Using Parametric Statistical Models to Estimate Mortality Structure: The Case of Taiwan

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Abstract†

A mixture parametric model is used to analyze the changing pattern of Taiwanese mortality from 1926 to 1991. Three different age ranges are modeled as mixtures of extreme value distributions, namely the Weibull, inverse Weibull, and Gompertz distributions. The results show a significant improvement of mortality over the years.

Key words and phrases: *mixture, extreme value distribution, Weibull, inverse-Weibull, and Gompertz*

1 Introduction

In a recent study of the mortality structure of the 1989 Taiwan Standard Ordinary Experienced Life Table, Chang (1995) observed that the mortality rates followed a parametric mixture model. Further, observing $\ln(q_x)$ from the published population life tables of Taiwan that were constructed between 1926 and 1991, a clear pattern emerges; see Figures 1 and 2.

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†The author wishes to thank the three anonymous referees and (especially) the editor for detailed comments and numerous suggestions on an earlier draft of this article.

Figure 1
Taiwanese Male Mortality Patterns From 1926 to 1991

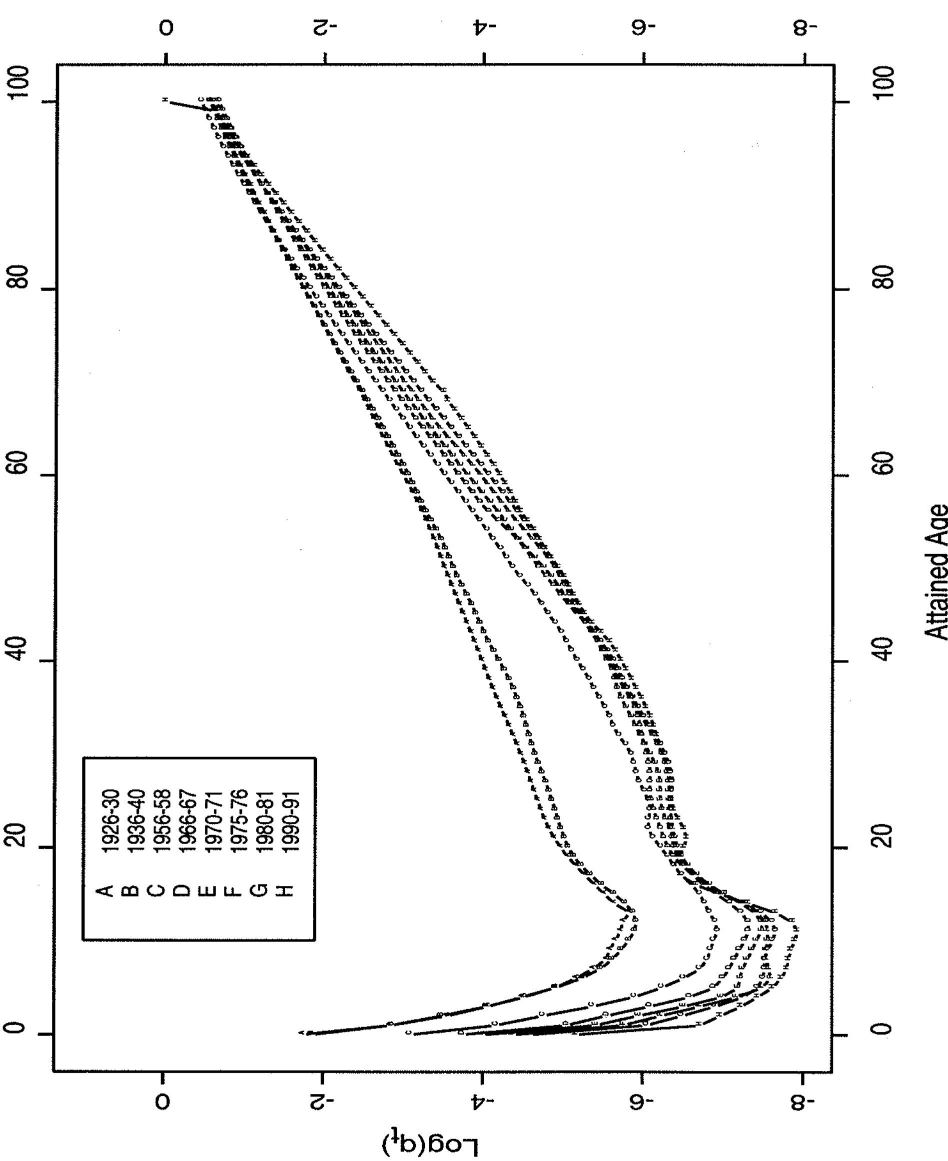
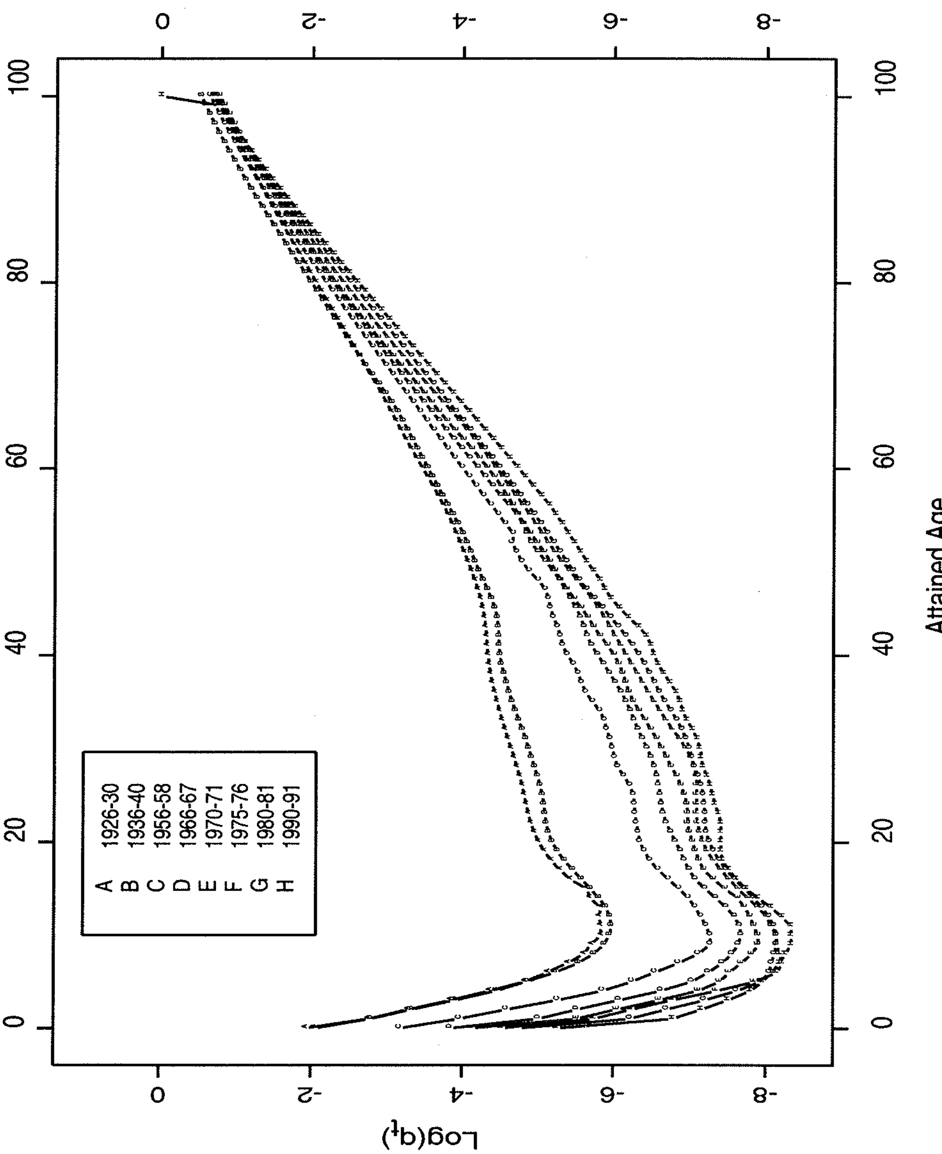


