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An effective hybrid variance reduction method for pricing the Asian options and its variants

King-Jeng Lu [a], Chiung-Ju Liang [a], Ming-Hua Hsieh [b], Yi-Hsi Lee [c, d]

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Abstract

In this paper, we propose a variance reduction method that combines importance sampling and control variates to price European Arithmetic Asian options and its variants (i.e., Asian options plus knock-in or knock-out options) under the Black-Scholes model. The numerical results show that the proposed methods are especially efficient under the following scenarios: in the money, low volatility, more sampling dates, and higher barrier thresholds.

Keywords

Asian options; Barrier options; Variance reduction; Importance sampling; Control variates

1. Introduction

Asian options (i.e., average options) are strong path-dependent contingent claims whose payoffs depend on the average price of the underlying asset over a given time period. These average-style options have been very successful in the marketplace because they offer improved hedging possibilities to holders with vast exposure, and they reduce the possibility of marketplace manipulations near the expiry. The smoothing mechanism causes Asian options to have lower options are structured to suit the specific needs of the end users (Fabozzi et al., 2008, Milevsky and Posner, 1998, Nelken, 1996).

The majority of Asian option trading types in the OTC marketplace are European-style options, which are based on the arithmetic average prices of the underlying assets. Pricing arithmetic Asian options has been a persistent issue when the price of the underlying asset is lognormally distributed. However, the sum of lognormally-distributed variables no longer have a known distribution; therefore, there is not a closed-form solution (i.e., the Black-Scholes formula) for Asian options. Several approaches have been attempted to obtain the pricing formulas of Asian options under geometric Brownian motion (GBM), including analytical approximation methods (Chen and Lyuu, 2007, Chung et al., 2003, Geman and Yor, 1993, Ju, 2002, Lévy, 1992, Milevsky and Posner, 1998, Nielsen and Sandmann, 2003, Rogers and Shi, 1995, Vorst, 1992, Zhang, 2003), lattices and closely related PDE methods (Dai and Lyuu, 2009, Hsu and Lyuu, 2007, Hsu and Lyuu, 2011, Hull and White, 1993, Klassen, 2001, Ritchken et al., 1993, Vecer, 2001, Zvan et al., 1999), and Monte Carlo and related quasi-Monte Carlo simulation methods (Kemna and Vorst, 1990, Broadie and Glasserman, 1996, Boyle et al., 1997, Lapeyre and Temam, 2001, Glasserman, 2004, Imai and Tan, 2006, Dagpunar, 2019, Asmussen and Glynn, 2007, Boyle and Potapchik, 2008, Achtsis et al., 2013, Mehrdoust, 2015, Müller, 2016).

The mass customization of OTC contracts makes the Monte Carlo approach a competitive and computational tool compared to other approaches. The arithmetic average schema is the most common type of Asian option, but it also has well-known and difficult computing issues. Therefore, in this paper, we develop two novel variance reduction algorithms for pricing European arithmetic Asian options under a GBM background. The remainder of this paper is organized as follows. Section 2 defines the problem to be solved. Section 3 elaborates on the simulation algorithm developed in this study. Section 4 presents numerical results for three cases. Section 5 concludes the paper.

2. Problem formulation

Asian options are one type of path-dependent contingent claim whose payoffs depend on the average price of the underlying asset during the prescribed period of time, which is at least a portion of the life of the option. In general, the payoff of Asian options can be distinguished into four types of call/put and fixed/floating strikes (i.e., the average price [rate]/average strike): 1) the payoff of a fixed strike call is $[A-K]^+$; 2) the payoff of a fixed strike put is $[K-A]^+$; 3) the payoff of a floating strike call is $[A-S]^+$; and 4) the payoff of a floating strike put is $[S-A]^+$. A represents the average price of the underlying asset, K represents the fixed strike price, S represents the floating strike price, and $[\cdot]^+$ represents the unit ramp function. The average can be determined arithmetically or geometrically, and each lead to different types of Asian options. Another popular choice is the exponentially weighted average, which results in recent prices that are weighted more than past prices via an exponentially decreasing function.

Under the Black-Sholes framework, the price of the underlying asset at time t, which is denoted by $S(t)$, follows the standard risk-neutral GBM model, represents the risk-free rate, q denotes the yield of the underlying asset, dt represents the small time interval, σ represents the volatility of the underlying asset price, and $B(t)$ denotes the Brownian motion.

