Journal
North American Actuarial Journal >
Latest Articles



Feature Articles

# Mortality Risk Management Under the Factor Copula Framework—With Applications to Insurance Policy Pools

Ming-Hua Hsieh, Chenghsien Jason Tsai & Jennifer L. Wang

Published online: 30 Jan 2020

Download citation https://doi.org/10.1080/10920277.2019.1653201 Check for updates

## Abstract



Mortality risk is one of the core risks that life insurers undertake. The uncertain future lifetime of each insured represents one risk factor, and the dependence structure among these risk factors determines the aggregate risk of an insurance policy pool. We propose using factor copulas to describe the dependence structure among the future lifetimes of numerous insureds. This differs from Chen, MacMinn, and Sun (2015) in that their focus is on pricing the securities linked to several mortality indexes. To mitigate the systematic mortality risk associated with an insurance pool, the insurer may purchase an asset exposed to similar systematic risk. We thus set up a two-factor copula framework

nvestment amount in the asset. In numerical illustrations, we

n a life insurer and a life settlement market maker involving

# 1. INTRODUCTION

Life insurance companies are exposed to both mortality risk and longevity risk. Benjamin and Soliman (1993) and McDonald et al. (1998) documented that unprecedented improvements in population longevity had occurred around the world and that decreasing mortality rates had created a major risk for life insurance companies selling annuities. On the other hand, we witnessed natural and human-made disasters as well as pandemics that had demanded sudden and significant payments of death benefits from life insurers. Different products sold by life insurers are thus exposed to mortality and longevity risks to different degrees. These uncertain cash flows may result in not only short-term liquidity shocks but also long-term solvency threats to life insurance companies.

The literature has proposed three major ways to mitigate the mortality risks of life insurers.[1] The first way is by adopting capital market solutions including mortality securitization (see, e.g., Cowley and Cummins 2005; Lin and Cox 2005; Blake et al. 2006a, 2006b; Cox et al. 2006), survivor bonds (e.g., Blake and Burrows 2001; Dowd 2003; Denuit et al. 2007), and survivor swaps (e.g., Dowd et al. 2006). The second way is through natural hedging that may take place within an insurer, as suggested by Cox and Lin (2007) and Wang et al. (2010). The third method is to build mortality projection models to predict mortality rates (e.g., Lee and Carter 1992; Renshaw and Haberm… 2003; Cairns, Blake, and Dowd 2006a, 2006b).



Mortality projection models inevitably have forecasting errors, and model risks are significant (Wang et al. 2010). With regard to natural hedging, life insurers would have difficulties in implementing the hedging because new sales may be insufficient and/or because life insurers do not have full control over the composition of new sales. Hedging the mortality risks from the asset side of the insurer's balance sheet may be more flexible and cost-effective than through the liability side, even though the market size of mortality-linked securities has been growing at a much slower speed than anticipated.



…) an insurer adopts, managing the mortality risks of its
… a counter portfolio to the chosen policy pool and modeling

complex to be modeled without finding a way to reduce the dimension of the structure and finding a way that better describes the dependence than using correlation coefficients.

We propose using factor copulas to model the dependence structure. A rapidly increasing number of papers since the early 2000s have adopted copulas to describe the dependence structure among financial assets—for example, Laurent and Gregory (2005), Cousin and Laurent (2008), and Chiang et al. (2007). As suggested by Asmussen and Glynn (2007, 53), copulas provide a way of modeling multivariate distributions in which one has a well-defined idea of the marginal distributions but only a vague one on the dependence structure. Copulas have been applied to examining dependencies among risks in the actuarial science and finance fields (e.g., Frees and Valdez 1998; Embrechts et al. 2003; Cherubini, Luciano, and Vecchiato 2004). More specifically, the literature reports examining the products covering the two lives of couples such as Frees et al. (1996), Carriere (2000), Shemyakin and Youn (2006), Youn and Shemyakin (1999, 2001), and Denuit et al. (2001) employed copulas to model the bivariate survival function. We have not seen the application of copulas to the dependence structure of an insurance pool and the corresponding risk management scheme.

The other issue that one needs to tackle when managing the mortality risk of an insurance pool is the curse of dimensionality. The future lifetime of each insured represents one risk factor. The number of risk factors to be modeled can easily reach many hundreds even for small pools of life insurance policies. We propose adopti approach of factor copulas to reduce the dimension of modeling the dependence structure. The factor copula approach imposes a structure on the copula in which the dependence among the latent variables of the copula is determined by common factors. Laurent and Gregory (2005) employed a factor approach to the pricing of basket credit derivatives and synthetic collateralized debt obligation (CDO) tranches. Oh and Patton (2015) presented flexible models for the copula of economic variables based on a latent factor structure. Chen, MacMinn, and Sun (2015) applied the method of Oh and Patton (2015) to six population mortality indexes and then used the estimated model to

ing and hedging associated with Swiss Re's Vita notes. We are copula approach to model the dependence structure of the analyze a case in which an insurer intends to mitigate the mortality risk of a life insurance policy pool. The pool consists of about 400 policies from a medium-size insurer. For the risk mitigation tool, we choose a portfolio of life settlements originated by Coventry, a major market maker in the United States. Life settlements are life insurance policies sold in the secondary market. The policyholder involved in a life settlement transaction receives a payment exceeding the surrender value but less than the death benefit. The investor in such a transaction assumes the role of paying premiums and the return depends on the quality of the life expectancy estimates provided by medical underwriters. Life settlements became an increasingly popular asset class because they seemed to render good returns and/or diversification benefits to major asset classes (Gatzert 2010; Braun, Gatzert, and Schmeiser 2012). When it is a life insurer buying life settlements, this emerging asset class may play an additional role—mitigating the mortality risk of the insurer's liabilities. Such investments may thus become a part of the insurer's asset-liability management. The case of AIG buying life settlements in the 2000s is therefore justifiable from this perspective.[2]

The motivation of our study is straightforward but meets real-world needs: to demonstrate how to mitigate the mortality risk of a life insurance pool consisting of many policies. The literature has not addressed this issue. Most papers have focused on mortality modeling (either single or multiple populations) and the pricing of mortality-linked securities (such as Chen, MacMinn, and Sun 2015; Zhu, Tan, and Wang 2017); some have investigated hedging issues involving few policies/products (e.g., Wang et al. 2010). Our approach is to use copulas to model the dependence structure of the insurance pool, as well as the structure between this pool and a risk mitigation portfolio. Since the risk factors involved are numerous, we employ the method of factor copulas to bypass the curse of dimensionality. We adopt a simple but reasonable two-factor framework in which the liabilities of the insurer and the risk mitigation portfolio are subject to one common risk factor, two risk factors associated with the liability pool and the mitigation portfolio, respectively, and numerous idiosyncratic risk factors associated with individual compositions of the pool and portfolio. Our approach is applicable to other risk mitigation vehicles such as a portfolio of annuities or other mortality-linked



serve as reference to a life insurer seeking to manage its
number of sold policies.

settlements as the secondary market of life insurance policies has been widely recognized in the literature (e.g., Giacalone 2001; Ingraham and Salani 2004; Ziser 2006, 2007; Smith and Washington 2006; Seitel 2006; Leimberg et al. 2008). The market of life settlements is as important as (if not bigger than) most mortality-linked securities, although the market growth seemed to be stalled by market conduct lawsuits recently. Life settlements are usually regarded as alternative investments in the financial markets since the underlying risk factor is mortality which has low or no correlation with financial risk factors. With mortality being the underlying risk factor, life settlements can certainly serve as one type of asset to hedge the life insurer's liabilities that are also subject to mortality risk in the asset-liability management framework. This aspect of life settlements has not yet been examined in the literature yet. Seeing the economic impacts of life settlements on life insurance markets and the size of the life settlement market, we think that it is meaningful to investigate the risk management role that may be played by life settlements for life insurers. Furthermore, life settlements are one of the few instruments that can play the roles of return-generating asset and risk-hedging tool for life insurers.

