Parameter Risks of Surplus Management Under a Stochastic Process*

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Abstract

To hedge the interest-rate risk against a firm's surplus, insurance companies commonly set the firm's asset duration equal to the debt ratio times the firm's liability duration. However, this strategy focuses only on the fluctuation of interest rates; it does not address any of the uncertainty in the underlined factors, which guide the changes in interest rates. This paper first identifies parameter risks against a firm's surplus. We further propose to use goal programming to integrate the traditional immunization strategy against interest-rate risk and the strategies against parameter risks. Since the goal programming suggested in our paper is an integrated model of immunization strategies against interest-rate risk and parameter risks, the immunization strategy suggested here includes classical immunization strategy as a special case. Moreover, the results of our simulation show that, compared to classical immunization, the goal programming proposed in this paper can reduce significantly the overall risks against an insurance company's surplus.

Keywords: asset and liability management, immunization strategy, parameter risks

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Ⅰ. Introduction

Many papers (Bierwag, 1987; Grove, 1974; and Reitano, 1992) have recommended using classical immunization—setting the duration of assets equal to the asset/liability ratio times the duration of liabilities—for immunizing interest-rate risk against an insurance company's surplus. To recognize the stochastic behavior of interest rates as found in the literature,[1] Briys and Varenne (1997) and Tzeng, Wang and Soo (2000) have extended the traditional research of surplus management to the case where interest rates follow a stochastic process. The researchers have found that, under a stochastic process of interest rates, the traditional measurement of duration may miscalculate the firm's risk and may require further modification.

Although this line of research has provided many insightful strategies for asset-liability management of insurance companies, most papers focus on changes in interest rates in the case of given parameters. However, an insurance company may usually need to cope with the environment in which both interest rates and other factors guiding interest rates could be uncertain simultaneously.

For example, the current interest rates may fluctuate because of mean-reverting as recognized by the literature (e.g., Vasicek, 1977; Cox, Ingersoll, and Ross, 1985). On the other hand, long-term interest rates could also shift due to changes in many macroeconomic policies. From an insurance company's point of view, a change in the trend of interest rates could cause even more significant impacts on a firm's surplus than the stochastic changes of the current interest rates. In this case, insurance companies may have not much information to characterize their parameter risks.

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[1] E.g., Vasicek, 1977; Dothan, 1978; Cox, Ingersoll, and Ross, 1979; Dothan and Feldman, 1986, Ho and Lee, 1986; Chan et al., 1992; and Heath, Jarrow, and Morton, 1992.

Another example is estimation error in parameter estimates, which insurance companies may have more information to evaluate their parameter risks. The managers in insurance companies typically use unbiased point estimators for parameters in the process. However, the managers also recognize that there exists estimation error in parameter estimates. Thus, the practitioners may like to further control the risk caused by estimates' standard errors, even they have already employed the unbiased estimators.

In the above two cases, the insurance company may have few or some information to measure their risk exposure on parameter risks. But, without any doubt, the managers in the insurance company should have a need to further control unexpected shock from parameter risks. Thus, this paper intends to investigate the parameter risks of surplus management when interest rates follow a stochastic process. We employ the model proposed by Tzeng, Wang, and Soo (2000) because their model is shown to be a general model of many other traditional models. However, unlike Tzeng, Wang, and Soo, who examine the effects of a stochastic change on current interest rates, we focus on the changes in the underlined parameter factors that guide the process of the interest rates. We first identify parameter risks against a firm's surplus and provide the methods for immunizing those risks. Furthermore, we propose a goal-programming algorithm to integrate traditional immunization strategy against interest-rate risk and the strategies against parameter risks. Since the goal programming suggested in our paper is an integrated model of immunization strategies against both interest-rate risk and parameter risks, this immunization strategy includes classical immunization strategy as a special case. Moreover, the results of our simulation show that, compared to classical immunization, the goal programming proposed in this paper can reduce significantly the downside parameter risks against an insurance company's surplus.

Ⅱ. Model of Parameter Risks

Let $CI(t)$ and $CO(t)$ denote the cash inflows and cash outflows of an insurance company at period t. Let us assume that the return of the interest rate follows the stochastic process suggested by Cox, Ingersoll, and Ross (1985)[2] and can be expressed as

$$dr_t = a(b - r_t)dt + \sigma\sqrt{r_t}dz, \tag{1}$$

where r_t is the spot rate at period t and a, b, and σ are parameters[3]. In the above stochastic process, dz follows a standard Brownian motion. $a(b - r_t)$ is the drift rate of the interest rate and characterizes a mean-reverting process, where a and b represent the momentum of the drift rate and the mean of the long-term interest rate, respectively. The standard deviation of the interest rate is proportional to $\sqrt{r_t}$ and is denoted by $\sigma\sqrt{r_t}$.

