Value at Risk of Life Insurance Policy Reserves
(壽險保單準備金之風險值)

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Abstract

We estimate the value at risk (VaR) of life insurance policy reserves in this paper. Since the market price of reserves does not exist, we construct a simulation model considering mortality rate risk, interest rate risk, surrender rate risk, and parameter estimation risks to estimate the VaR. Simulation results show that the VaR from mortality rate risk is small but interest rate risk as well as the parameter estimation risk of interest rate model significantly enlarges the VaR. On the other hand, surrender rate risk reduces reserve VaR. With regard to individual product, annuity and whole life insurance have the largest VaR, followed by pure endowment and endowment. Term life insurance has the smallest one.
Keywords: Value at risk; Policy reserves; Life Insurance

摘要

作者們於本文中估計數種保險商品準備金之風險值。由於準備金沒有市場價格，作者們建立了一個包含死亡率風險，利率風險，解約率風險，以及參數估計風險的模擬模型來估計風險值。我們發現死亡率所產生的風險值很低，利率風險以及利率模型的參數估計風險會使風險值顯著變大，而解約率風險則會降低準備金的風險值。保險商品中，年金與終身壽險的風險值最大，生存險與生死合險次之，而定期壽險的風險值則最小。
關鍵詞：風險值、保單準備金、人身保險

I. Introduction

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The use of value at risk (VaR) exploded in banking and securities industries during the last decade. It is becoming universal in the private sector (Dowd, 1998, p.19 – 20). In addition to getting popular in the private sector, VaR is gaining acceptance from regulators. For instance, the Basle Committee allowed certain banks to use their own VaR models in calculating required capital starting from 1997.

VaR is important to the insurance industries as well. Firstly, VaR can deal with underwriting and financial risks at the same time. It can cover risk factors embedded in the liabilities as well as the assets of an insurance company. It is also able to take into account the correlations among insurance lines, the correlations among invested assets, and the correlations between liabilities and assets.

Secondly, the trend of financial services integration will make VaR indispensable to the insurance industry. The integration among financial services industries brings the insurance company to compete with other financial institutions head to head in many aspects including risk management. Since VaR is a well-developed risk management system and has been adopted by many financial institutions, insurance companies will have to develop their own VaR systems to effectively manage their risks to remain competitive.

Thirdly, insurance regulators will probably adopt VaR in capital requirements. Bank regulators have accommodated VaR in addition to the Standardized method. Since the risk-based capital requirements currently used in the insurance industries are similar to the standardized method, insurance regulators might follow the track of bank regulators and adopt VaR in the future. Indeed, the International Association of Insurance Supervisors (IAIS) declares in the Principles on Capital Adequacy and Solvency "[insurance] supervisors may consider the use of internal capital models as a basis for a capital requirement."

Albeit the popularity and importance of VaR, we find only one paper studying VaR in the insurance literature. Panning (1999) identified two risk factors for property-casualty insurers' loss reserves: payout pattern and inflation. Panning used a regression model to decompose loss reserves into a loss growth function and a payout pattern function. From the variance-covariance matrix of the estimated parameters in these two functions, he was able to conduct Monte Carlo simulation to generate the distribution of loss reserves' present value and obtained VaR from this distribution.

Since no one has applied VaR to the liabilities of life insurance companies, we establish a valuation model and carry out Monte Carlo simulation to calculate the VaR of policy reserves for several life insurance products in this paper. To establish such a model we need to identify and model risk factors. The identified risk factors include mortality rate risk, interest rate risk, and surrender rate risk. The first two are easy to spot because mortality rates affect future cash flows and interest rate determines the discounting. Surrender rate risk deserves some discussions however. Most insurers include in their contracts a provision that grants the policyholder who elects to terminate the policy the right to a cash surrender value. The exercise for the surrender option would result in cash payments and decreases future premium income for life insurance companies. In addition, it would make the cash flows of life insurance be sensitive to the interest rate. Several recent actuarial studies in the *Transactions of Society of Actuaries Reports* documented that policyholders tend to surrender policies to reinvest in higher-yield alternatives as interest rate rises.[1] We also observed record numbers of surrenders during the high interest rate period of the 1980s.

The sensitivity of the cash flows to the interest rate could significantly alter the distribution of reserves. Babbel (1995), Briys and Varenne (1997), and Santomero and Babbel (1997) found that the sensitivity was critical to the duration and convexity of the insurance policy. Misspecifications about the sensitivity could cause large errors in the estimates of effective duration and even greater errors in the estimates of convexity. The disintermediation that happened to the U.S. life insurers during the 1980s also demonstrated the adverse effect of interest-rate-sensitive surrenders. Many life insurers experienced negative cash flows for the first time since the 1930s' depression and were forced to liquidate assets at depressed prices (Black and Skipper, 2000, p. 111). The consideration of interest-rate-sensitive surrender behavior is therefore crucial to the estimation for VaR.

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[1] For instance, Cox, Laporte, Linney, and Lombardi (1992) and the Annuity Persistency Study in the 1995-96 reports (p. 559-638).