Second, we applied factor copula to the dependence structure of the future lifetimes of the individuals underlying life settlements and life insurance policies. Hull and White (2004) and Laurent and Gregory (2005) applied factor copula to the dependence structure of basket default swaps and CDOs that are subject to default risk. Ten years later Oh and Patton (2015) applied factor copula to the constituents of S&P 100 that are subject to market risk. On the other hand, Chen, MacMinn, and Sun (2015) applied copula to mortality rate risk. We now extend the method of factor copula to the uncertainty of future lifetime of the insured. Applying the same method to different types of risk factor (e.g., default times, stock returns, mortality rates, and future lifetimes) can be regarded as contributions since the behaviors/models of different types of risk factor are different and demand different problem settings. The implications/applications of the results are different as well.[3]





…e insurance policies and life settlements in our analysis. We …ance of using these samples is greatly reduced by not being our case, no death has been reported by the time of securing the data. We thus have no way to estimate the parameters of our models. On the other hand, these portfolios are on the books of the investors and the insurer respectively. Our "sensitivity analysis" may serve as reference for them to assess the mortality risk and the degree of the hedging effect of life settlements on life insurance policies.

The remainder of this article is organized as follows: In Section 2 we introduce the valuation formulas and dependence structures between a life settlement fund and a life insurance pool. Then we delineate our data, the assumptions behind the numerical analyses, and how to solve for the optimal investment amount in the fund with respect to different risk measures in Section 3. Section 4 presents the numerical results along with justifications and robustness checks. Concluding remarks are provided in Section 5.

## 2. VALUATION, DEPENDENCE STRUCTURES, AND RISK MITIGATION ARRANGEMENT

### 2.1. Valuation

Suppose that a part of an insurer's liabilities contains *m* whole life policies and that the future lifetime of policy *j* is $\tau_j$, $j = 1, 2, \ldots, m$. Further assume that the insurer chooses to purchase a portion of a closed-end fund consisting of *n* life settlements with futur lifetime $T_i$ ($i = 1, 2, \ldots, n$) to mitigate the mortality risk associated with the aforemen insurance policy pool. The value of a life insurance policy and a life settlement can be expressed as

PDF
Help

$$V_{wl}^{j} = -\frac{A_j}{(1+r_{wl}(j))^{\tau_j}} + \sum_{t=\text{premium payment times before } \tau_j} \frac{Q_j^t}{(1+r_{wl}(j))^{t}}, \quad (1)$$

In this article

(2)

where $Q_j(t)$ and $A_j$ denote the premium payment at time $t$ and the death benefit associated with insurance policy $j$, respectively, $r_{wl}(j)$ indicates the annualized valuation rate of policy $j$, $P_i(t)$ and $B_i$ represent the premium payment at time $t$ and the death benefit associated with life settlement $i$, respectively, and $r_{ls}$ refers to the discount rate used to calculate the value of the life settlement.[6]

## 2.2. Dependence Structures

Stone and Zissu (2006, 2008) considered homogeneous life extensions or contractions, which imply that all life settlements and life insurance policies have the same deviation from expectancies. Such an assumption is inappropriate. One extension of the current article from theirs is that we model future lifetimes ($T_1, T_2, \ldots, T_n$) and ($\tau_1, \tau_2, \ldots, \tau_m$) using random vectors. Note that $V_{ls}$ and $V_{wl}$ will be random when $T_i$ and $\tau_j$ are random. Therefore, Equations (1) and (2) do recognize that the insured might die before the estimated life expectancy or live well beyond.

Let the marginal distribution function of $T_i$ and $\tau_j$ be denoted by $F_i(.)$ and $G_j(.)$, respectively. Assume that the joint dependence structure of $T_i$ and $\tau_j$ can be described by Gaussian factor copulas that include normal copulas and $t$ copulas.[7,8] This assumption is motivated by the similarity between defaultable bonds and life insurance.[9] For instance, Burtschell et al. (2009) selected normal factor copulas to model the dependence structure of the times to default for a bond portfolio subject to credit risk.

Under the framework of normal factor copulas, we first express $T_i$ by a corresponding latent variable $X_i$ as

$$T_i = F_i^{-1} N X_i, \tag{3}$$



In this article

...e distribution function (CDF) of the standard normal random

$$X_i = aM + bM_{ls} + \sqrt{1 - a^2 - b^2} Z_i, \quad (4)$$

where $M$, $M_{ls}$, $Z_1$, …, $Z_n$ are independent standard normal random variables and $a$, $b$ denote constant factor loadings. The common factor $M$ represents the global trend in the mortality improvements of the insured's underlying life settlements and life insurance policies, while $M_{ls}$ reflects the improvement trend in the mortalities of the insured underlying the life settlement fund only. On the other hand, $Z_1$, …, $Z_n$ are specific factors pertaining to each life settlement. The preceding setting reduces the modeling dimension on the dependence structure of life settlements from $n$ to 2 since simulating $X_i$ by Equation (4) requires two parameters only.

Similarly, we have the following expression for the whole life insurance policy:

$$\tau_j = G_j^{-1} N Y_j, \quad (5)$$

where $Y_j$ denotes the latent variables used to model the joint distributions of $\tau_j$. The dependence among the $Y_j$ is structured by common factors $M$ *and* $M_{wl}$ as follows:

$$Y_j = cM + dM_{wl} + \sqrt{1 - c^2 - d^2} W_j, \quad (6)$$

where $M_{wl}$, $W_1$, …, $W_m$ are independent standard normal random variables and the constants $c$, $d$ denote factor loadings. $M_{wl}$ represents the factors affecting the mortality improvements of the insured underlying the insurance pool and the $W_1$, …, $W_m$ are specific factors pertaining to each insurance policy. They are also independent of $M$, $M_{ls}$, $Z_1$, …, $Z_n$.





…t of a life settlement portfolio on a pool of life insurance …e fact that the future lifetimes of the insured's underlying systematic/accumulation risk of the life settlement fund and insurance pool individually) are reflected by group factors $M_{ls}$ and $M_{wl}$.