Let r and $P(t)$ denote the current interest rate and the current value of a one-dollar zero-coupon bond at period t. From Cox, Ingersoll, and Ross (1985),

$$P(t) = \alpha(t)\exp(-\beta(t)r), \tag{2}$$

where $\alpha(t) = [\frac{2\sqrt{a^2+2\sigma^2}\,e^{\frac{t}{2}(a+\sqrt{a^2+2\sigma^2})}}{(\sqrt{a^2+2\sigma^2}+a)(e^{t\sqrt{a^2+2\sigma^2}}-1)+2\sqrt{a^2+2\sigma^2}}]^{2ab/\sigma^2}$, and

$$\beta(t) = \frac{2(e^{t\sqrt{a^2+2\sigma^2}}-1)}{(\sqrt{a^2+2\sigma^2}+a)(e^{t\sqrt{a^2+2\sigma^2}}-1)+2\sqrt{a^2+2\sigma^2}}.$$

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[2] Many other stochastic models, such as Vasicek's (1979), can also be used. Although each model may have its own strength, it would be easier to apply the model with a closed form solution.

[3] In practices, how asset-liability managers can do in order to estimate these parameters correctly is an important issue. Chan et al. (1992) suggested a general moment method to estimate the parameters in the

Using Cox, Ingersoll, and Ross' method (1979),[4] we measure the present value of future cash flows of t periods by the amount of cash flows times the current price of a one-dollar zero-coupon bond, $P(t)$. Thus, the assets and liabilities of an insurance company, A and L, can be expressed as:

$$A = \sum_{t=0}^{n} CI(t)P(t) \text{, and}$$

$$L = \sum_{t=0}^{n} CO(t)P(t). \tag{3}$$

The surplus of insurance company S is then equal to

$$S = A - L. \tag{4}$$

Like many traditional papers, Tzeng, Wang, and Soo (2000) have proposed an immunization strategy by setting $\frac{\partial S}{\partial r} = 0$ (Interest Rate Immunization). Although the interest rate may change stochastically, the immunization strategy of $\frac{\partial S}{\partial r} = 0$ can protect the surplus of the firm, at least locally. However, the stochastic change in the interest rate is not the only source of risks against a firm's surplus. Let us recall Equation (1):

$$dr_t = a(b - r_t)dt + \sigma\sqrt{r_t}\,dz. \tag{1}$$

In the above stochastic process, a and b represent the momentum of the drift rate and the mean of the long-term interest rate, respectively. The level of long-term interest rates can be different from the one insurance company uses because of a change in government's financial

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interest rate process.

[4] Lai and Frees (1995) derived similar method of valuation to calculate the reserves for insurance contracts. This method of valuation was also used by Tzeng, Wang, and Soo (2000), who assumed that there is a spread between the discount rates of assets and liabilities. To focus on the integration of parameter risks and interest rate risk, for the sake of simplicity we assume that the discount rates of assets and liabilities are the same. However, the main results of the paper still hold after relaxing this assumption.

policies or simply because of estimation error. However, the strategy of $\frac{\partial S}{\partial r} = 0$ implicitly assumes that the parameters in the interest model do not change.

Specifically, the insurance company is assumed to face a situation, where managers of the company consider the assumption of the interest rate process is acceptable, but still worry about that the parameters in the interest rate process may change unexpectedly. In other words, the parameters in the interest rate process should be treated as variables, but unfortunately the managers of the insurance companies might have no idea or only little knowledge of the distribution, the pattern, or the process of those parameters.

