Journal of Actuarial Practice Vol. 10, 2002

Model Risks and Surplus Management Under a Stochastic Interest Rate Process

Jennifer L. Wang* and Rachel J. Huang†

Abstract‡

This paper uses simulations to explore the effects of incorrectly identifying the underlying interest rate process on assets, liabilities, and surplus levels. We show that mismodeling the interest rate (called model risk) could not only lead to a misstatement of the company's surplus, but could also cause a mismatch between the company's assets and liabilities. Our simulations demonstrate that three aspects of interest rates affect model risk: (i) volatility, (ii) level of long-term interest rate, and (iii) the speed at which the drift rate adjusts. We conclude that asset-liability managers should not ignore the impact of the model risks, regardless of the length of their planning horizon.

Key words and phrases: *asset and liability management, immunization strategy, interest rate risk, model risk*

---

*Jennifer L. Wang, Ph.D., is associate professor of risk management and insurance and deputy director of the Insurance Research and Education Center at the College of Commerce, National Chengchi University in Taiwan. She received a Ph.D in risk management and insurance from Temple University. Her research interests are in pensions, annuities, insurance finance, and insurance accounting. She has published many papers in various international journals including *Journal of Risk and Insurance, Journal of Insurance Issues*, and *Benefits Quarterly*.

Dr. Wang's address is: Department of Risk Management and Insurance, National Chengchi University, 64, Sec. 2, Chihnan Rd., Taipei, 116, TAIWAN. Internet address: *jenwang@nccu.edu.tw*

†Rachel J. Huang is an instructor of finance at Ming Chuan University and a Ph.D. student in finance at National Taiwan University.

Ms. Huang's address is: Department of Finance, National Taiwan University, 50, Lane 144, Sec. 4, Keelung Rd., Taipei, 106, TAIWAN. Internet address: *rachelhuang@mba.ntu.edu.tw*

‡The authors gratefully acknowledge the helpful comments of two anonymous referees and the editor of this journal and Dr. Larry Y. R. Tzeng and seminar participants at the 2001 Risk Management Seminar at National Chengchi University. We deeply appreciate Zn-Hong Chen's research assistance and the financial support from National Science Council of Taiwan.

1 Introduction

A serious problem insurance companies face is the problem of interest rate fluctuations on their assets and liabilities, the so-called C-3 risk by actuaries. Simply put, if assets have a longer maturity date than liabilities, a rise in interest rates will lead to a decrease in the net value of the insurer, while a fall in interest rates will lead to an increase in the net value of the insurer.

To deal with this problem, Redington (1952) introduced a so-called immunization strategy of setting the duration of assets equal to the asset/liability ratio times the duration of liabilities. Redington's approach is now a standard technique used by many authors, including Grove (1974), Bierwag (1987) and Reitano (1992), for immunizing the surplus of an insurance company against interest rate risk. See Panjer (1998, Chapter 3) for a detailed review of the actuarial approach to immunization.

Bellhouse and Panjer (1981), Beekman and Fuelling (1990), Frees (1990), Norberg (1995), and Lai and Frees (1995) have explored the impact of stochastic interest rates on the reserves of life insurance. On the other hand, Briys and Varenne (1997) and Tzeng, Wang, and Soo (2000) have extended the traditional duration approach to address the case where interest rates follow a stochastic process. Tzeng, Wang, and Soo (2000) show that, with certain adjustments, the classical immunization strategy still can be used for surplus management.

Although this line of research has provided some insightful strategies for asset-liability management of insurance companies, most papers focus on the change in interest rates and overlook the cost of mismodeling the interest rate process itself.[1]

Though many models of the stochastic behavior of interest rates have been proposed, two models are most popular: Vasicek (1977) and Cox, Ingersoll, and Ross (1985). The Vasicek model assumes the interest rate process is a mean-reverting process with constant volatility. The Cox, Ingersoll, and Ross model assumes the interest rate process is a mean-reverting process but with volatility that is proportional to the level of the interest rate. Other models have been proposed by Dothan and Feldman (1986); Ho and Lee (1986); Chan et al., (1992); and Heath, Jarrow, and Morton (1992).

[1] In practice, surplus managers are interested mostly in comparing how surplus levels change as strategies change. Although incorporating a stochastic interest model may not influence the decision in choosing an investment strategy, it certainly generates more accurate asset allocation in terms of immunization of surplus.

This paper considers a hypothetical insurance company and uses simulations and the Vasicek (1977) and Cox, Ingersoll, and Ross (1985) models[2] to measure the cost of misidentifying the interest rate model (i.e., the model risk) in two ways: (i) miscalculating the company's value, and (ii) mismatching the company's assets and liabilities. The paper is organized as follows: Section 2 describes some of the properties of the two interest rate models. Section 3 describes the relevant aspects of the hypothetical insurance company. Section 4 contains the results of the simulations.

2 The Vasicek and Cox, Ingersoll, and Ross Interest Rate Models

Although many alternative processes[3] have been suggested for modeling interest-rate behaviors, only a few of them have a closed-form solution for the price of a zero-coupon bond. Among these, Vasicek (1977) and Cox, Ingersoll, and Ross (1985) are most commonly used. Vasicek (1977) models the interest rate process, r_t, as

$$\mathrm{d}r_t = a_V(b_V - r_t)dt + \sigma_V \mathrm{d}z, \tag{1}$$

where a_V, b_V, and σ_V are constants and $\mathrm{d}z$ follows a standard Brownian motion. The term $a_V(b_V - r_t)$ is called the drift rate, and σ_V is the standard deviation of the interest rate process.

Vasicek (1977) solves equation (1) and shows that the current price of a one-dollar zero-coupon bond maturing in t periods, $P(t)$,

$$P(t) = P_V(t) = \alpha_V(t)\exp(-\beta_V(t)r), \tag{2}$$

[2]Although these two models are most commonly used interest rate models, they suffer certain limitations. Sometimes, the surplus manager would like to replicate the diverse nature of the yield curve. Neither the Vasicek nor the Cox, Ingersoll, and Ross model allows the yield curve to change from a positively sloped yield curve to a negatively sloped yield curve.