We have also tried using $t_v$ copulas to better capture tail dependence (Klugman et al. 2008). In particular,

$$T_i=F_i^{-1}\left(t_v\left(X_i\right)\right), \tag{7}$$

where $t_v(.)$ is the CDF of the $t$ distributed random variable with $v$ degrees of freedom. The dependence among the $X_i$ is induced through global factor $M$ and group factor $M_{ls}$ as follows:

$$X_i=\sqrt{\frac{v}{R}}\left(aM+bM_{ls}+\sqrt{1-a^2-b^2}\,Z_i\right), \tag{8}$$

where $R$ is an independent chi-squared random variable with $v$ degrees of freedom. Similarly, $\tau_j$ may be modeled as

$$\tau_j=G_j^{-1}\left(t_v\left(Y_j\right)\right), \tag{9}$$

and the dependence among the $Y_j$ is structured in the following way:

$$Y_j=\sqrt{\frac{v}{R}}\left(cM+dM_{wl}+\sqrt{1-c^2-d^2}\,W_j\right). \tag{10}$$



for fat tail dependence structure but does not allow for
…ore, we further tried using a skewed $t$-copula model based on

variable as

$$vRZ+\delta H, \tag{11}$$

where $Z$ is a standard normal random variable, $H$ is a normal random variable with mean $-2/\pi$ and variance 1 left truncated at $-2/\pi$, and $R$ is a chi-squared random variable with $v$ degrees of freedom. The random variables $Z$, $H$, and $R$ are independent. Note that the expected value of $H$ is zero and the skew $t$ random variable defined in Equation (11) reduces to $t$ random variable with $v$ degrees of freedom when $\delta=0$. Then, we can set

$$Ti=Fi-1H\delta Xi, \tag{12}$$

where $H\delta(.)$ is the CDF of the skew $t$ random variable defined in Equation (11). The dependence among the $X_i$ is modeled as follows:

$$Xi=vRaM+\delta H+bMIs+1-a2-b2Zi, \tag{13}$$



where $R$ is an independent chi-squared random variable with $v$ degrees of freedom. Similarly, $\tau_j$ can be set as

$$\tau j=Gj-1H\delta Yj, \tag{14}$$



ng the $Y_j$ is structured in the following way:

(15)

### 2.3. Risk Mitigation Arrangement

The total value of the positions including the value of the insurance policy pool and the investments in the life settlement fund, $V_h$, can be expressed as

$$V_h = V_{wl} + h\, V_{ls}, \quad (16)$$

in which $V_{wl} = \sum_{j=1}^{m} V_{wl_j}$, $h$ represents the so-called hedge ratio, and $V_{ls} = \sum_{i=1}^{n} V_{ls_i}$.[10] Since the life settlement fund is a closed-end fund, the insurer may choose to purchase a portion of the fund and $h$ will be smaller than 1 as a result. Note that $V_{wl}$ and $V_{ls}$ have opposite value changes to mortality shocks. For instance, $V_{wl}$ will decrease when mortality rates are higher than expected, but $V_{ls}$ will increase in these cases; $h$ should be positive as a result. A positive $h$ indicates that the life insurer is a direct investor in the life settlement fund. Life settlements may thus play the role of generating returns as well as the role of mitigating mortality risk when purchased by a life insurer.

## 3. DATA, SIMULATIONS, AND OPTIMAL MITIGATION

PDF Help

### 3.1. Data

A small pool of life insurance policies was obtained from a medium-sized life insurance company. These policies were sold during the years from 1999 to 2001. The sample consists of 418 whole life insurance policies, and the summary statistics are displayed in Table 1.

Statistics of the Life Insurance Policy Pool

In this article

The mean age of the insured is 64 years with a range from 55 to 76 as of 2011. These insureds are seniors and more vulnerable to pandemics than the population in general. The life insurer may thus have an incentive to hedge the mortality risk of this portfolio. Forty-six percent of the insureds were male. These policies had a wide range of death benefits, from $100,000 to $15 million, with an average of about half a million dollars.

Since the concerned insurance pool consists of seniors, one candidate for the risk mitigation tool is a portfolio of life settlements. We secure real-case data from Coventry, one of the major originators and market makers in the U.S. life settlement market. The samples that Coventry provided for us were a subset of the policies originated for an investor during a 21-month period (from July 2009 to April 2011). They are composed of 353 universal life insurance policies. Table 2 displays their summary statistics.

TABLE 2 Descriptive Statistics of the Life Settlement Portfolio

CSV Display Table

The insureds underlying these life settlements were seniors as expected, with ages ranging from 63 to 87 years and a mean age of 76 at the times when the policies were acquired by Coventry. Their life expectancies, as estimated by one of Coventry's major medical underwriters at those times, ranged from 6 years to 20 years with a mean of 13 years. Seventy-three percent of the insureds were male. The insurance policies we[re] acquired by Coventry during the early stages of the policies. The number average [of] years ago when the policies were bought was about 3; the youngest one was just 1 month old, while the oldest policy was in its 23rd policy year. Most policies had large amounts of death benefits: The average was 4 million dollars and the largest one reached as high as $20 million. The acquisition costs had a mean of about half a million dollars and a range from $20,000 to $6.8 million.



## 3.2. Simulations



[T]o simulate the death or survival of the insured underlying the [...] on those in the Social Security Administration (SSA) population mortality rates ($q_x$) in the SSA table so that the life expectancy for each life settlement would be equal to that in Coventry's dataset. This scaling is to reflect the differences in health conditions between the general population and the insureds underlying the life settlements assessed by the medical underwriter. More precisely, let $q_x$ be the standard SSA mortality rate at age $x$. We define a new mortality rate $\tilde{q}_x$ for a specific insured at age $x$ by setting

$$\tilde{q}_z = \text{scaling factor} \times q_z, \text{ for } z = x, x+1, \ldots, \text{maximum age}$$

The scaling factor is then chosen by a root-finding algorithm such that the life expectancy computed by $\tilde{q}_z$ equals the estimated life expectancy reported by Coventry. With these new mortality rates $\tilde{q}_z$, it is straightforward to derive the probability mass function of future lifetime for each insured. Note that each insured has his or her own scaling factor. The resulting scaling factors range from 0.53 to 3.55. The mortality rates applicable to the life insurance policies are provided by the insurer that supported us with the policy data.

To calculate the value of the life settlements and life insurance policies, we also need to specify discount rates. The discount rates used for the life insurance policies, $r_{wl}(j)$, range from 6.25% to 6.75% as advised by the data-providing insurer. Coventry did not m[…] suggestions on the discount rate for life settlements, $r_{ls}$. We resorted to the literatu[…] (Braun et al. 2012) and used the realized annual return of the life settlement index from December 2003 to June 2010, 4.85%, as $r_{ls}$.