To cope with the parameter risks in surplus management, asset-liability managers can mimic traditional immunization strategy and arrange the assets and liabilities of the firm as follows:

$$\frac{\partial S}{\partial a} = 0 \quad \text{(Momentum Immunization),} \tag{5}$$

$$\frac{\partial S}{\partial b} = 0 \quad \text{(Mean Immunization), and/or} \tag{6}$$

$$\frac{\partial S}{\partial \sigma} = 0 \quad \text{(Deviation Immunization).} \tag{7}$$

Mean immunization, momentum immunization, and deviation immunization, respectively, are used to hedge the risks of changes in the long-term interest rate level, the magnitude of the drift rate, and the variance in the interest rate. One advantage of using Cox, Ingersoll, and Ross' (1985) model is that the parameters in their model represent meaningful characteristics of the interest rate. Another advantage is that their model provides explicit solutions, such as Equation (2), for the price of a one-dollar zero-coupon bond. Thus, parameter risks can be measured easily by taking derivatives with respect to those parameters. However, it is very important to recognize that the existence of parameter risks in surplus management does not depend on the employment of any specific stochastic model for interest rates. Almost every model of stochastic interest rate is required to estimate certain parameters, which may have their own meanings in reality. Although we use Cox, Ingersoll, and Ross' (1985) model to demonstrate our methodology, the idea of this paper can be adjusted to fit into many other models.

It should be also recognized that different approach should be used, if the insurance company knows more information of parameters than we assume in the paper. If the distributions and/or the processes of the parameters are known, then this information should be integrated to re-derive Equation (2)[5]. Of course, the methodology to simultaneously cope with both interest rate risks and parameter risks should also be changed if Equation (2) is no longer valid.

Recalling Equations (2), (3), and (4), the surplus of an insurance company can be rewritten as

$$Surplus = S(r, a, b, \sigma), \tag{8}$$

where $S(.)$ denotes the surplus function.

Let Δ denote the difference. By Taylor's expansion series, the change in the surplus caused by the changes in interest rate and parameters in Equation (8) can be approximately expressed as:

[5] We appreciate that referees point out this critical remark.

$$\Delta S \approx \frac{\partial S}{\partial r}\Delta r + \frac{\partial S}{\partial a}\Delta a + \frac{\partial S}{\partial b}\Delta b + \frac{\partial S}{\partial \sigma}\Delta \sigma \text{ .}^{6} \tag{9}$$

Equation (9) is directly derived from Equation (8). However, it should be noted that change of interest rate could be a function of changes in parameters in the original interest rate process, Equation (1). Thus, Equation (9) should be only considered as an approximation of change in surplus when we do not have precise knowledge on the interaction between interest rate and parameters.

Equation (9) provides some rationales for the strategies suggested by Equations (5), (6), and (7). From Equation (9), we know that

$E(\Delta S) \approx \frac{\partial S}{\partial r}E(\Delta r) + \frac{\partial S}{\partial a}E(\Delta a) + \frac{\partial S}{\partial b}E(\Delta b) + \frac{\partial S}{\partial \sigma}E(\Delta \sigma)$ and

$std(\Delta S) \approx \frac{\partial S}{\partial r}std(\Delta r) + \frac{\partial S}{\partial a}std(\Delta a) + \frac{\partial S}{\partial b}std(\Delta b) + \frac{\partial S}{\partial \sigma}std(\Delta \sigma)$, if the changes in interest rate and parameters are all independent. Further assume that the firm keeps $\frac{\partial S}{\partial r} = 0$ to avoid interest rate risk, then $E(\Delta S) \approx \frac{\partial S}{\partial a}E(\Delta a) + \frac{\partial S}{\partial b}E(\Delta b) + \frac{\partial S}{\partial \sigma}E(\Delta \sigma)$ and

$std(\Delta S) \approx \frac{\partial S}{\partial a}std(\Delta a) + \frac{\partial S}{\partial b}std(\Delta b) + \frac{\partial S}{\partial \sigma}std(\Delta \sigma)$. If the insurance company have no idea of the change in parameters, then the insurance company may set $\frac{\partial S}{\partial a} = 0$, $\frac{\partial S}{\partial b} = 0$, and

[6] Equation (9) only provides the first order approximation of ΔS. The insurance companies can have a better performance on immunization if higher derivatives are also taken into consideration. Douglas (1990) and Christensen and Sorensen (1994) suggested, if managers expect the volatility of interest rates to be greater than what appears in the term-structure, then the firm's optimal objective would be to maximize its convexity of the surplus subject to the zero surplus duration and its budget constraints. However, Gagnon and Johnson (1994) and Barber and Copper (1997) have demonstrated that matching the convexities of asset and liability does not always improve the immunization results. In this paper, since we assume that the insurance company knows neither the distribution nor the process of those parameters, we do not model in both the interaction between change in interest rate and parameters and higher derivatives of changes in interest rate and parameters.

$\frac{\partial S}{\partial \sigma} = 0$ to make $E(\Delta S) \approx 0$ and $std(\Delta S) \approx 0$ to eliminate their parameter risks.