[3]Interest rate models such as those of Vasicek (1977); Cox, Ingersoll, and Ross (1985); Dothan and Feldman (1986); Ho and Lee (1986); Chan et al., (1992); and Heath, Jarrow, and Morton (1992) can be chosen by insurance companies for their own management purposes.

where r is the current level of interest rates,

$$\beta_V(t) = \frac{1 - \exp(-a_V t)}{a_V}, \quad \text{and} \tag{3}$$

$$\alpha_V(t) = \exp\left(\frac{(\beta_V(t) - t)(a_V^2 b_V - 0.5\sigma_V^2)}{a_V^2} - \frac{\sigma_V^2 \beta_V^2(t)}{4a_V}\right). \tag{4}$$

Under the Cox, Ingersoll, and Ross (1985), the interest rate process, r_t, is modeled as

$$\mathrm{d}r_t = a_I(b_I - r_t)dt + \sigma_I\sqrt{r_t}\,\mathrm{d}z \tag{5}$$

where a_I, b_I, and σ_I are constants and $\mathrm{d}z$ follows a standard Brownian motion. Here again, the drift rate is $a_I(b_I - r_t)$. The standard deviation, however, is now $\sigma_I\sqrt{r_t}$. Cox, Ingersoll, and Ross (1985) solve equation (5) and show that

$$P(t) = P_I(t) = \alpha_I(t)\exp(-\beta_I(t)r), \tag{6}$$

where r is the current level of interest rates,

$$\gamma_I^2 = a_I^2 + 2\sigma_I^2, \tag{7}$$

$$\alpha_I(t) = \left(\frac{2\gamma_I e^{t(a_I+\gamma_I)/2}}{(\gamma_I + a_I)(e^{t\gamma_I} - 1) + 2\gamma_I}\right)^{2a_I b_I/\sigma_I^2}, \quad \text{and} \tag{8}$$

$$\beta_I(t) = \frac{2(e^{t\gamma_I} - 1)}{(\gamma_I + a_I)(e^{t\gamma_I} - 1) + 2\gamma_I}. \tag{9}$$

It is important to recognize that, though they have the same functional form for the drift rate, the Vasicek (1977) and Cox, Ingersoll, and Ross (1985) models have different assumptions for interest-rate variations. Vasicek (1977) assumes a constant variation in the interest rate in each period, while Cox, Ingersoll, and Ross (1985) assume that the variation in the interest rate in a period is proportional to the square root of the interest rate in the period.

3 Model of the Hypothetical Insurance Company

Suppose a hypothetical insurance company has a current balance sheet (at the start of period 1) as shown in Table 1.

Table 1
Balance Sheet of
Hypothetical Insurance Company

Assets	Liabilities	Surplus
$9,045,110	$8,545,110	$500,000

Let $R(t)$ and $C(t)$ denote the cash inflows and cash outflows, respectively, of the hypothetical insurance company t periods in the future. Following the approach proposed by Tzeng, Wang, and Soo (2000), the assets and liabilities of an insurance company, A and L, satisfy the following equations:

$$A = \sum_{t=1}^{n} R(t)P^A(t), \quad \text{and} \tag{10}$$

$$L = \sum_{t=1}^{n} C(t)P^L(t), \tag{11}$$

where $P^A(t)$ and $P^L(t)$ are the current price of a one-dollar zero-coupon bond maturing in t periods based on the interest rate process followed by the assets and the liabilities, respectively. The surplus of insurance company, S, is then equal to

$$S = A - L = \sum_{t=1}^{n} R(t)P^A(t) - \sum_{t=1}^{n} C(t)P^L(t). \tag{12}$$

For simplicity, we further assume the company is a run-off case,[4] and the liabilities[5] are to be paid out over fifteen years, as shown in Table 2. This means that the present value, using discount rate $P^L(t)$, of cash outflows would be equal to the total liability. On the other hand, the

[4] A run-off case means that the company would not consider or implement any new business line over fifteen years.

[5] In practice, an insurance company's liability schedule is often hard to predict. Becker (1988) discusses the difficulty of correctly measuring the value of the liability of an insurance company. Recent research findings on the effective duration of insurance liabilities—see, for example, Babbel, Merril, and Planning (1997) and Briys and Varenne (1997)—can help to make more accurate predictions. We have made the liability schedule independent of the interest rate in order to concentrate on the analysis of model risk. In practice, however, interest rate changes do have a significant impact on lapse rates, policy loans, and surrenders, as documented in Briys and Varenne (1997) and hence on the duration of liabilities.

present value of cash inflows that the company pursues should satisfy the balance sheet condition, i.e.,

$$\sum_{t=1}^{15} [\alpha_V(t) \exp(-\beta_V(t) r)] R(t) = 9{,}045{,}110. \tag{13}$$

Table 2
Liabilities (Cash Outflows) of Hypothetical Insurance Company

t	$C(t)$	t	$C(t)$	t	$C(t)$
1	\$591,500	6	\$824,600	11	\$1,087,400
2	\$633,700	7	\$871,300	12	\$1,133,500
3	\$677,400	8	\$932,700	13	\$1,187,300
4	\$723,500	9	\$984,200	14	\$1,212,600
5	\$775,800	10	\$1,036,500	15	\$1,253,800

Let r_t^A and r_t^L denote the rate of return on assets and liabilities, respectively. Assume the insurance policies are interest-rate sensitive, and the company always maintains its interest rate for valuing liabilities as a fixed proportion of its rate for valuing assets. This means that

$$r_t^L = k r_t^A$$

where k is a positive constant. If the interest rate of assets follow Vasicek's (1977) model, i.e., $r_t^A = r_t$, then the interest rate for valuing liabilities would satisfy $r_t^L = k r_t$, i.e.,

$$\mathrm{d}r_t^L = a_V(k b_V - r_t^L)dt + k\sigma_V \mathrm{d}z. \tag{14}$$

This means that the long run level and the volatility of the liability rate of return are proportional to those of the asset rate of return. The adjustment speed for the liability rate of return to its long-term level is the same as that for the asset rate of return.

