

Optimal Pension Funding Incorporating Stochastic Simulations and Dynamic Programming*

應用隨機模擬與動態規劃決定最適退休金提撥

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摘 要



本文所建立之退休金隨機模擬系統乃結合動態最適化理論，用以決定固定給付退休計畫的最適基金提撥策略。結果顯示利用此種方法，較傳統確定性的退休金評價方法提供決策者更靈活的運用方式與財務資訊。於本研究中，隨機控制系統用於描述最適化評價機能，基於隨機模擬的人力結構及所給定的限制條件下，運用動態規劃的方法求得最適提撥策略，並以台灣公務人員退休撫卹計畫為實例進行分析。文中將仔細說明模型建構的過程及討論所得的結果。

關鍵字：退休金評價，隨機控制，動態規劃。

ABSTRACT

A simulation-based mechanism combining dynamic optimization is constructed to decide the optimal funding policy of the defined-benefit pension scheme. The results show a significant advantage and flexibility of this approach in projecting the optimal financial status over the traditional deterministic pension valuation. In this study, the valuation mechanism is formulated as a stochastic control system and the optimal funding strategies are calculated through dynamic programming under the simulated workforce and specified constraints. Taiwan public employees retirement system is studied for illustration. This article outlines the procedure of constructing the proposed mechanism and comments on the empirical findings.

Keywords: pension valuation; stochastic control; dynamic programming.

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I. Introduction

Some traditional problems in pension valuation are conveniently viewed in terms of dynamic and stochastic processes. We assume that, at any time, each plan participant is in one of a number of plausible working statuses. Changing working status of the plan member may have direct financial impact on the expected of future cash flows. In a recent study (Frees et al., 1997), a forecasting model has been constructed to assist policy-makers in projections of the Social Security system in United States. This forecasting model emphasizes on using statistical techniques to explore characteristics of the data. Based on the forecasts of plan assets, the policy-maker could monitor the plan financing and amend the funding policy. In this study, a different perspective based on the stochastic simulations of the workforce and the optimal funding strategy are considered.

Two types of risks concerning the stability and security of funding are introduced (see Haberman and Sung, 1994): the contribution rate risk and the solvency risk, which have characterized the trade off in plan decision process. A performance measurement associated with these two risks was then constructed to derive the optimal contributions subject to specific constraints through dynamic optimization. Based on the optimal funding policy, the policy-maker could attain the plan's target financing status by amending the funding schedule. In this study, the similar methodology is adopted and a solvency monitoring technique combining stochastic simulations and dynamic optimization is proposed to decide the optimal funding strategy of the defined benefit pension scheme. The accrued liabilities are estimated based on the realistic plan members' turnovers, while the plan investment performance is projected according to its stochastic returns. The results provide significant value added to the traditional pension valuation. Furthermore, stochastic simulation is used to explicitly characterize the participants' population based on its turnover and the investment return adjusted by its asset return associated with this scheme. The contribution rate risk and the solvency risk are minimized through dynamic optimization. Traditional valuation techniques, however, can merely examine the plan status at a specific valuation date.

Stochastic simulation and dynamic optimization are two of the best tools in policy-making, which can help the plan sponsor amend the benefit scheme or funding policy.

Stochastic simulations using time as the operational parameter in Bacinello (1988) are performed to obtain the best estimates of the projected workforce, while the projected cash flows are scrutinized through dynamic simulations. An extensive review of past pension cost analyses can be found in Shapiro (1985). Since pension plans are almost subject to some degree of interventions in their financial consideration. This has been much more the case since the substantial legislation was enacted in recent years. The proposed projection techniques can provide valuable inputs for planning decision.

This study emphasizes the importance of optimal control applied to pension research in optimizing the plan financial condition. Similar researches can be found in: Bowers et al. (1982), O'Brien (1986, 1987), Dufresne (1988, 1989), Haberman (1992, 1993, 1994), Daykin et al. (1994), Haberman and Sung (1994), Haberman and Wong (1997), Cairn and Parker (1997), Runggaldier (1998), Schäl (1998) and others. A discussion of a rigorous and tractable stochastic model for pension fund can be found in Janssen and Manca (1997). Our proposed pension financing basically involves projecting a series of cash flows for the future years through stochastic simulations according to the probabilistic experience set of actuarial assumptions. Then the optimal contributions are estimated subject to the specific performance measure. A brief summary of the advantages of this approach is listed below:

1. Through high-speed computers, the plan sponsor can forecast the plan's future cash flows and the turnover of the future workforce, which helps the fund management.
2. The optimal contribution can be estimated under systematic scenario based on specific plan investment and recruiting strategies.
3. The optimal funding strategy and funding status of the plan can be estimated under specific performance measurement implemented through a computerized system.
4. Running an extensive set of scenarios will shed light on the interaction between the plan liability and the investment performance.