Figure 2
Taiwanese Female Mortality Patterns From 1926 to 1991



Figures 1 and 2 show similar mortality patterns for both male and female populations. The mortality rates appear to have three humps. These humps can be described by a parametric mixture model. The early life tables (constructed between 1926-1930 and 1936-1940) show a pattern of higher mortality rates. This was due, in part, to the effects of World War II and lack of adequate medical care in Taiwan's early years.

Parametric modeling is an important technique often used in the graduation of mortality rates; see, for example, Tenenbein and Vanderhoff (1980), Heligman and Pollard (1980), Wetterstrand (1981), Siler (1983), Renshaw (1991), Carriere (1992, 1994), Haberman and Renshaw (1996), and Yuen (1997). These authors have shown that parametric models provide an excellent means to understand a population's mortality structure.

A brief summary of advantages of using parametric models is listed below:

- Factors that influence (do not influence) mortality can be added (removed) from the model;
- If the results are not consistent with the proposed model, the model can be revised until it produces reasonable results;
- The resulting mortality rates form a smooth progression;
- Under certain loss criteria in the parameter estimation a general law of mortality can be obtained; and
- The final and most important advantage is the ability to forecast future mortality rates.[1]

2 Constructing the Parametric Model

The Taiwanese population mortality rates from 1926 to 1991 can be placed into three distinct subgroups that account for the major proportion of deaths:

- The infant population (ages 0-3);
- The adult population (ages 18-64); and
- The elderly population (ages 65 and over).

[1] A discussion of time trends, modeling, and forecasting can be found in Renshaw, Haberman, and Hatzopolous (1996).

Each of these population subgroups can be modeled by a distinct probability distribution. By combining these distributions, a finite mixture model can be used to analyze the entire population mortality rates.

Heligman and Pollard (1980) propose an eight-parameter model containing three distributions that fits Australian mortality rates. Carriere (1992 and 1994) uses a mixture of extreme value survival functions to model population mortality rates for a U.S. life table.

The parametric model is constructed as follows:

Step 1: Several well-known parametric statistical distributions, such as the Gompertz, Weibull, and inverse Weibull distributions, are chosen to see if they fit the mortality data. From Chang (1995), mixtures of these distributions have generated satisfactory results in estimating the Taiwanese life table.

Step 2: Simple graphical techniques are used to select the appropriate form of parametric distributions. For example, plots of $\ln(\hat{\mu}_x)$ vs x are examined, where $\hat{\mu}_x$ is the estimate of force of mortality at age x. If the plot appears to show a straight line, the Gompertz distribution might be appropriate to model the mortality data. If the pattern is shown to be a straight line in plots of $\ln(\hat{\mu}_x)$ vs $\ln(x)$ the Weibull distribution might be a better candidate. See Elandt-Johnson and Johnson (1980, Chapter 7) for more details on the use of such plots.

Step 3: Choose a base time point ($t = 0$) from which time is measured (in years). In this case we set January 1, 1926 as $t = 0$.

Step 4: For $t = 1, 2, \ldots$, let $s_t(x) \geq 0$ denote the survival function at t. We assume that $s_t(x)$ is a mixture of n component survival functions, i.e.,

$$s_t(x) = \sum_{i=1}^{n} \rho_{it} s_{it}(x) \quad \text{for } t = 1, 2, \ldots \tag{1}$$

where, for $i = 1, 2, \ldots, n$, $s_{it}(x) \geq 0$ is the ith component of the survival function, and $\rho_{it} \geq 0$ is the ith component mixing probability. Note that

$$\sum_{i=1}^{n} \rho_{it} = 1.$$

Step 5: The mixing probabilities and the parameters of each component $s_{it}(x)$ are estimated using statistical techniques.

For $i = 1, 2, \ldots, n$, let θ_{it} denote the vector used to describe the parameters in $s_{it}(x)$, i.e.,

$$s_{it}(x) = s_{it}(x \mid \theta_{it}). \tag{2}$$

Once the θ_{it}s are estimated, Carriere's (1994) select and ultimate parametric model is used to express θ_{it} as a function of t, i.e.,

$$\theta_{it} = \theta_{i0} + (\theta_{i\infty} - \theta_{i0})\left(1 - \exp(-a_i t^{b_i})\right), \quad a_i > 0,\ b_i > 0 \tag{3}$$

where a_i and b_i are parametric constants that influence the rate of convergence (as $t \to \infty$) of θ_{it} to $\theta_{i\infty}$. Equation (3) specifies the non-linear relationship between the respective elements of the vector of parameters and year.