We calculate the VaR associated with mortality rate risk by the standard deviations of mortality rates derived within a binomial framework. To quantify interest rate risk, we employ the term structure model of the interest rate developed by Vasicek (1977) in which the instantaneous spot interest rate follows a mean-reverting Ornstein-Uhlenbeck process. To evaluate surrender rate risk, we establish a linear regression model of surrender rate to capture the relation between interest rate and surrender rate. We further consider the parameter estimation risk of the interest rate and surrender rate models. To assess the accuracy of our estimated VaRs, we use the kernel estimation method to estimate the confidence intervals of the VaRs. The above analyses are applied to endowment insurance, pure endowment insurance, term life insurance, whole life insurance, and deferred whole life annuity.

Our results show that the VaR generated from mortality rate risk is insignificant while the VaR from interest rate risk is substantial. These results are consistent with the observation that life insurers suffer more from interest rate risk than from mortality rate risk. On the other hand, the marginal contribution of surrender rate risk to VaR is negative. This result is understandable when we realize that the surrender option is analogue to the prepayment option in loans that has been shown to be able to decrease the effective duration. Such a result implies that life insurers would overestimate the VaR of insurance and over-hedge interest rate risk if they neglect the embedded surrender options in most policies.

We also find that the parameter risk of the interest rate model is considerable. With regard to product risk, annuity and whole life insurance have the largest VaR, term life insurance has the smallest one, and pure endowment and endowment insurance have medium VaRs.

This paper contributes to the literature in several aspects. First, we consider both stochastic interest rate and surrender rate when calculating the reserve VaR for several types of life insurance products. Second, we examine the impact on reserve VaR from several risk factors, including mortality rate, interest rate, surrender rate, and parameter estimation risk. Finally, we document the beneficial effect of surrender options on reserve VaR, which has implications to life insurer's risk management.

This paper consists of six sections. Following this introduction section is the simulation setting section in which we describe the set up of the simulation. We then analyze the reserve VaR for several types of life insurance from the identified three risk factors one after another. The accuracy check on the estimated VaR is provided in each section. The last section contains conclusions and discussions for future researches.

II. Simulation Setting

Consider a group of *N life-aged-x* policyholders. Assume that these policyholders have two causes of decrement: mortality and early surrender. For each of these policyholders, the probability of death and surrender in policy year t ($t \geq 1$) are specified by $q_{x+t-1}^{(m)}$ and $q_{x+t-1}^{(l)}$ respectively. Assume that *T-year* endowment policies, *T-year* pure endowment, *T-year* term life, whole life, and *T-year* deferred whole life annuities are available to these policyholders with net level annual premium of \$$P$ payable at the beginning of surviving years for at most T years. Let F_{end}, F_{term}, and F_{whole} be the death benefits of the endowment, term life, and whole life insurance payable at the end of the death year or at the end of the year $x+T$, F_{pure} be the living benefit of pure endowment insurance payable at the end of the year $x+T$ if the policyholder survives, and F_{ann} be the annual payment of the annuity payable at the beginning of surviving years. If policyholders surrender their policies during policy year t, they receive the amount of ${}_tS_x^i$ at the end of the year, where the up-script i denotes the type of policy. We assume that

$$\begin{aligned} {}_tS_x^i &= \left(0.8+0.2\times\frac{t}{T}\right)\times{}_tV_x^i \quad \text{if } t<T \\ &= {}_tV_x^i \quad \text{if } t \geqq T \text{ (for whole life insurance and annuity),} \end{aligned} \tag{1}$$

[2]

where ${}_tV_x^i$ is the policy reserve at the end of policy year t calculated using random future lifetime and deterministic interest rate as in the chapter 7 of Bowers et al. (1986). Let L_{end}, L_{term}, L_{whole}, L_{pure}, and L_{ann} be the random variable denoting the present value of the cash flows generated by these insurance policy pools, respectively. Then

$$\begin{aligned} L_{end} = \sum_{t=1}^{T}&\left\{\left[F_{end}\times\left(q_{x+t-1}^{(m)}\times l_{x+t-1}\right)+{}_tS_x^{end}\times\left(q_{x+t-1}^{(l)}\times l_{x+t-1}\right)\right]\times v_t - P\times l_{x+t-1}\times v_{t-1}\right\}, \\ &+F_{end}\times l_{x+T}\times v_T \end{aligned} \tag{2}$$

[2] The formula comes from the Model Life Insurance Policy Provisions of Taiwan. It possesses the general property of surrender charges: high at the beginning and descends as the policy matures.

$$L_{pure} = \sum_{t=1}^{T} \left\{ \left[{}_tS_x^{pure} \times \left(q_{x+t-1}^{(l)} \times l_{x+t-1} \right) \right] \times v_t - P \times l_{x+t-1} \times v_{t-1} \right\} + F_{pure} \times l_{x+T} \times v_T \,,$$

$$L_{term} = \sum_{t=1}^{T} \left\{ \left[F_{term} \times \left(q_{x+t-1}^{(m)} \times l_{x+t-1} \right) + {}_tS_x^{term} \times \left(q_{x+t-1}^{(l)} \times l_{x+t-1} \right) \right] \times v_t - P \times l_{x+t-1} \times v_{t-1} \right\},$$

$$L_{whole} = \sum_{t=1}^{\omega-x} \left\{ \left[F_{whole} \times \left(q_{x+t-1}^{(m)} \times l_{x+t-1} \right) + {}_tS_x^{whole} \times \left(q_{x+t-1}^{(l)} \times l_{x+t-1} \right) \right] \times v_t \right\} - \sum_{t=1}^{T} P \times l_{x+t-1} \times v_{t-1} \,,$$