With the preceding specifications, we may simulate $T_i$ and $\tau_j$ using different values of *a, b, c*, and *d*. One way to estimate these parameters is by applying the modified simulated method of moments (SMM) as demonstrated by Chen, MacMinn, and Sun (2015) to some dependence measures (such as the pair-wise Spearman's rho and quantile dependence) between the standardized residuals obtained from the time-series models

In this article

[…] models fitted to the historical mortality rates of the two […]the historical mortality rates of the insureds are unavailable.

For the purpose of illustration, we speculate on the values of *a, b, c*, and *d*. Li and Hardy (2011) observed, "In the second half of the twentieth century, there was, in general, a global convergence in mortality levels." Such an observation is consistent with the concept of coherence that was first proposed by Li and Lee (2005) and further investigated by Cairns et al. (2011). This well-accepted concept may serve as an indirect support for medium to large values of *a* and *c*. On the other hand, the insured targeted by life settlement originators might differ more from the general population than those of life insurance policies. We thus specify $c = 0.8$ and $a = 0.5$ as the initial speculation and will try other values later for robustness checks.

With regard to the values of *b* and *d*, we refer to a popular credit risk model: the KMV model. It specifies three common factors—global, geographic, industry—and their corresponding factor loadings, 0.25, 0.05, and 0.15, respectively (Glasserman et al. 2008). We speculate that the heterogeneity between the insured group of life settlements and that of life insurance is a little smaller than the heterogeneity among groups of loan borrowers or bond issuers across industries or geographic areas.[11] The values of *b* and *d* thus should be smaller than the loadings associated with the geographic and industry factors of the KMV model. On the other hand, we set our *b* and *d* to be a much larger number of 0.3. This would result in a conservative assessment regarding the mitigation effect of life settlements on insurance policies' mortality risk,[12] with robustness checks to be performed later.

Note that the correlation coefficient between life settlements *i* and *k* implied by Eq (4) is $a^2 + b^2$. The implied correlation coefficient between life policies *j* and *l* by Equ (6) is $c^2 + d^2$, and the correlation coefficient between life settlement *i* and insurance policy *j* is *ac*. The parameters specified as in the preceding thus imply that the correlation coefficients between a life settlement and an insurance policy, between two life policies, and between two life settlements are 0.4 (= 0.5 × 0.8), 0.73 (= 0.64 + 0.09), and 0.34 (= 0.25 + 0.09), respectively. These coefficients seem to be reasonable or a little bit conservative.

PDF
Help
In this article

eter values, we simulate 10,000 scenarios on the future
nderlying life settlements and insurance policies to construct

generate correlated uniform [0, 1] random variables NX1,..., NXn, NY1, ..., NYm, tvX1,..., tvXn, tvY1, ..., tvYm, and HδX1,..., HδXn, HδY1, ..., HδYm via normal, tv copulas and the skew *t*-copula, respectively.

The second step involves applying inverse CDF functions $F_i^{-1}(.)$ or $G_j^{-1}(.)$ to the generated random variables. We do not assume any specific functional forms for $F$ and $G$. Instead, we first derive the probability mass function of curtate future lifetime $K_x$ based on the mortality table calibrated to match the life expectancy of the insured. Then we produce the CDF functions of $T_i$ and $\tau_j$, with the additional assumption of a uniform distribution of death (UDD).

The last step to obtain future lifetimes is to apply the method of inversion (Asmussen and Glynn 2007) to the correlated uniform (0, 1) random variables. Suppose that a generated $N(X_i)$ is 0.11. If $P(K_x \leqq 5) = 0.1$ and $P(K_x \leqq 6) = 0.12$ based on the calibrated mortality table under the UDD assumption, then the corresponding $T_i$ would be 5.5.

### 3.3. Optimal Mitigation

Based on the resulting distributions, we may compute the optimal hedge ratio $h^*$ with respect to the standard deviation ($\sigma$), the value at risk (VaR) at the 95th percentile confidence level, and the expected shortfall (ES) for the 5% tail of the value of the mitigated portfolio $V_h$ given an objective function. When $\sigma$ is chosen to be the risk measure, the objective function is

PDF
Help

$$\min_h \sigma (V_{wl} + h V_{ls}). \tag{17}$$

Equation (17) implies that the mitigation effect of the life settlement fund on the insurance policy pool depends on $h$ as well as the variations in $V_{wl}$ and $V_{ls}$ caused by mortality shocks that in turn are affected by the dependence structures between insurance policies and life settlements.[13]

In this article

Table 3 indicates that life settlements could mitigate the mortality risk of a life insurance policy pool. More specifically, the values of the mortality risk of the insurance pool measured by $\sigma$, VaR, and ES under the normal factor copula framework are 34,143,483, 69,133,688, and 86,103,577, respectively. After purchasing 8.2%, 10.1%, and 10.3% of the life settlement fund, the risk of the mitigated pool reduces to 22,121,620, 38,578,204, and 49,255,396. The reductions in mortality risk are 35.2%, 44.2%, and 42.8%, respectively. Furthermore, the mitigation effect of life settlements on insurance seems to be more significant for downside risks than for the deviation risk (44.2% and 42.8% vs. 35.2%).

TABLE 3 Risk Mitigation Results: a = 0.5, c = 0.8, b = 0.3, d = 0.3, rls = 4.85%

CSV Display Table

Under the *t5* factor copula, the mortality risk of the insurance portfolio is a little bit larger than under the normal factor copula. The mitigation effects of life settlements remain significant: 34.6%, 44.6%, and 38.5% with regard to $\sigma$, VaR, and ES, respectively. The mitigation effect is again more significant for downside risks than for the deviation risk.

The asymmetry in tails does not cause material differences in the risk of the insura[nce] portfolio and the mitigation effects of life settlements either. The mortality risk of t[he] insurance portfolio is a little bit larger than the *t5* factor copula. So are the mitigation effects of life settlements: 41.9%, 50.0%, and 46.7% with regard to the risk measures of $\sigma$, VaR, and ES, respectively. The mitigation effect is also more significant for downside risks than for the deviation risk.

PDF
Help

An alternative way to interpret Table 3 is that life settlements are not a good hedging vehicle to an insurance pool. This is probably due to significant basis risk. The existence of significant basis risk is reasonable because the insureds and policies underlying life

In this article

...om those associated with an insurer's book in terms of ...ion, policy features, and other characteristics/features. For characteristics and policy features of insureds who are inclined toward surrendering their policies. Differences in these characteristics and features may result in different mortality risk profiles.

The significance of the basis risk in using life settlements to mitigate the mortality risk of life insurance policies is reflected by the values of *a*, *c*, and *b*, *d*. Smaller values of *b* and *d* indicate that the factors pertaining to the individual life settlement group and life insurance group ($M_{ls}$ and $M_{wl}$) are less significant. A larger portion of the mortality risk associated with life insurance could then be mitigated by life settlements or else diversified away. The mitigation takes place through the global factor *M* that determines the future lifetimes of the insureds underlying both life settlements and insurance policies; the diversification results from the independence among the specific factors pertaining to each insurance policy and life settlement (i.e., $W_1,\ldots, W_m$ and $Z_1,\ldots, Z_n$).