Let $\theta_t = (\theta_{1t}, \ldots, \theta_{nt})$ and $\rho_t = (\rho_{1t}, \ldots, \rho_{nt})$ denote the vectors of parameters used to define $s_t(x)$. The proposed parametric model is:

$$s(x \mid \theta_t, \rho_t) = \sum_{i=1}^{n} \rho_{it} s_{it}(x \mid \theta_{it}), \tag{4}$$

which characterizes a general surface of the mortality rates.

Also, $s(x \mid \theta_t, \rho_t)$ can be regarded as a generalization of the model proposed by Wetterstrand (1981), who uses a Gompertz distribution (with year considered as a covariate) to analyze the mortality data.

3 The Life Tables Used and the Model

3.1 The Life Tables Used

The population life tables used are those published by the Taiwanese Department of Statistics, Ministries of Interior up to 1994. These tables are summarized in Table 1.

Though a life table may be released in a certain year, the table usually spans several years. For the purposes of this paper, a single year is assigned to each table. So, for the rth table releases since 1926, let t_r denote the value of t assigned to the table.

Table 1
Taiwanese Life Tables

Release Order (r)	t_r	Collection Period	Release Date
1	1926	1926–1930	1936, November
2	1935	1935–1940	1947, June
3	1956	1956–1958	1965, September
4	1966	1966–1967	1972, June
5	1970	1970–1971	1977, September
6	1975	1975–1976	1982, June
7	1980	1980–1981	1992, June
8	1990	1990–1991	1994, December

3.2 The Model

This study uses a model with three components ($n = 3$), each component being an extreme value distribution with a location parameter m_{it} and dispersion parameter σ_{it}. Thus $\theta_{it} = (m_{it}, \sigma_{it})$.

For the ith component at time t, the force of mortality $\mu_{it}(x)$, survival function $s_{it}(x)$, and probability density function $f_{it}(x)$ are summarized below:

Infants: The infant population (ages 0–3) is denoted by $i = 1$. This population is modeled as a Weibull distribution with parameters $m_{1t} > 0$ and $\sigma_{1t} > 0$: For $x \geq 0$,

$$
\begin{aligned}
\mu_{1t}(x) &= \frac{1}{\sigma_{1t}}\left(\frac{x}{m_{1t}}\right)^{\frac{m_{1t}}{\sigma_{1t}}-1}, \\
s_{1t}(x) &= \exp\left[-\left(\frac{x}{m_{1t}}\right)^{\frac{m_{1t}}{\sigma_{1t}}}\right], \\
f_{1t}(x) &= \frac{1}{\sigma_{1t}}\left(\frac{x}{m_{1t}}\right)^{\frac{m_{1t}}{\sigma_{1t}}-1}\exp\left\{-\left(\frac{x}{m_{1t}}\right)^{\frac{m_{1t}}{\sigma_{1t}}}\right\}.
\end{aligned}
$$

Adults: The adult population (ages 18–64) is denoted by $i = 2$. This population is modeled as an inverse Weibull distribution with parameters $m_{2t} > 0$ and $\sigma_{2t} > 0$: For $x \geq 0$,

$$\begin{aligned}
\mu_{2t}(x) &= \frac{\frac{1}{\sigma_{2t}}\left(\frac{x}{m_{2t}}\right)^{-\frac{m_{2t}}{\sigma_{2t}}-1}}{\exp\left(\left(\frac{x}{m_{2t}}\right)^{-\frac{m_{2t}}{\sigma_{2t}}}\right)-1}, \\
s_{2t}(x) &= 1-\exp\left(-\left(\frac{x}{m_{2t}}\right)^{-\frac{m_{2t}}{\sigma_{2t}}}\right), \\
f_{2t}(x) &= \frac{1}{\sigma_{2t}}\left(\frac{x}{m_{2t}}\right)^{-\frac{m_{2t}}{\sigma_{2t}}-1}\exp\left(-\left(\frac{x}{m_{2t}}\right)^{-\frac{m_{2t}}{\sigma_{2t}}}\right).
\end{aligned}$$

Elderly: The elderly population (ages 65 and over) is denoted by $i = 3$. This population is modeled as a Gompertz distribution with parameters $m_{3t} > 0$ and $\sigma_{3t} > 0$: For $x \geq 0$,

$$\begin{aligned}
\mu_{3t}(x) &= \frac{1}{\sigma_{3t}}\exp\left(\frac{x-m_{3t}}{\sigma_{3t}}\right), \\
s_{3t}(x) &= \exp\left(e^{-\frac{m_{3t}}{\sigma_{3t}}}-e^{\frac{x-m_{3t}}{\sigma_{3t}}}\right), \\
f_{3t}(x) &= \frac{1}{\sigma_{3t}}\exp\left(\frac{x-m_{3t}}{\sigma_{3t}}+e^{-\frac{m_{3t}}{\sigma_{3t}}}-e^{\frac{x-m_{3t}}{\sigma_{3t}}}\right).
\end{aligned}$$

Finally, let

$$\mathring{e}^{(it)}_{x:\overline{110-x}|} = \int_0^{110-x}\frac{s_{it}(x+y)}{s_{it}(x)}dy \qquad (5)$$

denote the partial expectation of the future lifetime of a person between ages x and $110-x$ with survival function following $s_{it}(x)$. Because the tail probabilities of the Weibull distribution decrease to 0 slowly as $x \to \infty$, we choose the temporary complete life expectancy truncating at the limiting age at 110 instead of the complete future lifetime. For human lives, there have been few observations of age at death beyond 110; see, for example, Bowers et al., (1997, p. 86).

3.3 The Loss Function

In order to determine the parameters that best fit the data, a non-negative loss function, $L_t(\theta_t, \rho_t)$, is used to measure adequacy of the estimation. The loss function is based on the sum of squared deviations.

The parameter estimates are determined by minimizing the combined loss function.

Let ω denote the highest age in the population life table; $q_{x,t}$ denote the observed mortality rate at age x and time t; $\hat{q}_{x,t}(\theta_t)$ denote the fitted mortality rate at age x and time t; and $w_{x,t}$ denote the weight assigned to age x and time t. In addition, let T denote the set of years assigned to the population life tables. From Table 1, the elements of T are $t_1, \ldots, t_8$.