5. This approach will produce optimal solutions and is capable of inputting specific funding intervention.

II. Data Description

In this research, the Taiwan public employees retirement system (Tai-PERS) is studied and evaluated through the proposed dynamic optimization. In reality, fluctuations of the inflation rates, rate of returns, rate of increase in salaries and demographic factors subjected to recent economic conditions need to be taken into account when considering cost allocation and projection of the plan. Due to recent global financial crisis and the highly uncertainty of future economic predictions in the financial market, fund investment strategy and asset-liability related topics have taken on new significance and much attention has been focused on the implementation of a better monitoring system to oversee the entire pension

risks.

Tai-PERS is a large public retirement system that is designed to provide retirement and ancillary benefits to all government employees. There are 271,215 active members under the current benefit scheme. The present funding policy requires each member to contribute 2.8% of his covered salary while the sponsor of the plan contributes 5.2% of the plan member's covered payroll monthly to a public trust fund. A sample of 3,823 participants is used to evaluate the performance of the proposed approach. The average age of employee in our sample is 42.99 years and the average year of service is 15.6 years. Generally speaking, the workforce structure of these samples were relatively older than the overall population. Accordingly, the contribution rates obtained from the sample vary from that for Tai-PERS. The statistical summary of the sample is listed below.

Table 1. Basic Statistics of the Sample

attained age/year of service	0~4	5~9	10~14	15~19	20~24	25~29	30~34	35~39	40~
15~19	0	0	0	0	0	0	0	0	0
20~24	37	1	0	0	0	0	0	0	0
25~29	241	91	1	0	0	0	0	0	0
30~34	197	335	84	1	0	0	0	0	0
35	26	64	62	14	0	0	0	0	0
36~39	62	238	249	137	5	0	0	0	0
40~44	56	99	172	318	74	4	0	0	0
45	14	17	21	45	24	11	0	0	0
46~49	30	36	37	66	124	95	5	0	0
50~54	19	19	12	23	35	113	32	12	0
55	2	1	4	5	7	15	11	6	0
56~59	4	4	6	7	8	37	42	33	8
60~64	7	0	6	3	3	16	25	49	53
65	0	0	0	1	2	8	4	10	18
66~	0	0	0	2	4	5	6	12	31
Total	695	905	654	622	286	304	125	122	110

This system is a contributory defined benefit plan since the participant's retirement benefits are calculated according to the length of his service and his final salary upon retirement. To protect the retirement benefits against post-retirement inflation, the retirees have the option of a lump-sum retirement payment, monthly pension with the cost of living adjustments, or a mix between lump-sum payment and monthly pension.

A service table, which reflects all causes of member's turnover, is constructed based on the experience data collected from July 1, 1995 to June 30, 1996. Owing to the limitation of the data collection, further updating this table is necessary. In Section 3, the proposed procedure within this framework is formulated. In Section 4, the optimal contributions are estimated under the given performance measurement and the associated

stochastic cash flows are performed. Section 5 presents the model validation justifications and the final results of optimal plan status of Tai-PERS. Concluding comments are given in Section 6.

III. The Dynamic Procedure

A simulation-based optimization is proposed to assist policy-makers to evaluate the financing efficiency of the plan. Since management of the dynamic cash flows is critical to financial soundness of the plan, the model has emphasized on two elements that significantly affect the plan's financial balance. One is the annual optimal contribution of the plan members and sponsor, and the other is the fund actuarial status for the active members and pensioners. Every year, the cash inflow from plan members, plan sponsor and the investment returns should sustain the annual normal cost of this plan. Complexity of the actuarial valuation is sometimes viewed as another element of cost. Hence, the proposed simulation-based optimization procedure is

built into a user-oriented computerized system to reduce duplication of efforts. We use the Visual Basic 5.0 program in the calculation. Implementing the proposed approach into a computerized system can achieve more efficient policy-making.

A dynamic approach is proposed to decide the pension funding subject to specific constraints. Under certain risk criteria given in the estimation, an optimal contribution can be obtained. In our study, the size constrained population assumption is used to project the plan workforce. In the open system, to complete the specification of the model we must assume how new entrants are allocated to various states and age intervals (see Bartholomew, 1982). In our study, the new entrants are placed in the lowest job rank. The future plan workforce are predicted based on a size constrained assumption. Based on the previous experience of the new entrants into the system, a simplified dynamic model is assumed in this study to perform the simulations. The recruitment distributions of these new entrant are given as follows.

