Analysis of the Efficient Frontier for Life Settlements in the Presence of Longevity Risk

考慮長壽風險下保單貼現商品對效率前緣之影響

doi:10.6545/JFS.2015.23(1).1

財務金融學刊, 23(1), 2015

Journal of Financial Studies, 23(1), 2015

作者/Author： 楊曉文(Sharon S. Yang);葉俞昀(Yu-Yun Yeh)

頁數/Page： 1-29

出版日期/Publication Date：2015/03

Analysis of the Efficient Frontier for Life Settlements in the Presence of Longevity Risk

Sharon S. Yang*
Department of Finance, National Central University

Yu-Yun Yeh
Department of Finance, National Central University

Abstract

This study attempts to analyze the effect of portfolio selection when life settlements, which are considered to have zero or low correlation with traditional investment instruments, are taken into account. We utilize the efficient frontier to assess the investment performance of a portfolio that includes three assets, namely, stocks, bonds, and life settlements. Because mortality plays an important role in determining the prices of life settlements, we consider a stochastic mortality model in our pricing framework to reflect longevity risk. The impacts of age effect, mortality improvement, and transaction costs on life settlements and investment performance are investigated numerically.

Key words: Life settlements, efficient frontier

I. Introduction

A. Life Settlement Market

Life settlements have recently been introduced as a new asset class in the capital market. Moreover, following the financial crisis in 2008, they have received more attention from investors due to certain special characteristics, such as the performances of these life settlements being considered to be uncorrelated with other investment instruments (e.g., stocks or bonds). Modern portfolio theory has formulated the concept of diversification in managing portfolio risk. To achieve this aim, understanding the correlation between selected investment assets is very critical for the portfolio manager. During the financial crisis, global stock markets started to collapse, and like a contagious disease, such an effect quickly spread to every stock market around the world. As a result, many investors suffered serious financial losses and some institutional investors even filed for bankruptcy. Although various factors have been

---

* Corresponding author: Sharon S. Yang, E-mail: syang@ncu.edu.tw, Professor of Department of Finance, National Central University, No. 300, Jhongda Rd., Jhongli City, Taoyuan County 32001, Taiwan (R.O.C.), TEL: (886) 3-4227151, FAX: (886) 3-4226062.
The authors would like to thank the Ministry of Science and Technology (Taiwan) for the research funding (NSC 99-2410-H-008-019-MY3).

(Received: January 6, 2014; First Revision: March 12, 2014; Accepted: May 26, 2014)

attributed to this global crisis, the main reason for the huge chain reaction that resulted is the correlation among investment assets. The investment banks now know the danger of using highly correlated investment assets in managing their portfolios, and different underlying assets with low or potentially zero correlation are becoming more and more attractive. In this respect, the investment banks have turned their eyes towards an asset class with huge potential, namely, life settlements, in the U.S. In 2007, the life insurance industry had a total of USD5.1 trillion in assets (American Council of Life Insurers (2008)). When compared with the USD5.08 trillion in Treasury securities in the outstanding U.S. bond market debt in 2007 (Securities Industry and Financial Markets Association (2009)), the sheer size of the life insurance industry makes it a substantial untapped market.

A life settlement is a transaction in which an insurance policy owner sells a life insurance policy to a third party for an amount that exceeds the policy's cash surrender value, but is less than the expected death benefit of the policy. Life settlements originated from the viatical settlements in the late 1990s. At that time, the viatical industry faced many challenges because of the introduction of advanced therapy technology for the treatment of AIDS and medical breakthroughs, which not only significantly increased the survivability of AIDS patients but also increased the uncertainty of forecasting life expectancies for terminally ill policyholders. As a result, the viatical market effectively ceased to exist and was replaced by the life settlement market. In the U.S., the Life Settlements Model Act adopted by the National Conference of Insurance Legislators (the "NCOIL model act" (2007)) requires the structure of a life settlement transaction.[1] The differences between a life settlement and a viatical settlement are shown in Table I, in which the life settlement is structured to a higher policy size of over USD250,000 for seniors over the age of 65. Most insured individuals participating in today's life settlement market are seniors with a life expectancy of more than two years. A life settlement is usually accomplished through the efforts of a number of market intermediaries, of which each deals with a specific aspect of the settlement of a life insurance policy. Participants in a life settlement transaction generally include an insured individual or the owner of the policy, a producer who may be a financial advisor or an insurance agent, one or more settlement brokers who may also be insurance agents, one or more life expectancy underwriters, one or more providers who typically represent the party acquiring the policy, and one or more investors. The procedure of a life settlement is presented in Figure 1.

[1] The structure of a life settlement transaction includes (a) an assignment, transfer, sale, devise, or bequest of the benefit in a life insurance policy for value, (b) a loan or other lending transaction, secured by one or more life insurance policies, (c) certain premium finance loans made for a life insurance policy on or before the date of issuance of the life insurance policy, and (d) the transfer for compensation or value of the interest in a trust or other entity that owns a life insurance policy if the trust or other entity has been formed or availed of for the principal purpose of acquiring one or more life insurance contracts.