The loss function at time t, L_t and the combined loss function, L, are given by

$$L_t(\theta_t, \rho_t) = \sum_{x=0}^{\omega-1} w_{x,t}(\hat{q}_{x,t}(\theta_t) - q_{x,t})^2, \tag{6}$$

$$L = \sum_{t \in T} L_t(\theta_t, \rho_t). \tag{7}$$

Specifically, the weight function $w_{x,t} = 1/q_{x,t}^2$ is used in equation (6).[2] This weight function leads to the following loss function:

$$L_t(\theta_t, \rho_t) = \sum_{x=0}^{\omega-1} (1 - \frac{\hat{q}_{x,t}(\theta_t)}{q_{x,t}})^2 \quad \text{for } t \in T. \tag{8}$$

The minimization equation is:

$$\min L = \sum_{t \in T} L_t(\theta_t, \rho_t)$$

subject to $\sum_{i=1}^{3} \rho_{it} = 1$ for $t \in T$, $m_{it} > 0$, $\sigma_{it} > 0$ for $i = 1, 2, 3$ and $t \in T$. In the minimization process only the data between ages 0 and 90 are used because data over age 90 are difficult to obtain. Thus $\omega = 90$.

4 The Results

The computations in this paper were done using the software S-PLUS, which incorporates the S system developed at AT&T Bell Laboratories. Two basic references to S-PLUS are Becker, Chambers, and Wilks (1988) (for the programming aspects) and Chambers and Hastie (1992) (for the statistical modeling aspects).

[2]It is usual to use $w_{x,t} = 1/\sigma_{x,t}^2$ instead. So using $w_{x,t} = 1/q_{x,t}^2$ implies σ_x^2 is proportional to q_x^2, i.e., a constant coefficient of variation across age.

Table 2
Taiwan Male Population: Component 1
Estimates for Parameters and Partial Expectations of Life

r	Period	ρ_{1,t_r}	m_{1,t_r}	σ_{1,t_r}	$\overset{\circ}{e}{}^{(1,t_r)}_{0:\overline{110}\rceil}$	$\overset{\circ}{e}{}^{(1,t_r)}_{18:\overline{92}\rceil}$	$\overset{\circ}{e}{}^{(1,t_r)}_{65:\overline{45}\rceil}$
1	1926-30	0.2727	0.98	1.41	1.25	3.61	5.18
2	1936-40	0.2610	1.09	1.54	1.37	3.68	5.20
3	1956-58	0.0817	1.18	1.96	1.77	6.49	10.02
4	1966-67	0.0438	1.46	3.16	3.39	16.27	21.73
5	1970-71	0.0334	1.43	3.52	4.38	22.47	26.26
6	1975-76	0.0232	1.51	3.40	3.80	18.41	23.43
7	1980-81	0.0213	2.84	7.21	8.48	31.34	30.46
8	1990-91	0.0123	3.40	7.87	8.43	28.34	28.85

4.1 The Male Populations

Tables 2 to 4 present the parameter estimates and the partial expectations of life for the Taiwanese male population. The information in these tables is rearranged and displayed in Figures 3 through 8. In Table 2, the mixing probabilities $\rho_{1,t}$ decrease rapidly as t increases, while $m_{1,t}$ and $\sigma_{1,t}$ increase as t increases. Table 2 shows that the effect on the mortality rates from the infant population has diminished over time because $\rho_{1,t}$ decreases from 27.27 percent to 1.23 percent.

Table 3 shows that $\rho_{2,t}$ decreases from 3.54 percent to 2.16 percent gradually, while $m_{2,t}$ and $\sigma_{2,t}$ have shown no pattern over the years. In Table 4, $\rho_{3,t}$ shows a linear increasing trend from 69.19 percent to 96.61 percent over the years. In contrast, $m_{3,t}$ has increased from 61.87 to 79.86. There is no pattern for $\sigma_{3,t}$.

Figures 3 to 5 provide a better view of the parameter changes over the years. Figure 3 shows the mixture probabilities ($\rho_{i,t}$), Figure 4 shows the location parameters (m_{it}), and Figure 5 shows the dispersion parameters. Notice that the $\rho_{i,t}$ are decreasing by years in the infant population; are steady in the adult population; and, in the elderly population, increase to what appears to be their asymptotic values. In general, m_{it} and $\sigma_{i,t}$ have a tendency to increase both in the infant population and the elderly population, while $m_{i,t}$ remains level from about age 25 in the adult male population.

Table 3
Taiwan Male Population: Component 2
Estimates for Parameters and Partial Expectations of Life

r	Period	ρ_{2,t_r}	m_{2,t_r}	σ_{2,t_r}	$\mathring{e}^{(2,t_r)}_{0:\overline{110}\rceil}$	$\mathring{e}^{(2,t_r)}_{18:\overline{92}\rceil}$	$\mathring{e}^{(2,t_r)}_{65:\overline{45}\rceil}$
1	1926-30	0.0354	25.20	6.61	31.15	13.55	17.96
2	1936-40	0.0322	23.47	6.43	29.39	12.37	18.54
3	1956-58	0.0275	25.96	12.19	38.97	23.96	26.44
4	1966-67	0.0284	25.47	9.63	35.31	19.02	23.27
5	1970-71	0.0280	27.13	11.34	38.80	22.50	24.82
6	1975-76	0.0294	26.41	11.64	38.62	22.94	25.55
7	1980-81	0.0234	24.45	8.64	33.16	16.90	22.24
8	1990-91	0.0216	23.58	8.01	31.59	15.49	21.64

Table 4
Taiwan Male Population: Component 3
Estimates for Parameters and Partial Expectations of Life

r	Period	ρ_{3,t_r}	m_{3,t_r}	σ_{3,t_r}	$\mathring{e}^{(3,t_r)}_{0:\overline{110}\rceil}$	$\mathring{e}^{(3,t_r)}_{18:\overline{92}\rceil}$	$\mathring{e}^{(3,t_r)}_{65:\overline{45}\rceil}$
1	1926-30	0.6919	61.87	16.60	53.96	38.06	8.69
2	1936-40	0.7068	63.93	15.61	56.10	39.77	8.88
3	1956-58	0.8908	72.59	11.31	66.18	48.53	10.30
4	1966-67	0.9278	75.61	10.68	69.51	51.72	11.67
5	1970-71	0.9386	76.14	10.83	70.22	52.44	12.23
6	1975-76	0.9474	77.33	10.82	71.14	53.35	12.79
7	1980-81	0.9553	77.98	10.99	71.70	53.91	13.27
8	1990-91	0.9661	79.86	11.49	73.30	55.53	14.70