Table 2. Recruitment Distribution

age	20-24	25-29	30-34	35-39	40-44
percentage	20%	60%	18%	1%	1%

Based on the given assumptions, the projection of workforce demographics is given in Table 3 and

shown in Figure 1.

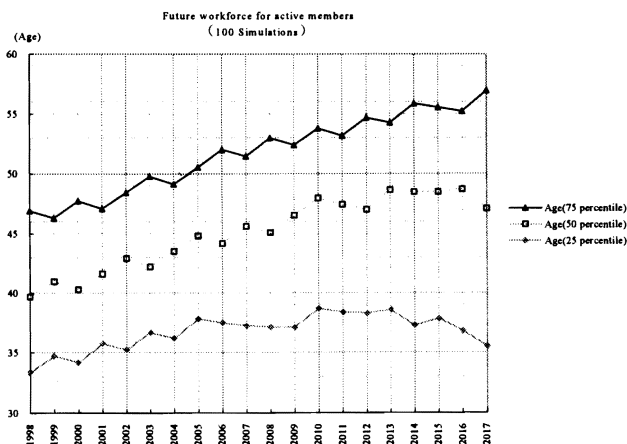


Figure 1. Workforce Projection

Table 3. Workforce Demographics

Future workforce between 1998-2017				
year	active employees		pensioner	
	number	age (50 percentile)	number	percentage by active employees
1998	3751	39.67	61	1.62%
1999	3750	40.98	81	2.15%
2000	3746	40.30	101	2.68%
2001	3755	41.61	116	3.10%
2002	3754	42.91	132	3.51%
2003	3754	42.20	147	3.90%
2004	3756	43.50	162	4.31%
2005	3754	44.79	176	4.69%
2006	3741	44.16	196	5.25%
2007	3739	45.60	217	5.80%
2008	3731	45.07	241	6.47%
2009	3737	46.52	261	6.99%
2010	3740	47.95	278	7.44%
2011	3741	47.44	295	7.87%
2012	3726	47.00	317	8.50%
2013	3710	48.65	345	9.29%
2014	3708	48.47	371	10.02%
2015	3684	48.48	407	11.05%
2016	3691	48.69	438	11.85%
2017	3681	47.07	471	12.81%

The actuarial notations used in stochastic simulation and fund recursive relationship are given as follows:

- N = the set of chosen time frame;
- $\{i_t\}_{t \in N}$ = return rate of pension fund at time t ;
- $\{C_t\}_{t \in N}$ = contribution at time t ;
- $\{v_t\}_{t \in N}$ = discount factor at time t ;
- $\{\beta_t\}_{t \in N}$ = risk weighted ratio at time t ;
- $\{NC_t\}_{t \in N}$ = normal cost at time t ;
- $\{F_t\}_{t \in N}$ = pension fund asset at time t ;
- $\{AL_t\}_{t \in N}$ = accrued liability at time t ;
- η = target fund ratio.

The procedure of our approach is constructed as follows:

Step 1:

The future information of the active and inactive members in the system is simulated through a series

of dynamic processes using Bernoulli trials. The working status of each member is simulated according to the decrement probabilities from the service table. Let $E_{x,t}$ denote the working status of the employee age x between t and $t+1$ year in the future. The process is shown as follows.

- (a) Simulate $E_{x,t}$ by generating a pseudo-random number from Bernoulli ($p_x^{(t)}$), where $p_x^{(t)}$ is obtained from the service table.
- (b) If $E_{x,t} = 1$ (i.e., this member is in working status), then go to step (e).
- (c) If $E_{x,t} = 0$ (i.e., this member is not in working status), then a pseudo-random number of multinomial distribution is generated from Multi

$$\left(\frac{q_x^{(d)}}{q_x^{(t)}}, \frac{q_x^{(i)}}{q_x^{(t)}}, \frac{q_x^{(l)}}{q_x^{(t)}}, \frac{q_x^{(r)}}{q_x^{(t)}} \right) \quad \text{where}$$

$q_x^{(t)} = q_x^{(d)} + q_x^{(i)} + q_x^{(l)} + q_x^{(r)}$. The superscript (i) stands for disability; (d) for death; (l) for layoff and withdrawal and (r) for retirement. Let $R_{x,t}$ denote the living status of the retiree aged x at

time t and $B_{x,t}$ denotes the option of his retirement benefits. If $R_{x,t} = 1$ (i.e., the member is retiring), a pseudo-random number $B_{x,t}$ is generated from Bernoulli($b_{x,t}$) to simulate the chosen benefit program where $b_{x,t}$ is estimated from past experience. If $R_{x,t} = 0$, the member is dead and the procedure ends.