$$L_{ann} = \sum_{t=1}^{\omega-x} \left[{}_tS_x^{ann} \times \left(q_{x+t-1}^{(l)} \times l_{x+t-1} \right) \right] \times v_t + \sum_{t=T}^{\omega-x} F_{ann} \times l_{x+T} \times v_{t-1} - \sum_{t=1}^{T} P \times l_{x+t-1} \times v_{t-1} \,,$$

where l_{x+t-1} is the number of survivors at the beginning of policy year *t*,

$$l_{x+t} = l_{x+t-1} \times \left(1 - q_{x+t-1}^{(m)} - q_{x+t-1}^{(l)} \right), \tag{3}$$

v_t is the discount factor for the cash flows at time *t*,

$$v_t = \frac{1}{(1+r_1)(1+r_2)\cdots(1+r_t)}, \tag{4}$$

r_t is the interest rate in policy year *t*, and

ω is the largest age in the mortality table.[3]

The random variable *L* represents the present value of insurers' liabilities associated with pools of policies at the time when the policies are sold. The statistical properties of *L* are critical to the risk management of reserves and are of great concerns to actuaries, insurance regulators, and various stakeholders of insurance companies. Our goal is to estimate the distribution and VaR of *L*.[4]

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[3] Notice that $l_x = N$ and $v_0 = 1$.

[4] The time horizon for the VaR of *L* is the coverage period of the policy because the uncertainty of *L* comes from the uncertainty in the timing of benefit payments and the interest rate over the entire policy life. For

In the following simulation, we specify that $N = 100{,}000$, $T = 20$, $P = 27.133$, $F_{end} = 1{,}000$, $F_{pure} = 1{,}110$, $F_{term} = 10{,}067$, $F_{whole} = 2{,}917$, $F_{ann} = 86$, $x = 30$, and $w = 100$.[5] In addition, $q^{(m)}_{x+t-1}$ is assumed to be distributed as the 1980 CSO male mortality table and the interest rate used in calculating ${}_tV^i_x$ is 6%. We ignore dividends, expenses, loadings, taxes, and new business in the simulation.

III. Mortality Rate Risk

Our focus in this section is on the risk arising from the uncertainty of mortality rates. We thus assume that the interest rate is fixed at 6% and there are no early surrenders in this section. When assessing mortality rate risk, we recognize that the number of deaths d_{x+t-1} = $q^{(m)}_{x+t-1} \times l_{x+t-1}$ follows a binomial distribution with parameters l_{x+t-1} and $q^{(m)}_{x+t-1}$. Hence,

$$E[\hat{q}^{(m)}_{x+t-1} \mid l_{x+t-1}] = \frac{1}{l_{x+t-1}} E[\hat{d}_{x+t-1}] = q^{(m)}_{x+t-1}, \text{ and} \quad (5)$$

$$Var[\hat{q}^{(m)}_{x+t-1} \mid l_{x+t-1}] = \frac{1}{l^2_{x+t-1}} V[d_{x+t-1}] = \frac{q^{(m)}_{x+t-1}(1-q^{(m)}_{x+t-1})}{l_{x+t-1}} \quad (6)$$

Since we do not observe $q^{(m)}_{x+t-1}$, we replace it with the estimate $\hat{q}^{(m)}_{x+t-1}$. Moreover, when l_{x+t-1} is large, $\hat{q}^{(m)}_{x+t-1}$ is distributed approximately as a normal distribution with mean $\hat{q}^{(m)}_{x+t-1}$ and variance $\dfrac{\hat{q}^{(m)}_{x+t-1}(1-\hat{q}^{(m)}_{x+t-1})}{l_{x+t-1}}$.

We then compute the VaR associated with mortality rate risk by the following steps. First, the standard errors of mortality rates are calculated. The results are in Table 1. Second, we assume that mortality rates are normally distributed with parameters estimated

instance, the risk of L_{end} originates from the uncertain cash flow (determined by uncertain mortality rate and surrender rate) and uncertain interest rate in the future 20 years. Therefore, the time horizon for L_{end}'s VaR is 20 years.

from the 1980 male CSO table. Third, we draw randomly a set of mortality rates from the above normal distributions and use them to calculate the number of deaths in every policy year. Fourth, cash inflows and outflows in every policy year are calculated according to the number of deaths and survivors. Fifth, under the assumption of fixed interest rate and zero surrender we calculate the policy reserve. We repeat step 3 to 5 for 10,000 times to construct the distribution of policy reserves and then estimate VaR. Our VaR is an absolute VaR and is defined as the 95th percentile of the simulated policy reserve distribution.