The results in Table 4 support the preceding reasoning.[14] When *b* and *d* decrease from 0.3 to 0.1, the value at risk of the mitigated pool declines from $40,008,071 to $24,007,962.[15] The risk mitigated by life settlements increases from 44.6% to 62.6%. We also observe that $h^*$ increases from 9.4% to 11%. The reason may be that smaller *b* and *d* imply less “noise” in the linkage between life settlements and life policies, which makes life settlements a risk mitigation tool worth being invested in more.

TABLE 4 Robustness Checks

CSV Display Table

Table 4 also indicates that a larger *a* increases the risk reduction resulting from purchasing life settlements.[16] For instance, the value at risk of the mitigated pool declines from $40,008,071 to $33,655,689, and the portion of the risk reduced by life settlements increases from 44.6% to 52.7%. These are reasonable because a larger *a* means that a larger portion of uncertainties about the future lifetimes of the insureds

… is subject to the risk factors common to those of life

…relation between the life settlement fund and the life insureds are mostly determined by one common factor such as the global mortality improvement trend.

We also observe from Table 4 that $h^*$ decreases with $a$. It declines to 7.2% from 9.4% when $a$ increases to 0.8 from 0.5. This is reasonable because when life settlements are more sensitive to the shocks on the global factor $M$, fewer life settlements would be needed to offset changes in life insurance reserves caused by the shocks. In other words, life settlements are a more effective mitigation tool when $a$ is larger.

The optimal hedge ratio $h^*$ also depends on the relative size of the life settlement fund to the insurance pool. The $h^*$ in Tables 3 and 4 is small, clustering within the range from 6% to 11%. This is because the size of the life settlement fund is large relative to that of the insurance pool. The average $V_{ls}$ is about $360 million when $r_{ls}$ = 4.85% and ($a$, $b$, $c$, $d$) = (0.5, 0.3, 0.8, 0.3), while the average $V_{wl}$ is nearly $72 million only.

The risk of the insurance pool, the optimal hedge ratio $h^*$, and the risk mitigation effect of life settlements on insurance all increase with $c$. For instance, $VaR(V_{wl})$ rises from $44,326,413 to $72,183,048 when $c$ changes from 0.5 to 0.8. This is reasonable since $c$ refers to the global/systematic risk of life insurance resulting from the shocks on the global factor $M$ that cannot be diversified away by the law of large numbers. Given the uncertainty about $M$, a larger $c$ would work like the $\beta$ in the CAPM (capital asset pricing model) framework and result in the larger risk of the insurance pool.

The global risk can only be mitigated using an instrument also affected by $M$. With exposure that the insurance pool has to the shocks on $M$, more life settlements are needed to mitigate this global risk. We therefore see that $h^*$ increases with $c$, from 5.9% to 9.4% when $c$ changes from 0.5 to 0.8, as shown in Table 4.



In addition, the portion of the life insurance pool's risk that can be reduced by investing in the life settlement fund increases from 30.7% to 44.6%. The rationale for the positive relationship between $c$ and the risk mitigation effect is the same as that for the relationship between $a$ and the mitigation effect. Increases in $a$ and/or $c$ imply higher



…ife settlement fund and the life insurance pool, and higher …nitigation abilities of life settlements to the life insurance pool.

expected returns. More specifically, we may choose the average of the expected IRR (internal rate of return) of the sampled life settlements as the discount rate. Table 4 shows that substituting an expected IRR of 8.6% for a realized 4.85% return has a minor impact on the risk mitigation abilities of life settlements.[17] The proportion of the insurance pool's risk that can be reduced by using life settlements is 46.8% when $r_{ls}$ is 8.6% while the proportion is 44.6% when $r_{ls}$ = 4.85%. The minor impact of $r_{ls}$ on the risk mitigation effect is reasonable since changes in the discount rate affect the valuation of life settlements but not the relation between life insurance and life settlements. The reduction in the values of life settlements due to an increase in the discount rate will be offset by a higher hedge ratio to achieve an equivalent level of risk reduction. We see from Table 4 that $h*$ increases a little from 9.4% to 10%.

## 5. CONCLUDING REMARKS

Life insurers indeed have few choices on the instruments that may hedge their mortality risks. Various types of mortality-linked security have emerged in the past two decades, but the aggregate market size is small relative to the policy reserves to be hedged. The worldwide life insurance industry as a whole therefore should make the best use of all sorts of instruments, including life settlements, to adequately manage its mortality risk exposure. Also, life settlements may also serve as alternative investments that bring in returns.



This article sets up a factor copula framework to deal with the mortality risk management of a life insurance pool consisting of many policies. The future lifetime of each insured underlying the pool is a risk factor, and the dependence structure among these insureds' future lifetimes determines the mortality risk of the pool. We contribute to the literature by introducing the factor copula approach to tackle the (dimensional) issue of modeling the dependence structure. The literature on managing/hedging mortality risks has seldom dealt with many risk factors. The analyzed objects have



ty indexes or several age-specific mortality rates, and the ing of the securities linked to several mortality indexes.

about the mortality risk of this pool since the insureds are all seniors. We notice that the insureds underlying life settlements are also seniors, and life insurers have invested in life settlements before. We thus choose a life settlement fund as the tool to mitigate the mortality risk of the life insurance pool, which is new to the literature as well.

In the illustration we have extended Stone and Zissu (2006, 2008) by allowing heterogeneous future lifetimes of the insured and have applied two-factor copula models to the dependence structure among the lifetimes of hundreds of the insured's underlying life insurance policies and life settlements. Then we have solved for the optimal investment amounts in the life settlement fund for the insurer to maximize the mitigation effect on its insurance pool. The parameters of the factor copula models are chosen based on speculation, but life insurers may pick up proxies of the underlying mortality rates that have adequate historical data and carry out parameter estimations by using methods like SMM.

We have investigated the potential mitigation effects of life settlements on the mortality risk of a life insurance pool with numerical examples. The effects were quantified under normal and *t* copula frameworks with respect to three risk measures: the standard deviation, value at risk, and expected shortfall. The numerical results imply that life settlements might have a role to play as a risk mitigation tool, in addition to being an investment object. The mortality risk of the insurance pool could be reduced by at least 30% and might reach 60%, depending on the parameters of the factor copulas as well as the chosen risk measures. Such results are reasonable in light of the trends in mo[PDF Help] improvements both nationally and globally. Furthermore, life settlements seem to provide more hedging benefits for downside risks than for the deviation risk. The insurer should, on the other hand, carefully assess other issues that have been raised in the literature or even by the news before making direct investments in life settlements for yield-enhancing and/or risk-mitigation purpose(s). Moreover, our numerical results have demonstrated that significant basis risk exists when the life settlement fund is used as a risk-mitigation tool.[18]



his article can be applied to natural hedging that employs insurer to mitigate the concerned pool. It is also useful in framework has many practical implications in addition to providing academic extensions.