- (d) If $B_{x,t} = 1$ (i.e., the retiree chooses the monthly annuity), the living status $P_{x,t}$ is generated from Bernoulli($q_{x,t}$) where $q_{x,t}$ is estimated from the post-employment life table of the retiree. If $B_{x,t} = 0$ (i.e., the retiree choose the lump sum payment), the benefit payment is then computed.
- (e) Let $t = t + 1$.

Step 2:

The contributions are assumed to be paid at the beginning of each year in proportion to the covered payroll of each active employee. Constant rate is paid by the active employees, whereas the remaining part is paid by the government. The major cash flows are defined as following:

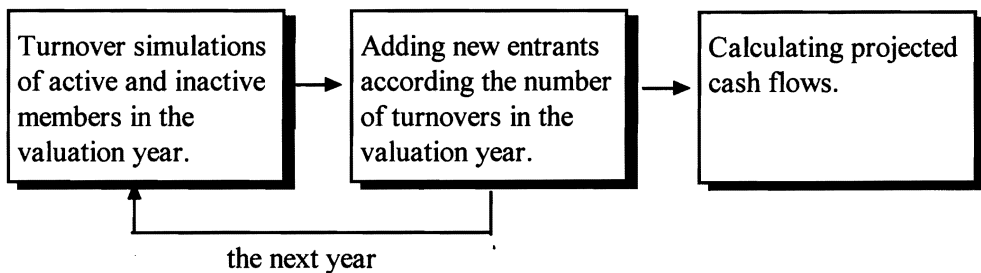
Benefit payments B_t including, disabled, death, layoff, retirement benefits and withdrawal refund. (also see Chang, 1998)

Contributions C_t from active employees, new entrants and the employer.

Returns R_t from pension fund and the **accumulated fund asset** F_t .

The Monte Carlo simulations can be carried out by repeating Step 1 and 2. The flow chart of the open group simulations is shown in Table 4.

Table 4. Flow Chart of the Open Group Simulations



Step 3:

Based on the projected benefits, C_t is estimated through minimizing the performance measure defined in Equation (1) from present to time $T-1$. Detailed clarification and explanation of this performance measure will be discussed in Section 4.

$$\Gamma(C_0, \dots, C_{T-1}) = E\left\{\sum_{t=0}^{T-1} \left[v_t \left(1 - \frac{C_t}{NC_t}\right)^2 + v_{t+1} \beta_{t+1} \left(1 - \frac{F_{t+1}}{\eta AL_{t+1}}\right)^2 \right] \right\} \quad (1)$$

During the year between time t and $t+1$ the fund balance will increase by the contribution C_t and decrease by the benefit outgo B_t at the beginning of

the year simulated from Step (2) and plus the investment return:

$$F_{t+1} = (F_t + C_t - B_t)(1 + i_t), \quad (2)$$

$$i_t \propto N(\theta, \sigma^2)$$

where the real return i_t between time t and $t+1$ are assumed to be a sequence of independent and identically distributed normal variables with mean θ and variance σ^2 . The return rates used in computing the accrued liability are simulated based on the model updated by the past investment performance. In this study, a simplified approach is used to model the flat term structure. For valuation and funding

purpose, relative conservative and optimistic assumptions are employed to investigate the financial impact on the future pension cost.

IV. The Model

After the results from these simulations are obtained, the benefit payments are estimated and recorded in the database. In our approach, time is assumed to be the operational parameter and the above steps are repeated as the fund development in time. We intend to forecast dynamically the overall

financial condition of the plan each year. Since bias may exist owing to variation in various realizations, we summarize the statistics of these estimates based on the simulations. For a particular j -th simulation, the projected benefit at time $t+1$ is denoted by ${}_j B_{t+1}$. From these simulations, a median estimate ${}_j \hat{B}_{t+1}$ is used to estimate the contribution at time $t+1$. The cash flows of the benefit payments, normal costs and the accrued liability in 20-years are projected. Then the optimal contributions are computed through the optimization procedure.

