Coupon	Trigger Underlying	Conversion Fraction
Bank of Cyprus	2011.4	Perpetual	€1.3 bn.	6.500%	Tier-1 core capital<5%	Conversion
Credit Suisse	2011.2	30	$2 bn.	7.875%	Tier-1 core capital <7%, Regulatory trigger	Conversion
Lloyds	2009.9	15~20	£7 bn.	9.125%	Tier-1 core capital<5%	Conversion
Rabobank	2010.3	10	€1.25 bn.	6.875%	Equity capital<7%	75% Write-down
Rabobank	2011.1	Perpetual	$2 bn.	8.375%	Equity capital<8%, Regulatory trigger	Write-down only
UBS	2012.2	10	$2 bn.	7.250%	Tier-1 core capital<5%, Regulatory trigger	Write-down only
Unicredit	2010.7	Perpetual	€0.5 bn.	9.375%	Total capital<6% , Regulatory trigger	Write-down only
ZKB	2012.1	Perpetual	CHF 0.59 bn.	3.500%	Tier-1 core capital<7%, Regulatory trigger	Write-down only

institutions may create a burden for their counterparts. Consequently, the financial market as a whole may become unstable, and investor confidence may weaken, which may then affect the whole economy. Third, if financial institutions are forced to clear their assets when on the verge of bankruptcy, the huge portion of their assets will usually cause drastic fluctuations in market prices. Therefore, Flannery (2005) proposes the idea of "reverse convertible debentures" (RCD) whereby, during financial crises, financial institutions could perform financial reorganization without additional funding. They would not be able to avoid default risk completely, but they could significantly lower its probability.

Flannery (2016) renames RCD the "contingent convertible certificate" (CCC), arguing that, as governments around the world took various bailout measures after the financial tsunami, they were clearly unwilling to see the close-down of financial institutions, thus causing ethical risks. Studies have reviewed these ethical risks since the first issue of CoCos at the end of 2009. Some scholars have examined how to design the issuing items, while others have focused on the development of evaluation models. Both of these approaches are discussed separately below.

B.1. Literature on CoCo Design

Flannery (2005) focuses on how to design the CoCo conversion mechanism, advocating that the conversion trigger be based on the "capital adequacy ratio," not to be measured by numbers in accounting statements but by assets and the market value of stockholders' equity, as accounting information usually lags behind the reality and can be manipulated by operators. For example, operators can embellish the statements by delaying the recognition of expenses or by asset impairment. The conversion mechanism proposed by Flannery (2005) has two other key points. (a) When the trigger is activated, only a portion of CoCos are converted to make the capital adequacy ratio meet the trigger perfectly. The author believes that determining which CoCo should be converted can be decided by lots, or the conversion order can be predetermined by subsection at the beginning of the issue. (b) The conversion price should be the stock price at the time of conversion instead of a predetermined price, to prevent traders from profiting from conversion by manipulating the stock price.

McDonald (2013) argues that CoCo conversion should meet a dual trigger. Aside from the stock price of issuing corporations, other variables and indexes should be referred to as well. CoCos should be converted only when both the stock price of the issuing corporations and the overall variables are lower than a certain level; otherwise, corporations may fail before CoCo conversion occurs. This dual trigger design would ensure that financial institutions fail only due to the financial problems they, themselves, cause, and the financial market will not be burdened by the failure of a few institutions, allowing the ethical risks stemming from high-risk investments conducted by firm managements to be avoided.

Sundaresan and Wang (2015) investigate how to design a CoCo trigger that would ensure that the fair value of the CoCo would be the only equilibrium point and that could be adjusted dynamically over time. The findings show that there will be multiple equilibria if the CoCo conversion price is too high because investors will assess the stock price when expecting conversion, but no conversion will occur. As a result, two equilibrium prices are created. If the CoCo conversion price is too low, there will be no unique equilibrium price. In the end, the authors point out that the unique equilibrium will exist as long as the conversion ratio is adjusted dynamically to the specific value of the CoCo price and conversion price.

Berg and Kaserer (2011) discuss CoCo design from another perspective. They believe that even the issue of CoCos by financial institutions would not help prevent speculation incentives, which can increase instead of decreasing under certain circumstances. Most scholars advocate adopting CoCos as a way of providing additional capital for financial institutions suffering financial difficulties in order to help them withstand losses; the ultimate purpose is to eliminate ethical risks resulting from financial institutions being too big to fail. However, the authors note that, if the conversion trigger is not well designed, not only will this purpose not be achieved, but company managers may also be encouraged to conduct high-risk investments or even refuse investment plans which have positive NPV. Berg and Kaserer (2011) analyze company investments in two extreme cases. In the first, all CoCos are given up once the trigger is reached; in the other, equities are substantially diluted or even given up while all CoCos are converted into equities. They point out that, under these circumstances, no matter whether the market is good or bad, managers will always choose low-risk investment plans.

Himmelberg and Tsyplakov (2012) also discuss the speculation incentives caused by poorly designed CoCos. They point out that the widely known advantage of the CoCo is that it might improve banks' ability to withstand losses during a financial tsunami, but few people are aware of this. If the CoCo is well designed, it can also restrain managers from conducting high-risk investments. Therefore, this study focuses on the extent to which equities have to be diluted by CoCo terms to effectively avoid speculation incentives. Himmelberg and Tsyplakov (2012) also consider real market conditions, including transaction costs, the costs of issuing new stocks, the asset jump process, and irrational conversion, and obtain the optimal arithmetic solution for conversion ratios by a "finite difference method."

B.2. Literature on CoCo Evaluation and Hedging

Research on CoCo evaluation can be divided into two types according to the conversion trigger. The first is the stock price trigger; conversion begins when the stock price falls to a predetermined level. In this conversion mechanism, the CoCo can be regarded as a combination of tripping in/out derivatives. The other is the capital adequacy ratio trigger. As its evaluation methods are relatively complex, most studies on CoCo evaluation are based on the stock price trigger.

Pennacchi (2010) is the first to propose a method of evaluating CoCos. Pennacchi (2010) uses the structural-form model, assuming that banks have three sources of capital—deposits, senior debts, and common stocks. He assumes that the term structure of the risk-free interest rate follows the CIR model, i.e., the asset value of banks complies with the geometric Brownian motion containing jumps, and the stockholders' equities and corporate bonds are considered options with the object of corporate assets. Pennacchi (2010) selects the stock price trigger for conversion, takes various conversion methods into account, and compares the interest margins of CoCo, deposit interest rate, and the value of equity under different conversion rules. He finds that the CoCo has the smallest interest margin when the number of converted stocks changes and stockholders' equity increases least when assets fluctuate more widely. Pennacchi (2010) also points out that the CoCo will have no default risks if the asset value process does not contain jumps, and the credit spread is zero. Pennacchi (2010) also discusses debt overhang and speculation incentives, pointing out that, as default risks decrease following the issuing of CoCo by banks, operators may conduct high-risk investments, creating another ethical risk. Pennacchi (2010) argues, however, that this ethical risk can be reduced as long as banks issue more first-lien debts or dilute more stockholders' equity.

Hilscher and Raviv (2014) use evaluation methods similar to Pennacchi's (2010). They assume that assets comply with the geometric Brownian motion and that senior debt and CoCos are the only types of corporate debt. The following three mutually exclusive events are considered: (a) CoCos are not converted, and corporations did not default before expiration; (b) CoCos have been converted, but corporations did not default before expiration; (c) CoCos have been converted, but corporations defaulted before expiration. According to these three situations, Hilscher and Raviv (2014) split CoCos into a barrier option portfolio with the object of corporate assets and then calculate the CoCos' theoretical price.[3]

Spiegeleer and Schoutens (2012) take the stock price as the trigger point of conversion and propose two other models, the credit derivatives approach (CDA) and the equity derivative approach (EDA), for the evaluation of CoCos. The CDA is a reduced-form model which can be utilized to evaluate common corporate bonds. CDA calculates the default chance and "default intensity" before bonds expire, then obtains the credit spread[4] bonds should have according to the default intensity and recovery rate, and finally calculates the bond price. The EDA considers the CoCo as a combined equity commodity, splitting CoCo into a portfolio: buy corporate bonds and forward contracts with the down-and-in object of company stock price and sell down-and-in digital options. The theoretical value is very easy to obtain, as the above three all have closed-form solutions.

Teneberg (2012) divides existing CoCo evaluation models into three types. The first is the Hilscher and Raviv (2014) method, "balance sheet pricing," and the other two are the CDA and the EDA proposed by Spiegeleer and Schoutens (2012). Teneberg (2012) believes that the EDA is the most suitable model. However, EDA assumes that stock returns are presented in the geometric Brownian motion, ignoring the reality that stock prices can plunge. Teneberg (2012) therefore revises this view. When stock prices have jumps, the closed-form solution of barrier options no longer applies, leaving only the numerical method to rely upon. Teneberg (2012) uses the "ternary tree" and adopts the "adaptive mesh model" (AMM) and Ritchken's technique to revise the ternary tree model in order to improve the efficiency of numeric calculation.

Glasserman and Nouri (2012) also evaluate the CoCo. Unlike the abovementioned evaluation methods, they adopt the accounting trigger for conversion for three main reasons. (a) All bank supervision mechanisms are based on accounting information; (b) All existing terms of issued CoCos use the capital adequacy ratio as the trigger value instead of the stock price; (c) Use of the market price trigger is easily manipulated by the market, and stock prices may fall due to irrationality in the market, causing unnecessary conversions; this goes against the principles of CoCo design, but it will not be influenced by the adoption of the accounting trigger. Ultimately, Glasserman and Nouri (2012) derive the theoretical price via a martingale evaluation.