Table 1: Statistics of Mortality Rates

Age x	n_x	d_x	$\hat{q}_x$	Standard Error of $\hat{q}_x$	Age x	n_x	d_x	$\hat{q}_x$	Standard Error of $\hat{q}_x$
30	9,579,998	16,573	0.1730%	0.0013%	65	7,329,740	186,322	2.5420%	0.0058%
31	9,563,425	17,023	0.1780%	0.0014%	66	7,143,418	198,944	2.7850%	0.0062%
32	9,546,402	17,470	0.1830%	0.0014%	67	6,944,474	211,390	3.0440%	0.0065%
33	9,528,932	18,200	0.1910%	0.0014%	68	6,733,084	223,471	3.3190%	0.0069%
34	9,510,732	19,021	0.2000%	0.0014%	69	6,509,613	235,453	3.6170%	0.0073%
35	9,491,711	20,028	0.2110%	0.0015%	70	6,274,160	247,892	3.9510%	0.0078%
36	9,471,683	21,217	0.2240%	0.0015%	71	6,026,268	260,937	4.3300%	0.0083%
37	9,450,466	22,681	0.2400%	0.0016%	72	5,765,331	274,718	4.7650%	0.0089%
38	9,427,785	24,324	0.2580%	0.0017%	73	5,490,613	289,025	5.2640%	0.0095%
39	9,403,461	26,236	0.2790%	0.0017%	74	5,201,588	302,680	5.8190%	0.0103%
40	9,377,225	28,319	0.3020%	0.0018%	75	4,898,908	314,461	6.4190%	0.0111%
41	9,348,906	30,758	0.3290%	0.0019%	76	4,584,447	345,209	7.5300%	0.0123%
42	9,318,148	33,173	0.3560%	0.0020%	77	4,239,238	326,930	7.7120%	0.0130%
43	9,284,975	35,933	0.3870%	0.0020%	78	3,912,308	328,243	8.3900%	0.0140%
44	9,249,042	38,753	0.4190%	0.0021%	79	3,584,065	326,329	9.1050%	0.0152%
45	9,210,289	41,907	0.4550%	0.0022%	80	3,257,736	321,995	9.8840%	0.0165%
46	9,168,382	45,108	0.4920%	0.0023%	81	2,935,741	314,858	10.7250%	0.0181%
47	9,123,274	48,536	0.5320%	0.0024%	82	2,620,883	307,299	11.7250%	0.0199%
48	9,074,738	52,089	0.5740%	0.0025%	83	2,313,584	296,740	12.8260%	0.0220%
49	9,022,649	56,031	0.6210%	0.0026%	84	2,016,844	282,862	14.0250%	0.0245%
50	8,966,618	60,166	0.6710%	0.0027%	85	1,733,982	265,213	15.2950%	0.0273%

[5] All *Fs* are calculated by the equivalence principle with P = \$27.133.

51	8,906,452	65,017	0.7300%	0.0029%	86	1,468,769	243,948	16.6090%	0.0307%
52	8,841,435	70,378	0.7960%	0.0030%	87	1,224,821	219,917	17.9550%	0.0347%
53	8,771,057	76,396	0.8710%	0.0031%	88	1,004,904	194,218	19.3270%	0.0394%
54	8,694,661	83,121	0.9560%	0.0033%	89	810,686	168,031	20.7270%	0.0450%
55	8,661,540	90,163	1.0410%	0.0034%	90	642,655	142,522	22.1771%	0.0518%
56	8,521,377	97,655	1.1460%	0.0036%	91	500,133	118,522	23.6981%	0.0601%
57	8,423,722	105,212	1.2490%	0.0038%	92	381,611	96,719	25.3449%	0.0704%
58	8,318,510	113,049	1.3590%	0.0040%	93	284,892	77,522	27.2110%	0.0834%
59	8,205,461	121,195	1.4770%	0.0042%	94	207,370	61,361	29.5901%	0.1002%
60	8,084,266	129,995	1.6080%	0.0044%	95	146,009	48,177	32.9959%	0.1231%
61	7,954,271	139,518	1.7540%	0.0047%	96	97,832	37,621	38.4547%	0.1555%
62	7,814,753	149,965	1.9190%	0.0049%	97	60,211	28,913	48.0195%	0.2036%
63	7,664,788	161,420	2.1060%	0.0052%	98	31,298	20,593	65.7965%	0.2682%
64	7,503,368	173,628	2.3140%	0.0055%	99	10,705	10,705	100%	0%

Since our VaR is a statistical estimate based on simulated distributions, we should assess its precision using certain measures such as the width of the confidence interval. We estimate the 95% confidence interval for our VaR by order statistics described in the following.

Let $Y_1 < Y_2 < \ldots < Y_n$ be the order statistics of a random sample of size n from a continuous-type distribution and let π_p denote the (100p)th percentile of the distribution. According to the section 10.2 in Hogg and Tanis (1997), the 100(1-α)% confidence interval for the unknown π_p can be derived using the following equation:

$$1-\alpha = P(Y_i < \pi_p < Y_j) = \sum_{k=i}^{j-1} \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k}$$

$$= P(i-0.5 < W < j-0.5) \approx \Phi\left(\frac{j-0.5-np}{\sqrt{np(1-p)}}\right) - \Phi\left(\frac{i-0.5-np}{\sqrt{np(1-p)}}\right), \quad (7)$$

where $1 \le i < j \le n$, W is binomially distributed with mean np and variance $np(1-p)$, and Φ is the cumulative normal distribution function. Once the samples are observed and the pair

(i, j) determined, the interval (Y_i, Y_j) could serve as a 100(1-α)% confidence interval for π_p.

Since equation (7) contains two unknown numbers, more than one pair of (i, j) can satisfy the equation. To assure the uniqueness of the confidence interval, we impose an additional constraint: the confidence interval should be as symmetric around π_p as possible. More specifically, (i, j) are chosen so that

$$p - \frac{i}{n} = \frac{j}{n} - p\,. \tag{8}$$

We specify that α = 0.05, p = 0.95, and n = 10,000 in this paper.