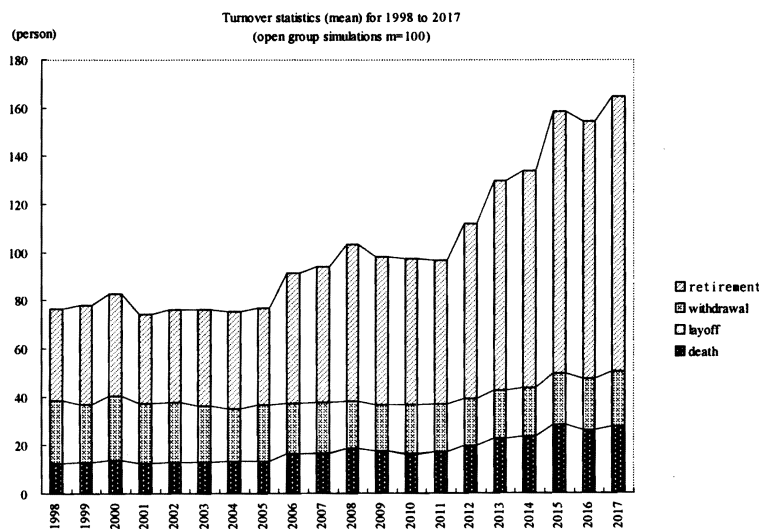


Figure 2. Turnover Statistics

In Figure 2, the averaged number of turnover using 100 simulations is shown. The number of the retiree has an increasing trend after year 2010, while the number of withdrawal for other causes remains steady. In order to determine the parameters that best fit the data, a performance measure, ${}_0 \Gamma_{T-1}$, is defined to measure the performance of the estimation between time 0 and time $T-1$. The performance measure is a nonnegative function based on the sum of weighted discounted squared deviations. The parameter estimates are determined by minimizing the combined performance measure in Equation (1).

$v_t(1 - \frac{C_t}{NC_t})^2$ is used to denote the contribution rate risk, while $v_{t+1}\beta_{t+1}(1 - \frac{F_{t+1}}{AL_{t+1}})^2$ is used to denote

the solvency risk where β_{t+1} is the relative weight in measuring the fund financial stability at time t . The relative ratios are used in measuring the discounted quadratic deviation over the chosen time horizon. The advantage of using this criterion is in decision making process, which is clearly requisite in predicting the performance ratio by years. Then the optimization could be formulated as:

$$\min_{c_0, \dots, c_{T-1}} {}_0 \Gamma_{T-1} = \min_{c_0, \dots, c_{T-1}} E \sum_{s=0}^{T-1} [v_s(1 - \frac{C_s}{NC_s})^2 + v_{s+1}\beta_{s+1}(1 - \frac{F_{s+1}}{\eta AL_{s+1}})^2 | H_s] \tag{3}$$

where H_t is the σ -field generated by $\{F_0, \dots, F_t\}$. $\{F_t\}_{t \in N}$ are assumed to follow a first order Markov process, which is the case that Tai-PERS internally

evaluate its financial status annually. Then

$$\begin{aligned} & E \sum_{s=0}^{T-1} \left[v_s \left(1 - \frac{C_s}{NC_s} \right)^2 + v_{s+1} \beta_{s+1} \left(1 - \frac{F_{s+1}}{\eta AL_{s+1}} \right)^2 \mid H_t \right] \\ & = E \sum_{s=0}^{T-1} \left[v_s \left(1 - \frac{C_s}{NC_s} \right)^2 + v_{s+1} \beta_{s+1} \left(1 - \frac{F_{s+1}}{\eta AL_{s+1}} \right)^2 \mid F_t \right] \end{aligned} \quad (4)$$

By defining $V_t(F_t)$ as:

$$\begin{aligned} & V_t(F_t) = \\ & \min_{C_t, C_{t+1}} E \sum_{s=t}^{T-1} \left[v_s \left(1 - \frac{C_s}{NC_s} \right)^2 + v_{s+1} \beta_{s+1} \left(1 - \frac{F_{s+1}}{\eta AL_{s+1}} \right)^2 \mid F_t \right] \end{aligned} \quad (5)$$

we have

$$\begin{aligned} & V_t(F_t) = \\ & \min_{C_t} E \left\{ v_t \left(1 - \frac{C_t}{NC_t} \right)^2 + v_{t+1} \beta_{t+1} \left(1 - \frac{F_{t+1}}{\eta AL_{t+1}} \right)^2 \mid F_t \right\} \\ & + E \{ V_{t+1}(F_{t+1}) \mid F_t \} \end{aligned} \quad (6)$$

then C_t can be estimated by induction. To solve the (Hamilton-Jacobi-) Bellman equation (for example, see Merton, 1990 or Fleming and Rishel, 1975), we assume that $V_t(F_t) = a_1(t)F_t^2 + a_2(t)F_t + a_3(t)$ for all $t, 0 \leq t \leq T$. Equation (6) could be written as $\Gamma(C_t)$ which is a second-order function of C_t . Then the optimal contribution \hat{C}_t satisfying the unique condition $a_1(t+1) > \frac{v_t}{((1+\theta)^2 + \sigma^2)NC_t^2} + \frac{v_{t+1}\beta_{t+1}}{(\eta AL_{t+1})^2}$ is obtained in Equation (7).