The process defined by Eq. (1) implies that the total return from the underlying asset, over any time period, is lognormally distributed (i.e., $\log S(t)$ is the normal distribution).

$$S(t)= S(0)e^{(v-q)t+\sigma B(t)}, \tag{2}$$

where $v = r - 1/2\sigma^2$ and

$$\log S(t)\sim\tilde{N}\left(\log S(t)+\left(r-q-\tfrac{1}{2}\sigma^2\right)t,\ \sigma^2 t\right). \tag{3}$$

As with most exotic options, Asian options can be priced in the familiar Black-Scholes framework. This means that the value of the option is equal to the discounted expectation of the payoff at maturity under risk-neutral measure. The premium of the arithmetic Asian call option is

$$Call = e^{-r\tau}\mathrm{E}^Q\left[\frac{1}{n+1}\sum_{i=0}^{n} S(t_i)-K,0\right]^+, \tag{4}$$

where $\mathrm{E}^Q[\cdot]$ represents the expectation under risk-neutral measure, K denotes the strike price, $t_0,\ t_1$,…, t_n represent the dates when the average is taken (i.e., the sampling dates), and $S(t_0),\ S(t_1)$,…, $S(t_n)$ represent the prices of the underlying asset at these dates. The payoff of the option occurs at date t_n. Hence, the valuation of the option amounts when evaluating the expectation is found with (4). From (3) it is clear that all $S(t_i)$ results have lognormal distributions, i.e., $\log S(t_i)$ has a normal distribution (2). However, the sum of the lognormally distributed variables, $\sum S(t_i)$, no longer has a known distribution. This is why analytic formulas, such as the Black-Scholes formula, do not exist for Asian options.

3. Methodology

This paper uses importance sampling plus control variates to price Asian options and its variants. Below we give the basic descriptions of variance reduction techniques that will be employed in this paper (Asmussen & Glynn, 2007) and elaborate on the simulation algorithm developed in this study.

3.1. Importance sampling (IS)

Suppose that we wish to estimate $\alpha = \mathrm{E}[X]$, where X represents the output of a complex stochastic system. A naïve Monte Carlo procedure would generate n independent copies of X and produce the standard estimate

$$\alpha_{\text{na\"i ve}} = \frac{1}{n}\sum_{i=1}^{n} X_i,$$

where $X_1, X_2, \ldots, X_n$ are independent copies of X.

Let P represent the probability measure governing the dynamics of X and let probability measure Q be an equivalent measure of P. Define where $X_Q^{(i)}$, $i = 1, \ldots, n$ are independent copies of X drawn from probability measure Q, and $L^{(i)}$, $i = 1, \ldots, n$ are Radon-Nikodym derivatives between probability measures P and Q. Therefore, it is clear that $\alpha_{IS}(Q)$ is also an unbiased estimate of α. It is possible to choose probability measure Q such that the variance of $\alpha_{IS}(Q)$ is much smaller than that of $\alpha_{\text{na\"i ve}}$. To apply importance sampling, it is vital to choose an appropriate probability measure (Q). In this paper, the main task is determining an appropriate probability measure, Q, for computing fair payout benefits.

3.2. Control variates (CV)

Suppose that we wish to estimate $\alpha = \mathbf{E}[X]$, where X represents the output of a complex stochastic system. Let $\mathbf{Y}$ be a $d \times 1$ random vector, where each component of $\mathbf{Y}$ is correlated with X. *Let* $\boldsymbol{\mu}$ **and** $\boldsymbol{\Sigma}$ denote the mean vector and the covariance matrix of $\mathbf{Y}$, respectively, where mean vector $\boldsymbol{\mu}$ is known. Suppose that the covariance between X and Y_i is c_i, and $\mathbf{c} = (c_1, c_2, \ldots, c_d)^T$. The control variates are defined as

$$\mathbf{C} = \mathbf{Y} - \boldsymbol{\mu}$$

It is clear that the mean vector of $\mathbf{C}$ is 0, the covariance matrix of $\mathbf{C}$ is $\boldsymbol{\Sigma}$, and the covariance between X and C_i is c_i. Now define

$$X_{\mathbf{C}}(\boldsymbol{\lambda}) = X - \boldsymbol{\lambda}^T \mathbf{C}.$$

It is clear that

$$\mathbf{E}[X_{\mathbf{C}}(\boldsymbol{\lambda})] = 0,$$

and

$$\mathrm{Var}[X_{\mathbf{C}}(\boldsymbol{\lambda})] = \sigma_X^2 - 2\boldsymbol{\lambda}^T \mathbf{c} + \boldsymbol{\lambda}^T \boldsymbol{\Sigma} \boldsymbol{\lambda}.$$