The simulation results for mortality rate risk are in Table 2. They indicate that the mortality rate risk defined in this paper is immaterial for all types of policies. Compared to annual premiums of $2,713,300, the VaRs of policies are insignificant. The insignificance is due to the small standard deviations of mortality rates that result from the large size of the insurance pool. The estimated VaRs have satisfactory accuracy since the widths of confidence intervals are about four to eight percent of the VaRs.

Table 2: Mortality Rate Risk (MR)

Policy	Mean	Standard Deviation	Skewness	Kurtosis	VaR	Confidence Interval Width of VaR
Endowment	37	3,071	-0.0014	-0.0324	5,141	228
Pure Endowment	2	2,113	0.0055	0.0094	3,470	181
Term Life	355	43,188	-0.0029	-0.0171	73,245	3,714
Whole Life	-2,246	12,517	0.0315	-0.0027	18,722	1,172

Annuity	-9,243	3,476	-0.0216	0.0134	-3,501	300

IV. Interest Rate Risk

1. The Term Structure Model and Its Estimation

In this section we incorporate an additional risk factor into the simulation: interest rate. To capture the dynamics of interest rate, we employ the term structure model developed by Vasicek (1977) and use statistical approaches to estimate the parameters of the model from historical interest rates. Vasicek's model assumes that the instantaneous spot rate follows a mean-reverting Ornstein-Uhlenbeck process:

$$dr_t = q(m - r_t)dt + v dW_t, \tag{9}$$

where W_t is a Wiener process, m is the long-run average of spot rates, q reflects the speed of mean reverting, and v is the volatility of the process. The Vasicek's model has a discrete AR(1) counterpart that is frequently used in the actuarial literature such as Beekman and Fuelling (1990; 1993), Parker (1994; 1997), and Marceau and Gaillardetz (1999).

The statistical method used to estimate the parameters of Vasicek's model is from Duan (1994). Assuming that the instantaneous spot rate follows equation (9), Duan derived the log-likelihood function in terms of the observed interest rates:

$$L(R_t(k),\ t = 1, 2, \ldots, n;\ \theta)$$

$$= (n\text{-}1)\ \ln(k) - (n\text{-}1)\ \ln(B(k;\ \theta))$$

$$-\frac{n-1}{2}[\ln(2\pi)+\ln V(v,q)]-\frac{1}{2V(v,q)}\sum_{t=2}^{n}[\hat{r}_t(\theta)-m-(\hat{r}_{t-1}(\theta)-m)e^{-q}]^2, \qquad (10)$$

where n = the observed sample size,

k = the time to maturity,

$B(k;\theta)=\frac{1}{q}[1-\exp(-qk)]$,

$V(v,q)=(\frac{v^2}{2q})(1-e^{-2q})$,

$\hat{r}_t(\theta)=\frac{1}{B(k;\theta)}[kR_t(k)+\ln A(k;\theta)]$,

$A(k;\theta)=\exp[(m+\frac{v\lambda}{q}-\frac{v^2}{2q^2})(B(k;\theta)-k)-\frac{v^2B^2(k;\theta)}{4q}]$, and

λ = the constant risk premium.

The data used for the estimation are the monthly rates of one-year U.S. Treasury bills from January 1960 to March 1999.[6] We use GAUSS to do the numerical optimization. The estimation results for the parameters are in Table 3.[7]

Table 3: Estimated Parameters of the Term Structure Model

Parameter	Estimate	Standard Error	p-value	Correlation Matrix of Estimated Parameters		
q	0.0151	0.0080	0.0293	1.000	-0.012	0.239
m	0.0602	0.0120	0.0000	-0.012	1.000	-0.003
v	0.0040	0.0001	0.0001	0.239	-0.003	1.000

2. Simulation of Interest Rates and *L*

6 We collect the data from the web site of Federal Reserve Bank of Minneapolis. The web site address is " http://minneapolisfed.org/economy/usindex.html." Interest rates are calculated based on ask quotations and then transformed to annualized continuously compound rates.

7 The results in Table 3 are obtained under the assumption that λ is zero because the preliminary maximum likelihood estimation involving four parameters shows that λ is not significantly different from zero.

We simulate 10,000 interest rate paths of monthly one-year T-bill rates for seventy years using 6% as the initial value.[8] Combining the 10,000 interest rate paths with the 10,000 sets of l_{x+t-1} simulated in Section III, we obtain the distribution of L under the consideration for stochastic interest rate as well as random mortality rates. The results are shown in Table 4.

Table 4: Interest Rate Risk (IR) + MR

Policy	Mean	Standard Deviation	Skewness	Kurtosis	VaR	Confidence Interval Width of VaR
Endowment	685,711	5,472,794	0.7142	0.5696	11,010,827	770,580
Pure Endowment	744,528	5,924,107	0.7172	0.5763	11,931,353	853,625
Term Life	152,405	1,459,783	0.5600	0.2540	2,830,904	159,111
Whole Life	1,659,562	8,592,566	1.2689	2.6985	17,848,179	1,142,640
Annuity	1,506,765	8,853,003	1.0507	1.7330	18,147,460	1,322,618

The interest rate risk is momentous for all types of policies. The VaRs of whole life insurance and deferred annuity are more than six times of annual premiums and the VaRs of endowment and pure endowment policies are more than four time of annual premiums. Even term life insurance has a VaR larger than annual premiums. These figures suggest that the insurer has to keep low premium-to-surplus ratio and tremendous amount of surplus to

Following Duan (1994), we re-estimate the other three parameters under the assumption that λ is zero.