$$\begin{aligned} \hat{C}_t = & \left[\frac{2v_t}{NC_t} + \frac{2v_{t+1}\beta_{t+1}H}{\eta AL_{t+1}} - 2 \left(\frac{v_{t+1}\beta_{t+1}}{\eta^2 AL_{t+1}^2} \right. \right. \\ & \left. \left. + a_1(t+1)K(F_t - B_t) - a_2(t+1)H \right) \right] \\ & / \left[\frac{2v_t}{NC_t} + \frac{2v_{t+1}\beta_{t+1}K}{\eta^2 AL_{t+1}^2} + 2a_1(t+1)K \right] \end{aligned} \quad (7)$$

where

$$H = 1 + \theta, \quad K = H + \sigma^2.$$

We set $a_1(T) = a_2(T) = a_3(T) = 0$ as the

boundary condition in the above equation which means no bias at time $t = T$. That is, the long term financial status of this pension plan is assumed to have no risk incurred. Then we could solve the recursive relationship for $a_1(t)$ and $a_2(t)$ for $0 \leq t \leq T$. Given

$$\begin{aligned} D_t = & \frac{v_t}{NC_t} + \frac{2v_{t+1}\beta_{t+1}H}{\eta AL_{t+1}} + \frac{2v_{t+1}\beta_{t+1}K}{\eta^2 AL_{t+1}^2} B_t \\ & + 2a_1(t+1)KB_t - a_2(t+1)H, \end{aligned}$$

$$E_t = -\frac{2v_{t+1}\beta_{t+1}K}{\eta^2 AL_{t+1}^2} - 2a_1(t+1)K,$$

$$F_t = \frac{2v_t}{NC_t} + \frac{2v_{t+1}\beta_{t+1}K}{\eta^2 AL_{t+1}^2} + 2a_1(t+1)K,$$

the recursive relationship for $a_1(t)$ and $a_2(t)$ are solved by the following equations.

$$\begin{aligned} a_1(t) = & \frac{v_t^2 E_t^2}{F_t^2 NC_t^2} + v_{t+1} \beta_{t+1} \frac{K}{\eta^2 AL_{t+1}^2} \left(1 + \frac{E_t}{F_t} \right)^2 \\ & + a_1(t+1)K \left(1 + \frac{E_t}{F_t} \right)^2 \end{aligned} \quad (8)$$

$$\begin{aligned} a_2(t) = & \frac{2E_t}{F_t NC_t} \left(\frac{D_t}{F_t NC_t} - 1 \right) - 2v_{t+1} \beta_{t+1} \frac{H}{\eta AL_{t+1}} \\ & \left(1 + \frac{E_t}{F_t} \right) + 2v_{t+1} \beta_{t+1} \frac{K}{\eta^2 AL_{t+1}^2} \left(1 + \frac{E_t}{F_t} \right) \left(\frac{D_t}{F_t} - B_t \right) \\ & + 2a_1(t+1)K \left(1 + \frac{E_t}{F_t} \right) \left(\frac{D_t}{F_t} - B_t \right) \\ & + a_2(t+1)H \left(1 + \frac{E_t}{F_t} \right) \end{aligned} \quad (9)$$

V. The Results and Analysis

We now use Tai-PERS to illustrate the results. As stated in previous sections, Tai-PERS presents a challenge for the pension actuary because of the complexity of its possible cash flow. There are certainly many directions in order to perform a reasonably realistic description of the actual behavior of a pension plan. Due to the specific and complex statutory requirements of Tai-PERS, we shall consider generalization where the return rate is described by a stochastic process. The actuarial

factors are reduced in performing the pension valuation. This restriction is more a reflection of our interest than an ambition to cover each detail. So the simplified procedure aims to reduce the computer run times and the outputs of the financial information stored in the database. Thus if we assume that:

Population: Tai-PERS service table based on 1995-1996; 1989 Taiwan Standard Ordinary Experience Life Table (1998) is used for the retiree's life table;

Actuarial Cost Method: Entry age normal (EAN) cost method (see Anderson, 1992);

Salary scale and inflation rate: 3.5% for annual salary increase and 3% for annual inflation rate;

Interest rate: a flat term structure model is assumed in discounting the cash flow. 8% is given for performance valuation, i.e., we assume $v_t = v' = (1.08)^{-t}$, i.e., a constant discount rate assumption;

Target Fund ratio: $\eta = 100\%$ for every year;

Risk measurement weight: $\beta_t = 100\%$ for every year;

Fund return rate: $\theta = 10\%$ and $\sigma^2 = 0.04\%$.