The above formula is minimized when

$$\boldsymbol{\lambda}^* = \boldsymbol{\Sigma}^{-1} \mathbf{c},$$

and

$$\mathrm{Var}[X_{\mathbf{C}}(\boldsymbol{\lambda}^*)] = \sigma_X^2 - 2\left(\boldsymbol{\Sigma}^{-1}\mathbf{c}\right)^T \mathbf{c} + \left(\boldsymbol{\Sigma}^{-1}\mathbf{c}\right)^T \boldsymbol{\Sigma} \left(\boldsymbol{\Sigma}^{-1}\mathbf{c}\right),$$

Hence,

$$\mathrm{Var}[X_{\mathbf{C}}(\boldsymbol{\lambda}^*)] = \sigma_X^2 - \mathbf{c}^T \boldsymbol{\Sigma}^{-1} \mathbf{c} < \sigma_X^2.$$

Let $X_{\mathbf{C}}^{(\mathrm{i})}(\boldsymbol{\lambda}^*)$, $i = 1, \ldots, n$ represent independent copies of $X_{\mathbf{C}}(\boldsymbol{\lambda}^*)$. Therefore, it is obvious that

$$\alpha_{control} = \frac{1}{n} \sum_{i=1}^{n} X_{\mathbf{C}}^{(\mathrm{i})}(\boldsymbol{\lambda}^*)$$

is a more efficient estimate of $\alpha_{\text{na\"i ve}}$. It is usually not possible to compute the exact value of $\boldsymbol{\lambda}^*$ because $\boldsymbol{\Sigma}$ and $\mathbf{c}$ are usually unknown. However, accurate estimates of $\boldsymbol{\Sigma}$ and $\mathbf{c}$ are easy to compute to find suitable control variates, C, for computing fair payout benefits.

3.3. The proposed algorithms

The proposed algorithm combines techniques from importance sampling and control variates. Let $\mathbf{X} = (X_0, X_1, \cdots, X_n)^T$ =(log $S(t_0)$, log$S(t_1)$, ..., log$S(t_n)$) be a Gaussian vector with mean vector $\boldsymbol{\mu} = (\mu_0, \mu_1, \cdots, \mu_n)^T$ and covariance matrix $\boldsymbol{\Sigma}$. We denote $\mathbf{X} \sim \mathbf{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ and set $Y_k = \exp(X_k)$ to define

$$g(X) = \frac{1}{n+1} \sum_{k=0}^{n} Y_k = \frac{1}{n+1} \sum_{k=0}^{n} e^{X_k}.$$

which results in the estimation of

$$V = e^{-rT} \mathrm{E}[g(X) - K]^+,$$

and its variants (for both knock-in barriers and knock-out barriers).

For the generation of random vector $\mathbf{X}$, we represent $\mathbf{X}$ as

$$\mathbf{X} = \boldsymbol{\mu} + \mathbf{CZ}, \tag{5}$$

where $\mathbf{C}$ represents a matrix satisfied by $\mathbf{CC}^T = \boldsymbol{\Sigma}$ **and** $\mathbf{Z} \sim \mathbf{N}(0, \mathbf{I}_{n+1})$. Let $(\lambda_1, \mathbf{q}_1), (\lambda_2, \mathbf{q}_2), \cdots, (\lambda_{n+1}, \mathbf{q}_{n+1})$ be eigenpairs of $\boldsymbol{\Sigma}$ and $\lambda_1 \geqslant \lambda_2 \geqslant \cdots \geqslant \lambda_{n+1}$. Suppose that the spectral decomposition of $\boldsymbol{\Sigma}$ is

$$\boldsymbol{\Sigma} = \sum_{k=1}^{d} \lambda_i \mathbf{q}_i \mathbf{q}_i^T = \mathbf{Q} \boldsymbol{\Lambda} \mathbf{Q}^T,$$

where $\boldsymbol{\Lambda} = diag(\lambda_1, \lambda_2, \ldots, \lambda_{n+1})$ is a diagonal matrix (i.e., eigenvalue matrix) and $\mathbf{Q} = (\mathbf{q}_1, \mathbf{q}_2, \ldots, \mathbf{q}_d)$ is an eigenvector matrix. It is easy to see that $\mathbf{Q}\boldsymbol{\Lambda}^{\frac{1}{2}}$ is a suitable candidate for $\mathbf{C}$.