Table I
Differences between Life Settlements and Viatical Settlements

	Viatical Settlement	Life Settlement
Policy Size	Less than USD100,000, and more than USD100,000 and usually between USD25,000 and USD50,000	More than USD100,000 and usually over USD250,000
Policyholder	AIDS patients in the 25-44 age band	Senior citizens over age 65
Life Expectancy	Less than 2 years and usually 12 months or less	More than 2 years to as high as 12-15 years

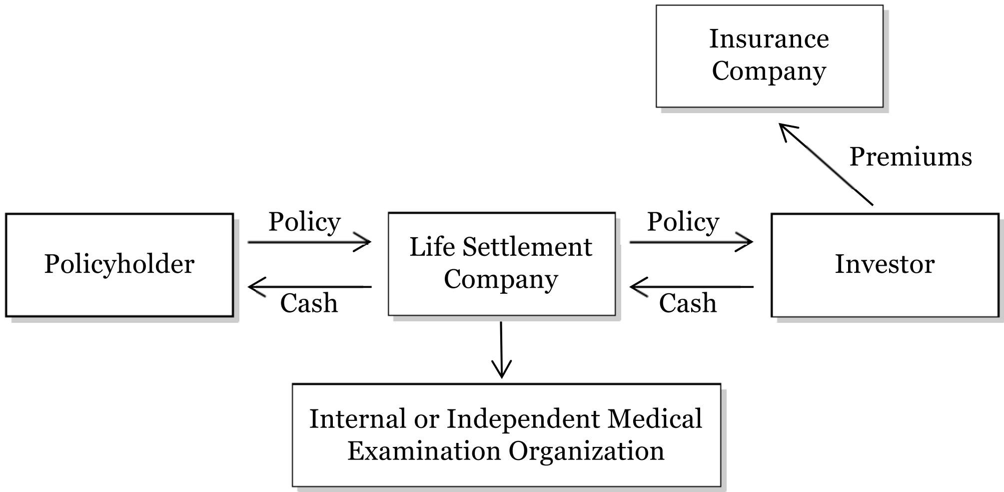

Figure 1. Life Settlement Transaction Flows

The success of the life settlement market depends on supply, demand, and regulatory and legal prerequisites (Gatzert (2010)). On the supply side, the size of the primary market is crucial so that an emerging secondary market needs to have a sufficient number of tradable policies. According to Rosenfeld (2009), the secondary market in life insurance policies is expected to reach between USD90 billion and USD140 billion by 2016, having risen from USD12 billion in 2007 and from close to USD0 in 2001 in the U.S. Moreover, the primary market for unwanted life insurance policies was estimated to be USD16 billion in 2008, and is expected to expand continuously. The growth of the life settlement industry is initially impeded primarily by a lack of liquidity, but the gathering momentum has overcome the difficulties. On the demand side, the economic crisis in 2008 is the major impediment to the growth of the life settlement market. Institutional investors are either seeking new investment opportunities or diversifying their existing investment portfolios, and life settlement may present new investment opportunities and a means to minimize risk in a new class of investment assets. However, the investors often do not have proficiency in understanding the life settlement transaction, such as obtaining and reviewing life expectancy reports and medical records. To help the success of the transaction between the buyer and the seller, the role of life settlement providers is getting more and more important. Life settlement providers normally have specific knowledge of life insurance policies from their initial acquisition in a life settlement transaction and can provide their services and expertise in life settlement transactions to the investor. Life settlement providers can help the investor be more informative and benefit from the development of the life settlement market. Therefore, a sound regulatory environment and sufficient transparency are crucial to encouraging market participation by sellers, customers, and investors.

B. Literature on Life Settlements

Only a few attempts have so far been made at life settlements because this issue is a rather new study. Earlier studies focus on the development of life settlements. Giacalone (2001) analyzes the viatical transactions and the secondary market for the life insurance policy industry. Doherty and Singer (2003a) show the benefits and welfare gains arising from the secondary market for life insurance policies. Kamath and Sledge (2005) describe the characteristics of the U.S. life settlement market and investigate the driving force behind the growth of this market. Moreover, Seitel (2006, 2007) observes the industry from an institutional investor's and a life settlement provider's viewpoints, respectively. The regulatory and tax aspects are studied by Doherty and Singer (2003b), Kohli (2006), and Gardner, Welch, and Covert (2009). Gatzert, Hoermann, and Schmeiser (2009) analyze the effects of a secondary market on the surrender profits of life insurance providers, and Katt (2008) discusses direct sales without intermediaries. Gatzert (2010) provides a comprehensive overview of the benefits and risks of the secondary markets for life insurance in the U.K., Germany, and the U.S. and points out that the U.S. market has considerable growth potential but the U.K. and German markets are somewhat limited in this respect due to a loss of tax advantages and a decreasing target policy volume.