The actuarial factors are given above in performing the optimization. This restriction is more a reflection of our interest than an ambition to cover each possible scenario. So the given values aims to illustrate the results based on our baseline estimates. With the above assumptions, the estimated actuarial accrued liabilities, normal costs and benefit payments are simulated based on Steps 1 and 2.

We assume that the fund provides benefits for 3,823 employees in Tai-PERS and the initial fund is set to be 373,211,585 NT dollars. This corresponds to 1.41% of the 271,215 plan participants in 1996. The future optimal fund status are forecasted through the recursive relation in Equation (8) and (9). With the boundary condition being to have no risk at the end of the forecast period, the explicit optimal contributions could be projected by recursively computing estimates C_t using Equation (7).

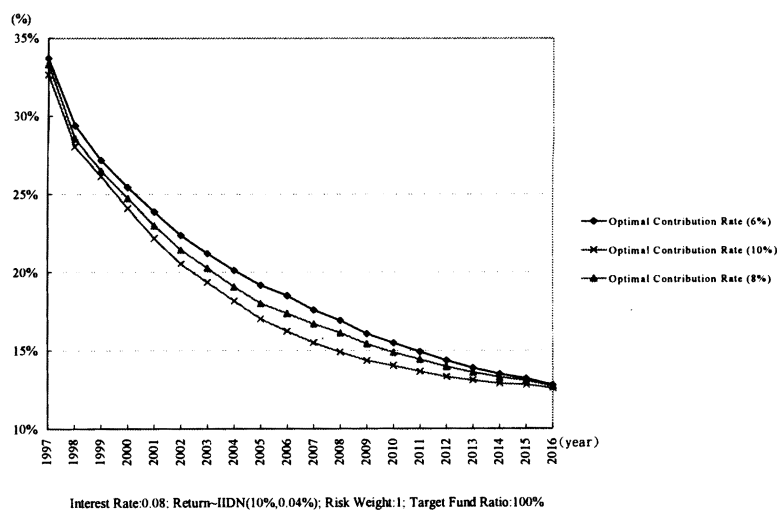


Figure 3. Optimal Contribution Rates in 1997-2016 under Various Fund Return Rate Scenarios

Figure 3 compares the optimal contribution rates from 1997 to 2016 under various fund return scenarios. It shows that the estimates of the contribution are decreasing by years. Since the EAN cost method is used in this study, the fund accrued

liabilities from the simulations have much larger values than the estimates from other cost methods. Hence the optimal contribution rates computed from the dynamic programming are the most conservative among other actuarial cost methods.

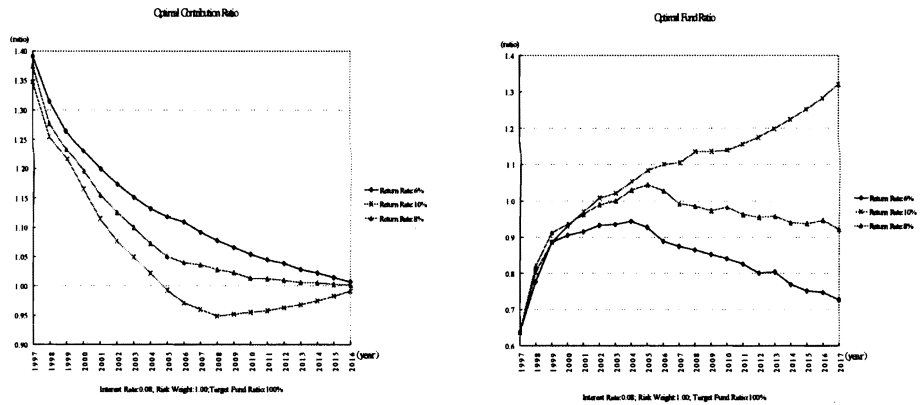


Figure 4. Optimal Contribution Ratios and Fund Ratios in 1997-2016 under Various Fund Return Rates

Figure 4 compares the optimal contribution ratios and optimal fund ratios from 1997 to 2016 under various fund returns. It shows that the estimated contribution ratios vary by years, suggesting that the variable contribution ratios are influenced by the fund return rates significantly. The optimal fund ratios are gradually increasing given return = 10% by years from 1998 to 2017, while show decreasing trends given return set at 6% and

8%. The optimal fund ratios significantly deviate from 1 between 1997 and 2017 might be partially due to the assigned return rates. Further analysis is needed to explore this. In this study, a set of given actuarial assumptions are chosen to illustrate the optimal procedure. By varying the assumptions, the plan manager could foresee the optimal plan financial status.