We then separate $\mathbf{C}$ into $\left(\mathbf{c}_1, \widetilde{\mathbf{C}}\right)$. Submatrices $\mathbf{c}_1$ and $\widetilde{\mathbf{C}}$ have dimensions $(n+1) \times 1$ and $(n+1) \times n$, respectively. Additionally, $\mathbf{Z}$ is separated into $\left(Z_1, \widetilde{\mathbf{Z}}\right)^T$. Subvectors Z_1 and $\widetilde{\mathbf{Z}}$ have dimensions 1 and n, respectively. We can rewrite Equation (7) as

$$\mathbf{X} = \boldsymbol{\mu} + \mathbf{c}_1 Z_1 + \widetilde{\mathbf{C}} \widetilde{\mathbf{Z}}, \tag{6}$$

We can utilize Eq. (8) to provide a two-step procedure for generating replicates of $\mathbf{X}$. In particular, we can generate $\widetilde{\mathbf{Z}}$ first and then generate Z_1. In this way, we only select the importance sampling distribution for Z_1 given the sampling value of $\widetilde{\mathbf{Z}}$.

Given any simulated value of $\widetilde{\mathbf{Z}}$, $g(X)$ is a strictly monotone increasing function in Z_1. By solving a single variate root-finding problem, we can find a threshold value of b for Z_1, such that $g(X) > K$ for $Z_1 > b$. Therefore, if we generate Z_1 as a truncated standard normal with a truncated region $(-\infty, b)$, the generated $g(X)$ is always greater than K and Z_1 has a simple likelihood ratio $= 1 - \Phi(b) < 1$, *for*

We outline the algorithm of the proposed IS estimator below.

1. Input $\boldsymbol{\mu}, \boldsymbol{\Sigma}$, and N, where N represents the sample size
2. Compute $\mathbf{c}_1$ and $\widetilde{\mathbf{C}}$3. For $i = 1, 2, \ldots, N$
   I. Generate $\widetilde{\mathbf{Z}}$
   II. Compute the value of b such that $g\left(\boldsymbol{\mu} + \mathbf{c}_1 b + \widetilde{\mathbf{C}}\widetilde{\mathbf{Z}}\right) = K$
   III. Set $L^{(i)} = \Phi(-b) = 1 - \Phi(b)$, , where $L^{(i)}$ represents the likelihood ratio
   IV. Given the value of $\widetilde{\mathbf{Z}}$, simulate Z_1 as a truncated standard normal with a truncated region $(-\infty, \mathrm{b})$. Compute $V\left(\widetilde{\mathbf{Z}}, Z_1\right) = e^{-rT}\left[g\left(\boldsymbol{\mu} + \mathbf{c}_1 Z_1 + \widetilde{\mathbf{C}}\widetilde{\mathbf{Z}}\right) - K\right]^+$
   V. Set $L^{(i)} V\left(\widetilde{\mathbf{Z}}, Z_1\right)$ as an IS sample

The variability of the generated sample $V\left(\widetilde{\mathbf{Z}}, Z_1\right)$ in above algorithm can be further reduced by computing its expectation with respect to Z_1. To be precise, we compute

$$V\left(\widetilde{\mathbf{Z}}\right) = e^{-rT} E\left[g\left(\boldsymbol{\mu} + \mathbf{c}_1 Z_1 + \widetilde{\mathbf{C}}\widetilde{\mathbf{Z}}\right) - K\right]^+$$

for any simulated value of $\widetilde{\mathbf{Z}}$. Note that above expectation is easy to compute, because it involves only integrating one-dimensional lognormal random variables (all depends on Z_1 only) from b to ∞. The statistical efficiency of the algorithm can therefore be enhanced by the following modification:

1. Input $\boldsymbol{\mu}, \boldsymbol{\Sigma}$, and N, where N represents the sample size
2. Compute $\mathbf{c}_1$ and $\widetilde{\mathbf{C}}$
3. For $i = 1, 2, \ldots, N$

I. Generate $\widetilde{\mathbf{Z}}$

II. Compute the value of b such that $g\left(\boldsymbol{\mu} + \mathbf{c}_1 b + \widetilde{\mathbf{C}}\widetilde{\mathbf{Z}}\right) = K$

III. Set $V\left(\widetilde{\mathbf{Z}}\right) = e^{-rT} E\left[g\left(\boldsymbol{\mu} + \mathbf{c}_1 Z_1 + \widetilde{\mathbf{C}}\widetilde{\mathbf{Z}}\right) - K\right]^+$ as an sample