On the other hand, if the asset rate of return follows Cox, Ingersoll, and Ross's (1985) model, we have

$$\mathrm{d}r_t^L = a_I(k b_I - r_t^L)dt + \sqrt{k}\sigma_I \sqrt{r_t^L}\mathrm{d}z. \tag{15}$$

Here, the long-term level of the liability interest rate is still k times that of the asset return, as in Vasicek's model. In the Cox, Ingersoll, and

Ross (1985) model, however, the standard deviation of the liability rate of return is $\sqrt{k}\sigma_I\sqrt{r_t^L}$.

Assume that the current interest rate of asset is $r = 6\%$. The parameters of the Vasicek (1977) and Cox, Ingersoll, and Ross (1985) models are obtained from Chan et al., (1992), who estimate them from U.S. Treasury yield data from June 1964 to December 1989.[6] Thus, we can generate the parameters as follows: $a_V = 0.1779$, $b_V = 0.0866$, $\sigma_V = 0.02$; and $a_I = 0.2339$, $b_I = 0.0808$, $\sigma_I = 0.0854$.[7] We assume $k = 80\%$, so that the adjustment speed, long-term level, and standard error of r_t^L are 0.1779, 0.0693, and 0.0160, respectively.

4 The Immunization Equations

Let us suppose that the hypothetical company manages surplus by assuming that the interest-rate process follows Vasicek's (1977) model with the parameters given at the end of Section 3. We further assume, however, that interest rates actually follow the Cox, Ingersoll, and Ross (1985) model with the parameters given at the end of Section 3. The deviation of expected surplus from the actual surplus is referred to as mismodeling cost.

Tzeng, Wang, and Soo (2000) show that, if a closed-form solution of $P^A(t)$ and $P^L(t)$ exists, an immunization strategy can be generated by

$$\frac{\mathrm{d}S}{\mathrm{d}r} = 0,$$

where r is the spot rate. For this hypothetical company, the above immunization strategy can be expressed as

$$\frac{\mathrm{d}S}{\mathrm{d}r} = \sum_{t=1}^{n} R(t)\frac{\mathrm{d}P^A(t)}{\mathrm{d}r} - \sum_{t=1}^{n} C(t)\frac{\mathrm{d}P^L(t)}{\mathrm{d}r} = 0. \tag{16}$$

[6]The proxy of the short-term interest rate in their model is the Treasury yield, which is generated from Fama (1984) and maintained by the Center for Research in Security Prices (CRSP). The one-month yield is the average of the bid-and-ask price for Treasury bills and is normalized as a standard month with 30.4 days. It should be recognized that, besides the prices of short-term bonds, the prices of long-term bonds and the price of interest options could provide additional information for interest rate volatility especially when more sophisticated models [such as Heath, Jarrow and Morton (1992)] are adopted.

[7]In Chan et al.'s (1992) Table III, the expectation of the short-term interest rate under Vasicek's setting is $E[r_{t+1} - r_t] = 0.0154 - 0.1779r_t$. Therefore, we have $a_V = 0.1779$, $b_V = \frac{0.0154}{0.1779} = 0.0866$. By the same token, under the Cox, Ingersoll, and Ross model, $E[r_{t+1} - r_t] = 0.0189 - 0.2339r_t$. Thus, we have $a_I = 0.2339$, and $b_I = \frac{0.0189}{0.2339} = 0.0808$.

Substituting the cash outflows and parameters chosen for the Vasicek (1977) model, the above equation is equivalent to

$$\sum_{t=1}^{15} \alpha_V(t)\beta_V(t)\exp(-\beta_V(t)r)R(t) = 26,049,488. \qquad (17)$$

From equations (2) to (9), it is obvious that the immunization strategies under Vasicek (1977) and Cox, Ingersoll, and Ross (1985) can be substantially different. Moreover, given the same set of cash in-flows and out-flows, the value of a company's surplus depends on the interest rate model used. Thus, the model risks associated with surplus management actually stems from two sources: misevaluation and mismatch. Misevaluation of the company's surplus refers to incorrectly calculating the surplus due to mismodeling the interest rates, i.e., incorrectly identifying the underlying interest rate model. Mismatch refers to the lack of immunization of a company's assets and liabilities due to mismodeling interest rates.

In practice, insurance companies must satisfy certain statutory regulations such as minimum solvency margins and restrictions against borrowing. If there is a minimum solvency margin of $M(t)$ in period t and the insurance company can reinvest its net cash flows in each period in the same investment portfolio, then the solvency constraints for the insurance company can be expressed as

$$\sum_{t=1}^{j} (R(t) - C(t))\frac{P_V^A(t)}{P_V^A(j)} \geq M(j), \qquad j = 1, ..., 15, \qquad (18)$$

and $R(t) \geq 0$ for $t = 1, \dots, 15$. There may exist multiple solutions that satisfy equations (13), (17), and (18). To keep all the comparisons on an equal basis, we choose a maximum-convexity strategy[8] as the optimal strategy for the insurance company. If we assume that the solvency margin, $M(t)$, is \$10,000, the company's optimal immunization strategy can be modeled as

[8]Douglas (1990) and Christensen and Sorensen (1994) suggested that if asset-liability managers expect the volatility of interest rates to be greater than what appears in the term-structure, then the company's optimal objective would be to maximize its convexity of the surplus subject to the zero surplus duration and its budget constraints. Gagnon and Johnson (1994) and Barber and Copper (1997), however, have demonstrated that matching the convexities of asset and liability does not always improve the immunization results.