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3 In the first scenario, CoCos can be seen as buying a binary down-and-out barrier call option; the other two events can be seen as a combination of common down-and-in options. As the barrier option has closed-form solutions, the theoretical price of the CoCo can be obtained by totaling the values of this portfolio.

4 In the reduced-form model, the relationship among the default rate (λ), the recovery rate (R), and the credit spread (cs) is $cs = (1-R)\times\lambda$.

Wilkens and Bethke (2014) conduct the first empirical study of evaluation models. The study compares the accuracy of three models using CoCos issued by LBG and Credit Suisse. The models follow the structural-form model in Pennacchi (2010), CDA, and EDA. Wilkens and Bethke (2014) show that the prices derived by these three models roughly match the practical data. However, the overall hedging errors in EDA are the smallest when Wilkens and Bethke (2014) conduct the dynamic hedging with hedging parameters. Therefore, the authors argue that EDA is the most practical for risk management.

Hedging and evaluation are closely related, and investors can take corresponding hedging measures with accurate evaluation models. However, most studies focus on evaluation methods instead of CoCo-related hedging. Cheridito and Xu (2015) point out that there are three CoCo risk sources (i.e., interest rate risks, conversion risks, and stock price risks) and that dynamic hedging can be conducted through interest rate exchange, CDS, and underlying stocks. Cheridito and Xu (2015) evaluate the CoCo using the structural-form model and the reduced-form model and calculate related dynamic hedging combinations via the two models. They compare the degrees of fitness to the models with CoCos issued by LBG and Rabobank, and they find that both fitting results are identical.

III. Methodology

This study assumes the stock price trigger for CoCo conversion and uses a model framework similar to the EDA in Spiegeleer and Schoutens (2012), for several reasons. First, as Teneberg (2012) suggests, other evaluation methods have higher model risks. For example, the structural-form model has to assume that bank assets are tradable when, in fact, it is impossible to hedge by trading (copying) bank assets from the investors' perspective. The reduced-form model focuses too heavily on the nature of CoCos as bonds and may thus underestimate the effect of stock price changes on them. Second, Wilkens and Bethke (2014) study the factors affecting CoCo returns and find that the returns of bank stock price have the strongest explanatory power for CoCo returns (minimum p-value), while the CDS' price differences and interest rates vary very slightly. Thus, the CoCo clearly has the strongest correlation with the stock price, and it is the most appropriate to use models related to equity. Third, when we use delta, gamma, and rho for hedging, it is usually most effective to obtain results by conducting dynamic hedging with hedging parameters calculated by the equity derivative approach, but the hedging parameters of the structural-form model must rely on the accuracy of the estimation of banks' asset value. This section first introduces the principles of model establishment and then describes relevant risk management methods, as well as the indexes of measuring hedging performance. Then, this study simulates the performance of hedging strategies via a Monte Carlo simulation.

A. Default-Free Models

There are several important hypotheses in the EDA of Spiegeleer and Schoutens (2012). (a) The conversion trigger is the trigger of stock price, meaning that conversion will be triggered when the stock price falls to a certain level (S^*). (b) The conversion proportion is known; (c) The CoCo will not default before conversion. Assuming that the expiration of the CoCo is T, then the end-of-period payoff function P_T of CoCo is

$$P_T = \begin{cases} C_r S_T, & \textit{if triggered} \\ F, & \textit{if not triggered.} \end{cases} \tag{2}$$

Assuming that the conversion time-point of CoCo is τ_C and an indicator function is introduced, the time-point will be one if the CoCo is converted before expiration; otherwise, zero. This indicator function can be expressed by the following mathematical expression:

$$1_{\{CoCo\ has\ been\ converted\}} = 1_{\{\tau_C < T\}}, \quad \text{where} \quad \tau_C = \inf_{0<t<T}\left\{S_t \le S^*\right\}.$$

Given the indicator function as above, Equation (2) can be rewritten as

$$P_T = F + C_r(S_T - C_P)1_{\{\tau_C \le T\}}. \tag{3}$$

Equation (3) implies that the end-of-period profits and losses can be duplicated by zero-coupon bonds with the par value of F and a "down-and-in forwards" of C_r, whose underlying assets are stocks and the barrier point is S^*;[5] however, we must consider the interest income for completely duplicating the cash flow of the CoCo. The duplication method is to buy all the present values of CoCos' interest payments and then sell the "binary down-and-in calls" of every period of interest, because investors will lose their interest incomes after the CoCo is converted into stocks. The interest can be expressed by the following mathematical expression:

$$\sum_{i=1}^{n} c_i e^{-rt_i} - \sum_{i=1}^{n} c_i e^{-rt_i} 1_{\{\tau_C < T\}}. \tag{4}$$

In Equation (4), c_i represents the interest paid in the i^{th} period; $t_1 < t_2 < \ldots < t_n = T$ represents the payday of the interest of the i^{th} period; n represents the number of interest payments before expiration.

We can obtain the closed-form solution of the CoCo via the hypothesis of the Black-Scholes (BS) model: The stock price dynamics comply with the geometric Brownian motion, $dS_t = rS_t dt + \sigma S_t dW_t$ in which r and σ are known constants representing the risk-free interest rate and stock price fluctuation respectively. Under this hypothesis, combined with Equations (3) and (4), the CoCo can be

5 The end-of-period profits and losses of down-and-in forward contracts can also be duplicated by buying down-and-in call options and emptying them.

broken down into three parts: common coupon-paying bonds (A), down-and-in forward contracts (B), and selling n binary down-and-in calls (C). As all three have closed-form solutions, the closed-form solution of the CoCo values can be expressed as follows:

$$P_{CoCo} = A + B + C. \tag{5}$$

In the equation,

$$A = Fe^{-rT} + \sum_{i=1}^{n} c_i e^{-rt_i}$$

$$B = C_r\left[S(\tfrac{S^*}{S})^{2\lambda} N(y_1) - Xe^{-rT}(\tfrac{S^*}{S})^{2\lambda-2} N(y_2) - Xe^{-rT} N(-x_2) + SN(-x_1)\right]$$

$$C = -\sum_{i=1}^{n} c_i e^{-rt_i}\left[N(-x_{2i}) + (\tfrac{S^*}{S})^{2\lambda-2} N(y_{2i})\right],$$

$$x_1 = \tfrac{\ln(S/S^*)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T};\; x_2 = x_1 - \sigma\sqrt{T}$$

$$y_1 = \tfrac{\ln(S^*/S)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T};\; y_2 = y_1 - \sigma\sqrt{T}$$

$$x_{1i} = \tfrac{\ln(S/S^*)}{\sigma\sqrt{t_i}} + \lambda\sigma\sqrt{t_i};\; x_{2i} = x_{1i} - \sigma\sqrt{t_i}$$

$$y_{1i} = \tfrac{\ln(S^*/S)}{\sigma\sqrt{t_i}} + \lambda\sigma\sqrt{t_i};\; y_{2i} = y_{1i} - \sigma\sqrt{t_i}$$

$$\lambda = \tfrac{r+\sigma^2/2}{\sigma^2};\; X = C_p.$$

Part A in Equation (5) is discounted by the risk-free interest rate instead of the market return rate. Since we can duplicate return forms with risk-free assets and stocks, no additional risk premiums are required.

B. Fixed Default Intensity Models

In Section III.A., we assume that stock price change is a geometric Brownian motion and obtain the closed-form solution of CoCo; in reality, however, financial institutions can go bankrupt without warning (e.g., Lehman Brothers in 2008). This means that stock price changes are discontinuous and may drop to zero at a certain time-point. This section explains how to improve the abovementioned models to describe this phenomenon.

Assuming that stock price changes involve a Wiener process containing jumps, such jumps can make the stock price fall to zero instantly (when a corporation goes bankrupt without warning).[6] Therefore, if the risk is neutral, the stock price process can be expressed as

$$S_t = S_0 \exp\left((r - \eta - \tfrac{1}{2}\sigma^2)t + \sigma W(t)\right) 1_{\{\tau_D > t\}}. \tag{6}$$

---

6 As the characteristic of the stock price instantly falling to zero is equivalent to the jumps in the Merton model, the terms “jump” and “bankruptcy” are used interchangeably.

In the equation, $\eta = \dfrac{\ln P(\tau_D > t)}{t}$ is the adjustment item for risk premium, τ_D is the default time-point, and $P(\tau_D > t)$ stands for the probability that no jumps occur (i.e., corporations do not go bankrupt) at time *t*.

Assume that banks' bankruptcy events have the nature of the Poisson process. If the fixed constant λ is the default intensity, then the probability of banks having no default before time *t* is

$$P(No\ defauts\ in[0,t]) = P(\tau_D > t) = e^{-\lambda t}. \tag{7}$$

Assume that default events are independent from the Brownian motion; it can then be proven that the stock price is risk-neutral under this hypothesis because the expected value of S_t is

$$\begin{aligned} E(S_t) &= E\left[S_0 \exp\left((r-\eta-\tfrac{1}{2}\sigma^2)t + \sigma W(t)\right)1_{\{\tau_D>t\}}\right] \\ &= S_0 \exp((r-\eta)t)E(1_{\{\tau_D>t\}}) \\ &= S_0 e^{rt} e^{-\eta t} P(\tau > t) = S_0 e^{rt}, \end{aligned}$$

which means that the stock price grows by risk-free interest rates.