[8] The choice of 6% is referred to the policy credit rate, the long-term interest rate in our sampling period, and

maintain an acceptable solvency probability.

The large VaRs result from the possibilities of low interest rates and the long-term nature of policies. When interest rates are low, the present value of the insurer's future obligation increases. The long maturity of the insurance aggravates such effect and makes reserves much larger than expected. Whole life insurance and annuity have the largest VaRs because they have the longest expected maturities. Endowment and pure endowment policies have larger VaRs than term life because they have larger expected payments toward the end of the contracts. Pure endowment has a slightly larger VaR than endowment because it pays living benefits at the end only and thus has higher percentage of cash outflows incurred later. The substance of interest rate risk versus the insignificance of mortality rate risk is consistent with the literature and the experience of the life insurance industry.

3. Parameter Estimation Risk of the Interest Rate Model

To evaluate the parameter estimation risk, we first use the covariance matrix resulting from the estimation for the interest rate model to generate sets of parameters. Specifically, we assume that the parameters $(\hat{q}, \hat{m}, \hat{v})$ are from a multivariate normal distribution with expected values and covariance matrix as specified in Table 3. We then draw 10,000 sets of $(\hat{q}, \hat{m}, \hat{v})$ from this multivariate normal distribution. For each of these 10,000 sets of

the average interest rate in our simulations.

parameters, we simulate one interest rate path and get 10,000 paths altogether.[9] Combining these interest rate paths with the 10,000 sets of l_{x+t-1} simulated in Section III, we obtain the distribution of L under random mortality rates, stochastic interest rate, and parameter estimation errors of the interest rate model. Relevant results are in Table 5.

Table 5: Parameter Estimation Risk of Term Structure Model (ETS) + IR + MR

Policy	Mean	Standard Deviation	Skewness	Kurtosis	VaR	Confidence Interval Width of VaR
Endowment	1,028,786	7,252,410	0.8683	0.9468	15,139,663	982,409
Pure Endowment	1,118,007	7,853,112	0.8726	0.9549	16,445,554	1,122,512
Term Life	219,813	1,868,950	0.6681	0.5547	3,709,120	235,813
Whole Life	4,457,253	17,202,571	2.3276	7.9248	37,705,427	3,402,726
Annuity	3,248,548	14,737,277	1.6510	3.9987	32,381,437	3,374,174

The estimating errors of term structure parameters substantially increase the VaRs. Such increases are reasonable since we now have one more type of uncertainty and thus have more chances to get low interest rates. Our results indicate that insurers should pay attention to the potential parameter estimation errors whenever they employ a term structure model. The results further confirm the importance of the risk derived from the randomness of interest rate. The accuracy of our VaRs deteriorates a little bit: the widths of confidence

[9] We are aware that simulating only one interest rate path for each parameter set might not correctly reflect the parameter estimation risk; we might either under-estimate or over-estimate the risk. Ideally, we should generate many paths for each parameter set. The computing time for such simulation is prohibitively long,

intervals increase to six to ten percent of VaRs, which are still acceptable.

V. Surrender Rate Risk

1. The Surrender Rate Model and Its Estimation

To assess surrender rate risk, we have to establish a surrender rate model first. Two hypotheses have been proposed to explain the behavior of surrenders (Outreville, 1990). The emergency fund hypothesis contends that policyholders utilize the cash values of policies as emergency funds when facing personal financial distresses. Furthermore, policyholders may be unable to maintain premium payments for insurance coverage in these difficult times. Policyholders therefore incline to terminate their insurance policies during personal financial distresses. A testable implication of this hypothesis is that surrenders would increase during pervasive economic recessions. As a competing counterpart, the interest rate hypothesis conjectures that surrender rate rises when market interest rate increases because the latter acts as the opportunity cost of owning insurance contracts. More specifically, policyholders tend to surrender their policies to exploit higher yields available in the financial markets. These two hypotheses serve as the starting point in establishing an empirical surrender rate model.

Following Outreville (1990), we use linear regression to model the surrender rate. We utilize the stepwise regression method to select independent variables from the set of variables qw_{t-1}, qw_{t-2}, r_t, r_{t-1}, r_{t-2}, u_t, u_{t-1} and u_{t-2}, where qw_t, r_t, and u_t represent the surrender rate of ordinary life policies, the one-year T-bill rate, and the unemployment

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however.

rate in year t, respectively. The surrender rate data are from the 1998 *Life Insurance Fact Book*, an annual statistical report of the American Council of Life Insurance, and contain annual voluntary termination rates of ordinary life insurance policies in force from 1965 to 1995.[10] We obtain the U.S. unemployment rate from the Taiwan Economic Journal Database.[11]

The stepwise regression chooses qw_{t-1} and r_{t-1} as the independent variables. Unemployment rates have little power in explaining the variation of surrender rates. The regression results show that surrender rate increases with the surrender rate and interest rate in the previous one period. Table 6 contains relevant estimation statistics.

Table 6: Estimated Parameters of the Surrender Rate Model

Parameter	Estimate	Standard Error	t-Statistics	p-value
β_1	0.8161	0.0252	32.33	0.0000
β_2	0.2185	0.0280	7.80	0.0000

Correlation of Estimated Parameters: -0.00069

Intercept term is insignificant with p-value 0.8910.