If the source of parameter risk come from estimation errors. Given that the point estimators in the process are unbiased, we could consider that $E(\Delta a) = E(\Delta b) = E(\Delta \sigma) = 0$. Thus, if the firm does not take any risk on the changes in interest rate, then the firm can keep $\frac{\partial S}{\partial r} = 0$ and make $E(\Delta S) = 0$. Assume that the changes in interest rate and parameters are independent, then the standard deviation of the change of the surplus could be approximated by $\frac{\partial S}{\partial a} std(\Delta a) + \frac{\partial S}{\partial b} std(\Delta b) + \frac{\partial S}{\partial \sigma} std(\Delta \sigma)$ at $\frac{\partial S}{\partial r} = 0$. If the firm would like to further control any risk on the changes in parameters, then the best strategy is to set $\frac{\partial S}{\partial a} = 0$, $\frac{\partial S}{\partial b} = 0$, and $\frac{\partial S}{\partial \sigma} = 0$.

Thus, if the firm does not like to take any risk on the changes in interest rate and parameters, then the best strategy is to keep $\frac{\partial S}{\partial r} = 0$ as well as $\frac{\partial S}{\partial a} = 0$, $\frac{\partial S}{\partial b} = 0$, and $\frac{\partial S}{\partial \sigma} = 0$. Separately, it may not be difficult for managers to cope with each risk, such as $\frac{\partial S}{\partial r} = 0$ or $\frac{\partial S}{\partial b} = 0$. However, immunization strategies may conflict with each other and/or may not even be completely compatible. To integrate the immunization strategies against interest-rate risk and parameter risks, we propose using the goal-programming algorithm as follows:

$$\min_{CI(t)} \quad d \tag{10}$$

$$s.t. \quad \left|\frac{\partial S}{\partial a}\right| \leq d \cdot w_a \,,$$

$$\left|\frac{\partial S}{\partial b}\right| \le d \cdot w_b \ ,$$

$$\left|\frac{\partial S}{\partial \sigma}\right| \le d \cdot w_\sigma \ ,$$

$$\left|\frac{\partial S}{\partial r}\right| \le d \cdot w_r \ .$$

where d is the risk position the firm takes and w_a, w_b, w_σ, and w_r are the weights of parameter risks and interest-rate risk, respectively.

Given the insurance company's liability schedule $CO(t)$, it is worth noting that $\frac{\partial S}{\partial a} = 0$, $\frac{\partial S}{\partial b} = 0$, $\frac{\partial S}{\partial \sigma} = 0$, and $\frac{\partial S}{\partial r} = 0$ are all linear functions of asset allocation $CI(t)$, which is the decision variable of Equation (10). Thus, management can solve Equation (10) by linear programming.

The rationale of Equation (10) is that managers make the optimal allocation of a firm's assets and liabilities to cope simultaneously with parameter risks and interest-rate risk against a firm's surplus. If the optimal solution of Equation (10) is $d^* = 0$, then the strategies against parameter risks and interest-rate risk are completely compatible. If the optimal solution of Equation (10) is greater than zero, then managers can also easily know how much risk they take under various risk factors.

By means of their experience and judgment, asset-liability managers can further adjust the weights between parameter risks and interest-rate risk accordingly. The smaller the value of the weight given in a risk, the stricter the immunization strategy against the underlined risk the managers intend to take.[7] For example, managers can use the strategy setting $w_r = 0$

and $w_a = w_b = w_\sigma = \infty$ to implement the classical immunization against interest-rate risk. Furthermore, by setting $w_r = 0$ along with the appropriate weights for other parameter risks, managers not only immunize a firm's interest-rate risk but also control the firm's parameter risks. Thus, the model suggested by Equation (10) can be considered as a general model of traditional classical immunization strategy, since it includes classical immunization strategy as a special case.

Ⅲ. Simulation

To investigate parameter risks in surplus management, we construct a hypothetical insurance company given expected claims. The balance sheet and claims schedule for the hypothetical insurance company are shown as Exhibits 1 and 2.

Exhibit 1: Balance Sheet of a Hypothetical Insurance Company

Assets	Liabilities	Surplus
$14,130,274	$13,630,274	$500,000

For the sake of simplicity, we further assume the firm is a run-off case, and the liabilities are to be paid out over ten years, as shown in Exhibit 2.

Exhibit 2: Claims Schedule of a Hypothetical Insurance Company

Periods	Cash Outflows
1	$354,000
2	$675,000
3	$989,000
4	$1,417,000
5	$1,732,000

[7] One way to determine the weights is to set them proportional to the standard errors of estimators.