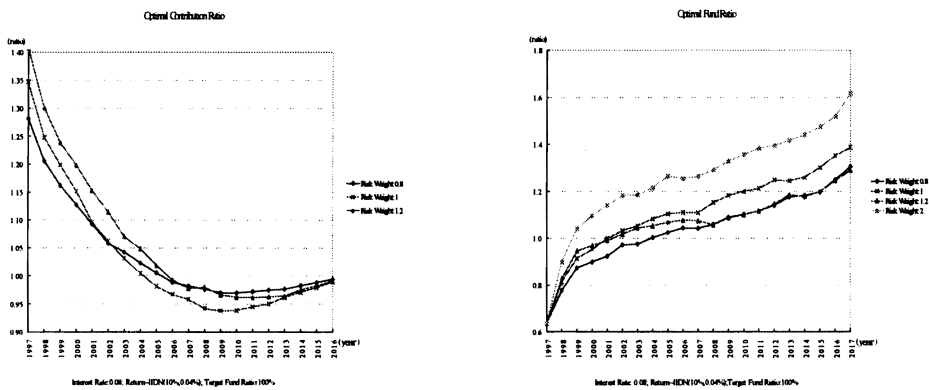


Figure 5. Optimal Contribution Ratios and Fund Ratios in 1997-2016 under Various Risk Weight Measurements

Figure 5 compares the optimal contribution rates under various risk weights. The risk ratios are modified to reflect the user's subjective risk measurement in performing the optimization. It shows that the optimal fund ratios increase with an

increase in risk weight measurements, while the optimal contribution rate ratios have shown a similar pattern. Through minimizing the performance measure, the explicit fund information could be obtained.

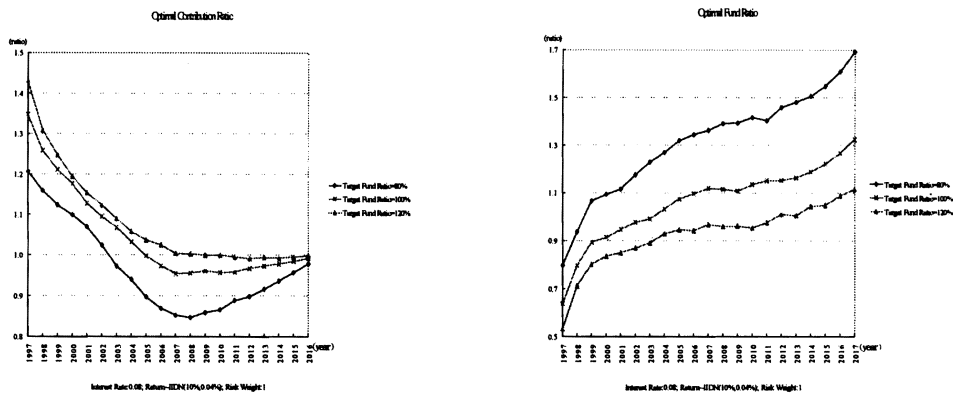


Figure 6. Optimal Contribution Ratio and Fund Ratios in 1997-2016 under Various Target Fund Ratios

Figure 6 compares the optimal contribution rates under various target fund ratios. The target fund ratios are given at various levels with respect to the attained accrued liabilities to reflect the user's subjective management requirements. It shows that the optimal contribution rate ratios increase at higher target fund ratios, while the optimal fund ratios have shown decreasing trends in Figure 6.

VI. Conclusions

Most models of pension valuation have been based on deterministic approaches. While a dynamic-stochastic model in control framework is constructed. This paper has described algorithms that are used to compute the optimal funding strategy. One purpose of this paper is to provide a flexible mechanism which allows the plan manager to incorporate his own performance measurement in the decision processes.

The approach in this model has integrated two relatively recent innovations in pension valuation: the stochastic simulations in predicting the factors in the performance function and the dynamic programming in calculating the optimal funding strategies. This approach could be adopted in assisting decision-making and acted as a benchmark to compare the funding schedule from the many actuarial cost methods employed in traditional valuation.

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