$$\max_{R(t)} \frac{d^2 S}{dr^2} = \sum_{t=1}^{15} [\alpha_V(t)\beta_V^2(t)\exp(-\beta_V(t)r)]R(t) \tag{19}$$

such that

$$\sum_{t=1}^{15} [\alpha_V(t)\exp(-\beta_V(t)r)]R(t) = 9,045,110,$$

$$\sum_{t=1}^{15} [\alpha_V(t)\beta_V(t)\exp(-\beta_V(t)r)]R(t) = 26,049,488,$$

$$\sum_{t=1}^{j} (R(t) - C(t))\frac{P_V^A(t)}{P_V^A(j)} \geq 10,000, \quad j = 1, \ldots, 15, \quad \text{and}$$

$$R(t) \geq 0, \quad t = 1, \ldots, 15.$$

Notice that when the company's surplus (S), liability schedule ($C(t)$), and the parameters of the stochastic interest rate processes are given, equations (13), (17), and (18) are all linear functions with respect to $R(t)$. Therefore, equation (19) can be solved by linear programming, and the optimal allocation of cash flows is shown in Table 3.

Table 3
Optimal Income Stream (Cash Inflows)
Of Hypothetical Insurance Company

t	$R(t)$	t	$R(t)$	t	$R(t)$
1	$5,035,935	6	$0	11	$1,086,624
2	$0	7	$0	12	$1,132,644
3	$0	8	$0	13	$1,186,887
4	$0	9	$331,756	14	$1,211,622
5	$0	10	$1,035,246	15	$5,437,539

5 The Results of the Simulation

The simulation is divided into two parts: First, we compare the differences between α_V and α_I, and between β_V and β_I.[9] Then we evaluate the cost of mismodeling.

5.1 Differences in Vasicek (1977) and Cox, Ingersoll, and Ross (1985)

As mentioned earlier, the model risks actually result from the differences in the α and β terms in the two models. Therefore, it is important that we examine these differences under different parameters values for a, b, σ, and t as it will help to identify the severity of the model risks.

Tables A1, A2, A3, and A4 in the appendix display $\alpha_V - \alpha_I$, $(\alpha_V - \alpha_I)/\alpha_V$, $\beta_V - \beta_I$, and $(\beta_V - \beta_I)/\beta_V$, respectively, for $b = 3$, various time periods, and various levels of a and σ.

Table A1 shows that $|\alpha_V - \alpha_I|$ increases as σ increases, but decreases as a increases. In addition, it is important to recognize that $|\alpha_V - \alpha_I|$ approaches zero as σ approaches zero because the Vasicek (1979) and Cox, Ingersoll, and Ross (1985) models collapse into the same model when the variance of the interest rate process approaches zero. Table A2 shows that the relative difference, $|(\alpha_V - \alpha_I)/\alpha_V|$, increases as σ increases. For large a, $|(\alpha_V - \alpha_I)/\alpha_V|$ is an increasing convex function with respect to t. For small a, however, there is no clear impact pattern on $|(\alpha_V - \alpha_I)/\alpha_V|$. Table A3 shows $|\beta_V - \beta_I|$ decreases as a increases, but increases as σ or t increases. As the same pattern observed in $|\alpha_V - \alpha_I|$, we find $|\beta_V - \beta_I|$ also will approach zero when σ is sufficiently small . In Table A4 we find $|(\beta_V - \beta_I)/\beta_V|$ also decreases as a increases, but increases as σ or t increases.

Further results obtained by varying b, but not reported in these tables, show that:

- $|\alpha_V - \alpha_I|$ decreases as b increases;
- For large b, $|(\alpha_V - \alpha_I)/\alpha_V|$ is an increasing convex function with respect to t. For small b, however, it shows no clear impact pattern; and,
- As expected, b has no impact on $|\beta_V - \beta_I|$ because in both the Vasicek (1977) and Cox, Ingersoll, and Ross (1985) models, β is independent of the level of the long-term interest rate b.

[9] Although the results of this paper depend on model forms in the simulation, they still serve as a case for demonstrating the severity of model risks.

Based on the results of the simulation, we can conclude that low long-term interest rate levels, high interest rate volatility, or low drift rate momentum increases model risk. We do not have a conclusive finding with respect to an increase in the time horizon. Thus, we would caution financial planners at insurance companies to not ignore the possible effects of model risks, regardless of the length of their planning horizon.

5.2 Costs of Mismodeling

The costs of model risks are measured in two ways: miscalculation of a company's value and mismatch of a company's assets and liabilities. Given the cash outflows and inflows in Tables 2 and 3, we then calculate the values of a company's assets, liabilities, and surplus under the Vasicek (1977) and Cox, Ingersoll, and Ross (1985) models. Table 4 shows the estimated values of a company's assets, liabilities, and surplus for each model. Notice that miscalculation of the surplus value is roughly 5 percent, which is a substantial amount.

Table 4
The Cost of Miscalculating the Company's Value

	Assets	Liabilities	Surplus
Expected:	$9,045,000	$8,545,000	$500,000
Actual:	$9,138,000	$8,613,000	$525,000
Cost:	$93,000	$68,000	$25,000
% Change:	1.0%	0.8%	5.0%

Notes: Expected refers to the Vasicek (1977) model; Actual refers to the Cox, Ingersoll, and Ross (1985) model; Cost = Actual − Expected; % Change = Cost / Expected; Numbers are rounded to the nearest $1,000.

To measure the mismatch cost caused by the model risks, we further assume that the current interest rate immediately shifts from $r = 6\%$ to $r = i\%$, where $i = 2, 3, \ldots, 10$.[10] The estimated surplus values under

[10]The shift in the interest rate is assumed to be non-stochastic, although our simulation can be applied to both stochastic and non-stochastic changes in interest rates. In practice, company managers may be more concerned with the non-stochastic changes in interest rates in the short run, although they may recognize the underlying stochastic structure of interest rates in the long run. In addition, if the interest rate is allowed to vary within two standard deviations, then a maximum 4 percent shock may be acceptable.