6	\$2,057,000
7	\$2,480,000
8	\$2,803,000
9	\$3,129,000
10	\$3,550,000

Let us assume that the interest rate follows Cox, Ingersoll, and Ross' process (1985), where $r = 5\%$, $a = 0.1$, $b = 0.05$, and $\sigma = 0.03$. Since insurance companies may need to fulfill minimum solvency margins and may not be able to borrow money in real practice, our simulations consider two additional solvency constraints and are expressed as:

$$\sum_{t=0}^{j}(CI(t) - CO(t))\frac{P(t)}{P(j)} \geq 10{,}000,\ j = 1,...,10\ ,^{8} \text{ and} \tag{11}$$

$$CI(t) \geq 0, t = 0,...,10. \tag{12}$$

Let us set $w_r = 0$ and $w_a = w_b = w_\sigma = 1$. Thus, the optimal allocation of cash flows can be generated by Equation (13) and is shown in Exhibit 3.

$$\min_{CI(t)} \quad d$$

$$s.t. \quad \left|\frac{\partial S}{\partial a}\right| \leq d\ ,$$

$$\left|\frac{\partial S}{\partial b}\right| \leq d\ ,$$

$$\left|\frac{\partial S}{\partial \sigma}\right| \leq d\ ,$$

$$\left|\frac{\partial S}{\partial r}\right| \leq 0\ ,$$

$$S = A - L\ ,$$

[8] The insurance company is assumed to be able to reinvest its net cash flows at each period in the same investment portfolio and the minimum solvency margin is 10,000.

$$\sum_{t=0}^{j}(CI(t)-CO(t))\frac{P(t)}{P(j)} \geq 10{,}000,\ j=1,...,10\text{ , and}$$

$$CI(t) \geq 0, t=0,...,10. \tag{13}$$

Exhibit 3: Optimal Cash Inflows Allocation Using the Goal-Programming Method

Periods	Cash Inflows
1	$1,348,892
2	$33,268
3	$1,040,038
4	$1,575,733
5	$1,155,584
6	$3,587,633
7	$1,963,884
8	$2,265,836
9	$2,446,475
10	$4,267,016

The classical immunization strategy can be implemented by the solutions that satisfied Equation (4) and $\frac{\partial S}{\partial r} = 0$. However, to avoid the problems of multiple solutions, the optimal cash flows under classical immunization are generated by maximizing the convexity[9] of the firm subject to Equations (4), (11), and (12) and $\frac{\partial S}{\partial r} = 0$. The solution of the classical immunization strategy is shown in Exhibit 4.

Exhibit 4: Optimal Cash Inflows Allocation Using the Immunization Strategy

Periods	Cash Inflows
1	$862,447
2	$600,442
3	$609,342
4	$3,163,553
5	$186,618
6	$3,947,679
7	$388,629
8	$2,569,774
9	$3,650,238
10	$3,684,377

[9] The main results still hold if the classical immunization strategy is chosen by other criteria.

Furthermore, a direct comparison of the changes in a firm's surplus value between the goal programming method and the immunization strategy under different parameters a, b, σ, and r will help to compare the performance of these two strategies for alternative circumstances, respectively. In real practice, insurance company usually has no idea or only little knowledge of the distribution, the pattern, or the process of those parameters. Thus, in our simulation, we assume the insurance company knows the current value of parameters (a =0.1, b =0.05 and σ =0.03) but does not know how they will change statistically. To demonstrate the point of "the unexpected changes", we try to simulate alternative cases (such as a changes from 0.1 to 0.01 or 0.02； b changes from 0.1 to 0.05 or 0.07；and σ changes from 0.03 to 0.01 or 0.05). To keep the internal consistency of the paper, we do not simulate our results by assuming a, b and σ follow certain distributions. If the insurance company knows the distribution of a, b and σ, they should take it into consideration to derive the pricing formula. Since we employ equation (2) through the paper, we make our simulation consistent to our model. The insurance company can use our proposed goal programming to control the risk caused by the unexpected changes of the parameters. Table Sets 1 and 2 in the Appendix report the percentage difference in changes in a firm's surplus value between the goal-programming method and the immunization strategy. The results indicate that the performance of the goal-programming method does not universally dominate that of the classical immunization strategy[10]. However, it is worth noting that the strategies of both classical immunization and goal programming are conservative investment strategies, which intend to lower the downside risk instead of gaining profits. Thus, the results of Table Sets 1 and 2 could be considered as a trade-off between