When default events are taken into account, conversion forms can be divided into two categories according to whether default events happen. If no defaults happen, we execute the conversion process in accordance with the conversion proportion stipulated in contracts or write-downs of par value; if defaults take place, we use the conversion form with zero as the write-down of par value. As bankruptcy events have the nature of a Poisson process, bankruptcies occur independently in non-overlapping sections. Thus, we discrete Equation (6) as follows.

$$S_{t+\Delta t} = S_t \exp\left((r-\eta-\tfrac{1}{2}\sigma^2)\Delta t + \sigma\varepsilon\sqrt{\Delta t}\right)1_{\{\tau_D>\Delta t\}}. \tag{8}$$

In the equation, $1_{\{\cdot\}}$ is the random variable Bernoulli:

$$1_{\{\tau_D>\Delta t\}} = \begin{cases} 1, & with\ prob.\ p \\ 0, & with\ prob.\ 1-p, \end{cases}$$

$p = P(\tau_D > \Delta t) = \exp(-\lambda \Delta t)$ represents the probability of success in Bernoulli experiments. Here, it refers to the probability of bankruptcy within a unit interval time.

The stock price is still risk-neutral because, when the information of period *t* is given, the conditional expected value of $S_{t+\Delta t}$ is as follows:

$$\begin{aligned} E(S_{t+\Delta t} \mid \mathcal{F}_t) &= S_t e^{r\Delta t} P(\tau_D > t)^{-\Delta t/t} P(\tau_D > \Delta t) \\ &= S_t e^{r\Delta t} e^{-\lambda t \cdot (-\Delta t/t)} \cdot e^{-\lambda \Delta t} = S_t e^{r\Delta t}. \end{aligned}$$

After variable Bernoulli is multiplied by consecutive Brownian motions, the models can capture the risks of bank defaults. However, the stock price will become discontinuous, and it is impossible to obtain the closed-form solution of the down-and-in forward contract and binary down-and-in call, causing the

CoCos to have no closed-form solutions. Therefore, in a Monte Carlo simulation, this study simulates the price of CoCos when defaults occur. The simulation's steps are presented in Section IV.

C. Two-Stage Default Intensity Models

To highlight that CoCo conversion can reduce the probability of default by issuing banks in order to reduce the unfair situations in which governments save banks by funding them, the models in this section follow the defaultable models in the previous section, assuming that banks' default intensity will be affected by CoCo conversion or write-down. If CoCos are converted into common stocks, the probability of default can be lowered by providing banks with a capital buffer or reducing debt overhang. Therefore, assume that the default intensity of bank bankruptcies will decrease after CoCo conversion:

$$\lambda = \lambda(S_t, S^*) = \begin{cases} \lambda_1, & \textit{if not triggered}, \\ \lambda_2, & \textit{if triggered}. \end{cases} \tag{9}$$

In the equation, λ_1 and λ_2[7] are both fixed constants, and $\lambda_1 > \lambda_2$. Under the hypothesis of this default probability model, the stock price dynamic is a two-stage function:

$$S_t = \begin{cases} S_0 \exp\left((r - \eta_1 - \tfrac{1}{2}\sigma^2)t + \sigma W(t)\right) 1_{\{\tau_D > t\}}, & \textit{if } \tau_C > t, \\ S_0 \exp\left((r - \eta_2 - \tfrac{1}{2}\sigma^2)t + \sigma W(t)\right) 1_{\{\tau_D > t\}}, & \textit{if } \tau_C \le t. \end{cases} \tag{10}$$

In the equation, $\eta_1 = \dfrac{\ln P(\tau_D > t \mid \tau_C > t)}{t}$ and $\eta_2 = \dfrac{\ln P(\tau_D > t \mid \tau_C \le t)}{t}$ are risk premium terms. We can prove that stock price changes are still risk-neutral under the hypothesis of this risk premium:

$$\begin{aligned} E(S_t) &= E\begin{bmatrix} S_0 \exp\left((r - \eta_1 - \tfrac{1}{2}\sigma^2)t + \sigma W(t)\right) 1_{\{\tau_D > t,\ \tau_C > t\}} \\ + S_0 \exp\left((r - \eta_2 - \tfrac{1}{2}\sigma^2)t + \sigma W(t)\right) 1_{\{\tau_D > t,\ \tau_C \le t\}} \end{bmatrix} \\ &= S_0 \exp((r - \eta_1)t) E(1_{\{\tau_D > t,\ \tau_C > t\}}) + S_0 \exp((r - \eta_2)t) E(1_{\{\tau_D > t,\ \tau_C \le t\}}) \\ &= S_0 \exp((r - \eta_1)t) P(\tau_D > t,\ \tau_C > t) + S_0 \exp((r - \eta_2)t) P(\tau_D > t,\ \tau_C \le t) \\ &= S_0 e^{rt} e^{-\eta_1 t} P(\tau_D > t \mid \tau_C > t) P(\tau_C > t) + S_0 e^{rt} e^{-\eta_2 t} P(\tau_D > t \mid \tau_C \le t) P(\tau_C \le t) \\ &= S_0 e^{rt} P(\tau_C > t) + S_0 e^{rt} P(\tau_C \le t) \\ &= S_0 e^{rt}. \end{aligned}$$

7 For the setting of λ_1 and λ_2, in regard to λ_1, its default intensity rate can be inferred from the rating provided by credit rating companies or estimated using the CDS in the credit-derived commodity market. As for λ_2, the relationship between λ_1 and λ_2 can also be determined through the ratings of financial institutions issuing CoCos after conversion or the changes in CDS default intensity. Too few samples of issued CoCos are available to study the relationship between default intensities before and after activation. Thus, this study assumes that λ_2 is a fixed constant smaller than λ_1.

Under the hypothesis of the model in this section, the discrete process of the stock price is identical to the fixed default intensity model, while the prices of CoCos also have no closed-form solution. The prices of CoCos in this model are analyzed via the Monte Carlo simulation later on in the paper.

D. Risk Management Strategy

First, this study discusses the hedging methods of CoCos. Under the EDA framework, there are two common hedging strategies. The first is "dynamic hedging," obtaining hedging parameters such as delta and vega by using Equation (5) and emptying the corresponding underlying shares and option contracts for hedging. However, the weight of hedging portfolio must be continuously adjusted to achieve the effect of dynamic hedging, which is practically impossible because traders and brokers adjust portfolio only once in a while. However, this will cause hedging errors that may accumulate over time; moreover, human and transaction costs also have to be taken into practical consideration, especially as CoCo expiration can easily be a decade away, and the associated costs are remarkably high.

The second strategy is "static hedging," buying or selling derivatives for hedging. The greatest risk of CoCos derives from the down-and-in forward contract while the greatest loss caused by selling data calls is only the coupon rate c_i and its influence is relatively small; therefore, to avoid downside risks, we need only buy down-and-in puts (PDI) of unit C_r at the beginning of the period to fix the end-of-period earnings at the predetermined strike price (K).

Given the disadvantages of dynamic hedging and difficult practical operations, this study adopts "static hedging" as the hedging strategy, but there are two problems. First, there may be no barrier options of underlying stocks in the market. Second, even though there is a PDI of underlying stocks, exchanges have no contracts that last for such a long period.

The first problem is not very difficult to solve. Derman, Ergener, and Kani (1995) propose the method of statically duplicating barrier options with the strike price of K and the limit of B via standard European options. However, the second problem cannot be solved by duplication, unless we can find counterparties in OTC (which would cause credit risks for those counterparties). Otherwise, we have to roll over by the due date of the old options, recover the end-of-period profits and losses $(K-S)^+$ of the old contracts, and buy options of the next period, but this might influence hedging accuracy. To explore this problem, we consider the effect of roll-over in the subsequent hedging simulations and compare it with that in non-roll-over circumstances.

For risk measurement, this study consults hedging effectiveness (HE) indexes in Demirer and Lien (2003) and Cotter and Hanly (2006) and measures hedging effectiveness using four indexes: reduction degree of variances, VaR reduction, expected loss, and expected loss variation. The reduction degree of variances is used to compare the reduction degree of portfolio value variances before and after hedging. Ederington (1979) creates this index, which can be expressed by the following mathematical expression:

$$HE_1 = 1 - \frac{Variance_{hedged\ portfolio}}{Variance_{unhedged\ portfolio}}. \quad (11)$$

However, when there is an asymmetrical distribution (when the coefficient of skew is not zero) in the portfolio value, this index might overestimate or underestimate the hedging performance. Therefore, the VaR reduction is calculated to measure the extent of reduced losses at a certain confidence level. This index can be expressed by the following mathematical expression:

$$HE_2 = 1 - \frac{1\%\ VaR_{hedged\ portfolio}}{1\%\ VaR_{unhedged\ portfolio}}. \quad (12)$$

In addition, as the purpose of hedging is to avoid downside risks, aside from the traditional standard deviation, we also take into account two other risk indexes, "mean loss" and "downside risks," defined as

$$\text{Mean Loss(ML)} = E(\text{L} \mid \text{L} > 0), \quad (13)$$

$$\text{Downside Risk(DR)} = Var(L \mid L > 0); \quad (14)$$

$$HE_3 = 1 - \frac{\text{ML}_{hedged\ portfolio}}{\text{ML}_{unhedged\ portfolio}},\ HE_4 = 1 - \frac{\text{DR}_{hedged\ portfolio}}{\text{DR}_{unhedged\ portfolio}}. \quad (15)$$

In the equation, L stands for loss, $L = E(V_0) - V_0$. These two indexes measure how huge the expected loss and risk will be when portfolio has already suffered from losses. These two indexes are used because, after hedging, a portion of the downside risks have been eliminated, and the variances measured by the traditional standard deviation may be "upside risks" that are, however, favored by investors. Therefore, the hedging performance may not be correctly measured by the standard deviation alone. We can divide the four efficiency indexes into two categories. HE_2 and HE_3 measure the damping of "loss;" the greater the value is, the better the hedging performance will be at controlling losses. HE_1 and HE_4 measure the damping of "fluctuation;" the greater the value is, the more effectively it can reduce its uncertainty. In the next section, this study simulates hedging effectiveness under different parameter settings with a Monte Carlo simulation to further our understanding of the effect of static hedging.