Vasicek (1977) and under Cox, Ingersoll, and Ross (1985) are shown in Table 5.

Table 5
The Cost of Mismatch Due to Mismodeling

(1)	(2)	(3)	(4)	(5)	(6)
	Vasicek	IMMUZ	CIR	Differences	% Cost
r	(1977)	Effect	(1985)	$\frac{(4)-(2)}{500,000}$	$\frac{(4)-525,000}{525,000}$
2%	$500,000	100%	$503,000	0.6%	-4.3%
3%	$500,000	100%	$509,000	1.7%	-3.2%
4%	$500,000	100%	$513,000	2.8%	-2.3%
5%	$500,000	100%	$519,000	3.9%	-1.2%
6%	$500,000	100%	$525,000	5.0%	0%
7%	$500,000	100%	$531,000	6.2%	+1.0%
8%	$500,000	100%	$537,000	7.5%	+2.2%
9%	$500,000	100%	$544,000	8.7%	+3.5%
10%	$500,000	100%	$550,000	9.9%	+4.7%

Notes: IMMUZ = Immunization; CIR = Cox, Ingersoll, and Ross; and % Cost = Percentage cost of mismodeling. Numbers are rounded to the nearest $1,000.

Columns (2) and (4) in Table 5 show the estimated surplus values at different interest rates under the processes of Vasicek (1977) and Cox, Ingersoll, and Ross (1985), respectively. Column (3) demonstrates the immunization effect of surplus management if the immunization strategy is derived from Vasicek (1977) and the underlined interest rate follows Vasicek (1977). On the other hand, Columns (5) and (6) demonstrate the percentage difference in the surplus value if the immunization strategy is derived from Vasicek (1977), whereas the underlined interest rate follows Cox, Ingersoll, and Ross (1985).

The results of Table 5 show that mismodeling causes a mismatch of a company's assets and liabilities and exposes the company's surplus to interest-rate risk. Although the cost of mismodeling is not as high as misevaluation in our simulation, a one-percent change in the interest rate could still influence a company's surplus value by more than one percent. Moreover, all other risks, such as equity, operational, liquidity, etc., remain unchanged in the simulation. The simulation shows that it could cost the company ± 5 percent of its surplus purely because of mismodeling.

6 Summary and Conclusions

In practice, asset-liability managers often rely on sophisticated models to develop risk management strategies. The over-reliance on such models may cause unpredictable crises when the real world does not behave according to the models. This paper investigates the impact of interest rate model risks on an insurance company's surplus using two popular interest-rate models: Vasicek (1977) and Cox, Ingersoll, and Ross (1985).

We find that differences in parameters between Vasicek (1977) and Cox, Ingersoll, and Ross (1985) are higher when the long-term interest-rate level is low, the volatility of the interest rate is high, and the momentum of the drift rate is low. In other words, a low level of the long-term interest rate, high volatility of the interest rate, and low momentum of the drift rate increase the model risks. We do not have a conclusive finding with respect to an increase in the time horizon. Thus, managers in insurance companies should not ignore the possible impact of the model risks whether they are engaged in short-term or long-term financial planning. We further show that the cost of failing to recognize model risks can be extremely high. Because of mismodeling, misevaluation could cause about a 5 percent shock on a company's surplus. A mismatch of a company's assets and liabilities also could cause at least a one-percent fluctuation for a one percentage change in the interest rate.

In this paper we focus on estimating the cost of model risk for a yearly adjustment surplus management strategy; thus, the liability schedules of an insurance company are assumed to be independent of interest rate, and the shock of interest rate is a one-time shock. In the real world, however, many factors–such as surrender rate, lapse rate, and policy loan as suggested in Briys and Varenne (1997)–could make a liability schedule sensitive to the path of the interest rate. A dynamic immunization relaxing the above two assumptions could provide further understanding for asset-liability management in future studies.

References

Babbel, D.F., Merril, C. B. and Planning, W.."Default Risk and the Effective Duration of Bonds." *Financial Analysts Journal* 53 (1997): 35–44.

Barber, J.R., and Copper, M.L. "Is Bond Convexity a Free Lunch?" *The Journal of Portfolio Management*, Fall 1997: 113-119.

Barney, L.D. "The Relation Between Capital Structure, Interest Rate Sensitivity, and Market Value in the Property-Liability Insurance Industry: Comment." *The Journal of Risk and Insurance* 64 (1997): 733-738.

Becker, D.N. "A Generalized Profit Released Model for the Measurement of the Return on Investment for Life Insurance." *Transactions of the Society of Actuaries* 40 (1988): 61-114.

Beekman, J.A., and Fuelling, C.P. "Interest and Mortality Randomness in Some Annuities." *Insurance: Mathematics and Economics* 9 (1990): 185-196.

Bellhouse, D.R., and Panjer, H.H. "Stochastic Modeling of Interest Rates with Applications to Life Contingencies—Part II." *Journal of Risk and Insurance* 48 (1981): 275-287.

Bierwag, G.O. "Duration and the Term Structure of Interest Rate." *The Journal of Financial and Quantitative Analysis* 12 (1977): 725-742.

Bierwag, G.O. *Duration Analysis: Managing Interest Rate Risk.* Cambridge, Mass.: Ballinger Publishing Company, 1987.

Bierwag, G.O., Corrado, C.J. and Kaufman, G.G. "Duration for Portfolios of Bonds Priced on Different Term Structures." *Journal of Banking and Finance* 16 (1992): 705-714.

Bierwag, G.O., Kaufman, G.G. and Toevs, A. "Bond Portfolios Immunization and Stochastic Process Risk." *Journal of Bank Research*, Winter 1993: 282-291.