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[10] Intuitively the more volatile these parameters are, the more likely that the goal programming approach will dominate the classical immunization approach. We appreciate that referee point out this critical remark.

controlling risk and taking a risk for making profits. Our simulation results support that the proposed goal programming model has the advantages over the classical immunization model as long as a, b and σ are random variable that will change unexpectedly. Moreover, the results of Table Sets 1 and 2 show that the performance of the goal-programming method generally dominates that of the classical immunization strategy when a and b are small as well as when σ is large. Thus, we can conclude that the goal-programming method could help insurance companies to reduce the impact of unfavorable parameter shocks significantly.

Ⅳ. Discussions

Model risks as one type of parameter risks

We demonstrate further that model risks could be considered as one type of parameter risks. According to Wang and Huang (2002), who have investigated the so-called model risk in surplus management, model risk is evaluated when the manager of a firm implements the immunization strategy under the assumption that the interest rate follows Vasicek's process (1977), when the interest rate actually follows Cox, Ingersoll, and Ross' process (1985). In fact, both processes can be integrated into a more general stochastic process as follows:

$$dr_t = a(b - r_t)dt + \sigma r_t^{\,p} dz \tag{14}$$

Thus, it is obvious that the model risk investigated by Wang and Huang (2002) is the parameter risk of p.

Although Equation (14) includes several well-known stochastic processes for interest Rate,[11] not every model can provide a closed-form solution for $P(t)$, which is essential for

[11] E.g., Vasicek, 1977; Dothan, 1978; Cox, Ingersoll, and Ross, 1979; Dothan and Feldman, 1986; Ho and Lee, 1986; Chan et al., 1992; and Heath, Jarrow, and Morton, 1992.

implementing the algorithm in Equation (10). If $P(t)$ does not have a closed-form solution, we cannot use derivatives—such as Equations (5), (6), and (7)—to measure the parameter risks. Thus, we further propose a method to bypass the non-closed-form-solution barrier caused by Equation (14).

If the interest rate is assumed to follow a more general process as Equation (14), the surplus of the firm can be rewritten as

$$S=\sum_{t=0}^{n}[CI(t)-CO(t)]P(t)\,. \tag{15}$$

Thus, the goal-programming algorithm in Equation (10) can be rearranged as

$$\min_{CI(t)} \quad d \tag{16}$$

$$\begin{aligned}
s.t.\quad & \sum_{t=0}^{n}CO(t)\frac{\partial P(t)}{\partial r}-d\cdot w_a \le \sum_{t=0}^{n}CI(t)\frac{\partial P(t)}{\partial r} \le \sum_{t=0}^{n}CO(t)\frac{\partial P(t)}{\partial r}+d\cdot w_a\ ,\\
& \sum_{t=0}^{n}CO(t)\frac{\partial P(t)}{\partial b}-d\cdot w_b \le \sum_{t=0}^{n}CI(t)\frac{\partial P(t)}{\partial b} \le \sum_{t=0}^{n}CO(t)\frac{\partial P(t)}{\partial b}+d\cdot w_b\\
& \sum_{t=0}^{n}CO(t)\frac{\partial P(t)}{\partial \sigma}-d\cdot w_\sigma \le \sum_{t=0}^{n}CI(t)\frac{\partial P(t)}{\partial \sigma} \le \sum_{t=0}^{n}CO(t)\frac{\partial P(t)}{\partial \sigma}+d\cdot w_\sigma\,,\\
& \sum_{t=0}^{n}CO(t)\frac{\partial P(t)}{\partial p}-d\cdot w_p \le \sum_{t=0}^{n}CI(t)\frac{\partial P(t)}{\partial p} \le \sum_{t=0}^{n}CO(t)\frac{\partial P(t)}{\partial p}+d\cdot w_p\,,\\
& \sum_{t=0}^{n}CO(t)\frac{\partial P(t)}{\partial r}-d\cdot w_r \le \sum_{t=0}^{n}CI(t)\frac{\partial P(t)}{\partial r} \le \sum_{t=0}^{n}CO(t)\frac{\partial P(t)}{\partial r}+d\cdot w_r\,.
\end{aligned}$$

Obviously, the goal programming can be operated if we know $\frac{\partial P(t)}{\partial a}$, $\frac{\partial P(t)}{\partial b}$, $\frac{\partial P(t)}{\partial \sigma}$, $\frac{\partial P(t)}{\partial p}$, and $\frac{\partial P(t)}{\partial r}$. Although we may not have a closed-form solution of $P(t)$ to derive those coefficients directly, they can be estimated by simulation. Thus, by the forth constraint in Equation (16) we can include the model risk indicated by Wang and Huang (2002) as one type of parameter risks.