For the method of risk control, given the aforementioned static hedging framework, we buy down-and-in PDI at the beginning of the period to create a CoCo hedging portfolio and fix the end-of-period CoCo earnings at the predetermined strike price. However, if we buy PDI with a different strike price at the beginning of the period, it will affect the end-of-period value of the portfolio. Theoretically, the higher the strike price is, the higher the end-of-period earnings will be, which can reduce the downside risks of the portfolio. However, given the higher hedging costs, profits may be reduced at the same time. Therefore, we can try to control the risks of portfolio using different PDI points to fulfill the purpose of risk control.

IV. Numerical Analysis

A. Simulation Process

First, we look at the default-free event. The following hedging analysis will take three circumstances into account: hedging-free (the value of investing only in CoCos), hedging without roll-over, and hedging with roll-over. The analysis compares their investment performance differences. Hedging performance is simulated using the Monte Carlo simulation as follows:

(a) According to one stock price path generated by the geometric Brownian motion $S_t = S_0 \exp\left(\left(r - \frac{1}{2}\sigma^2\right)t + \sigma\varepsilon\sqrt{t}\right), \varepsilon \sim N(0,1)$, check whether CoCo once fell to the trigger value and activated conversion before the due date, and calculate the conversion time-point τ_C if CoCo was converted.

(b) Calculate the end-of-period profits and losses of the hedging portfolio. If CoCo was converted, the profits and losses will be the stock value plus the put option value; otherwise, they will be the face value of the CoCo. This can be expressed by the following mathematical expression:

$$P_T = \begin{cases} F, & \text{if } \tau_C > T \\ C_r\left(S_T + (K - S_T)^+\right), & \text{if } \tau_C \le T. \end{cases} \tag{16}$$

When no hedging is involved, P_T can be calculated by Equation (2).

(c) Calculate the present value of interest incomes. If CoCo was converted, the interest after the conversion time-point must be deducted. Assuming that t_m is the latest interest payment date before the conversion, the total present value of interests will be

$$PV(Coupon) = \sum_{i=0}^{m} c_i \exp(-rt_i).$$

(d) Calculate the present value of the hedging cost. When no roll-over is involved, the hedging cost is simply PDI itself. When roll-over is involved, assuming that the roll-over occurs once a year, the cost of each roll-over date is the value produced when the price of the new contract is subtracted by the cash flow of the previous contract:

$$CF_t = \begin{cases} C_r \cdot PDI(S_t, K, B, T = 1; \sigma, r), & \text{if } \tau_C > t, \\ C_r\left(Put^{BS}(S_t, K, T = 1; \sigma, r) - (K - S_t)^+\right), & \text{if } \tau_C \le t. \end{cases}$$

Then, the total present value of the hedging cost can be expressed as

$$PV(Cost) = \sum_{t=0}^{T} CF_t \exp(-rt).$$

(e) Calculate the value of the portfolio at the beginning of the period (V_0). Discount the end-of-period value P_T first, then add the present value of interest. Then, deduct the present value of hedging costs:

$$V_0 = P_T \exp(-rT) + PV(Coupon) - PV(Cost). \tag{17}$$

(f) Repeat Steps (a) to (e) N times. Calculate the mean value and standard deviation of these V_0. The former represents the estimated value of the expected value of the portfolio, and the latter represents the risks in portfolio.

When default events are considered, the hedging simulation will be divided into two phases. In the first, it is necessary to simulate the value of PDI under different parameters. This method is similar to the simulation of standard put options: First, calculate the value of PDI at the due date according to the multiple stock price paths generated by Equation (8) or (10):

$$PDI_T = \begin{cases} (K - S_T)^+, & \text{if } S^* \text{ has been reached.} \\ 0, & o.w. \end{cases}$$

Then, discount all PDI_T by the risk-free interest rate and calculate the mean value—the simulation value of PDI. In the second phase, repeat Steps (a) to (f), but replace the stock price process in Step (a) with the stock price process containing jumps, and replace the hedging cost in Equation (17) with the simulation value of PDI.

When default events are involved, the continuous simulation with put option roll-over will increase the calculation load substantially. For example, if the roll-over cycle is a year, we first need to know the stock price after the year to estimate the value of the new PDI or standard put option value at the time of roll-over after one year. Assume that there are 100,000 simulation paths, and the due date is in 10 years. There will be one million stock prices at roll-over date, and one million simulations need to be done to obtain the estimated value of these PDI. If 10,000 additional simulations (i.e., in the first phase) are needed to simulate a PDI, 10 billion simulations will be needed to simulate a roll-over strategy. Thus, the calculation of the Monte Carlo simulation is unfeasibly complex. Therefore, when default events (stock price jumps) are involved, this study only compares the performance differences of "no hedging" and "hedging without roll-over", and the analysis on the performance of hedging with roll-over is excluded in this study.

B. Simulation Results

In this section, this study observes the value changes and risk management performances of CoCos under different parameter settings and stock price dynamic models. First, define a basic parameter group: the face value of the CoCo (F)=\$1,000; the stock price at the beginning of the period S_0=\$100; the conversion price C_P is the stock price at the beginning of the period (hence, the conversion proportion C_r=10); the strike price of PDI K=\$70; the stock price trigger of the CoCo S^*=\$70; the due date T=10 years; the stock price fluctuation σ =25%; the risk-free interest rate r_f=3%; and the coupon rate c=3%. Then, observe the hedging performance by changing one parameter once each time. All simulation values in this section are the calculated results of 100,000 simulations.

First, the study discusses the value changes of the portfolio under the default-free model. Table II shows the statistics of the portfolio when σ changes. The summary of efficiency in Table III can be obtained by sorting or compiling the data in Table II according to the hedging effectiveness equation defined in Section III.B. Combining Tables II and III, we can obtain the following results:

Table II
The Statistics of the Portfolio Value under the Default-Free Model Changes in σ

σ	15%	20%	25%	30%	35%
Expected Value	911.5407	854.8274	814.3822	786.0064	765.7169
Unhedged Portfolio					
Mean	911.4765	854.8427	815.6211	786.2502	765.1801
S.D.	185.4393	292.5830	422.0137	576.4241	781.9426
Min.	147.2303	72.4830	29.3132	11.5425	6.6919
Q_1	851.4761	653.0751	515.9217	400.1990	309.3331
Q_2	1000	1000	899.1153	741.4540	625.0251
Q_3	1000	1000	1000	1000	1000
Max.	3529.9710	5580.9091	9003.0812	15655.0680	26878.6460
$VaR_{0.01}$	549.3685	620.5680	668.1775	693.6836	707.7421
$E(L\|L>0)$	261.3153	288.7020	325.4382	363.8153	397.8850
$SD(L\|L>0)$	146.2356	168.7354	182.6399	189.4456	192.6728
Hedged Portfolio & No Roll-Over					
Mean	911.5464	854.9845	815.2322	785.4511	765.1436
S.D.	157.1519	243.8460	360.4352	506.5461	710.0717
Min.	506.8758	485.3113	457.4728	426.3783	393.8883
Q_1	838.6226	656.0753	556.4460	484.1940	423.2266
Q_2	987.1465	965.5820	836.8588	661.4667	557.5009
Q_3	987.1465	965.5820	937.7435	906.6490	874.1590
Max.	3517.1175	5546.4912	8940.8247	15561.7170	26752.8060
$VaR_{0.01}$	375.3267	354.7377	357.7594	359.6281	371.8286
$E(L\|L>0)$	223.8871	225.7248	240.0481	261.2636	285.9833
$SD(L\|L>0)$	98.1956	97.2955	94.6721	92.0323	91.4042
Hedged Portfolio & Roll-Over					
Mean	911.5055	855.1572	815.1572	785.5737	765.9507
S.D.	166.0945	254.9740	370.6756	515.0784	716.7196
Min.	385.0527	281.6609	185.4806	54.3769	-28.8279
Q_1	813.0470	645.4117	567.1323	500.2541	442.6093
Q_2	996.9883	973.5080	796.5861	649.1678	589.7327
Q_3	999.6605	996.6540	988.4666	972.9217	946.6737
Max.	3507.7285	5574.0301	8975.7354	15546.5410	26834.0662
$VaR_{0.01}$	413.7981	446.2915	487.2403	533.4128	593.0384
$E(L\|L>0)$	237.0348	238.7502	251.9552	269.5055	289.8804
$SD(L\|L>0)$	102.1369	108.8570	115.8435	124.7496	136.1843

(a) These three have roughly equal expected values for portfolio, and their results meet the intuitive expectation because, on average, the hedging earnings (the value of due PDI) will be offset by hedging costs at the beginning of the period.