Bierwag, G.O., Fooladi, I. and Roberts, G.S. "Designing an Immunized Portfolio: Is M-squared the Key?" *Journal of Banking and Finance* 17 (1993): 1147-1170.

Briys, E., and de Varenne, F. "On the Risk of Insurance Liabilities: Debunking Some Common Pitfalls." *The Journal of Risk and Insurance* 64 (1997): 673-694.

Chan, K.C., Karolyi, G.A., Longstaff, F.A., and Sanders, A.B. "An Empirical Comparison of Alternative Models of the Short-Term Interest Rate." *The Journal of Finance* 47 (1992): 1209-1227.

Christensen, P.O., and Sorensen, B.G. "Duration, Convexity, and Time Value." *The Journal of Portfolio Management*, Winter 1994: 51-60.

Cox, J.C., Ingersoll, J.E. and Ross, S.A. "Duration and the Measurement of Basis Risk." *Journal of Business* 52 (1979): 51-61.

Cox, J.C., Ingersoll, J.E. and Ross, S.A. "A Re-Examination of Traditional Hypotheses About the Term Structure of Interest Rates." *The Journal of Finance* 36 (1981): 769-799.

Cox, J.C., Ingersoll, J.E. and Ross, S.A. "A Theory of the Term Structure of Interest Rates." *Econometrica* 53 (1985): 385-407.

Dothan, L.U. "On the Term Structure of Interest Rates." *Journal of Financial Economics* 6 (1978): 59-69.

Dothan, M.U., and Feldman, D. "Equilibrium Interest Rates and Multiperiod Bonds in a Partially Observable Economy." *The Journal of Finance* 41 (1986): 369-382.

Douglas, L.G. *Bond Risk Analysis: A Guide to Duration and Convexity.* New York, N.Y.: New York Institute of Finance, 1990.

Fama, E.F. "Term Premiums in Bond Returns." *Journal of Financial Economics* 13 (1984): 529-546.

Fong, H.G., and Vasicek, O.A. "A Risk-Minimizing Strategy for Portfolio Immunization." *The Journal of Finance* 39 (1984): 1541-1546.

Frees, E.W. "Stochastic Life Contingencies with Solvency Considerations." *Transactions of the Society of Actuaries* 42 (1990): 91-129.

Gagnon, L., and Johnson, L.D. "Dynamic Immunization Under Stochastic Interest Rates." *The Journal of Portfolio Management*, Spring 1994: 48-54.

Grove, M.A. "On Duration and Optimal Maturity Structure of the Balance Sheet." *Bell Journal of Economics and Management Sciences* 5 (1974): 696-709.

Heath, D.C., Jarrow, R.A., and Morton, A. "Bond Pricing and the Term Structure of Interest Rates: A New Methodology for Contingent Claims Valuation." *Econometrica* 60 (1992): 77-105.

Ho, T.S., and Lee, S.B. "Term Structure Movements and Pricing Interest Rate Contingent Claims." *The Journal of Finance* 41 (1986): 1011-1029.

Ingersoll, J.E., Jr., and Ross, S.A. "Waiting to Invest: Investment and Uncertainty." *Journal of Business* 65 (1992): 1-29.

Lai, S.L., and Frees, E.W. "Examining Changes in Reserves Using Stochastic Interest Models." *Journal of Risk and Insurance* 62 (1995): 535-574.

Lee, S.B., and Cho, H.Y. "A Rebalancing Discipline for an Immunization Strategy." *The Journal of Portfolio Management*, Summer 1992: 56-62.

Moller, C.M. "Duration, Convexity, and Time Value." *The Journal of Portfolio Management*, Winter 1994: 51-60.

Moller, C.M. "A Counting Process Approach to Stochastic Interest Rates." *Insurance: Mathematics and Economics* 17 (1995): 181-192.

Norberg, G. "A Continuous-Time Markov Chain Interest Model with Applications to Insurance." *Journal of Applied Stochastic Models and Data Analysis* 11 (1995): 245-256.

Panjer, H.H. (ed), Boyle, P.P., Cox, S.H., Dufresne, D., Gerber, H.U., Mueller, H.H., Pedersen, H.W., Pliska, S.R., Sherris, M., Shiu, E.S., and Tan, K.S. *Financial Economics with Applications to Investments, Insurance and Pensions.* Schaumburg, Ill.: The Actuarial Foundation, 1998.

Redington, F. M. "Review of the Principle of Life Office Valuations." *Journal of the Institute of Actuaries* 18 (1952): 286-340.

Reitano, R.R. "Non-Parallel Yield Curve Shifts and Spread Leverage." *The Journal of Portfolio Management*, Spring (1991): 82-87.

Reitano, R.R. "Non-Parallel Yield Curve Shifts and Immunization." *The Journal of Portfolio Management*, Spring (1992): 36-43.

Reitano, R.R. "Non-Parallel Yield Curve Shifts and Stochastic Immunization." *The Journal of Portfolio Management* 22 (1996): 71-78.

Staking, K.B., and D.F. Babble. "The Relation Between Capital Structure, Interest Rate Sensitivity, and Market Value in the Property-Liability Insurance Industry." *The Journal of Risk and Insurance* 62 (1995): 690-718.

Tzeng, L.Y., Wang, J.L. and Soo, J. "Surplus Management Under a Stochastic Process." *Journal of Risk and Insurance* 67 (2000): 451-462.

Vasicek, O. "An Equilibrium Characterization of the Term Structure." *The Journal of Financial Economics* 5 (1977): 177-188.

Vetzal, K.R. "A Survey of Stochastic Continuous Time Models of the Term Structure of Interest Rates." *Insurance: Mathematics and Economics* 14 (1994): 139-161.

Zenios, S.A., Holmer, M.R., McKendall, R., and Vassiadouzeniou, C. "Dynamic Models for Fixed-Income Portfolio Management Under Uncertainty." *Journal of Economic Dynamics and Control* 22 (1998): 1517-1541.