Recently, studies on life settlements have paid more attention to the pricing method and investment performance. Investment banks regard a life settlement as an investment product. Such a product has existed in the market for a long time, but there is no consensus regarding the pricing methods. Lubovich, Sabes, and Siegert (2008), Mason and Singer (2008), and Erkmen (2011) propose different pricing models for life settlements. Three approaches, namely, the deterministic, probabilistic, and stochastic approaches, are found to determine the value of life settlements. In practice, most of the banks use the deterministic or probabilistic model. Mason and Singer (2008) value life settlement transactions with a real options approach. Erkmen (2011) first considers the stochastic process of the mortality rate for pricing life settlements. As for the return on the life settlements, Perera and Reeves (2006) show that the return on a life settlement is sensitive to life expectancy estimates. Smith and Washington

(2006) focus on transactional aspects, such as the diversification of life settlement portfolios, in order to reduce risks, and they anticipate that investors will be able to maximize opportunities to improve their performance because of the existence of an active secondary market for life insurance policies today. Stone and Zissu (2007) describe the possibilities of risk diversification and price life settlement contracts. Smith and Washington (2006) and Dorr (2008) examine the performance of portfolios with life settlements. Braun, Gatzert, and Schmeiser (2012) analyze open-end funds dedicated to investing in U.S. life settlements and find that life settlements offer good returns with near-zero betas. Wang, Hsieh, and Tsai (2012) propose that life settlements can be an effective hedging tool to reduce the insurance companies' mortality risk. Hsieh et al. (2012) investigate the spread determinants of life settlements using real-case data on life settlements.

C. Purpose and Contribution of this Study

The above literature has studied the investment performance of life settlements and concluded that the life settlements can be good investments as well as an effective hedging tool for life insurance companies. However, the most important factor influencing the return and risk of life settlements is the uncertainty of life expectancy. Nowadays, due to the progress made in medical resources, people live longer than expected, i.e., there is longevity risk. Longevity risk can affect the investment performance of a life settlement, and the above studies do not consider longevity risk in evaluating the investment performance of life settlements. In addition, Braun, Gatzert, and Schmeiser (2012) point out that even though the empirical results suggest that life settlement funds offer attractive returns paired with low volatility and are virtually uncorrelated with other asset classes, longevity risk did generally not materialize in the past and hence is largely not reflected by the historical data. They cannot be captured by classical performance measures. To fill this gap, we extend the existing literature to consider a simulation study to examine the performance of life settlements and take into account a stochastic mortality model for capturing the longevity for pricing life settlements. To be specific, we utilize the efficient frontier to assess the investment performance of the investment portfolio with the life settlement and further find the optimal investment portfolio based on the Sharpe ratio.

An efficient frontier is a useful tool to measure the investment performance of a portfolio in terms of both the risk and return perspectives. Following the introduction of modern portfolio theory by Markowitz (1952), investors made efforts to increase their return given the particular risk and tried to look for potential underlying assets. After the economic downturn in 2007, they tried desperately to count life insurance products in and utilize them to reduce the high correlation in existing portfolios. Some studies have discovered that a mortality-linked security, such as a life settlement that does not fluctuate with the stock market, is a vital property for diversifying the portfolio risk. Berketi (1999) applies a mean-variance approach to analyze the effect of longevity-linked products. This study attempts to analyze the effect of portfolio selection when a life settlement, which is considered to have zero or no correlation with the traditional investment instruments, is taken into account. In addition to a life settlement, we also consider the inclusion of a stock and a bond in the portfolio. We assume that the stock price follows the geometric Brownian motion (GBM) process and the bond price is valued by the CIR interest rate model introduced by Cox, Ingersoll, and Ross (1985). To find the price of a life settlement, we employ both the Lee and Carter's (1992) model and Cairns, Blake, and Dowd's (2006) mortality model (hereafter LC model and CBD model respectively). The LC model is the pioneering work in measuring the dynamics of mortality improvement. The application of the LC model has been widely adopted. In recent years, Cairns, Blake, and Dowd (2006) have developed a stochastic mortality model that allows a quadratic age effect to deal with the mortality rate for elders. For comparison purposes, we take both mortality models into account in our pricing framework for life settlements. Life settlements constitute a new asset class that has not been introduced in the Taiwan insurance market. To evaluate the effect of longevity risk on the life settlement market, we demonstrate the mortality experience in Taiwan. The goodness-of-fit of the mortality models is examined for both in-sample and out-of-sample.

The remainder of this paper is organized as follows. In Section II, we introduce our pricing methodology for life settlements and the mortality dynamics used for calculating the price of life settlements. The financial dynamics used for calculating the price of stocks, bonds for constructing the efficient frontier, and the calculation of the Sharpe ratio are presented in Section III. We conduct the model fitting for the mortality and financial models in Section IV. In Section V, we carry out the simulation study and analyze the efficient frontier for life settlements. In Section VI, we conclude the paper.

II. Pricing Framework and Mortality Dynamics for Life Settlements

A. Life Settlement Pricing Model

The economic value of the life settlement is determined by computing the difference of expected present value between the policy face value and future premium payments until the estimated time of death. There are different types of life insurance, which can be the life settlement contracts.[2] We consider a

[2] For example, whole life insurances, universal life insurances, variable life insurances, and joint or survivorship insurances.

whole life insurance policy because it is the most common life insurance policy in the life settlement market. Assume that the death benefit is paid at the end of the year and premiums are received at the beginning of the same year. Let A_x represent the actuarial present value of a whole life insurance issued to the policy aged x. Assume that the actuary prices the insurance contract under the deterministic assumption,[3] thus

$$A_x = \sum_{n=0}^{w-1} {}_n p_x \cdot q_{x+n} \cdot (1+\bar{r})^{-(n+1)}, \tag{1}$$

where ${}_n p_x$ is the survival probability for a person aged x who survives to $x+n$ and $\bar{r}$ is the constant pricing interest rate. $q_{x+n}=1-p_{x+n}$ is the mortality rate at age $x+n$.