Although Equation (14) represents a family of interest-rate models in the literature, those models are classified as one-factor models, since they model only the stochastic behavior of the short rates. To capture possible changes in parameters in one-factor models, many papers have proposed multi-factor interest rate models.[12] With more degrees of freedoms multi-factor models can usually explain the change in interest rates more precisely. However, most multi-factor models are often difficult to apply because they usually do not have closed-form solutions and an estimation of parameters in those models could be tedious. Thus, the goal-programming algorithm in our paper could serve as a compromise between theory and practice. On one hand, to make the strategy technically tractable, the model employs a one-factor model to characterize the behavior of interest rates. On the other hand, goal programming ensures that all parameter shifts are under control.

When market prices are different from the predicted value according to the theoretical stochastic process

To apply the algorithm of this paper, it is important for asset-liability managers to have information of the process on the interest rate and the prices of zero-coupon bonds. However, it may not be necessary to have the information in this order. In the simulation, we assume that insurance companies can allocate their future cash flows by prices expected as theoretical predictions. Unfortunately, this may not be the case in reality. Let us assume that a firm uses historical data on interest rates to estimate the underlined stochastic process of the interest rate, as suggested by Chan et al. (1992). The firm further employs the stochastic process to calculate the prices of zero-coupon bonds. However, the market prices

[12] E.g., Brennan and Schwartz, 1982; Fong and Vasicek, 1991; Longstaff and Schwartz, 1992; Chen and Scott, 1992; and Anderson and Lund, 1996.

of the bonds may not be as predicted in theory and, therefore, the firm may not be able to transfer cash flows from one period to another as theoretical prices.

One way to cope with this issue is to use the market prices of bonds instead of the theoretical prices to discount cash flows in the budget constraint. Rather than using the estimated interest rate by the stochastic process to calculate the prices of zero-coupon bonds, we can use the market prices of zero-coupon bonds to estimate the stochastic process of the interest rate. Since the discount factors in Equation (15) come from market prices, it, indeed, makes the allocation of the cash flows feasible. The firm further uses bond prices to estimate the underlined stochastic process of the interest rate taking the form of $dr = a(b - r)dt + \sigma r^{p} dz$. The problem with this approach is that the estimated interest-rate process may not have a close form for evaluating the comparative statics for alternative risks. Fortunately, using the simulation suggested in the above section, we can overcome this possible barrier.

The stochastic process of discount factors for liabilities

For the sake of simplicity, we have assumed so far that there is no spread between the discount rates of assets and liabilities. Thus, we employ the same set of zero-coupon bond prices to discount future cash inflows and outflows. However, it may not be appropriate for insurance companies to evaluate future cash outflows as negative values of cash inflows, since this involves all kinds of uncertainties. Furthermore, cash outflows generated by different lines of business may not be appropriate for using the same discount rates because different types of insurance businesses may involve dramatically different risks. Thus, it could improve the effects of immunization to estimate the stochastic processes of rates of returns for assets and liabilities separately, as suggested by Tzeng, Wang, and Soo (2000).

However, there may not exist a complete secondary market for the cash outflows of insurance companies. Thus, what we call bond prices for cash outflows may not be observable for managers in insurance companies. Therefore, we may need to estimate the stochastic process of rates of returns of a firm's liabilities first. Fortunately, the insurance industry usually records loss ratios and combined ratios, which can be employed as proxies for the rates of returns of a firm's liabilities.

Ⅴ. Conclusions

In this paper, we proposed to use goal programming to integrate the traditional immunization strategy against interest-rate risk and the strategies against parameter risks. Since the goal programming suggested in our paper is an integrated model of immunization strategies of interest-rate risk and parameter risks, this immunization strategy includes classical immunization strategy as a special case. We have also demonstrated that the algorithm of goal programming can be extended to cope with model risk in surplus management. Several practical issues in implementing the immunization strategy have been discussed and possible solutions have been proposed. Moreover, the results of our simulation show that, compared to classical immunization, the goal programming proposed in this paper can reduce significantly downside parameter risks against an insurance company's surplus.