(b) As Table II shows, the value of CoCo will decrease as σ increases, while risk indexes, such as the standard deviation and VaR, will increase progressively. Comparing these three portfolios, however, we find that the standard deviation, the VaR, and average loss of the combination with hedging are all smaller than those of the combination without hedging; they also increase more slightly as σ increases, indicating that the hedging strategy may be unable to improve the value of portfolio but may lower the risk and expected loss.

(c) Comparing the hedging performance in Table III shows that, when no roll-over is involved, all hedging performances improve as σ increases, except for HE_1. The damping can reach nearly 50%, especially when 1% VaR fluctuates wildly; when roll-over is involved, however, hedging performances increase and then decrease.

Table III

The Hedging Effectiveness of the Default-Free Model Changes in *σ*

The hedging effectiveness is calculated by the value of CoCo's portfolio. HE_1 is the reduction degree of variances; HE_2 is the 1% VaR reduction; HE_3 is the expected loss; and HE_4 is the expected loss variation. The higher value represents the better hedging effectiveness.

σ	Hedging Effectiveness	Unhedged Portfolio vs. Hedged Portfolio & No Roll-Over	Unhedged Portfolio vs. Hedged Portfolio & Roll-Over
15%	HE_1	28.18%	19.78%
	HE_2	31.68%	24.68%
	HE_3	14.32%	9.29%
	HE_4	54.91%	51.22%
20%	HE_1	30.54%	24.06%
	HE_2	42.84%	28.08%
	HE_3	21.81%	17.30%
	HE_4	66.75%	58.38%
25%	HE_1	27.66%	23.47%
	HE_2	46.55%	26.77%
	HE_3	26.49%	23.13%
	HE_4	73.05%	59.78%
30%	HE_1	22.78%	20.15%
	HE_2	48.16%	23.10%
	HE_3	28.19%	25.92%
	HE_4	76.40%	56.64%
35%	HE_1	17.54%	15.99%
	HE_2	47.46%	16.21%
	HE_3	28.12%	27.14%
	HE_4	77.49%	50.04%

Tables IV and V show the statistics and hedging effectiveness of the portfolio when T changes. T increases by five years every time, since T=10 years. These two tables present the following results:

(a) The value of CoCo will decrease progressively as T increases but progressively more slowly in range, while various risk indexes will increase progressively. As for hedging without roll-over, the risk is smaller than that without

Table IV
The Statistics of the Portfolio Value under the Default-Free Model Changes in T

T	10 Years	15 Years	20 Years	25 Years	30 Years
Expected Value	814.3822	801.3075	794.4350	790.3438	787.7215
Unhedged Portfolio					
Mean	813.7388	801.6085	794.1915	791.1034	787.8670
S.D.	420.4500	578.1913	753.5113	937.8420	1112.0860
Min.	25.4210	12.4151	7.1008	5.0797	2.2450
Q_1	513.7884	424.8176	362.7615	312.2607	271.9202
Q_2	893.3538	790.0467	719.4258	661.1599	602.7148
Q_3	1000	1000	1000	1000	1000
Max.	10394.0521	20772.5020	49314.7600	51934.0490	66078.7990
$VaR_{0.01}$	667.6884	702.9806	723.2762	733.9526	744.9290
$E(L\|L>0)$	325.6178	365.2638	393.6279	417.2470	439.4138
$SD(L\|L>0)$	182.6642	192.7927	198.5082	202.1054	202.1624
Hedged Portfolio & No Roll-Over					
Mean	814.2327	802.3388	793.8653	790.6748	787.9366
S.D.	357.6030	524.5552	709.9451	902.6570	1083.5050
Min.	457.4728	378.9033	316.5279	265.8762	224.1886
Q_1	553.3850	477.8765	415.5011	364.8494	317.0222
Q_2	831.0973	724.7557	676.7655	628.2597	581.1654
Q_3	937.7435	931.0696	930.6443	933.3730	937.6814
Max.	10331.7960	20703.5720	49245.4000	51867.4230	66016.4806
$VaR_{0.01}$	356.9094	422.4042	477.9071	524.4676	563.5329
$E(L\|L>0)$	239.3517	281.8047	318.3619	350.4856	382.9486
$SD(L\|L>0)$	94.8350	115.6342	134.3960	149.5187	157.5623
Hedged Portfolio & Roll-Over					
Mean	814.6031	803.4142	793.8624	789.8438	787.8120
S.D.	367.8050	529.9627	711.0627	900.7967	1079.1390
Min.	189.0190	12.7269	-141.8857	-216.3867	-277.6121
Q_1	566.7260	506.0590	454.4600	409.4726	367.8744
Q_2	791.1481	700.1298	659.8658	634.4201	611.4211
Q_3	988.3416	986.3740	984.1785	982.1006	978.8160
Max.	10366.6160	20699.7530	49250.1900	51894.3020	66053.8768
$VaR_{0.01}$	488.9793	590.5977	676.4256	743.5447	806.9051
$E(L\|L>0)$	250.3084	286.0148	316.8359	345.3929	371.5021
$SD(L\|L>0)$	115.8377	144.8411	168.7717	188.1485	201.9672

hedging; it is roughly the same when there is roll-over, except that the risk of VaR is greater than that without hedging when T=30 years.

(b) Unlike the changes in fluctuation, hedging performances decrease progressively as T increases. Negative performances occur when there is roll-over, meaning that hedging effectiveness is poor when the period is long, and the risk may even be greater than it is without hedging.

By comparing the results of the changes in σ and T, we can conclude that all the risk indexes will increase when either of the two parameters increases, but the changes in these two parameters have counter effects on hedging performance: Efficiency will decrease if the due date increases, likely because, as these two increase, the probability of CoCo conversion grows, and CoCo performance becomes more like a stock price, while risk indexes increase. Nevertheless, under risk-neutrality, the stock price grows exponentially by the risk-free interest rate; thus, the mean value of the stock price due date will increase over time, and downside risks will be offset, worsening hedging performances. However, when the fluctuation range changes, the mean value of the stock price at the due date will be a fixed value, and risk indexes are influenced only by the fluctuation. Hedging performances will therefore improve as the fluctuation increases.

Table V

The Hedging Effectiveness of the Default-Free Model Changes in *T*

T (Years)	Hedging Effectiveness	Unhedged Portfolio vs. Hedged Portfolio & No Roll-Over	Unhedged Portfolio vs. Hedged Portfolio & Roll-Over
10	HE_1	27.66%	23.47%
	HE_2	46.55%	26.77%
	HE_3	26.49%	23.13%
	HE_4	73.05%	59.78%
15	HE_1	17.69%	15.99%
	HE_2	39.91%	15.99%
	HE_3	22.85%	21.70%
	HE_4	64.03%	43.56%
20	HE_1	11.23%	10.95%
	HE_2	33.92%	6.48%
	HE_3	19.12%	19.51%
	HE_4	54.16%	27.72%
25	HE_1	7.36%	7.74%
	HE_2	28.54%	-1.31%
	HE_3	16.00%	17.22%
	HE_4	45.27%	13.33%
30	HE_1	5.07%	5.84%
	HE_2	24.35%	-8.32%
	HE_3	12.85%	15.46%
	HE_4	39.26%	0.19%

Next, this study examines the effect of parameter changes on hedging performances if banks go bankrupt without warning. First, the study analyzes the fixed default intensity model, whose basic parameter group is identical to that of the default-free model. We must also assume the default chance of the banks, however. We take the success probability of the Bernoulli experiments, p =1%, as the reference. The probability of bank defaults is thus 1% within a year. As with the simulation method described in Section IV.A., we must first simulate the value of PDI at the beginning of the period in order to calculate the hedging cost in Equation (10). The simulation results of the PDI are listed in Table VI. The estimated values of PDI under different fluctuations, due dates, and default chances are also listed and one parameter is changed once each time. In Table VI, the values in parentheses are simulation standard errors, and the results of 100,000 simulation iterations are sufficiently accurate. It is obvious that the PDI grows as p increases, meaning that corporations might suddenly go bankrupt; if they do, hedging costs will be higher.

Table VI

The PDI Price under Fixed Default Intensity Model

p=1−$P(\tau > 1)$ is default probability in one year. The PDI price is estimated by the Monte Carlo simulation and the number in the parentheses is standard errors. The basic parameters are S_0=\$100, C_P=\$100, PDI strike price K=\$70, S^*=\$70, T=10, σ=25%, risk-free interest rate r_f=3%, and coupon rate c=3%.