Under the actuarial equivalence principle, the fair yearly pure premium of the whole life policy P with the face value or death benefit Z is calculated as

$$P = Z \cdot \frac{A_x}{\ddot{a}_{x:m\rceil}}, \tag{2}$$

where $\ddot{a}_{x:m\rceil}$ denotes the actuarial present value of a m-year temporary life annuity-due of 1 per year for an individual's age x, which can be expressed by

$$\ddot{a}_{x:m\rceil} = \sum_{n=0}^{m-1} {}_n p_x \cdot (1+\bar{r})^{-n}.$$

In practice, the insurer charges a gross premium for the insurance policy according to the premium loading γ. Thus, the gross premium GP for the insurance policy can be obtained as follows:

$$GP = \frac{P}{(1-\gamma)}. \tag{3}$$

Consider a whole life insurance policy for life settlements. Let $V_x(0,s)$ denote the fair value of a life settlement at the time that the insurance policy is sold for the transaction and s is the remaining year for paying the insurance premium. The fair value of the life settlement is determined by the present value of the difference between the death benefit and premium paid. The pricing model of the life settlement can be defined as follows.

$$\begin{aligned} V_x(0,s) = Z \cdot \mathrm{E}\left[\sum_{n=0}^{w-1} {}_n \tilde{p}_{x,t} \cdot \tilde{q}_{x+n,t+n} (1+r_{t+n})^{-(n+1)}\right] \\ -GP \cdot \sum_{n=0}^{s-1} \mathrm{E}\left[{}_u \tilde{p}_{x,t} (1+r_{t+n})^{-n}\right], \end{aligned} \tag{4}$$

where ω is the upper limit age, and s is the final year when a premium payment is made. ${}_n\tilde{p}_{x,t}$ is the risk-neutral probability valued in year t for a

3 According to a deterministic mortality table and constant interest rate assumption.

person aged x who survives to $x+n$, and $\tilde{q}_{x+n,t}$ is the risk-neutral mortality rate at age $x+n$ in year t, $\tilde{q}_{x+n,t} = 1 - \tilde{p}_{x+n,t}$. r_t is the risk free interest rate in year t, and $\mathrm{E}(\cdot)$ denotes the expected value.

The crucial factors in pricing life settlements are the assumptions of life expectancy. To capture longevity risk in measuring life expectancy, the use of a stochastic mortality model is more suitable for describing the conditional probability density function of life expectancy. We use the mortality models developed by Lee and Carter (1992) and Cairns, Blake, and Dowd (2006) to capture the pattern of mortality in our pricing framework in Equation (4).

B. Mortality Dynamics

We employ both the LC model and CBD model to capture the mortality dynamics for policyholders. Lee and Carter (1992) use statistical time series techniques to model mortality rates where the mortality level is described by a single index. The logarithm of the central mortality rate $\ln(m_{x,t})$ is represented as a linear function of the time-varying index κ_t and two parameters of sets of age-specific constants α_x and β_x.

$$\ln(m_{x,t}) = \alpha_x + \beta_x \cdot \kappa_t + \varepsilon_{x,t}, \tag{5}$$

where $m_{x,t}$ is defined as the central death rate for age x in year t, α_x is the general shape of the hazard rate across different ages, and β_x indicates how different ages react to changes in time trend κ_t such that

$$\frac{d}{dt}\ln(m_{x,t}) = \beta_x \cdot \frac{d}{dt}\kappa_t. \tag{6}$$

Basically, κ_t is a decreasing trend according to time, indicating the trend of mortality improvement. The error term $\varepsilon_{x,t}$, with mean 0 and variance σ_ε^2, reflects particular age-specific historical influences not captured by the model. According to the central death rate projected by the LC model, we can obtain the future one-year survival rate as $p_{x,t} = e^{-m_{x,t}}$ and mortality rate as $q_{x,t} = 1 - e^{-m_{x,t}}$. Therefore, the future u-year survival rate can be calculated as ${}_u p_{x,t} = p_{x,t} \cdot p_{x+1,t+1} \cdot \dots \cdot p_{x+u-1,t+u-1}$.

For a comparison purpose, we also adopt another mortality model, the CBD model. Different to the LC model, the CBD model is designed to capture the mortality dynamics for the elders. It allows a quadratic age effect in their model, which is defined as

$$\mathrm{logit}(q_{x,t}) = A_t^1 + A_t^2(x - \bar{x}) + \varepsilon_{x,t}, \tag{7}$$

where $\mathrm{logit}(q_{x,t}) = \ln(\frac{q_{x,t}}{1-q_{x,t}})$, and $\bar{x}$ is the average of a range of age groups. In this model, A_t^1 will be a downward trend which reflects general improvements in mortality over time at all ages. A_t^2 presents an increasing trend which means that the curve is getting slightly steeper over time. Thus, mortality improvements have been greater at lower ages.

The parameter estimations in the LC and CBD models are based on the maximum likelihood estimation (MLE). The details are described in Appendix A.