The above procedure can combine control variate techniques. In step 3, $\widetilde{\mathbf{Z}} \sim \mathbf{N}(0, \mathbf{I}_n)$ and $\widetilde{\mathbf{C}}\widetilde{\mathbf{Z}} \sim \mathbf{N}\left(0, \widetilde{\mathbf{C}}\widetilde{\mathbf{C}}^T\right)$. Each component of $\widetilde{\mathbf{Z}}$ is positively correlated with $V\left(\widetilde{\mathbf{Z}}\right)$, which results in a natural control variate. The exponential of each component for $\widetilde{\mathbf{C}}\widetilde{\mathbf{Z}}$ is a lognormal random variable, which is also obviously positively correlated with $V\left(\widetilde{\mathbf{Z}}\right)$. Therefore, we select two sets of control variates for numerical experiments. The first set consists of exponentials for each component of $\widetilde{\mathbf{C}}\widetilde{\mathbf{Z}}$ (hereafter denoted as the H1 algorithm). The second set consists of exponentials for each component of $\widetilde{\mathbf{C}}\widetilde{\mathbf{Z}}$ and each component of $\widetilde{\mathbf{Z}}$ (hereafter denoted as the H2 algorithm). The means of control variates in the H1 algorithms are zeros. The means of control variates in the H2

4. Numerical results

We examine the accuracy of our proposed algorithms under a GBM framework with three cases in Glasserman, Heidelberger, and Shahabuddin (1999) below, which are the pure Asian option, the Asian option with a knock-in barrier and the Asian option with a knock-out barrier. Table 1, Table 2, Table 3 show the comparative results between our methods and the methods of Glasserman et al. (1999) by variance reduction ratios (VRs). We assume that the expected CPU times of all methods except naïve Monte Carlo method for one replication are about equal. This is a conservative assumption. The modest additional effort required to decompose the covariance matrix in our method should be less than the additional effort required in the methods of Glasserman et al. (1999) for solving optimization problem, finding the optimal eigenvector, and determining the optimal allocation of replications in each stratum.

Table 1. Estimated Variance Reduction Ratios for the Asian Option.

n	σ	K	Premium	VR_H1 [1]	VR_H2 [2]	VR_G1 [3]	VR_G2 [4]	VR_D [5]	[1]/[3]	[1]/[4]	[2]/[3]	[2]/[4]	[1]/
16	0.1	45	6.05	9,469,812	55,990,097	1097	1246	6,294,659	8632	7600	51,039	44,936	1.5
		50	1.92	73,790	403,483	4559	5710	33,026	16	13	89	71	2.2
		55	0.20	31,813	139,014	15,520	17,026	258	2.0	1.9	9.0	8.2	12[illegible]
	0.3	45	7.15	88,870	337,969	1011	1664	7890	88	53	334	203	11
		50	4.17	45,840	147,489	1304	1899	2047	35	24	113	78	22
		55	2.21	37,669	105,905	1746	2296	519	22	16	61	46	73
64	0.1	45	6.00	14,657,913	67,010,338	967	1022	11,102,373	15,158	14,342	69,297	65,568	1.3
		50	1.84	87,680	405,431	4637	5665	29,757	19	15	87	72	2.9
		55	0.17	39,286	161,588	16,051	17,841	88	2.4	2.2	10.1	9.1	44[illegible]
	0.3	45	7.02	110,143	423,787	1016	1694	8745	108	65	417	250	13
		50	4.02	55,337	189,300	1319	1971	1820	42	28	144	96	30
		55	2.08	45,704	141,317	1767	2402	345	26	19	80	59	13[illegible]

All cases use $S(0)= 50$, $r = 0.05$, and $T = 1$; $S(0)$: the current price of the underlying asset; r: the risk-free rate; T: the time to maturity; n: the total number of sampling dates; Premium: the expected price by the naïve Monte Carlo simulation using 1,000,000 runs; VR_H1: variance reduction ratios by the 1st algorithm using 1000 runs;

(1999); VR_G2: variance reduction ratios by importance sampling combined with stratified sampling (100 stratum) and optimal eigenvectors from Glasserman et al. (1999) using 1,000,000 runs; VR_D: variance reduction ratios by the improved importance sampling approach from Dagpunar (2019).