2. Simulation of Surrender Rates and *L*

[10] Strictly speaking, we should establish a surrender rate model for each type of policies since the surrender behaviors of different insurance may respond differently to unemployment rate and interest rate. We are unable to do so due to the lack of the data, however.

[11] Since unemployment rates are reported monthly in the database, we use the average of monthly rates as the unemployment rate in that year. The one-year Treasury bill rates are also recorded monthly, we transform monthly interest rate to annual rate by the following compounding method:

$$\text{annual interest rate} = (1+\frac{m_1}{12})(1+\frac{m_2}{12})...(1+\frac{m_{12}}{12})-1,$$

where m_i denotes the interest rate in month i, i = 1, 2, …, 12.

Using the above simple regression model and the 10,000 interest rate paths from Section IV 3, we simulate 10,000 surrender rate paths with the mean of the sampled surrender rate as the initial surrender rate.[12] These surrender rates are then combined with simulated mortality rates in Section III to determine the number of survivors as well as deaths in each policy year and the corresponding cash flows. Combining the resulting 10,000 cash flow paths with the 10,000 interest rate paths, we obtain the distribution of *L* under the consideration of random mortality rates, stochastic interest rate, random parameters of the interest rate model, and interest-rate-sensitive surrender rate. The simulation results are in Table 7.

Table 7: Surrender Rate Risk (LR) + ETS + IR + MR

Policy	Mean	Standard Deviation	Skewness	Kurtosis	VaR	Confidence Interval Width of VaR
Endowment	-461,184	2,830,127	1.3170	2.3918	5,084,695	490,631
Pure Endowment	-973,346	2,976,969	1.4240	2.7820	4,879,183	544,607
Term Life	-192,551	856,483	0.9072	1.1395	1,444,830	113,158
Whole Life	-324,687	4,538,632	3.2996	16.6294	7,966,843	1,100,364
Annuity	45,394	4,518,288	2.3492	8.3472	8,948,829	1,005,501

The right to surrender dramatically changes the reserve distribution. It makes the

[12] We use the word 'simulate' instead of 'obtain' because we do not simply plug numbers into the regression model and calculate surrender rates directly. Instead, we simulate a random error from the error distribution of the regression first and then plug it into the regression model to generate a surrender rate.

distribution narrower as we can tell from the higher kurtosis (e.g., 2.39 vs. 0.95 for endowment policy) and lower standard deviation (e.g., \$2.8 million vs. \$7.3 million for endowment). The distribution is more positively skewed (e.g., 1.32 vs. 0.87) and its mean shifts leftwards from \$1 million to \$-0.46 million in the case of endowment.

More importantly, the VaRs decrease significantly, which implies that the right to surrender benefits the insurance company. An obvious reason for such decreases is the surrender charge that is almost 20% for the first year and gradually descends with the policy period. Since the insurer does not have to pay full amount of reserves to the policyholders who surrender the policies, its liabilities decrease.

The drops of the VaRs are also due to the surrenders that happen during the periods of low interest rates. Without surrenders, the insurer bears the entire adverse consequence of low interest rates; with surrenders, the insurer suffers less because some policyholders voluntarily terminate their polices during low interest rate era and such termination releases the insurer from the 6% credit rate guarantee offered at the policy inception. In other words, policyholders who choose to surrender their policies when interest rates are low indeed relinquish a valuable guarantee offered by the insurance company and hence benefit the insurer. This seemingly irrational behavior of surrender does exist in the history. For instance, during the early 1960s, one-year rates are very low but surrender rates are still higher than 5%. Even in Japan where recent interest rates were extremely low, more than ten percent of policies surrendered in 1997, 1998 and 1999.[13] Our explanation for such surrender behavior is that policyholders might keep or surrender their policies for reasons other than the returns on insurance policies, e.g., changed demand. If return is not the only reason for the policyholder to purchase a life insurance policy, the surrender behavior would not solely depend on interest rate level. We thus would observe surrenders incurred in the periods of low interest rate. The insurance company, on the other hand, always benefits from the surrenders happening in low interest rate periods no matter which reasons the policyholder surrenders for. Therefore, surrenders could reduce VaR and moderate interest rate risk.

We can also explain the reduction in VaR due to surrenders by the concept of incremental VaR discussed in Jorion (2001, p. 155 - 159). Incremental VaR, defined as the change in VaR due to a new position, would be negative if the return of asset *i* is negatively correlated with that of portfolio. We may consider our reserve VaR as a portfolio VaR with respect to three fundamental risk factors: mortality rate, interest rate, and surrender rate. When we consider only mortality rate risk, the positions in the other two risk factors are assumed to be zero. The marginal impact of an additional risk factor is then similar to incremental VaR. Hence, the marginal impact of a new risk factor could be negative if the effects of risk factors are negatively correlated. Since low interest rates are harmful to

[13] The number is estimated with the data from the Life Insurance Business in Japan (http://www.seiho.or.jp/english/index.html).

insurers but surrenders during these times are beneficial, they have opposite effects. The marginal impact of surrender rate risk could be negative therefore, as long as the benefits resulting from surrenders outweigh the harms. Furthermore, the negative effect of surrenders on VaR is consistent with the findings about the decreased effective duration of reserves resulting from surrenders documented in Babbel (1995), Briys and Varenne (1997), and Santomero and Babbel (1997).