To calculate the fair value of life settlements, we need to risk neutralizing the mortality rates in our pricing framework. Wang (2000) proposes a transformation for pricing contingent claims that can be traded or not. Because contracts contingent on mortality rates are usually not traded on financial markets, Wang's transformation helps value mortality-linked securities (Lin and Cox (2005), Dowd et al. (2006), Liao, Yang, and Huang (2007), Denuit, Devolder, and Goderniaux (2007), and Yang and Wang (2013)). To convert the real-world mortality rates (without tildes) into risk-neutral ones (with tildes), we apply the Wang transformation. The Wang transform converts ${}_{u}q_{x,t}$ into the risk-neutral measure function ${}_{u}\tilde{q}_{x,t}$ with a distortion operator:

$$ {}_{u}\tilde{q}_{x,t} = g\left({}_{u}q_{x,t}\right) = \Phi(\Phi^{-1}\left({}_{u}q_{x,t}\right) - \lambda), \tag{8} $$

where g is a distortion function with $g(0) = 0$, $g(1) = 1$, and $g'(0) = \infty$. Furthermore, Φ is the standard normal cumulative distribution function, and λ is the market price of risk for insurance products. Thus, we can obtain the risk neutral survival probability of ${}_{u}\tilde{p}_{x,t}$ by $1 - {}_{u}\tilde{q}_{x,t}$.

III. Financial Dynamics and Portfolio Performance

A. Financial Dynamics

We consider three types of products in the portfolio set, equity, bond, and life settlements. In addition to life settlements, we need to understand the dynamics of bond price and stock price. Furthermore, in consistent with life settlements, our paper uses simulation to construct the portfolios. We assume that stock prices follow the geometric Brownian motion (GBM) and are based on the CIR model. The GBM stochastic process is defined as

$$ dS_t = \mu S_t dt + \sigma_s S_t dW_t. \tag{9} $$

In this stochastic process, S_t is the stock price at time t, μ is the expected return of stock, σ_s is the standard deviation of stock, and W_t is a standard Brownian motion. We adopt this formula to capture the pattern of stock price in the risk-neutral measure and calculate the return and variance of the stock price under different scenarios.

The CIR model developed by Cox, Ingersoll, and Ross (1985) specifies that the instantaneous interest rate follows the stochastic differential equation as

$$ dr(t) = a(b - r(t))dt + \sigma_r \sqrt{r(t)} dW_t^r, \tag{10} $$

where a is the speed of reversion of short-term interest, b represents the long-term mean level, σ_r is the volatility of interest rate, and the drift factor $a(b - r(t))$ makes sure that the mean reversion of the interest rate $r(t)$ regroups toward the long run level b with the adjusted speed of strictly positive parameter a. Moreover, the volatility factor $\sigma_r\sqrt{r}$ prevents the interest rate from negative value, and W_t^r is a standard Brownian motion.

Then, the price at time t of a zero-coupon bond with maturity T is

$$P(t,T) = A(t,T)\exp[-B(t,T) \times r(t))], \tag{11}$$

where

$$B(t,T) = \frac{2(\mathrm{e}^{\gamma(T-t)} - 1)}{(a+\gamma)(\mathrm{e}^{\gamma(T-t)} - 1) + 2\gamma}$$

and

$$A(t,T) = \left[\frac{2\gamma \mathrm{e}^{(a+\gamma)(T-t)/2}}{(a+\gamma)(\mathrm{e}^{\gamma(T-t)} - 1) + 2\gamma}\right]^{2ab/\sigma_r^2}$$

with $\gamma = \sqrt{a^2 + 2\sigma_r^2}$.

Under the risk-neutral measure Q, the bond price dynamics can be easily obtained via Ito's formula:

$$dP(t,T) = r(t)P(t,T)dt - B(t,T)P(t,T)\sigma_r\sqrt{r(t)}dW_t^r. \tag{12}$$

By inverting the bond-price formula, thus deriving r from P, we obtain

$$d\ln P(t,T) = \left(\frac{1}{B(t,T)} - \frac{1}{2}\sigma_r^2 B(t,T)\right)[\ln A(t,T) - \ln P(t,T)]dt$$
$$-\sigma_r\sqrt{B(t,T)[\ln A(t,T) - \ln P(t,T)]}dW_t^r. \tag{13}$$

Moreover, we take the correlation between stocks and bonds into account. Let $dW_t dW_t^r = \rho dt$, $\rho \in [-1,1]$.

B. Portfolio Performance

We attempt to evaluate the performance of the portfolio that includes life settlements. The portfolio performance is evaluated according to the returns and volatility of stocks, bonds, and life settlements, and the correlation among the assets. We construct the efficient frontier and calculate the Sharpe ratio for the portfolio and each asset. The Sharpe ratio is used to measure the excess return (or risk premium) per unit of deviation in the investment. In other words, the Sharpe ratio is defined as a ratio of the excess return to the risk. For a portfolio, the Sharpe ratio is defined as

$$\text{Sharpe ratio } = \frac{\mathrm{E}(R_p - r_f)}{\sigma_p}, \tag{14}$$

where R_p is an annualized return of asset, σ_p is an annualized volatility of portfolio, and r_f is risk-free rate. We can replace the return and volatility for an individual asset to get the Sharpe ratio for the individual asset.