Appendix

Table A1
The Differences $\alpha_V - \alpha_I$ for $b = 3$

	$a = 0.1$			$a = 0.2$			$a = 0.3$		
t	$\sigma = 0.1$	$\sigma = 0.2$	$\sigma = 0.3$	$\sigma = 0.1$	$\sigma = 0.2$	$\sigma = 0.3$	$\sigma = 0.1$	$\sigma = 0.2$	$\sigma = 0.3$
1	1.2E-03	1.1E-02	3.2E-02	9.3E-04	8.4E-03	2.4E-02	6.9E-04	6.3E-03	1.8E-02
2	5.6E-03	5.4E-02	1.7E-01	2.5E-03	2.4E-02	7.5E-02	1.1E-03	1.1E-02	3.5E-02
3	8.5E-03	9.3E-02	3.8E-01	1.8E-03	2.1E-02	8.5E-02	4.3E-04	4.9E-03	2.1E-02
4	7.3E-03	1.0E-01	7.1E-01	6.7E-04	9.9E-03	6.5E-02	6.2E-05	1.1E-03	7.9E-03
5	4.2E-03	9.2E-02	1.5E+00	1.5E-04	3.4E-03	4.2E-02	1.1E-06	1.4E-04	2.3E-03
6	1.9E-03	7.3E-02	4.1E+00	2.0E-05	9.1E-04	2.6E-02	-1.1E-06	7.0E-06	5.8E-04
7	6.5E-04	5.5E-02	1.7E+01	1.6E-06	2.1E-04	1.7E-02	-2.5E-07	-1.3E-06	1.4E-04
8	1.9E-04	4.3E-02	1.1E+02	-2.5E-08	4.4E-05	1.1E-02	-3.4E-08	-4.6E-07	3.0E-05
9	4.6E-05	3.5E-02	1.2E+03	-3.1E-08	8.4E-06	7.8E-03	-3.7E-09	-9.1E-08	6.5E-06
10	9.7E-06	3.1E-02	2.1E+04	-6.2E-09	1.5E-06	5.9E-03	-3.4E-10	-1.4E-08	1.4E-06
11	1.8E-06	2.9E-02	6.0E+05	-8.6E-10	2.6E-07	4.7E-03	-2.8E-11	-2.0E-09	2.9E-07
12	3.1E-07	3.1E-02	2.7E+07	-1.0E-10	4.3E-08	4.0E-03	-2.1E-12	-2.7E-10	6.1E-08
13	4.6E-08	3.6E-02	1.8E+09	-1.0E-11	7.0E-09	3.5E-03	-1.5E-13	-3.4E-11	1.3E-08
14	6.4E-09	4.6E-02	1.9E+11	-9.7E-13	1.1E-09	3.3E-03	-1.1E-14	-4.1E-12	2.6E-09
15	8.0E-10	6.5E-02	2.8E+13	-8.5E-14	1.7E-10	3.1E-03	-7.2E-16	-4.8E-13	5.3E-10

Table A2

Relative Differences $(\alpha_V - \alpha_I)/\alpha_V$ for $b = 3$

	$a = 0.1$			$a = 0.2$			$a = 0.3$		
t	$\sigma = 0.1$	$\sigma = 0.2$	$\sigma = 0.3$	$\sigma = 0.1$	$\sigma = 0.2$	$\sigma = 0.3$	$\sigma = 0.1$	$\sigma = 0.2$	$\sigma = 0.3$
1	1.4E-03	1.3E-02	3.5E-02	1.2E-03	1.1E-02	3.0E-02	1.0E-03	9.4E-03	2.6E-02
2	9.8E-03	8.5E-02	2.2E-01	7.0E-03	6.3E-02	1.7E-01	4.9E-03	4.5E-02	1.2E-01
3	2.8E-02	2.3E-01	5.3E-01	1.7E-02	1.5E-01	3.8E-01	8.9E-03	8.5E-02	2.5E-01
4	5.5E-02	4.2E-01	8.0E-01	2.7E-02	2.4E-01	5.9E-01	8.8E-03	1.1E-01	3.6E-01
5	9.0E-02	6.0E-01	9.4E-01	3.3E-02	3.3E-01	7.6E-01	1.4E-03	9.4E-02	4.5E-01
6	1.3E-01	7.6E-01	9.9E-01	3.2E-02	3.9E-01	8.7E-01	-1.6E-02	3.9E-02	5.2E-01
7	1.7E-01	8.7E-01	1.0E+00	2.0E-02	4.5E-01	9.4E-01	-4.4E-02	-6.5E-02	5.7E-01
8	2.0E-01	9.4E-01	1.0E+00	-3.1E-03	4.8E-01	9.7E-01	-8.4E-02	-2.3E-01	6.1E-01
9	2.3E-01	9.7E-01	1.0E+00	-4.1E-02	5.1E-01	9.9E-01	-1.4E-01	-4.7E-01	6.5E-01
10	2.6E-01	9.9E-01	1.0E+00	-9.5E-02	5.2E-01	1.0E+00	-2.0E-01	-8.1E-01	6.8E-01
11	2.8E-01	1.0E+00	1.0E+00	-1.7E-01	5.2E-01	1.0E+00	-2.8E-01	-1.3E+00	7.1E-01
12	2.9E-01	1.0E+00	1.0E+00	-2.6E-01	5.2E-01	1.0E+00	-3.7E-01	-1.9E+00	7.3E-01
13	2.9E-01	1.0E+00	1.0E+00	-3.8E-01	5.1E-01	1.0E+00	-4.7E-01	-2.8E+00	7.5E-01
14	2.8E-01	1.0E+00	1.0E+00	-5.3E-01	5.0E-01	1.0E+00	-6.0E-01	-3.9E+00	7.7E-01
15	2.6E-01	1.0E+00	1.0E+00	-7.1E-01	4.8E-01	1.0E+00	-7.3E-01	-5.5E+00	7.9E-01