Panel A. σ=25%

	T=10	T=15	T=20	T=25	T=30
p=0	6.21977 (0.03473)	6.89283 (0.03541)	6.93754 (0.03380)	6.66759 (0.03114)	6.22846 (0.02832)
p=0.001	6.55526 (0.03728)	7.30906 (0.03770)	7.41435 (0.03594)	7.18171 (0.03322)	6.71845 (0.03012)
p=0.005	7.89717 (0.04521)	8.92369 (0.04516)	9.28952 (0.04301)	9.06272 (0.03937)	8.71985 (0.03575)
p=0.01	9.69408 (0.05332)	11.03679 (0.05242)	11.47727 (0.04896)	11.38456 (0.04453)	10.86545 (0.03973)
p=0.02	12.94022 (0.06402)	15.01409 (0.06136)	15.58205 (0.05585)	15.51401 (0.04944)	14.88854 (0.04295)
p=0.05	22.05141 (0.07883)	24.9142 (0.06888)	25.28994 (0.05697)	24.34554 (0.04598)	22.73112 (0.03629)

Panel B. T=10

	σ=15%	σ=20%	σ=25%	σ=30%	σ=35%
p=0	1.28737 (0.01359)	3.43148 (0.02465)	6.21977 (0.03473)	9.34549 (0.04339)	12.60602 (0.05035)
p=0.001	1.71558 (0.02041)	3.85502 (0.02884)	6.55526 (0.03728)	9.65306 (0.04510)	12.86004 (0.05156)
p=0.005	3.55446 (0.03683)	5.41217 (0.04033)	7.89717 (0.04521)	10.89369 (0.05089)	13.90417 (0.05548)
p=0.01	5.79345 (0.04894)	7.38515 (0.05041)	9.69408 (0.05332)	12.31513 (0.05642)	15.26433 (0.05983)
p=0.02	9.90294 (0.06350)	11.14791 (0.06328)	12.94022 (0.06402)	15.24498 (0.06526)	17.81711 (0.06638)
p=0.05	20.88933 (0.08045)	21.33781 (0.07994)	22.05141 (0.07883)	23.37754 (0.07763)	24.96908 (0.07627)

As the value of PDI is estimated, the second phase of the simulation analyzes the effect of various parameters' changes on hedging performances. We discuss below the effect when there are changes in the fluctuation, due date, and default chance. Tables VII and VIII present statistics on and the hedging effectiveness of the portfolio under different σ when the default chance is 1% annually. We can draw a conclusion similar to that drawn when no jump is involved. The difference is that hedging performances improve sharply when fluctuation is slight and then decrease progressively as the fluctuation increases. Nevertheless, on average, hedging effectiveness is always superior to that when there is no default. For example, the VaR reduction is higher than 50% on average.

Table VII
The Statistics of the Portfolio Value under the Fixed Default Intensity Model Changes in σ (p=0.01)

σ	15%	20%	25%	30%	35%
Unhedged Portfolio					
Mean	859.7451	813.9465	779.6637	758.4871	740.5329
S.D.	290.0180	360.4347	470.3403	631.6481	850.2708
Min.	0	0	0	0	0
Q_1	810.2993	603.6897	456.5430	345.5935	253.8800
Q_2	1000.0000	1000.0000	885.2893	725.3629	599.7461
Q_3	1000.0000	1000.0000	1000.0000	1000.0000	1000.0000
Max.	3373.9060	7790.4990	11404.0200	26043.7400	37658.8800
$VaR_{0.01}$	844.9668	799.1682	764.8854	758.4871	740.5329
$E(L\|L>0)$	401.9705	369.9435	378.5997	398.3693	422.8279
$SD(L\|L>0)$	280.1959	246.6158	227.3517	215.9773	207.9838
Hedged Portfolio & No Roll-Over					
Mean	858.9191	813.6938	779.5503	758.0581	740.1363
S.D.	167.5601	253.2860	371.9458	536.1417	767.9314
Min.	461.7948	445.8778	422.7885	396.5780	367.0860
Q_1	752.3648	609.4904	508.2463	454.3938	396.4242
Q_2	942.0655	926.1485	788.3485	620.1927	521.0379
Q_3	942.0655	926.1485	903.0592	876.8487	847.3567
Max.	3315.9717	7716.6471	11307.0780	25920.5880	37506.2330
$VaR_{0.01}$	382.3460	353.0377	356.7618	361.4801	373.0503
$E(L\|L>0)$	234.5126	230.4876	242.4006	264.0172	288.0423
$SD(L\|L>0)$	96.8997	95.5545	92.8204	91.4546	90.8111

Table VIII
The Hedging Effectiveness of Fixed Default Intensity Model Changes in σ (p=0.01)

σ	15%	20%	25%	30%	35%
Hedging Effectiveness					
HE_1	66.62%	50.62%	37.46%	27.95%	18.43%
HE_2	54.75%	55.82%	53.36%	52.34%	49.62%
HE_3	41.66%	37.70%	35.97%	33.73%	31.88%
HE_4	88.04%	84.99%	83.33%	82.07%	80.94%

Tables IX and X show the statistics and hedging effectiveness when T changes. Effectiveness is slightly higher and decreases progressively as the due date increases, as before. It is worth noting that, as Table IX shows, from the 15th year, VaR with no hedging is equal to the value at the beginning of the period; in other words, all principals can be lost in the coming 15 years, while VaR with hedging is approximately 55% of the value at the beginning of the period, roughly half of the principal. Therefore, we can infer that hedging performance decreases progressively because the downside risks of the portfolio without hedging no longer increase (as maximum loss has been reached) when the due date is more than 15 years, while the combination with hedging continues to increase, resulting in lower efficiency.

Tables XI and XII present the statistics and hedging effectiveness when the default chance changes. As Table XI shows, when the default chance is extremely low (p=0.001), the performance will be very close to that without jumps; the difference is that the combination without hedging can lose all its principal (as the minimum value is zero) without warning, while the combination with hedging cannot. As Table XII indicates, hedging performances will improve as p increases, especially when the default chance is very high; nearly all indexes can reduce the efficiency by 50%, unlike in the scenario where other parameters change.

Table IX
The Statistics of the Portfolio Value under the Fixed Default Intensity Model Changes in T (p=0.01)

T	10	15	20	25	30
Unhedged Portfolio					
Mean	779.6637	757.2955	744.7361	738.0168	727.6196
S.D.	470.3403	639.0267	829.9937	1017.5780	1235.3750
Min.	0	0	0	0	0
Q_1	456.5430	326.1636	261.6204	211.9690	175.9730
Q_2	885.2893	753.9320	643.2999	546.5239	481.9108
Q_3	1000.0000	1000.0000	1000.0000	1000.0000	1000.0000
Max.	11404.0200	29776.3500	49349.4400	61562.4400	74644.2200
$VaR_{0.01}$	764.8854	757.2955	744.7361	738.0168	727.6196
$E(L\|L>0)$	378.5997	415.0729	435.2966	451.2094	458.3344
$SD(L\|L>0)$	227.3517	217.1888	209.1551	204.4016	200.9405
Hedged Portfolio & No Roll-Over					
Mean	779.5503	757.9174	744.6671	737.6531	728.8693
S.D.	371.9458	556.1433	763.7506	963.6242	1192.3100
Min.	422.7885	337.4658	271.1109	218.6577	177.8527
Q_1	508.2463	422.9236	356.5687	304.1155	252.1351
Q_2	788.3485	664.2795	600.3645	541.5257	488.6468
Q_3	903.0592	889.6321	885.2273	886.1544	891.3455
Max.	11307.0780	29665.9850	49234.6640	61453.7840	74535.5690
$VaR_{0.01}$	356.7618	420.4516	473.5562	518.9954	551.0166
$E(L\|L>0)$	242.4006	283.6333	318.4252	349.4191	376.1095
$SD(L\|L>0)$	92.8204	114.2206	132.7513	149.0244	156.0618

Table X
The Hedging Effectiveness of Fixed Default Intensity Model Changes in *T* (*p*=0.01)

T / Hedging Effectiveness	10	15	20	25	30
HE_1	37.46%	24.26%	15.33%	10.32%	6.85%
HE_2	53.36%	44.48%	36.41%	29.68%	24.27%
HE_3	35.97%	31.67%	26.85%	22.56%	17.94%
HE_4	83.33%	72.34%	59.72%	46.84%	39.68%

Table XI
The Statistics of the Portfolio Value under the Fixed Default Intensity Model Changes in *p*

p	0.001	0.005	0.01	0.02	0.05
Unhedged Portfolio					
Mean	810.0890	797.4114	779.6637	744.0816	637.6816
S.D.	424.0974	448.2233	470.3403	510.4127	607.5133
Min.	0	0	0	0	0
Q_1	509.3965	490.1095	456.5430	357.5661	98.9732
Q_2	894.0374	892.5547	885.2893	862.0372	675.9586
Q_3	1000.0000	1000.0000	1000.0000	1000.0000	1000.0000
Max.	11054.9300	10545.9700	11404.0200	11991.4600	11217.5100
$VaR_{0.01}$	697.8001	782.6331	764.8854	744.0816	637.6816
$E(L\|L>0)$	329.3975	350.4555	378.5997	425.7975	483.7874
$SD(L\|L>0)$	188.9843	210.6300	227.3517	234.9441	163.1633
Hedged Portfolio & No Roll-Over					
Mean	810.4805	797.7165	779.5503	744.8701	638.0286
S.D.	357.8982	367.5530	371.9458	384.6262	442.2449
Min.	454.1767	440.7576	422.7885	390.3271	299.2152
Q_1	547.1115	533.3397	508.2463	475.7849	370.9549
Q_2	826.2084	816.1527	788.3485	732.6350	525.1331
Q_3	934.4474	921.0283	903.0592	870.5978	779.4859
Max.	10989.3740	10467.0030	11307.0780	11862.0530	10996.9910
$VaR_{0.01}$	356.3038	356.9589	356.7618	354.5430	338.8134
$E(L\|L>0)$	239.6493	240.6099	242.4006	243.5444	238.5273
$SD(L\|L>0)$	94.0349	93.6727	92.8204	90.8218	81.6136

Table XII
The Hedging Effectiveness of Fixed Default Intensity Model Changes in *p*

p / Hedging Effectiveness	0.001	0.005	0.01	0.02	0.05
HE_1	28.78%	32.76%	37.46%	43.21%	47.01%
HE_2	48.94%	54.39%	53.36%	52.35%	46.87%
HE_3	27.25%	31.34%	35.97%	42.80%	50.70%
HE_4	75.24%	80.22%	83.33%	85.06%	74.98%