IV. Model Fitting for Mortality and Financial Models

We use simulations to construct the efficient frontier for the portfolio based on the mortality and financial dynamics. To reflect the market condition, the parameters of these models are fit based on the empirical data. We describe the goodness fit of the models as follows.

A. Model Fitting for Mortality Models

To fit the mortality dynamics, we use the Taiwan mortality data from the Human Mortality Database for the elders (ages 65-99). The data are divided into the training period (in-sample) and testing period (out-of-sample), where the period of 1970 to 2005 represents the training period and that of 2006 to 2010 is the testing period.

A.1. In-Sample Model Fitting for Mortality Models

We first discuss the model fitting based on the training period. Since more parameters usually reduce fitting errors, we adapt the Akaike information criterion (AIC) and Bayesian information criterion (BIC)[4] for accuracy comparison. These two criteria consist of the likelihood function and number of parameters and are frequently used in model selection. The parameters estimated for both the LC model and the CBD model are plotted in Figure 2 and Figure 3. As expected, κ_t in the LC model could be reducing over time, and in the CBD model, A_t^1 could be a downward trend and A_t^2 could be an increasing trend. The results of parameter estimation are consistent with our expectation. Moreover, these results indicate that mortality is decreasing and longevity risk is increasing over time. The corresponding fitting results of the training data for these two models are shown in Table II. In terms of the log-likelihood, AIC, and BIC, we find that the LC model performs better than the CBD model for Taiwan mortality experience.

Table II
Fitting Accuracy of the LC and CBD Models for the Elderly

The numbers in bold indicate a better fit.

Model	Gender	Log-Likelihood	AIC	BIC
LC	Male	**-12,451.2**	**25,114.3**	**25,659.1**
	Female	**-11,150.7**	**22,513.3**	**23,058.0**
CBD	Male	-14,874.8	29,893.5	30,263.5
	Female	-13,731.8	27,607.5	27,977.5

4 AIC is defined by the following equation: $\mathrm{AIC} = (-2 \cdot \mathrm{LLF}) + (2 \cdot d)$, and BIC is defined by the following equation: $\mathrm{BIC} = (-2 \cdot \mathrm{LLF}) + d \cdot \ln(N)$, where LLF is the vector of optimized log-likelihood objective function values, d is the number of estimated parameters, and N is the number of values in the estimation data set.