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Appendix

Table Set 1. The Percentage Difference in Changes in Surplus Using the Classical Immunization Strategy

		a=0.01			a=0.1			a=0.2		
		σ=0.01	σ=0.03	σ=0.05	σ=0.01	σ=0.03	σ=0.05	σ=0.01	σ=0.03	σ=0.05
	R=0.03	-1.3760%	-1.3959%	-1.4397%	-1.3767%	-1.4007%	-1.4494%	-1.3761%	-1.3956%	-1.4344%
b=0.03	R=0.05	-1.3959%	-1.4397%	-1.3125%	-1.4007%	-1.4494%	-0.7603%	-1.3956%	-1.4344%	-0.3395%
	R=0.07	-1.4397%	-1.3125%	-1.3438%	-1.4494%	-0.7603%	-0.7978%	-1.4344%	-0.3395%	-0.3707%

		a=0.01			a=0.1			a=0.2		
		σ=0.01	σ=0.03	σ=0.05	σ=0.01	σ=0.03	σ=0.05	σ=0.01	σ=0.03	σ=0.05
	R=0.03	-1.2342%	-1.2543%	-1.2985%	-0.1246%	-0.1504%	-0.2025%	0.6991%	0.6767%	0.6321%
b=0.05	R=0.05	-1.2543%	-1.2985%	-1.1705%	-0.1504%	-0.2025%	0.4918%	0.6767%	0.6321%	1.7048%
	R=0.07	-1.2985%	-1.1705%	-1.2020%	-0.2025%	0.4918%	0.4514%	0.6321%	1.7048%	1.6688%

		a=0.01			a=0.1			a=0.2		
		σ=0.01	σ=0.03	σ=0.05	σ=0.01	σ=0.03	σ=0.05	σ=0.01	σ=0.03	σ=0.05
	R=0.03	-1.0918%	-1.1121%	-1.1566%	1.1274%	1.0994%	1.0429%	2.7104%	2.6847%	2.6336%
b=0.07	R=0.05	-1.1121%	-1.1566%	-1.3854%	1.0994%	1.0429%	-1.3847%	2.6847%	2.6336%	-0.6774%
	R=0.07	-1.1566%	-1.3854%	-1.4216%	1.0429%	-1.3847%	-1.4180%	2.6336%	-0.6774%	-0.7222%

Table Set 2 The Percentage Difference in Changes in Surplus Using the Goal-Programming Method

		a=0.01			a=0.1			a=0.2		
		σ=0.01	σ=0.03	σ=0.05	σ=0.01	σ=0.03	σ=0.05	σ=0.01	σ=0.03	σ=0.05
	R=0.03	0.0088%	0.0590%	0.1529%	0.0048%	0.0240%	0.0602%	0.0031%	0.0091%	0.0205%
b=0.03	R=0.05	0.0590%	0.1529%	0.0072%	0.0240%	0.0602%	-0.0378%	0.0091%	0.0205%	0.0697%
	R=0.07	0.1529%	0.0072%	0.0807%	0.0602%	-0.0378%	-0.0082%	0.0205%	0.0697%	0.0798%

		a=0.01			a=0.1			a=0.2		
		σ=0.01	σ=0.03	σ=0.05	σ=0.01	σ=0.03	σ=0.05	σ=0.01	σ=0.03	σ=0.05
	R=0.03	-0.0323%	0.0183%	0.1130%	-0.0724%	-0.0513%	-0.0113%	0.1786%	0.1861%	0.2004%
b=0.05	R=0.05	0.0183%	0.1130%	-0.0328%	-0.0513%	-0.0113%	-0.0686%	0.1861%	0.2004%	0.3368%
	R=0.07	0.1130%	-0.0328%	0.0413%	-0.0113%	-0.0686%	-0.0367%	0.2004%	0.3368%	0.3483%

		a=0.01			a=0.1			a=0.2		
		σ=0.01	σ=0.03	σ=0.05	σ=0.01	σ=0.03	σ=0.05	σ=0.01	σ=0.03	σ=0.05
	R=0.03	-0.0715%	-0.0205%	0.0749%	-0.0602%	-0.0380%	0.0044%	0.5291%	0.5370%	0.5521%
b=0.07	R=0.05	-0.0205%	0.0749%	0.0190%	-0.0380%	0.0044%	0.0079%	0.5370%	0.5521%	0.0165%
	R=0.07	0.0749%	0.0190%	0.0670%	0.0044%	0.0079%	0.0240%	0.5521%	0.0165%	0.1007%