Table 2. Estimated Variance Reduction Ratios for the Asian Option with a Knock-in Barrier.

n	σ	*K*	Premium	VR_H1 [1]	VR_H2 [2]	VR_G1 [3]	[1]/[3]	[2]/[3]
50	0.1	60	0.53	1582	6137	25	63	245
		70	0.02	1262	5079	992	1.3	5.1
		80	0.00	49,708	95,400	195,055	0.3	0.5
	0.3	60	3.14	629	4517	14	45	323
		70	2.07	2031	3063	16	127	191
		80	1.17	3308	8036	34	97	236
		100	0.30	1274	16,983	167	7.6	102
55	0.1	60	0.15	1387	4301	43	32	100
		70	0.01	2288	14,307	787	2.9	18.2
		80	0.00	58,463	117,829	154,406	0.4	0.8
	0.3	60	1.94	768	3690	41	19	90
		70	1.44	1029	3090	18	57	172
		80	0.89	3410	3931	34	100	116
		100	0.25	1797	31,935	157	11	203

All cases use $S(0)$= 50, $r = 0.05$, $T = 1$, and $n = 16$; $S(0)$: the current price of the underlying asset; r: the risk-free rate; T: the time to maturity; n: the total number of sampling dates; B: the barrier level; Premium: the expected price by the naïve Monte Carlo simulation using 1,000,000 runs; VR_H1: variance reduction ratios by the 1st algorithm using 1000 runs; VR_H2: variance reduction ratios by the 2nd algorithm using 1000 runs; VR_G1: variance reduction ratios by importance sampling combined with stratified sampling (100 stratum) and optimal μ from Glasserman et al. (1999) using 1,000,000 runs.

Table 3. Estimated Variance Reduction Ratios for the Asian Option with a Knock-out Barrier.

				[1]	[2]	[3]		
50	0.1	60	1.38	1323	5530	6.1	217	907
		70	1.90	26,137	114,078	240	109	475
		80	1.92	99,373	440,040	3864	26	114
	0.3	60	1.02	112	448	2.4	47	187
		70	2.10	944	1237	4.1	230	302
		80	2.99	2541	7977	8.9	286	896
		100	3.86	5192	40,982	46	113	891
55	0.1	60	0.05	247	519	9.1	27	57
		70	0.19	12,528	67,629	351	36	193
		80	0.20	44,924	174,712	12,988	3.5	13
	0.3	60	0.27	50	174	4.5	11	39
		70	0.77	304	636	4.5	68	141
		80	1.32	1954	2642	9.2	212	287
		100	1.96	4986	43,081	51	98	845

All cases use $S(0)= 50,\ r = 0.05,\ T = 1$, and $n = 16$; $S(0)$: the current price of the underlying asset; r: the risk-free rate; T: the time to maturity; n: the total number of sampling dates; B: the barrier level; Premium: the expected price by the naïve Monte Carlo simulation using 1,000,000 runs; VR_H1: variance reduction ratios by the 1st algorithm using 1,000 runs; VR_H2: variance reduction ratios by the 2nd algorithm using 1000 runs; VR_G1: variance reduction ratios by importance sampling combined with stratified sampling (100 stratum) and optimal μ from Glasserman et al. (1999) using 1,000,000 runs.

4.1. The Asian option

This case is extracted from Table 5.1 in Glasserman et al. (1999). The payoff of the Asian option is $[A - K]^+$. Table 1 shows that our H1 and H2 algorithms are superior to the G1 and G2 algorithms in Glasserman et al. (1999) and the D algorithm in Dagpunar (2019). Our algorithms relative to the Glasserman et al. (1999) algorithms are particularly efficient under in-the-money, low-volatility, and more sampling date scenarios. In addition, our algorithms are better than that of Dagpunar (2019) using the out-of-the-money scenario. The H2 algorithm exhibits a higher variance reduction performance than that of the H1 algorithm because it utilizes more correlated control variates than the H1 algorithm.

4.2. The Asian option with a Knock-in barrier

barrier feature $1_{\{S(t_n)\leqslant B\}}$ gives a buffer zone to both the holder and the writer; therefore, it can reduce the hedging cost (i.e., option premium). Table 2 shows the calculation results of the Asian option with a knock-in barrier[1]. The numerical results show that our proposed methods (H1 and H2) are superior to that (G1) proposed by Glasserman et al. (1999), except for the scenario which has a lower volatility ($\sigma = 0.1$) and a higher barrier level ($B = 80$). Glasserman et al. (1999) argues that the importance sampling method is particularly effective when pricing the knock-in option with a large threshold. If B is large, reaching B is a rare event. However, our approach outperforms theirs under higher volatility ($\sigma = 0.3$) and high threshold level ($B = 100$) scenarios. We did not compare the performance of algorithm G2 or algorithm D, because they are not applicable for Asian options with knock-in or knock-out features.