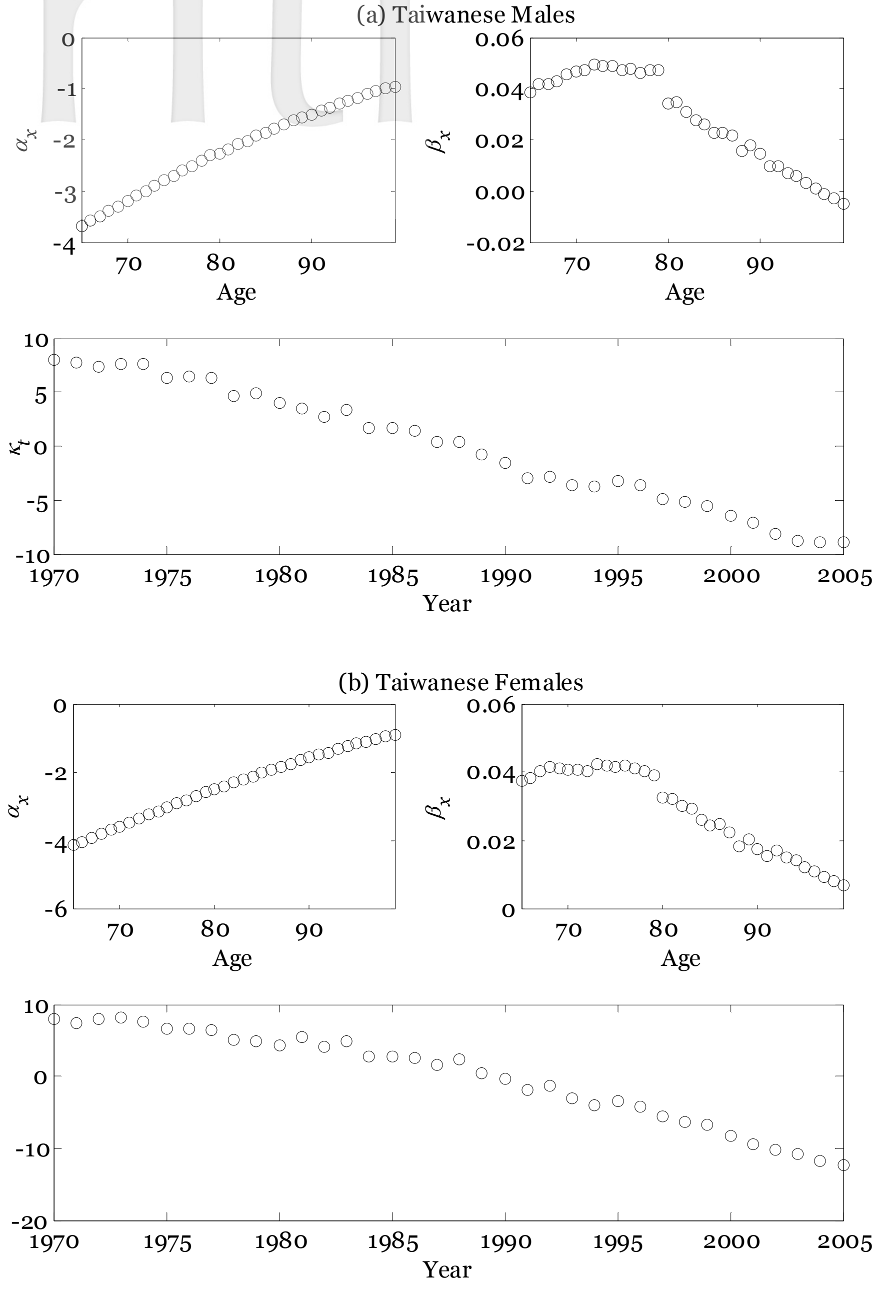

Figure 2. Parameter Estimates for the LC Model (Taiwan 65+)

A.2. Out-of-Sample Mortality Forecasting for Mortality Models

We continue evaluating these models with respect to mortality prediction for the testing data. The key to predicting mortality rates for these models is to decide the future values of period effects. There are quite a few choices, and applying the time series model to the period effects is one of them. The autoregressive integrated moving average (ARIMA) process is a common method for projecting

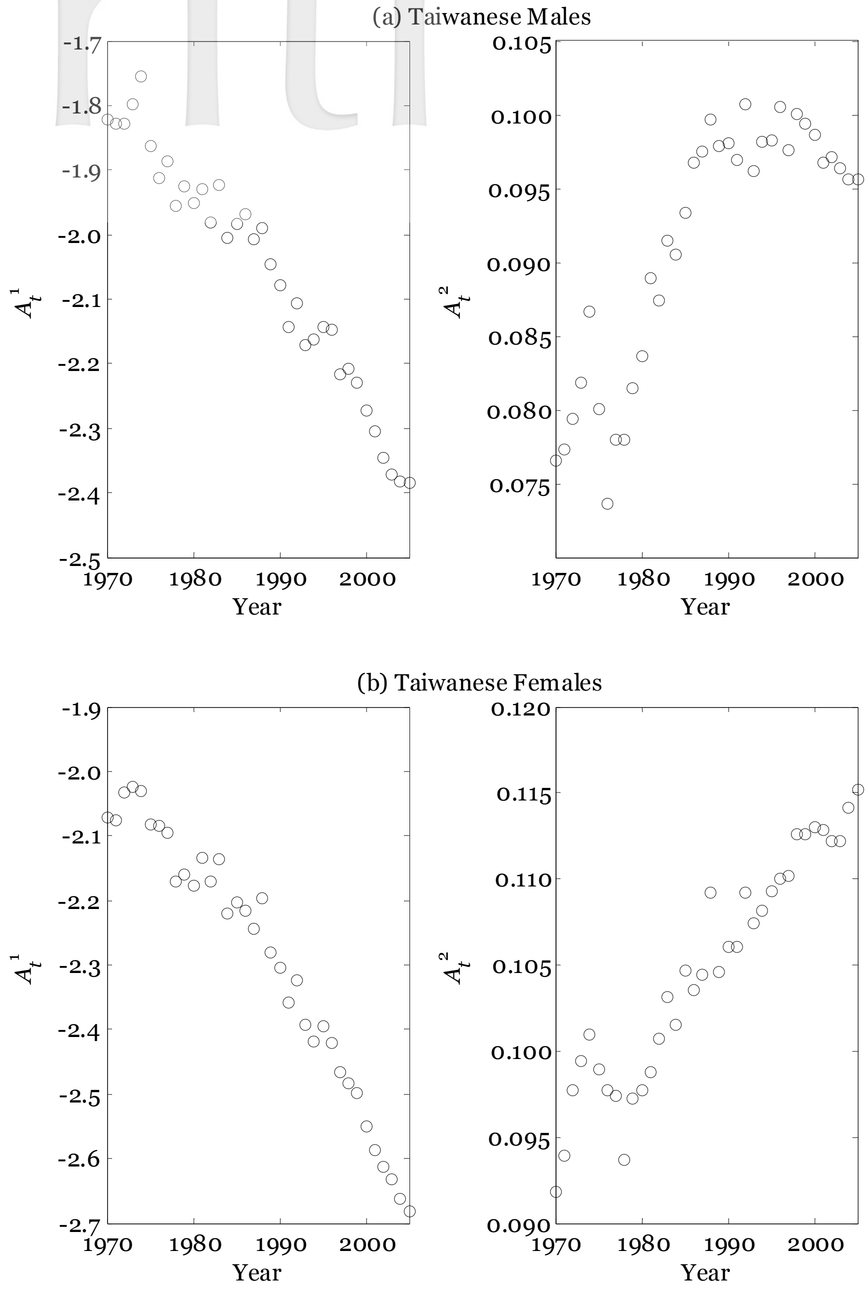

Figure 3. Parameter Estimates for the CBD Model (Taiwanese 65+)

the period effect for a single population under the LC (Renshaw and Haberman (2003) and Koissi, Shapiro, and Högnäs (2006)) and the CBD model (Cairns, Blake, and Dowd (2006) and Cairns et al. (2009)). We also use the ARIMA process to model the period effects in two models.[5] For a comparison, we