Figure 3
Mixture Probabilities for Taiwanese Male Tables 1926-1991

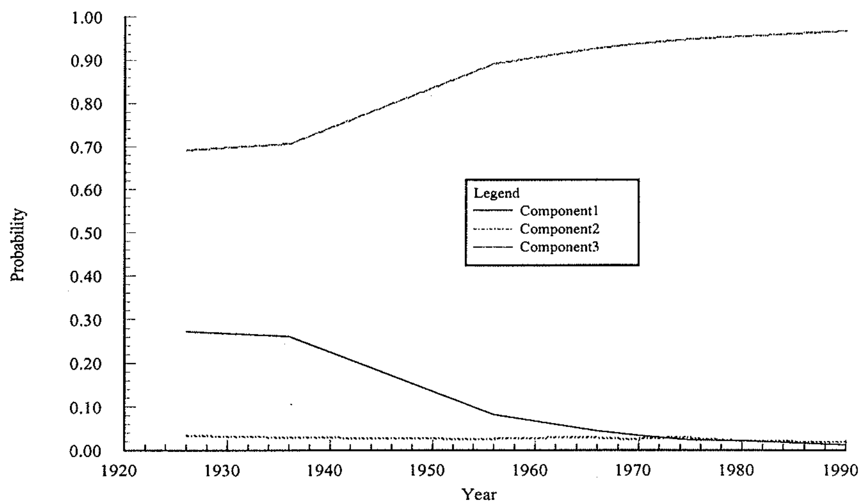


Figure 4
Location Parameters for Taiwanese Male Tables 1926-1991

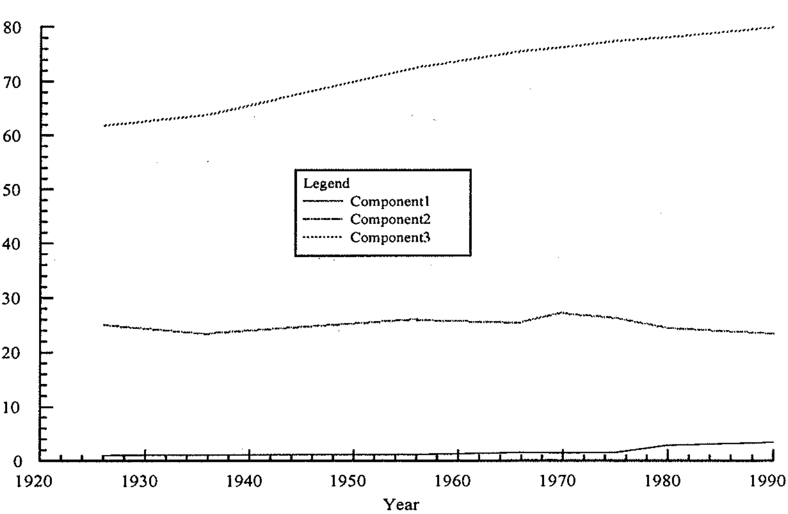
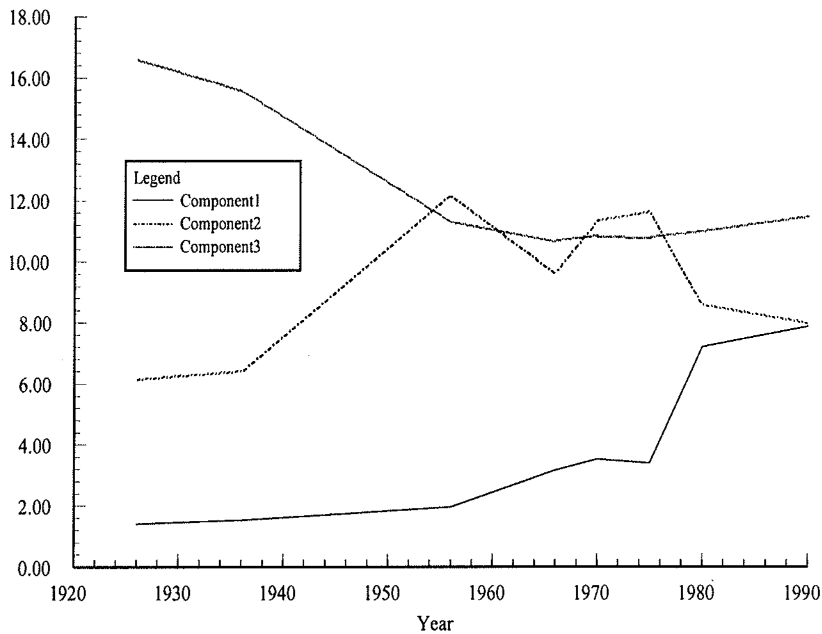


Figure 5
Dispersion Parameters for Taiwanese Male Tables 1926–1991



The partial expectations of life for the male population in Table 2 to Table 4 have shown increasing trends over the years both in component 1 and component 3, while the same pattern does not appear in component 2. Table 4 displays that the partial expectations of life in component 3 increase gradually from 53.96 to 73.30 at age 0; 38.06 to 55.33 at age 18; and 8.69 to 14.70 at age 65 .

Figures 6 through 8 compare the partial expectations of life over the years with each component. In component 1, it has shown an increasing trend indicating that the future lifetime is increasing at different ages. This increasing pattern in component 3 has explained the aging pattern in Taiwan, while the pattern in component 2 is not clearly shown.