4.3. The Asian option with a Knock-out barrier

This case is extracted from Table 5.2 in Glasserman et al. (1999). The payoff is $[A - K]^{+}1_{\{S(t_n)>B\}}$, where B represents the barrier level, and $1_{\{\cdot\}}$ represents the indicator function. Table 3 shows the calculation results of the Asian option with a knock-out barrier[2]. Additional knock-out barrier feature $1_{\{S(t_n)>B\}}$ gives the protection threshold to the option writer; therefore, it can reduce the hedging cost (i.e., option premium). The numerical results show that our proposed methods (H1 and H2) are superior to that (G1) proposed by Glasserman et al. (1999). Our approach is particularly efficient in low-volatility and high barrier threshold scenarios.

5. Conclusions

This study has developed a novel variance reduction method, which combines importance sampling and control variates to price pure Asian options and its variants (i.e., Asian options plus knock-in or knock-out options). Our method has a conceptually convincing mechanism advantage for implementation compared to existing benchmark methods. The numerical results show that the proposed algorithms are especially efficient under in the money, low volatility, more sampling dates, and higher barrier threshold scenarios.

Appendix A. Supplementary data

The following are the Supplementary data to this article:

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Supplementary data 1.

References

Achtsis et al., 2013 N. Achtsis, R. Cools, D. Nuyens
Conditional sampling for barrier option pricing under the LT method
SIAM Journal on Financial Mathematics, 4 (1) (2013), pp. 327-352
CrossRef View Record in Scopus Google Scholar

Asmussen and Glynn, 2007 S. Asmussen, P.W. Glynn
Stochastic simulation: Algorithms and analysis: Algorithms and Analysis
Springer (2007)
Google Scholar

Boyle et al., 1997 P. Boyle, M. Broadie, P. Glasserman
Monte Carlo methods for security pricing
Journal of Economic Dynamics and Control, 21 (8–9) (1997), pp. 1267-1321
Article Download PDF View Record in Scopus Google Scholar

Boyle and Potapchik, 2008 P. Boyle, A. Potapchik
Prices and sensitivities of Asian options: A survey
Insurance: Mathematics and Economics, 42 (1) (2008), pp. 189-211
Article Download PDF View Record in Scopus Google Scholar

Broadie and Glasserman, 1996 M. Broadie, P. Glasserman
Estimating security price derivatives using simulation
Management Science, 42 (2) (1996), pp. 269-285
CrossRef View Record in Scopus Google Scholar

Chen and Lyuu, 2007 K.W. Chen, Y.D. Lyuu
Accurate pricing formulas for Asian options
Applied Mathematics and Computation, 188 (2) (2007), pp. 1711-1724
Article Download PDF View Record in Scopus Google Scholar

Chung et al., 2003 S.L. Chung, M. Shackleton, R. Wojakowski
Efficient quadratic approximation of floating strike Asian option values
Finance, 24 (1) (2003), pp. 49-62
View Record in Scopus Google Scholar

Dagpunar, 2019 Dagpunar, J. S. (2007). Novel importance sampling for the valuation of basket and Asian options, Available at SSRN: https://ssrn.com/abstract=1017836 or https://doi.org/10.2139/ssrn.1017836.
Google Scholar

Dai and Lyuu, 2009 T.S. Dai, Y.D. Lyuu
Accurate and efficient lattice algorithms for American-style Asian options with range bounds

Fabozzi et al., 2008 F.J. Fabozzi, R. Fuss, D.G. Kaiser
The handbook of commodity investing
Wiley (2008)
Google Scholar

Geman and Yor, 1993 H. Geman, M. Yor
Bessel processes, Asian options, and perpetuities
Mathematical Finance, 3 (4) (1993), pp. 349-375
CrossRef View Record in Scopus Google Scholar

Glasserman, 2004 P. Glasserman
Monte Carlo methods in financial engineering
Springer (2004)
Google Scholar

Glasserman et al., 1999 P. Glasserman, P. Heidelberger, P. Shahabuddin
Asymptotically optimal importance sampling and stratification for pricing path-dependent options
Mathematical Finance, 9 (2) (1999), pp. 117-152
View Record in Scopus Google Scholar