Two other results are especially noteworthy. First, apart from the four risk indexes, the changes in the minimum value and Q_1 in the statistics tables indicate that, when defaults are not involved and the due date is very long (or the fluctuation is strong), the value of the combination with hedging can still be $300 to $400 in the worst-case scenarios, even when the minimum value of CoCo is a single digit. When defaults are involved, regardless of the due date and fluctuation hypothesis, the minimum value of the combination without hedging is always zero, while the value of the combination with hedging can still reach $200 to $300 in the worst-case scenarios. In some circumstances, Q_1 of the combination without hedging is even smaller than the minimum value of the combination with hedging. For example, when σ=35%, Q_1 of the combination without hedging is $253.88, while the minimum value of the combination with hedging is $367.09. This fact indicates that, through hedging, we can eliminate the numerous tail risks and fix the loss at a certain level to avoid losing the entire principal, meaning that hedging by derivatives is effective. Second, we see according to the changes in the maximum value or Q_3 that the combination with hedging performs slightly poorer than the combination without hedging because the stock price will rise after CoCos are converted into stock, or the CoCos will not default or will not be converted before the due date. At this moment, the PDI for hedging will become expenses of portfolio instead of being executed to reduce their value. In other words, if banks' business operations keep improving, investors will gain fewer profits if they perform hedging, which can be regarded as a sacrifice investors must make to obtain the benefits of hedging.

Finally, we analyze the two-stage default intensity model. Table XIII presents the value statistics of the portfolio under different reduced default chances after CoCo conversion under the hypothesis that the issuing banks' annual default chance before conversion is 5%. As Table XIII shows, when CoCo receives no hedging, the average loss gradually decreases, and the Q_1 of the CoCo value gradually increases as the post-conversion default chance decreases. Under the risk-neutral measurement, the lowering of default chances is accompanied by a decline in the risk premium; thus, the maximum value of CoCo will decrease as the default chance decreases as well. Considering the hedging portfolio of CoCo, we find that the average loss and downside risk are both significantly smaller than the value of CoCo without hedging, while both the minimum value and Q_1 are significantly higher than the value without hedging, but Q_3 and the maximum value are smaller than the value of CoCo without hedging, indicating that the hedging portfolio can indeed achieve the effect of hedging, while the hedging cost will reduce CoCo profits when there is no default.

Although the setting of CoCo conversion can lower the issuing banks' default chance, the expected CoCo price and the hedging portfolio remain at the same level because the value of converted CoCo is determined by the stock price; given the neutral risk characteristic of the stock price, the value of the CoCo will not be affected by the reduced default intensity after the default intensity is adjusted by the corresponding risk premium, and only the price variation will be affected.

Table XIII
The Statistics of the Portfolio Value under the Two-Stage Default Intensity Model Changes in *p*

p before Conversion	0.05	0.05	0.05	0.05	0.05
p after Conversion	0.05	0.02	0.01	0.005	0.001
Unhedged Portfolio					
Mean	636.0863	635.8477	636.4145	636.5042	636.6918
S.D.	603.1781	527.3226	503.8415	492.4226	483.7085
Min.	0	0	0	0	0
Q_1	98.97321	151.0668	175.9730	183.0362	188.1501
Q_2	670.3802	666.5651	659.8181	655.6240	652.6854
Q_3	1000.0000	1000.0000	1000.0000	1000.0000	1000.0000
Max.	15067.3800	11777.3700	10867.7200	10442.8900	10116.5500
$VaR_{0.01}$	636.0863	635.8477	636.4145	636.5042	636.6918
$E(L\|L>0)$	483.6654	438.3665	420.8723	412.0122	404.9498
$SD(L\|L>0)$	161.7996	179.9631	182.9609	183.3440	183.2298
Hedged Portfolio & No Roll-Over					
Mean	637.5651	637.8368	636.6071	636.9447	636.6919
S.D.	435.7423	362.3261	341.4469	331.5232	324.3074
Min.	299.2152	322.0505	328.1983	332.2541	334.9138
Q_1	370.9549	393.7902	399.9380	403.9938	406.6535
Q_2	522.8299	545.6652	551.8130	551.5175	549.1224
Q_3	779.4859	802.3212	808.4690	812.5248	815.1845
Max.	14846.8670	11579.6870	10676.1850	10255.4190	9931.7299
$VaR_{0.01}$	338.3499	315.7863	308.4088	304.6906	301.7781
$E(L\|L>0)$	237.7723	214.6149	207.3773	203.8837	201.0904
$SD(L\|L>0)$	81.8339	79.7091	78.9675	78.5444	78.2841

Table XIV
The Hedging Effectiveness of the Two-Stage Default Intensity Model Changes in *p*

p after Conversion	0.05	0.02	0.01	0.005	0.001
Hedging Effectiveness					
HE_1	47.81%	52.79%	54.07%	54.67%	55.05%
HE_2	46.81%	50.34%	51.54%	52.13%	52.60%
HE_3	50.84%	51.04%	50.73%	50.52%	50.34%
HE_4	49.42%	55.71%	56.84%	57.16%	57.28%

Concerning hedging performance, Table XIV shows that four hedging indexes have hedging performances of close to over 50%. HE_3 does not change as the post-conversion default chance changes, while the other three hedging indexes have better hedging effects as the post-conversion default chance changes more markedly.

C. Risk Control

To highlight that the CoCo market can reduce investment risks and even further control both risks and rewards through derivatives, this section explains how to achieve the objective of risk control with different hedging points. The risk management strategy adopted in this paper is designed to create a hedging portfolio with CoCos by using the PDI. The situational analysis in the previous section assumes a PDI strike price of K=70 because the conversion trigger is set at 70; thus, hedging begins instantly after CoCos are converted. In fact, we can regulate and control the CoCo's risk and rewards by selecting different FDI strike prices when establishing the hedging portfolio. This paper analyzes the effect of different strike prices on the value of portfolio under the basic parameter set by using the default-free risk model and the fixed default intensity model respectively.

Table XV presents the statistics of the portfolio when the PDI strike price changes under the default-free model. The table shows that the change in PDI strike price is irrelevant to the value of CoCos when there is no hedging. When there is hedging but no roll-over, the higher the strike price is, the greater the minimum value of the hedging portfolio will be, and the smaller the VaR and downside risks will be, indicating that the downside risks of the portfolio are reduced. On the other hand, although the expected value of the hedging portfolio decreases only slightly, Q_2 , Q_3, and the maximum value decrease significantly as the strike price increases, suggesting that the maximum profit of the CoCo portfolio has an inverse relationship with the strike price. When there is hedging with roll-over, the higher the PDI strike price is, the lower the expected value and maximum profit of the portfolio will be. For risks, when the strike price is lower than the conversion price, the higher the strike price is, the smaller the downside risks will be. However, when the strike price is higher than the conversion price, the damping of downside risks will gradually become smaller based on the roll-over cost.

Table XVI shows the statistics of the portfolio under different PDI strike prices by the fixed default intensity model when the default chance is p=0.01. The result is the same: The higher the strike price is, the lower the maximum profit and downside risks will be. However, the expected value of the portfolio does not change significantly, indicating that the PDI strike price is used mainly to regulate and control the overall risks of the portfolio.

Table XV
The Statistics of the Portfolio Value under the Default-Free Model Changes in *K*

K	90	80	70	60	50
Unhedged Portfolio					
Mean	815.6211	815.6211	815.6211	815.6211	815.6211
S.D.	422.0137	422.0137	422.0137	422.0137	422.0137
Min.	29.3132	29.3132	29.3132	29.3132	29.3132
Q_1	515.9217	515.9217	515.9217	515.9217	515.9217
Q_2	899.1153	899.1153	899.1153	899.1153	899.1153
Q_3	1000.0000	1000.0000	1000.0000	1000.0000	1000.0000
Max.	9003.0812	9003.0812	9003.0812	9003.0812	9003.0812
$VaR_{0.01}$	668.1775	668.1775	668.1775	668.1775	668.1775
$E(L\|L>0)$	325.4382	325.4382	325.4382	325.4382	325.4382
$SD(L\|L>0)$	182.6399	182.6399	182.6399	182.6399	182.6399
Hedged Portfolio & No Roll-Over					
Mean	814.8693	815.0952	815.2322	815.3060	815.4187
S.D.	321.8656	340.9943	360.4352	378.9102	395.105
Min.	548.1399	504.7571	457.4728	405.4409	348.2966
Q_1	619.8796	586.5694	556.4460	524.7158	507.4898
Q_2	794.5325	809.8961	836.8588	859.0739	876.1767
Q_3	879.9166	910.7808	937.7435	959.9586	977.0614
Max.	8882.9978	8913.8621	8940.8247	8963.0398	8980.1427
$VaR_{0.01}$	266.7293	310.3380	357.7594	409.8652	467.1221
$E(L\|L>0)$	174.1577	208.0779	240.0481	268.5791	291.7234
$SD(L\|L>0)$	73.1393	81.2652	94.6721	113.1810	134.5359
Hedged Portfolio & Roll-Over					
Mean	812.2197	814.3478	815.1572	815.2804	815.4688
S.D.	324.7929	344.0251	370.6756	394.2205	409.4661
Min.	111.6821	175.8355	185.4806	176.0839	154.2559
Q_1	648.6798	615.5518	567.1323	519.2227	487.5795
Q_2	786.0616	763.9034	796.5861	845.0064	879.2684
Q_3	930.2920	963.2516	988.4666	998.1004	999.8468
Max.	8936.7406	8932.5309	8975.7354	8996.0730	9002.1046
$VaR_{0.01}$	452.9619	456.5202	487.2403	516.2241	544.0072
$E(L\|L>0)$	169.0785	204.5960	251.9552	288.0838	311.4227
$SD(L\|L>0)$	117.7656	112.9990	115.8435	129.0020	147.4730