Figure 6
$\mathring{e}_{0:\overline{110}|}$ for Taiwanese Male Tables 1926–1991

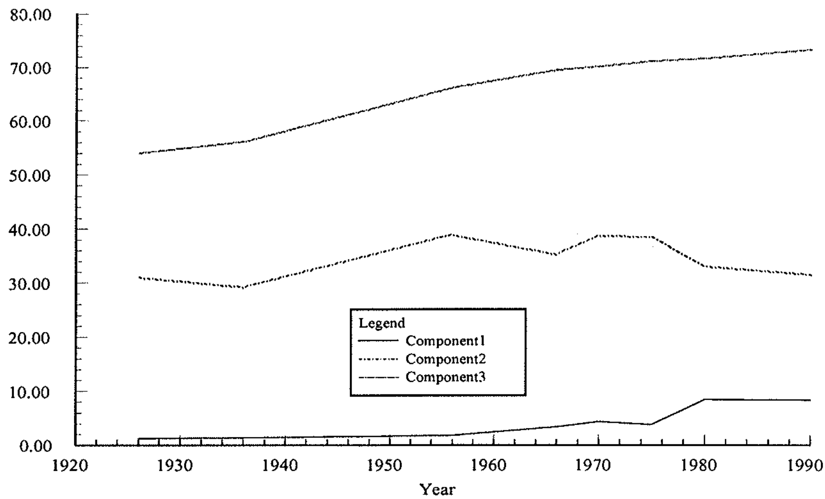


Figure 7
$\mathring{e}_{18:\overline{92}|}$ for Taiwanese Male Tables 1926–1991

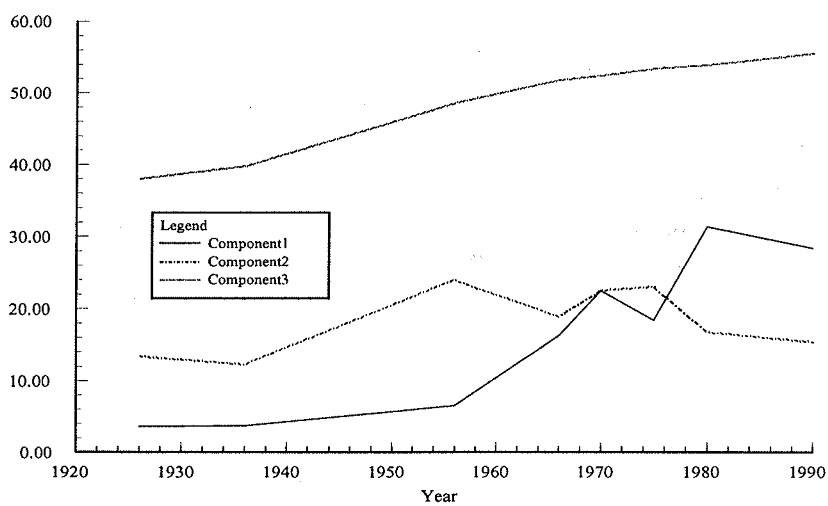
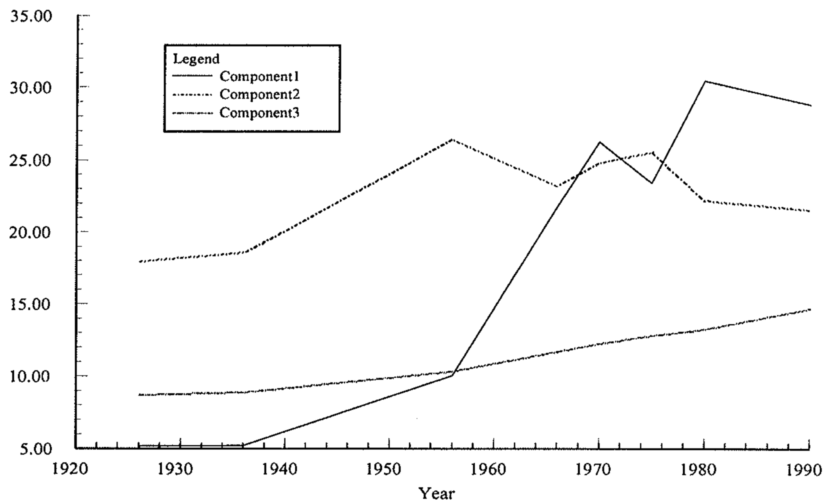


Figure 8
$\mathring{e}_{65:\overline{45}|}$ for Taiwanese Male Tables 1926-1991



4.2 The Female Populations

The parameter estimates for Taiwanese females are listed in Tables 5 to 7 and displayed in Figures 9 through 14.

The results for females are largely similar to those for males, especially for the infant and elderly populations. In contrast to the results obtained for the males, however, $m_{2,t}$ and $\sigma_{2,t}$ for the female populations are more unstable. Notice that $m_{2,t}$ has increased in the 1975-1976 table, then decreased in the 1990-1991, and $\sigma_{2,t}$ shows the same pattern; (see Figures 9 to 11).[3]

Like the male population, the partial expectations of life for the female population in Tables 5 to 7 also show an increasing trend over the years in components 1 and 3 and, to a lesser extent, in component 2.

4.3 Parameter Asymptotics

Notice that the population life tables in 1926-1930 and 1936-1940, which were constructed before and during World War II, yield signif-

[3]This pattern may be due to the change in the socioeconomic status of women. Further analysis is needed to explore this.

Table 5
Taiwan Female Population: Component 1
Estimates for Parameters and Partial Expectations of Life

r	Period	ρ_{1,t_r}	m_{1,t_r}	σ_{1,t_r}	$\mathring{e}^{(1,t_r)}_{0:\overline{110}\rceil}$	$\mathring{e}^{(1,t_r)}_{18:\overline{92}\rceil}$	$\mathring{e}^{(1,t_r)}_{65:\overline{45}\rceil}$
1	1926-30	0.2756	1.24	1.97	1.75	5.85	8.85
2	1936-40	0.2549	1.33	1.99	1.77	5.15	7.45
3	1956-58	0.0817	1.35	1.84	1.64	3.85	5.27
4	1966-67	0.0388	1.29	2.04	1.82	5.93	8.93
5	1970-71	0.0272	1.14	2.12	2.01	9.02	14.01
6	1975-76	0.0186	1.22	1.71	1.52	3.89	5.46
7	1980-81	0.0153	1.36	2.85	2.97	14.29	20.01
8	1990-91	0.0124	4.59	11.57	12.16	36.38	32.31

icantly higher mortality than the other tables for both males and females. So, to determine the parameters that fit equation (3), we only consider the estimates from the life tables after 1940 (to obtain more consistent results). The parameters of equation (3) are estimated using *NLMIN*, the nonlinear optimization procedure in S-PLUS.[4] The estimates and asymptotes (as $t \to \infty$) are given in Table 8.

The Kolmogorov-Smirnov goodness-of-fit test is used with the null hypothesis that the distributions of these two samples are the same.[5] The critical value for D_n with a 5 percent significance level is approximately $1.36/\sqrt{90} = 0.14335$. The D_ns for the fitted model are summarized in Table 9. Because the D_ns are less than 0.14335, the results again support the use of the mixture parametric model to fit the population mortality rates.

[4] *NLMIN* is based on a quasi-Newton method using double dogleg step with BFGS secant update to the Hessian. For more details, see Dennis, Gay, and Welsch (1981) and Dennis and Mei (1979).