Hsu and Lyuu, 2007 W.W.Y. Hsu, Y.D. Lyuu
A convergent quadratic-time lattice algorithm for pricing European-style Asian options
Applied Mathematics and Computation, 189 (2) (2007), pp. 1099-1123
Article Download PDF View Record in Scopus Google Scholar

Hsu and Lyuu, 2011 W.W.Y. Hsu, Y.D. Lyuu
Efficient pricing of discrete Asian options
Applied Mathematics and Computation, 217 (24) (2011), pp. 9875-9894
Article Download PDF View Record in Scopus Google Scholar

Hull and White, 1993 J.C. Hull, A.D. White
Efficient procedures for valuing European and American path-dependent options
The Journal of Derivatives, 1 (1) (1993), pp. 21-31
CrossRef Google Scholar

Imai and Tan, 2006 J. Imai, K.S. Tan
A general dimension reduction technique for derivative pricing
Journal of Computational Finance, 10 (2) (2006), p. 129
CrossRef View Record in Scopus Google Scholar

Ju, 2002 N. Ju
Pricing Asian and basket options via Taylor expansion
Journal of Computational Finance, 5 (3) (2002), pp. 79-103
CrossRef View Record in Scopus Google Scholar

Journal of Banking & Finance, 14 (1) (1990), pp. 113-129
Article Download PDF View Record in Scopus Google Scholar

Klassen, 2001 T.R. Klassen
Simple, fast, and flexible pricing of Asian options
Journal of Computational Finance, 4 (3) (2001), pp. 89-124
CrossRef View Record in Scopus Google Scholar

Lapeyre and Temam, 2001 B. Lapeyre, E. Temam
Competitive Monte Carlo methods for the pricing of Asian options
Journal of Computational Finance, 5 (1) (2001), pp. 39-58
CrossRef View Record in Scopus Google Scholar

Lévy, 1992 E. Lévy
Pricing European average rate currency options
Journal of International Money and Finance, 11 (5) (1992), pp. 474-491
Article Download PDF View Record in Scopus Google Scholar

Mehrdoust, 2015 F. Mehrdoust
A new hybrid Monte Carlo simulation for Asian options pricing
Journal of Statistical Computation and Simulation, 85 (3) (2015), pp. 507-516
CrossRef View Record in Scopus Google Scholar

Milevsky and Posner, 1998 M.A. Milevsky, S.E. Posner
Asian options, the sum of lognormals, and the reciprocal gamma distribution
Journal of Financial & Quantitative Analysis, 33 (3) (1998), pp. 409-421
CrossRef Google Scholar

Müller, 2016 A. Müller
Improved variance reduced Monte-Carlo simulation of in-the-money options
Journal of Mathematical Finance, 6 (3) (2016), pp. 361-367
CrossRef View Record in Scopus Google Scholar

Nelken, 1996 I. Nelken
The handbook of exotic options: Instruments, analysis, and applications
Irwin (1996)
Google Scholar

Nielsen and Sandmann, 2003 J.A. Nielsen, K. Sandmann
Pricing bounds on Asian options
Journal of Financial and Quantitative Analysis, 38 (2) (2003), pp. 449-473
CrossRef View Record in Scopus Google Scholar

Ritchken et al., 1993 P. Ritchken, L. Sankarasubramanian, A.M. Vijh
The valuation of path dependent contracts on the average
Management Science, 39 (10) (1993), pp. 1202-1213

The value of an Asian option
Journal of Applied Probability, 32 (4) (1995), pp. 1077-1088
CrossRef Google Scholar

Vecer, 2001 Jan Vecer
A new PDE approach for pricing arithmetic average Asian options
Journal of Computational Finance, 4 (4) (2001), pp. 105-113
CrossRef View Record in Scopus Google Scholar

Vorst, 1992 T. Vorst
Prices and hedge ratios of average exchange rate options
International Review of Financial Analysis, 1 (3) (1992), pp. 179-193
Article Download PDF View Record in Scopus Google Scholar

Zhang, 2003 J.E. Zhang
Pricing continuously sampled Asian options with perturbation method
Journal of Futures Markets, 23 (6) (2003), pp. 535-560
View Record in Scopus Google Scholar

Zvan et al., 1999 R. Zvan, P. Forsyth, K. Vetzal
Discrete Asian barrier options
Journal of Computational Finance, 3 (1) (1999), pp. 41-67
CrossRef View Record in Scopus Google Scholar

1 There is no in the money scenario in Glasserman et al. (1999).

2 There is no in the money scenario in Glasserman et al. (1999).

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