Table XVI
The Statistics of the Portfolio Value under the Fixed Default Intensity Model Changes in K (p=0.01)

K	90	80	70	60	50
Unhedged Portfolio					
Mean	779.5447	779.5447	779.5447	779.5447	779.5447
S.D.	468.6014	468.6014	468.6014	468.6014	468.6014
Min.	0	0	0	0	0
Q_1	458.2073	458.2073	458.2073	458.2073	458.2073
Q_2	884.8927	884.8927	884.8927	884.8927	884.8927
Q_3	1000.0000	1000.0000	1000.0000	1000.0000	1000.0000
Max.	8956.1130	8956.1130	8956.1130	8956.1130	8956.1130
$VaR_{0.01}$	764.7664	764.7664	764.7664	764.7664	764.7664
$E(L\|L>0)$	377.2508	377.2508	377.2508	377.2508	377.2508
$SD(L\|L>0)$	227.1311	227.1311	227.1311	227.1311	227.1311
Hedged Portfolio & No Roll-Over					
Mean	780.5347	780.7964	780.8272	780.8654	780.9101
S.D.	331.3487	350.5669	370.4425	389.8792	407.9422
Min.	511.8403	469.2806	423.4478	373.5630	319.4083
Q_1	583.5799	541.0203	508.9056	485.8518	457.7410
Q_2	750.7511	760.1970	788.6112	812.9734	833.0658
Q_3	843.6169	875.3043	903.7185	928.0807	948.1730
Max.	8799.7302	8831.4176	8859.8318	8884.1940	8904.2863
$VaR_{0.01}$	267.7045	310.2641	356.0970	405.9818	460.1365
$E(L\|L>0)$	175.4369	209.5439	242.6206	272.7882	299.4623
$SD(L\|L>0)$	73.0696	80.5531	92.3361	109.4148	129.6468

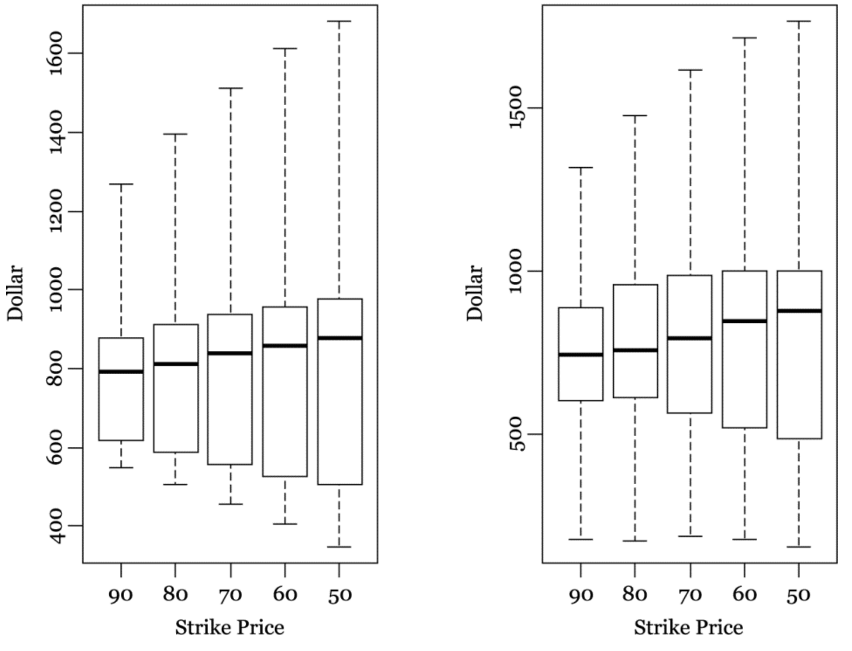

Figure 1. The Growth in the Risk of CoCo Portfolio under the Default-Free Model Following the Changes in PDI Strike Prices

The left panel presents the simulation results in the case of hedged and no roll-over; the right panel shows the simulation results in the case of hedged with roll-over.

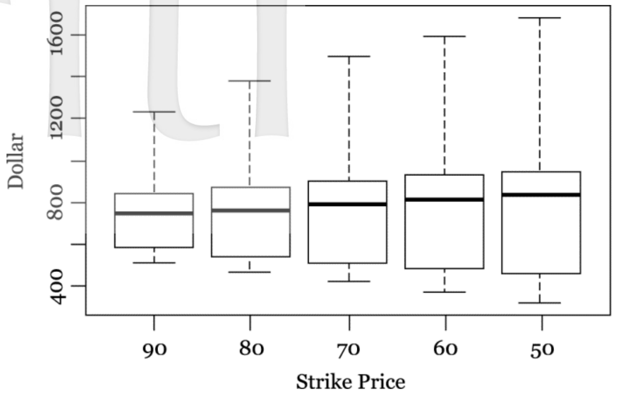

Figure 2. The Growth in the Risk of CoCo Portfolio under the Fixed Default Intensity Model Following the Changes in PDI Strike Prices

Figures 1 and 2 present the relationship of growth in the risk of CoCo portfolio following the changes in the PDI strike price, which is ignoring the extreme values[8] of the simulation results in Tables XV and XVI. They clearly show that different hedging points can indeed influence the value of CoCo portfolio, indicating that CoCos can manage the risk of CoCos through derivatives, allowing investors with different risk attitudes to control risks according to their own risk preference. This can attract more investors as well as help promote and develop the CoCo market, thus stabilizing the overall financial market.

V. Conclusion

This study discusses the CoCo risk management strategy from the standpoint of investors. Taking the EDA as its basic framework, this paper conducts an efficiency analysis of CoCo hedging with derivatives using a Monte Carlo simulation and later introduces jumps to allow for sudden bank bankruptcies and observes the changes in hedging performance. Bankruptcy jumps are introduced because the stock price never reaches zero under the traditional Black-Scholes model; this means that banks never go bankrupt, which is obviously inconsistent with the reality in financial markets. Investors will overestimate the value of CoCos and underestimate the real hedging value if they ignore the risk of default. This paper also divides default-allowed models into fixed default rate models and fluctuant default rate models, thereby describing a price analysis that can improve the operating performance of banks and reduce the chance of default after CoCos reach their conversion trigger.

By analyzing risk management performances under different parameters, this study finds that reducing the standard deviation and value-at-risk through hedging by buying down-and-in puts is effective. When there is no default,

8 The extreme value here is a sample point greater than $Q_3 + 1.5IQR$ and smaller than $Q_1 - 1.5IQR$, in which $IQR = Q_3 - Q_1$.

hedging effectiveness will improve as the fluctuation increases. When default occurs, hedging effectiveness will be much higher than that when there is no default, and its performance will also improve as the default chance increases. Moreover, hedging can significantly reduce tail risks when defaults are allowed because, when corporations conduct defaults, CoCo investors will lose the principal entirely, but the end-of-period profits and losses can be locked up by put option hedging. This study also finds that, under some extreme hypotheses, where the due date is very long or the fluctuation is very wide, hedging will not be significantly effective if the roll-over strategy is considered and will not be cost-effective if transaction costs are also taken into account. Thus, considering that investors are unlikely to find long-dated options, they should decide whether to conduct static hedging according to current market conditions and the CoCo's provisions. It is inappropriate to conduct static hedging if the fluctuation is too wide or the due date is too long; other hedging strategies may have to be considered.

The development of the CoCo is crucial for the stabilization of the financial system, though its downside risks are higher than are those of common corporate bonds. Excessively speculative commodities attract only investors with certain risk preferences, and hinder the development of the CoCo market as a whole. The risk management results in this study indicate that investors can reduce the risks of CoCos through derivatives and that investors can even control risks according to their own risk preferences when investing in CoCos, which may encourage investors to participate in the CoCo market by hedging. Meanwhile, we recommend that underwriting institutions should establish several portfolio contracts with various risks and rewards using down-and-in put options and roll-over frequencies of different strike prices and then sell the contracts separately according to clients' varying risk preferences in order to promote investors' motivation to buy, market liquidity, and the future of the CoCo market.

Given the trend of using CoCos for financial supervision, the CoCo market is very likely to improve. Future studies should take an empirical direction. The accuracy of their evaluation models should be consistently tested, and the risk management strategies derived from the models should also be tested in real-world conditions. As long as the evaluations are accurate and the hedging tools are secure, the high interests of CoCos will attract more investors. Banks will then be able to improve their operational performance, and supervisory institutions will be able to overcome the "too big to fail" problem, creating a triple-win situation.

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或有可轉債之風險管理策略分析

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摘 要

或有可轉換債券(CoCo)是新興結構式商品。本文以投資人立場出發，探討 CoCo 風險管理策略。以股權衍生性法為架構，本文藉由蒙地卡羅分析選擇權靜態避險的績效，隨後加入跳躍項允許銀行突然違約，並觀察避險績效的變化。透過情境分析，本研究發現 CoCo 可透過股權衍生性商品調控其投資風險，且靜態避險能有效降低標準差與風險值。

關鍵詞：或有可轉換債券、靜態避險、避險績效、風險值

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