*Discussions on this article can be submitted until October 1, 2020. The authors reserve the right to reply to any discussion. Please see the Instructions for Authors found online at http://www.tandfonline.com/uaaj for submission instructions.*

## Notes

1 For brevity, we use the term “mortality risks” in a broad sense to include both mortality and longevity risks hereafter. Note that the risks concerned are systematic risks.

2 The case had legal disputes later over conduct / ethical issues.

3 The literature applies copula to two types of risk factors associated with rates (such as stock returns and mortality rates) and time (e.g., time of default and future lifetime) respectively. Oh and Patton (2015) apply a factor copula to the returns of S&P 100 stocks, while Chen, MacMinn, and Sun (2015) apply a factor copula to the mortality rates of six countries. The formal Oh and Patton paper filters 100 time series with AR(1)–GJR-GARCH models and then derives a proposition to determine the number of common factors in the copula; the latter fits the mortality improvement rates of six countrie an ARMA-GARCH model and imposes the assumption of one common factor upon copula. The SMM (simulated method of moments) method used by Chen, MacMinn, and Sun (2015) is from Oh and Patton (2013). The pricing method used by Chen, MacMinn, and Sun (2015) is from Stutzer (1996) and had also been used in a few previous studies on pricing mortality-linked securities (see, e.g., Li 2010; Kogure and Kurachi 2010; Li and Ng 2011). In short, Chen, MacMinn, and Sun (2015) build upon the previous literature by providing new pricing and risk mitigation techniques for mortality bonds.





ers in the literature dealing with default risk apply a copula to vith time. In this research the risk factor is future lifetime.

and White (2004) in showing that most complex multiname credit derivatives can be priced in a semi-analytical way under a one-factor copula model; we apply the methodology of Hull and White (2004) to the future lifetimes of insureds under the two-factor copula framework for the purpose of analyzing the risk mitigation effects of life settlements. The Hull–White model is for financial instruments, while our model is for new life market instruments. The implications of our results are thus different from those of other results in the literature. In short, we build upon the previous literature by showing the risk mitigation effects of life settlements.

4 Note that Laurent and Gregory (2005) do not conduct parameter estimation either, since they do not have data on the defaults of individual names. On the other hand, Chen, MacMinn, and Sun (2015) applied the factor copula to the residuals of the ARMA-GARCH processes fitted to the mortality rate time series of several countries. Wang, Yang, and Huang (2015) also applied copulas to filtered mortality rate time series, while Zhu, Tan, and Wang (2017) applied copulas to the residuals of fitting Lee–Carter models. Difficulties in model/parameter estimation arise when one tried to apply copulas to individuals' mortality (as Laurent and Gregory [2005] encountered with individual names' defaults).

5 Note that $V_{wl}(j)$ is equivalent to negative policy reserves. Policy reserves are equal to the sum of the present values of expected cash outflows minus that of expected cash inflows, while $V_{wl}(j)$ is the sum of the present values of expected cash inflows minus that of expected cash outflows.

PDF
Help

6 Note that $\tau_j$, $T_i$, and $t$ need not be integers. The premium payment schedules of life settlements are usually in months.

7 An alternative assumption could be conditional independence in a Cox-process setup as in survival analysis (Hougaard 2000). These types of models correspond to the Clayton copula (a special case of an Archimedean copula) when the marginal distributions are some well-known parametric distributions. Since the marginal distributions are deduced from mortality tables and are thus nonparametric, employing such types of models

In this article

e thus choose the $t$-copula as the alternative model to the endence structure.

only conjecture that tail dependence among the future lifetimes of these insureds may be caused by pandemics, catastrophes, breakthroughs in gene therapies, etc., like the tail dependence among the future lifetimes of the general public.

9 A default on a bond is similar to the death of a policyholder. Both are usually rare events with significant consequences. Applying default risk models to insurance policies can also be seen in Chaplin, Aspinwall, and Venn (2011). They did not, however, employ factor copula to model dependence.

10 We prefer the term "risk mitigation" to "hedging" in this article since we are dealing with liabilities, while standard hedging is usually for owned assets. This preference is also because the setup involves significant basis risk as reflected by group factors $M_{ls}$ and $M_{wl}$.

11 The reasoning for the heterogeneity between two groups of insureds being smaller than that between two groups of company borrowers is as follows. Personal mortality is determined by population biological factors, personal genes, social–economic factors, and personal lifestyles; personal default risk depends on macroeconomics and personal financial conditions. The lack of the biological similarity factor thus may results in the higher heterogeneity across (groups of) corporate borrowers.

12 We may refer to Li and Hardy (2011) for educated guesses on whether the global trend, the trend in a specific insurance pool, or the idiosyncratic risk dominates the mortality dynamics. They applied the augmented common factor model to the fen populations of Canada and the United States. From their Figure 8, we may roughly compare B(x) × K(t) with b(x,i) × k(t,i) and say that the global trend seems to be more significant than the population-specific factors. The idiosyncratic risk is probably immaterial since the model has a very high explanation ratio of 0.9931. The forces driving the dynamics of mortality rates may also make the dynamics of future lifetimes subject to significant global trends, moderate population-specific factors, and insignificant idiosyncratic risk.





ve to establish time-series models for the global factor and
not aim to engage in dynamic hedging. Our proposed strategy

14 For the sake of brevity, we report the results of one risk measure under one factor copula framework only. All results are consistent across risk measures and copulas.

15 The value at risk of the insurance pool is reduced as well: from $72,183,048 to $64,172,539. Such a reduction is understandable since a smaller *d* represents a smaller sensitivity to the shocks on the group factor $M_{wl}$.

16 On the other hand, VaR($V_{wl}$) should be independent of *a* since the risk of the life insurance pool should have nothing to do with the risk of life settlements pertaining to the global factor *M*. Table 4 does display such a result: $72,183,048 vs. $71,127,485. The small difference is due to random variations in simulations.

17 The value 8.6% is the average of the expected IRRs on individual life settlements. The data from Coventry contain the acquisition cost, life expectancy, scheduled premiums, and death benefit for each policy. We are thus able to solve for the expected IRR.

18 Life settlements are probably at the same level of (in)effectiveness as other mortality-linked securities because the underlying populations of these hedging vehicles differ from life insurance policies carried by life insurers and the markets of these vehicles are still at their early development stage. In terms of product form/format, life settlements are like longevity bonds to life insurers and can be used as a mortality risk management tool in the asset-liability management (ALM) framework. Mortality swaps (including q-forwards), futures, and options are used as derivatives that have small or no value contract inception, on the other hand.