[5] The Kolmogorov-Smirnov goodness-of-fit statistic (D_n) is defined as

$$D_n = \sup_{0 \le x \le 89} [\,|\, s(x) - s_n(x) \,|\,]$$

where n is 90. See Hogg and Klugman (1984) or Hogg and Tanis (1983) for a detailed discussion on this statistic.

Table 6
Taiwan Female Population: Component 2
Estimates for Parameters and Partial Expectations of Life

r	Period	ρ_{2,t_r}	m_{2,t_r}	σ_{2,t_r}	$\mathring{e}^{(2,t_r)}_{0:\overline{110}\rceil}$	$\mathring{e}^{(2,t_r)}_{18:\overline{92}\rceil}$	$\mathring{e}^{(2,t_r)}_{65:\overline{45}\rceil}$
1	1926-30	0.1068	27.08	10.61	37.85	21.20	23.87
2	1936-40	0.0914	26.58	11.12	38.10	22.00	24.80
3	1956-58	0.0582	33.49	21.39	52.89	37.78	31.14
4	1966-67	0.0537	39.44	29.48	60.94	45.75	33.46
5	1970-71	0.0385	40.34	30.74	42.87	24.87	6.61
6	1975-76	0.0376	43.48	38.93	66.01	51.84	35.60
7	1980-81	0.0211	33.31	22.43	53.45	38.93	31.79
8	1990-91	0.0197	32.80	19.94	51.39	36.04	30.46

Table 7
Taiwan Female Population: Component 3
Estimates for Parameters and Partial Expectations of Life

r	Period	ρ_{3,t_r}	m_{3,t_r}	σ_{3,t_r}	$\mathring{e}^{(3,t_r)}_{0:\overline{110}\rceil}$	$\mathring{e}^{(3,t_r)}_{18:\overline{92}\rceil}$	$\mathring{e}^{(3,t_r)}_{65:\overline{45}\rceil}$
1	1926-30	0.6176	70.62	14.49	62.84	45.83	11.12
2	1936-40	0.6537	71.71	13.52	64.29	47.03	11.09
3	1956-58	0.8601	77.38	10.89	71.16	53.36	12.85
4	1966-67	0.9075	79.14	10.04	73.37	55.49	13.61
5	1970-71	0.9343	79.92	10.07	74.13	56.24	14.14
6	1975-76	0.9454	81.17	9.78	75.54	57.63	14.90
7	1980-81	0.9659	82.14	9.92	76.43	58.52	15.64
8	1990-91	0.9679	83.57	9.40	78.15	60.20	16.54

Table 8
Parameters from the Nonlinear Estimation

Parameter	1956-58	Asymptote	a	b
$\rho_{3,t}$(%) (female)	77.38	85.64	0.0072	1.50
$m_{3,t}$ (female)	86.01	97.02	0.0115	1.69
$m_{3,t}$ (male)	89.07	99.10	0.0571	0.91

Table 9
Kolmogorov-Smirnov Statistic D_n

Year	Male	Female
1926-30	0.00835	0.012511
1936-40	0.00272	0.012900
1956-58	0.00139	0.006344
1966-67	0.01995	0.002678
1970-71	0.00732	0.004283
1975-76	0.00204	0.004531
1980-81	0.00618	0.003248
1990-91	0.01338	0.005294

5 Closing Comments

Extreme value distributions are the underlying distributions of the parametric mixture model used to analyze the mortality structure in Taiwan from 1926 to 1991. This approach provides a more detailed model to understand the changing mortality pattern over the years and may form a better basis for projecting future mortality rates.

The mixture model points out the different patterns of mortality between Taiwanese male and female populations in different age components. The gender differences may be explained by the different socioeconomic roles men and women play in Taiwanese society.

Further research is needed to apply these results to premium and reserve calculations and to construct appropriate parametric models that include the effects of specific demographic impacts on mortality.