Table A3
The Differences $\beta_V - \beta_I$ for $b = 3$

	$a = 0.1$			$a = 0.2$			$a = 0.3$		
t	$\sigma = 0.1$	$\sigma = 0.2$	$\sigma = 0.3$	$\sigma = 0.1$	$\sigma = 0.2$	$\sigma = 0.3$	$\sigma = 0.1$	$\sigma = 0.2$	$\sigma = 0.3$
1	1.5E-03	1.3E-02	3.6E-02	1.4E-03	1.2E-02	3.3E-02	1.2E-03	1.1E-02	3.0E-02
2	1.1E-02	9.2E-02	2.3E-01	8.9E-03	7.6E-02	1.9E-01	7.4E-03	6.4E-02	1.6E-01
3	3.3E-02	2.6E-01	6.0E-01	2.5E-02	2.0E-01	4.7E-01	1.9E-02	1.5E-01	3.6E-01
4	7.0E-02	5.2E-01	1.1E+00	4.8E-02	3.7E-01	7.9E-01	3.4E-02	2.6E-01	5.8E-01
5	1.2E-01	8.4E-01	1.6E+00	7.8E-02	5.6E-01	1.1E+00	5.1E-02	3.8E-01	7.7E-01
6	1.9E-01	1.2E+00	2.1E+00	1.1E-01	7.5E-01	1.4E+00	6.8E-02	4.8E-01	9.4E-01
7	2.7E-01	1.6E+00	2.6E+00	1.5E-01	9.3E-01	1.6E+00	8.4E-02	5.7E-01	1.1E+00
8	3.6E-01	2.0E+00	3.1E+00	1.8E-01	1.1E+00	1.9E+00	9.8E-02	6.4E-01	1.2E+00
9	4.6E-01	2.3E+00	3.5E+00	2.2E-01	1.2E+00	2.0E+00	1.1E-01	7.0E-01	1.2E+00
10	5.7E-01	2.7E+00	3.9E+00	2.5E-01	1.4E+00	2.2E+00	1.2E-01	7.4E-01	1.3E+00
11	6.8E-01	3.0E+00	4.2E+00	2.8E-01	1.5E+00	2.3E+00	1.3E-01	7.8E-01	1.3E+00
12	7.9E-01	3.3E+00	4.5E+00	3.1E-01	1.6E+00	2.4E+00	1.4E-01	8.1E-01	1.4E+00
13	9.0E-01	3.6E+00	4.8E+00	3.4E-01	1.6E+00	2.5E+00	1.4E-01	8.3E-01	1.4E+00
14	1.0E+00	3.8E+00	5.1E+00	3.6E-01	1.7E+00	2.6E+00	1.5E-01	8.5E-01	1.4E+00
15	1.1E+00	4.0E+00	5.3E+00	3.8E-01	1.8E+00	2.6E+00	1.5E-01	8.6E-01	1.4E+00

Table A4
The Relative Differences $(\beta_V - \beta_I)/\beta_V$ for $b = 3$

	$a = 0.1$			$a = 0.2$			$a = 0.3$		
t	$\sigma = 0.1$	$\sigma = 0.2$	$\sigma = 0.3$	$\sigma = 0.1$	$\sigma = 0.2$	$\sigma = 0.3$	$\sigma = 0.1$	$\sigma = 0.2$	$\sigma = 0.3$
1	1.6E-03	1.4E-02	3.8E-02	1.5E-03	1.3E-02	3.6E-02	1.4E-03	1.3E-02	3.4E-02
2	6.0E-03	5.1E-02	1.3E-01	5.4E-03	4.6E-02	1.2E-01	4.9E-03	4.2E-02	1.1E-01
3	1.3E-02	1.0E-01	2.3E-01	1.1E-02	8.9E-02	2.1E-01	9.5E-03	7.8E-02	1.8E-01
4	2.1E-02	1.6E-01	3.3E-01	1.8E-02	1.3E-01	2.9E-01	1.5E-02	1.1E-01	2.5E-01
5	3.1E-02	2.1E-01	4.1E-01	2.5E-02	1.8E-01	3.5E-01	2.0E-02	1.4E-01	3.0E-01
6	4.2E-02	2.7E-01	4.7E-01	3.2E-02	2.1E-01	4.0E-01	2.4E-02	1.7E-01	3.4E-01
7	5.4E-02	3.1E-01	5.2E-01	3.9E-02	2.5E-01	4.4E-01	2.9E-02	1.9E-01	3.6E-01
8	6.6E-02	3.6E-01	5.6E-01	4.6E-02	2.8E-01	4.7E-01	3.2E-02	2.1E-01	3.8E-01
9	7.8E-02	3.9E-01	5.9E-01	5.2E-02	3.0E-01	4.9E-01	3.6E-02	2.2E-01	4.0E-01
10	9.0E-02	4.2E-01	6.1E-01	5.8E-02	3.2E-01	5.1E-01	3.8E-02	2.4E-01	4.1E-01
11	1.0E-01	4.5E-01	6.3E-01	6.4E-02	3.3E-01	5.2E-01	4.1E-02	2.4E-01	4.2E-01
12	1.1E-01	4.7E-01	6.5E-01	6.9E-02	3.5E-01	5.3E-01	4.3E-02	2.5E-01	4.2E-01
13	1.2E-01	4.9E-01	6.6E-01	7.3E-02	3.6E-01	5.4E-01	4.4E-02	2.5E-01	4.3E-01
14	1.3E-01	5.1E-01	6.7E-01	7.7E-02	3.6E-01	5.4E-01	4.5E-02	2.6E-01	4.3E-01
15	1.4E-01	5.2E-01	6.8E-01	8.0E-02	3.7E-01	5.5E-01	4.6E-02	2.6E-01	4.3E-01