# REFERENCES

1. Asmussen, S., and P. W. Glynn. 2007. *Stochastic simulation: Algorithms and analysis*, New York, NY: Springer. [Crossref], [Google Scholar] Find it@NCCU

of distributions which includes the normal ones. *Scandinavian*
71–78. [Web of Science ®], [Google Scholar] Find it@NCCU

4. Bahna-Nolan, M., and C. Ritzke. 2008. Valuation basic table (VBT) report. Society of Actuaries. http://www.soa.org/research/experience-study/ind-life/valuation/2008-vbt-report-tables.aspx [Google Scholar] Find it@NCCU

5. Benjamin, B., and A. S. Soliman, eds. 1993. *Mortality on the move*: *Methods of mortality projection*. Oxford, UK: Institute of Actuaries. [Google Scholar] Find it@NCCU

6. Blake, D., and W. Burrows. 2001. Survivor bonds: Helping to hedge mortality risk. *Journal of Risk and Insurance* 68: 339–48. [Crossref], [Web of Science ®], [Google Scholar] Find it@NCCU

7. Blake, D., A. J. G. Cairns, and K. Dowd. 2006a. Living with mortality: Longevity bonds and other mortality-linked securities. *British Actuarial Journal* 12: 153–97. [Crossref], [Google Scholar] Find it@NCCU

8. Blake, D., A. J. G. Cairns, and K. Dowd. 2006b. Longevity bonds: Financial engineering, valuation, and hedging. *Journal of Risk and Insurance* 73: 647–72. [Crossref], [Web of Science ®], [Google Scholar] Find it@NCCU

9. Braun, A., N. Gatzert, and H. Schmeiser. 2012. Performance and risks of open-end life settlement funds. *Journal of Risk and Insurance* 79: 193–230. [Crossref], [Web of Science ®], [Google Scholar] Find it@NCCU

0. Burtschell, X., J. Gregory, and J. P. Laurent. 2009. A comparative analysis of CDO pricing models under the factor copula framework. *Journal of Derivatives* 16: 9–37. [Crossref], [Web of Science ®], [Google Scholar] Find it@NCCU

1. Cairns, A. J. G., D. Blake, and K. Dowd. 2006a. Pricing death: Frameworks for the valuation and securitization of mortality risk. *ASTIN Bulletin* 36: 79–120. ...nce ®], [Google Scholar] Find it@NCCU

3. Cairns, A. J. G., D. Blake, K. Dowd, G. D. Coughlan, D. Epstein, and M. Khalaf-Allah. 2011. Mortality density forecasts: An analysis of six stochastic mortality models. *Insurance: Mathematics and Economics* 48: 355–67. [Crossref], [Web of Science ®], [Google Scholar] Find it@NCCU

4. Carriere, J. F. 2000. Bivariate survival models for coupled lives. *Scandinavian Actuarial Journal* 1: 17–31. [Taylor & Francis Online], [Google Scholar] Find it@NCCU

5. Chan, J. C. C., and D. P. Kroese. 2010. Efficient estimation of large portfolio loss probabilities in *t*-copula models. *European Journal of Operational Research* 205: 361–67. [Crossref], [Web of Science ®], [Google Scholar] Find it@NCCU

6. Chaplin, G., J. Aspinwall, and M. Venn. 2011. *Life settlements and longevity structures: Pricing and risk management*. Hoboken, NJ: Wiley Finance. [Google Scholar] Find it@NCCU

7. Chen, H., R. MacMinn, and T. Sun. 2015. Multi-population mortality models: A factor copula approach. *Insurance: Mathematics and Economics* 63: 135–46. [Crossref], [Web of Science ®], [Google Scholar] Find it@NCCU

8. Cherubini, U, E. Luciano, and W. Vecchiato. 2004. *Copula Methods in Finance*. West Sussex, UK: John Wiley & Sons. [Crossref], [Google Scholar] Find it@NCCU

9. Chiang, M. H., M. L. Yueh, and M. H. Hsieh. 2007. An efficient algorithm for basket default swap valuation. *Journal of Derivatives* 15: 8–19. [Crossref], [Web of Science ®], [Google Scholar] Find it@NCCU

10. Cousin, A., and J. P. Laurent. 2008. An overview of factor models for pricing CDO tranches. In *Frontiers in quantitative finance. Credit risk and volatility modeling*, ed. R. Cont, 185–216. Chichester, UK: John Wiley & Sons. [Google Scholar] Find it@NCCU

…mmins. 2005. Securitization of life insurance assets and … *and Insurance* 72: 193–226. [Crossref], [Web of Science ®], *American Actuarial Journal* 11: 1–15. [Taylor & Francis Online], [Google Scholar] Find it@NCCU

3. Cox, S. H., Y. Lin, and S. N. Wang. 2006. Multivariate exponential tilting and pricing implications for mortality securitization. *Journal of Risk and Insurance* 73: 719–36. [Crossref], [Web of Science ®], [Google Scholar] Find it@NCCU

4. Denuit, M., J. Dhaene, C. Le Bailly de Tilleghem, and S. Teghem. 2001. Measuring the impact of dependence among insured lifelengths. *Belgian Actuarial Bulletin* 1: 18–39. [Google Scholar] Find it@NCCU

5. Denuit, M., P. Devolder, and A. C. Goderniaux. 2007. Securitization of longevity risk: Pricing survivor bonds with Wang transform in the Lee–Carter framework. *Journal of Risk and Insurance* 74: 87–113. [Crossref], [Web of Science ®], [Google Scholar] Find it@NCCU

6. Dowd, K. 2003. Survivor bonds: A comment on Blake and Burrows, *Journal of Risk and Insurance* 70: 339–48. [Crossref], [Web of Science ®], [Google Scholar] Find it@NCCU

7. Dowd, K., D. Blake, A. J. G. Cairns, and P. Dawson. 2006. Survivor swaps. *Journal of Risk and Insurance* 73: 1–17. [Crossref], [Web of Science ®], [Google Scholar] Find it@NCCU

8. Eling, M., and D. Kiesenbauer. 2014. What policy features determine life insurance lapse? An analysis of the German market. *Journal of Risk and Insurance* 81: 241–69. [Crossref], [Web of Science ®], [Google Scholar] Find it@NCCU

9. Embrechts, P., F. Lindskog, and A. McNeil. 2003. Modeling dependence with copulas and applications to risk management. In *Handbook of heavy tailed distributions in finance*, ed. S. Rachev. 331–84. Amsterdam, The Netherlands: Elsevier. [Crossref], [Google Scholar] Find it@NCCU

e, and E. A. Valdez. 1996. Annuity valuation with dependent
*k and Insurance* 63: 229–61. [Crossref], [Web of Science ®],

*American Actuarial Journal* 2: 1–25. [Taylor & Francis Online], [Google Scholar] Find it@NCCU

2. Gatzert, N. 2010. The secondary market for life insurance in the U.K., Germany, and the U.S.: Comparison and overview. *Risk Management and Insurance Review* 13: 279–301. [Crossref], [Google Scholar] Find it@NCCU

3. Giacalone, J. A. 2001. Analyzing an emerging industry: Viatical transactions and the secondary market for life insurance policies. *Southern Business Review* 27: 1–7. [Google Scholar] Find it@NCCU

4. Glasserman, P., W. Kang, and P. Shahabuddin. 2008. Fast simulation of multifactor portfolio credit risk. *Operations Research* 56: 1200–17. [Crossref], [Web of Science ®], [Google Scholar] Find it@NCCU