3. Parameter Estimation Risk of the Surrender Rate Model

To estimate the parameter estimation risk of the surrender rate model, we first assume that the parameters $\left(\hat{\beta}_1, \hat{\beta}_2\right)$ have a bivariate normal distribution with expected values and covariance structure as shown in Table 6. We then draw 10,000 samples of $\left(\hat{\beta}_1, \hat{\beta}_2\right)$ from this bivariate normal distribution. From these 10,000 sets of parameters and the 10,000 interest rate paths from Section IV 3, we simulate 10,000 surrender rate paths. We then use the same methodology as the one in Section V 2 to generate the distribution of L. Such distribution is constructed under relatively comprehensive consideration: random mortality rates, stochastic interest rate, interest-rate-sensitive cash flows, and parameter estimation risks for both the interest rate and surrender rate models. We believe that this distribution of L could render us valuable information about the risks of insurance policies. The statistics of the distribution are in Table 8.

Table 8: Parameter Estimation Risk of Surrender Rate Model + LR + ETS + IR + MR

Policy	Mean	Standard Deviation	Skewness	Kurtosis	VaR	Confidence Interval Width of VaR
Endowment	-651,380	2,540,448	0.8275	1.0945	4,070,062	299,554
Pure Endowment	-1,195,002	2,580,961	0.8447	1.1730	3,565,619	323,476
Term Life	-231,440	798,307	0.5839	0.4988	1,234,650	106,943
Whole Life	-926,461	3,088,948	1.7815	6.0683	4,805,048	493,176
Annuity	-446,892	3,568,499	1.3081	3.1089	6,308,442	606,786

The randomness of the parameters of the surrender rate model further decreases the VaRs of insurance policies. Such decrease is reasonable because the randomness of parameters enlarges the variations of surrender rate, results in more policyholders surrendering their policies at the times of low interest rates, and thus leads to the VaR decreases. The impact of the parameter uncertainty however is moderately significant only: it makes merely ten percent changes in VaRs on average. The moderate significance comes from the relatively small standard errors in Table 6.

Since the numbers in Table 8 are calculated by taking various relevant risks of policies into consideration, they are reasonable estimates about reserves. They indicate that the 95th percentile maximum shortfall of the reserve ranges from 0.5 to 2.3 times of annual premiums for different types of policies, which is rather significant. Term life insurance is the safest one, pure endowment and endowment policies are the next, and whole life insurance and annuity have the largest risk.[14]

Table 9 displays the margin impact of risks on VaR. It shows that mortality rate risk is unimportant, interest rate risk is the largest risk faced by insurers, and surrender rate serves to alleviate interest rate risk. Furthermore, parameter estimation risks as derived risks matter but are not as important as underlying risks. Our results suggest that insurers should pay the most attention to interest rate risk, but should also be aware of the moderation effect of surrender rate to avoid over-hedging.

Table 9: Marginal Impact of Risks on VaR

Policy	Estimation Risk of Mortality Rates	Interest Rate Risk	Parameter Risk of Term Structure Model	Surrender Rate Risk	Parameter Risk of Surrender Rate Model
Endowment	5,141	11,005,686	4,128,836	-10,054,968	-1,014,633
Pure Endowment	3,470	11,927,883	4,514,201	-11,566,371	-1,313,564
Term Life	73,245	2,757,659	878,216	-2,264,290	-210,180
Whole Life	18,722	17,829,457	19,857,248	-29,738,584	-3,161,795
Annuity	-3,501	18,150,961	14,233,977	-23,432,608	-2,640,387

VI. Summaries and Conclusions

In this paper, we estimate the VaR of reserves for prevailing types of life insurance

[14] The differences in risks among policies might even be larger if policies have different surrender tendency toward interest rate. More specifically, if annuity has more interest-rate-sensitive surrender rate than term life has, their VaRs will have larger differences than the one shown in Table 8 since the surrenders that are not sensitive to interest rate act to mitigate interest rate risk.

policies. To perform such estimation, we construct a simulation model featured with five risk layers: the mortality rate risk from random mortality rates, the interest rate risk from stochastic interest rate, the parameter estimation risk of the assumed interest rate model, the early surrender risk from interest-rate-dependent surrender rate, and the parameter estimation risk of the established surrender rate model. We also provide the confidence intervals of the VaRs to check the accuracy of our estimation.

Consistent with the literature, our results show that the mortality risk is negligible but the interest rate risk is substantial. We also find that the parameter estimation risk of the interest rate model is noteworthy. The simulation further shows that the surrender option may indeed benefit the insurance company: the emergence of early surrender reduces the VaR of reserves. Probable explanations for such findings are the surrender charge and the surrenders happening in low interest rate periods. The parameter uncertainty embedded in the surrender rate model further augments the impacts of early surrender a little bit. The above estimations are reasonably accurate, according to the widths of the confidence intervals.

Several research topics might be worthy to be further pursued. First, other types of mortality rate risk can be considered, e.g., the trend of decreasing mortality rates. Second, future studies may want to assess the model risk of interest rate by trying alternative term structure models. Third, researchers could move toward dynamic analysis (going-on business) from our static analysis (closed-block business). Fourth, the asset/investment side of business could be incorporated to generate surplus VaR that is the focal point of solvency regulation and insurers' risk management. The optimal asset allocation in terms of surplus VaR given compositions of business would also be interesting.

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