Figure 9
Mixture Probabilities for Taiwanese Female Tables 1926–1991

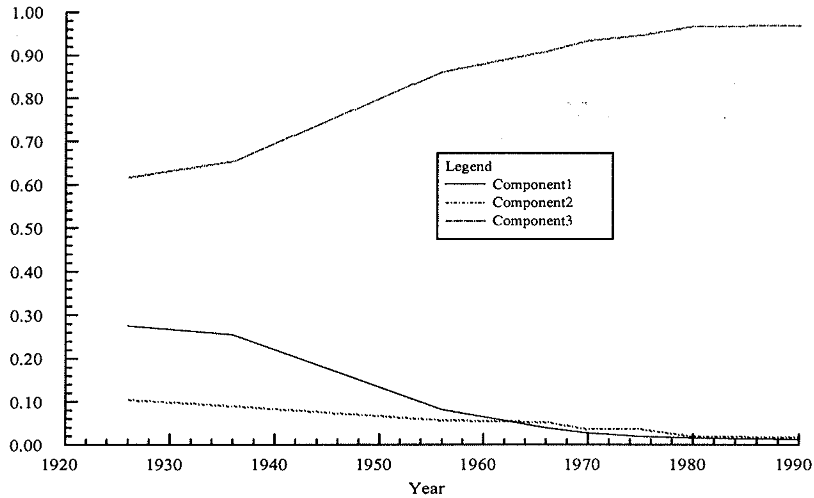


Figure 10
Location Parameters for Taiwanese Female Tables 1926–1991

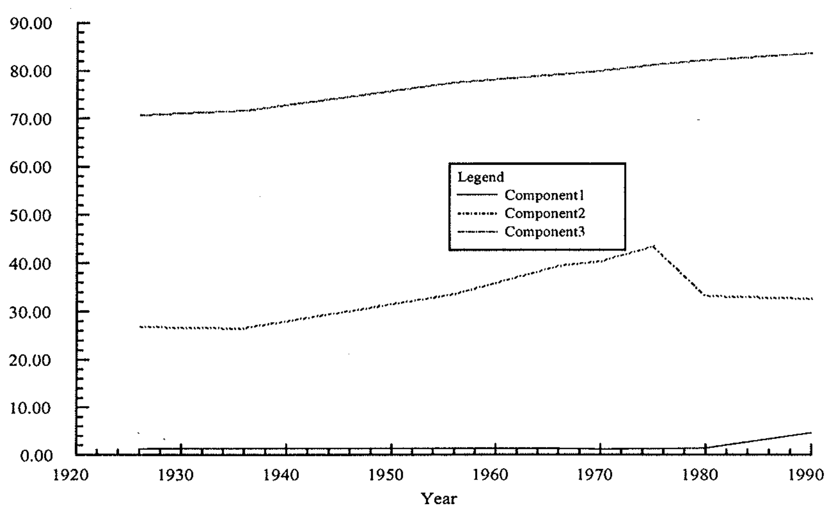


Figure 11
Dispersion Parameters for Taiwanese Female Tables 1926–1991

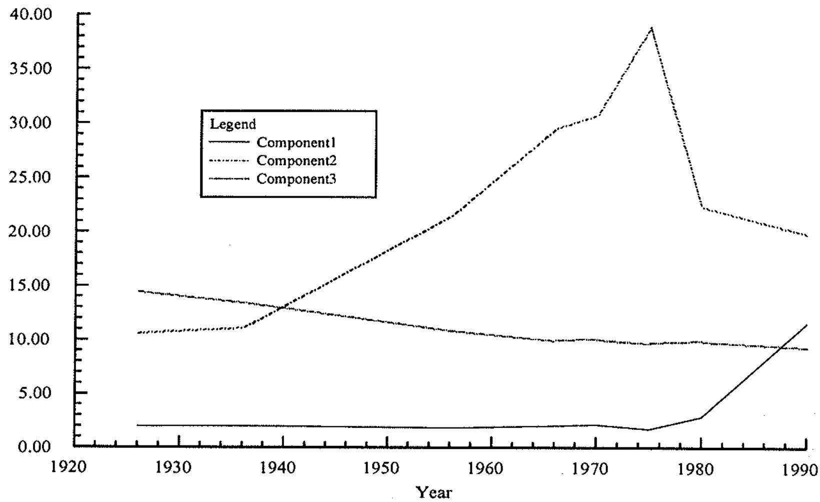


Figure 12
$\mathring{e}_{0:\overline{110}|}$ for Taiwanese Female Tables 1926–1991

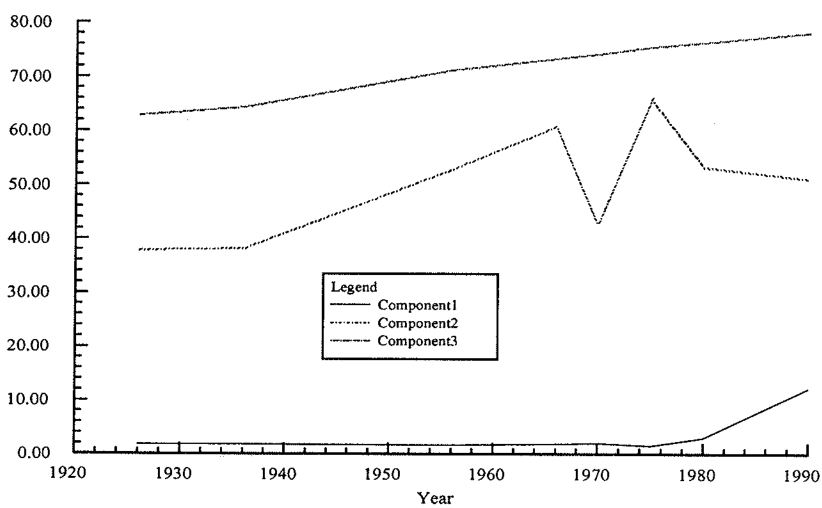


Figure 13
$\mathring{e}_{18:\overline{92}|}$ for Taiwanese Female Tables 1926–1991

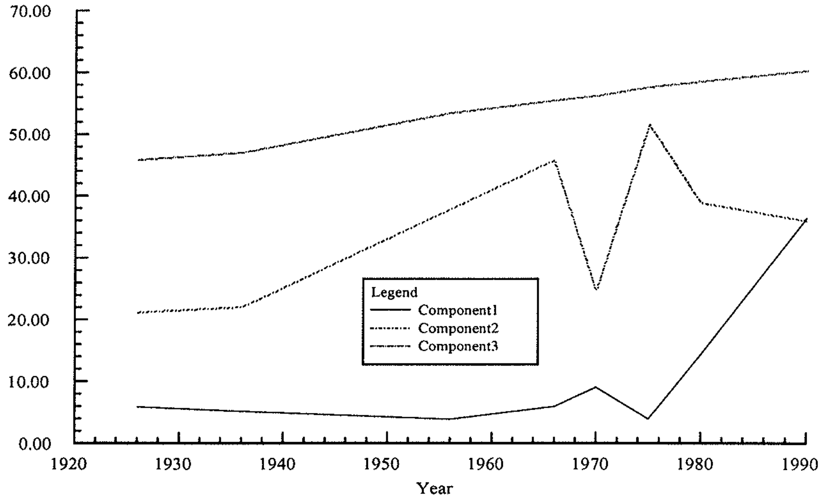
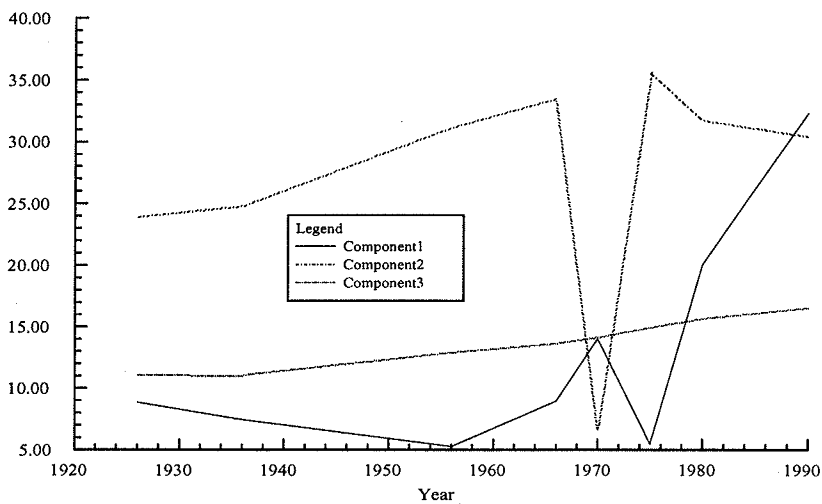


Figure 14
$\mathring{e}_{65:\overline{45}|}$ for Taiwanese Female Tables 1926–1991



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