5. Hougaard, P. 2000. *Analysis of multivariate survival data (Statistics for biology and health).* New York, NY: Springer. [Crossref], [Google Scholar] Find it@NCCU

6. Hull, J., and A. White. 2004. Valuation of a CDO and an nth to default CDs without Monte Carlo simulation. *Journal of Derivatives* 12: 8–23. [Crossref], [Google Scholar] Find it@NCCU

7. Ingraham, H. G., and S. S. Salani. 2004. Life settlements as a viable option. *Journal of Financial Service Professionals* 58: 72–6. [Google Scholar] Find it@NCCU

8. Klugman, S., H. Panjer, and G. Willmot. 2008. *Loss models: From data to decisions*, 3rd ed. Hoboken, NJ: Wiley. [Crossref], [Google Scholar] Find it@NCCU

9. Kogure, A., and Y. Kurachi. 2010. A Bayesian approach to pricing longevity risk based on risk-neutral predictive distributions. *Insurance: Mathematics and Economics* 46: ... of Science ®], [Google Scholar] Find it@NCCU

1. Lee, R. D., and L. R. Carter. 1992. Modeling and forecasting U.S. mortality. *Journal of the American Statistical Association* 87: 659–71.
[Taylor & Francis Online], [Web of Science ®], [Google Scholar] Find it@NCCU

2. Leimberg, S. R., M. D. Weinberg, B. T. Weinberg, and C. J. Callahan. 2008. Life settlements: Know when to hold and know when to fold. *Journal of Financial Service Professionals* 62: 61–72. [Google Scholar] Find it@NCCU

3. Li, J. S., and M. Hardy. 2011. Measuring basis risk in longevity hedges. *North American Actuarial Journal* 15: 177–200. [Taylor & Francis Online], [Google Scholar] Find it@NCCU

4. Li, J. S. H. 2010. Pricing longevity risk with the parametric bootstrap: a maximum entropy approach. *Insurance: Mathematics and Economics* 47: 176–86.
[Crossref], [Web of Science ®], [Google Scholar] Find it@NCCU

5. Li, J. S. H., and A. C. Y. Ng. 2011. Canonical valuation of mortality-linked securities. *Journal of Risk and Insurance* 78: 853–84. [Crossref], [Web of Science ®],
[Google Scholar] Find it@NCCU

6. Li, N., and R. Lee. 2005. Coherent mortality forecasts for a group of population: An extension of the Lee–Carter method. *Demography* 42: 575–94.
[Crossref], [PubMed], [Web of Science ®], [Google Scholar] Find it@NCCU

7. Lin, Y., and S. H. Cox. 2005. Securitization of mortality risks in life annuities. *Journal of Risk and Insurance* 72: 227–52. [Crossref], [Web of Science ®], [Google Scholar]
Find it@NCCU

8. McDonald, A. S., A. J. G. Cairns, P. L. Gwilt, and K. A. Miller. 1998. An international comparison of recent trends in the population mortality. *British Actuarial Journal* 3: 3–141. [Crossref], [Google Scholar] Find it@NCCU

d V. Maume-Deschamps. 2011. Surrender triggers in life
and risk predictions. *Bulletin Francais d'Actuariat* 22: 5–48.

copulas. *Journal of Business & Economic Statistics* 35: 139–54.
[Taylor & Francis Online], [Web of Science ®], [Google Scholar] Find it@NCCU

1. Oh, D.H., and A. J. Patton. 2013. Simulated method of moments estimation for copula-based multivariate models. *Journal of the American Statistical Association* 108: 689–700. [Taylor & Francis Online], [Web of Science ®], [Google Scholar] Find it@NCCU

2. Pinquet, J., M. Guillen, and M. Ayuso. 2011. Commitment and lapse behavior in long-term insurance: A case study. *Journal of Risk and Insurance* 78: 983–1002. [Crossref], [Web of Science ®], [Google Scholar] Find it@NCCU

3. Renshaw, A. E., and S. Haberman. 2003. Lee–Carter mortality forecasting with age specific enhancement. *Insurance: Mathematics and Economics* 33: 255–72. [Crossref], [Web of Science ®], [Google Scholar] Find it@NCCU

4. Seitel, C. L. 2006. Inside the life settlement industry: An institutional investor's perspective. *Journal of Structured Finance* 12: 38–40. [Google Scholar] Find it@NCCU

5. Shemyakin, A., and H. Youn. 2006. Copula models of joint last survivor analysis. *Applied Stochastic Models in Business and Industry* 22: 211–24. [Crossref], [Web of Science ®], [Google Scholar] Find it@NCCU

6. Smith, B. B., and S. L. Washington. 2006. Acquiring life insurance portfolios: Diversifying and minimizing risk. *Journal of Structured Finance* 12: 41–5. [Google Scholar] Find it@NCCU

7. Stone, C. A., and A. Zissu. 2006. Securitization of senior life settlements: Managing extension risk. *Journal of Derivatives* 13: 66–72. [Crossref], [Google Scholar] Find it@NCCU

ı. 2008. Using life extension-duration and life extension-
r life settlement contracts. *Journal of Alternative Investments*

*Journal of Finance* 51: 1633–52. [Crossref], [Web of Science ®], [Google Scholar] Find it@NCCU

0. Wang, J. L., H. C., Huang, S. S. Yang, and J. T. Tsai. 2010. An optimal product mix for hedging longevity risk in life insurance companies: The immunization theory approach. *Journal of Risk and Insurance* 77: 473–97. [Crossref], [Web of Science ®], [Google Scholar] Find it@NCCU

1. Wang, C. W., S. W. Yang, and H. C. Huang. 2015. Modeling multi-country mortality dependence and its application in pricing survivor index swaps - A dynamic copula approach. *Insurance: Mathematics and Economics* 63: 30–9. [Google Scholar] Find it@NCCU

2. Youn, H., and A. Shemyakin. 1999. Statistical aspects of joint life insurance pricing. *1999 Proceedings of the Business and Statistics Section of the American Statistical Association*, 34–38. [Google Scholar] Find it@NCCU

3. Youn, H., and A. Shemyakin. 2001. Pricing practices for joint last survivor insurance. *Actuarial Research Clearing House* 2001. 1. [Google Scholar] Find it@NCCU

4. Ziser, B. 2006. Life settlements today: A secret no more. *Journal of Structured Fi*
12 (2): 35–7. [Google Scholar] Find it@NCCU

5. Ziser, B. 2007. An eventful year in the life settlement industry. *Journal of Structured Finance* 13 (2): 40–3. [Google Scholar] Find it@NCCU

6. Zhu, W., K. S. Tan, and C. W. Wang. 2017. Modeling multi-country longevity risk with mortality dependence: A Levy subordinated hierarchical Archimedean copulas approach. *Journal of Risk and Insurance* 84: 477–93. [Crossref], [Web of Science ®], [Google Scholar] Find it@NCCU

The authors are grateful to the Risk and Insurance Research Center of National Chengchi University for its financial support and for financial support from the Ministry of Science and Technology (project number: 102-2410-H-004-027-MY3).