5 In the CBD model, we follow Cairns, Blake, and Dowd (2006) to use the Cholesky decomposition to reflect the correlation between the A_t^1 and A_t^2 forecasting processes.

evaluate the fitting results of the ARIMA process by the AIC and BIC, and we used the BIC as the main criterion for model selection in order to forecast future mortality. The results are shown in Table III and Table IV. Table III shows that ARIMA(0,1,0) is selected to model the period effects (κ_t) because the smaller the standard of the AIC and BIC, the better. Table IV shows A_t^1 and A_t^2 fitting results. In this table, there are difference results between the AIC and BIC. The standard of our model selection is the BIC because the BIC takes the number of parameters in the estimation data set into account, whereas the AIC does not. According to the results, we find that ARIMA(0,1,0) is better for A_t^2, ARIMA(0,1,1) is better for A_t^1 male, and ARIMA(1,1,0) is better for A_t^1 female.

Table III
Fitting Results of Period Effects in the LC Model

The numbers in bold indicate a better fit.

LC	AIC		BIC	
	κ_t		κ_t	
ARIMA	Male	Female	Male	Female
(0,1,0)	**-77.3**	**-36.7**	**-74.5**	**-33.9**
(1,1,0)	-77.1	-34.8	-72.9	-30.6
(0,1,1)	-76.9	-34.8	-72.7	-30.5
(1,1,1)	-75.1	-33.2	-69.5	-27.6

Table IV
Fitting Results of Period Effects in the CBD Model

The numbers in bold indicate a better fit.

CBD	AIC				BIC			
	A_t^1		A_t^2		A_t^1		A_t^2	
ARIMA	Male	Female	Male	Female	Male	Female	Male	Female
(0,1,0)	-122.4	-131.5	**-306.7**	-331.8	-119.3	-128.4	**-303.6**	**-328.7**
(1,1,0)	-126.7	**-134.9**	-305.6	**-333.0**	-122.0	**-130.2**	-300.9	-328.4
(0,1,1)	**-127.1**	-134.1	-305.5	-332.7	**-122.4**	-129.4	-300.9	-328.0
(1,1,1)	-125.2	-133.0	-304.5	-331.1	-119.0	-126.8	-298.2	-324.9

In addition to the AIC and BIC, we also use the MAPE (Mean absolute percentage error)[6] to measure the forecasting accuracy. We use the best fitted model for the training data to predict the mortality rates for the testing data. The MAPE is used to measure the forecasting accuracy and the results are in Table V.

[6] MAPE $= \frac{1}{N}\sum_{i=1}^{n}\frac{|Y_i-\hat{Y}_i|}{Y_i}\times 100\%$, where Y_i and $\hat{Y}_i$ are the actual values and estimated (or predicted) values of mortality.

In the LC model, the MAPE for males and females is 13.30% and 6.55%, respectively. The results are consistent with in Table II (training data), the LC model shows better prediction accuracy than the CBD model.

Table V
MAPE Comparison for Forecasting the Elderly Mortality (Ages 65-99)

The numbers in bold indicate a better fit.

Model	Male	Female
LC	**13.30%**	**6.55%**
CBD	13.49%	13.32%

B. Estimation of Parameters in Financial Models

We evaluate a portfolio with three assets: stocks, bonds, and life settlements. To reflect the dynamics of bond and stock prices, we use the GBM to simulate stock dynamic process and the CIR interest rate model to demonstrate the bond dynamic process. The Taiwan Stock Exchange (TAIEX) data from the Taiwan Economic Journal (TEJ) database are used as proxy for the Taiwan equity market, and the 10-year government bond from the Central Bank of the Republic of China (Taiwan) is applied as proxy for the Taiwan bond market. To be consistent with the market condition, we use the period of 1996 to 2010 for parameters estimation.[7] The estimates of parameter values in the financial models are shown in Table VI.

Table VI
Parameter Values in the Financial Models

The parameter values are measured annually.

Model	Notation	Definition	Parameter Value
GBM	μ	Mean of Stock	0.037
	σ_s	Volatility of Stock	0.325
CIR	a	Mean Reverting Rate	0.111
	b	Mean Reverting Level	0.007
	σ_r	Volatility of Interest Rate	0.029
	$r(0)$	Initial Short Rate	3.560%
Correlation	ρ	Correlation between Stock and Bond	-0.106

7 The equity data are from the TEJ and the bond data are from the Central Bank of the Republic of China (Taiwan). There are three steps of parameters estimation as follows. First, we calculate the historical correlation of the TAIEX and Taiwan 10-year government bond yield. Second, we use the historical TAIEX data to calculate the mean and standard deviation, the two parameters of the GBM. Third, we can direct the noncentral χ^2 probability density function to be an implement to solve the MLE and get parameters of the CIR model.

V. Analysis of Efficient Frontier for Life Settlements

A. Assumption and Price of Life Settlements

We analyze the efficient frontier for life settlements by using simulations. To conduct the simulation study, we demonstrate with a whole life insurance contract with death benefits of USD100,000 for a person aged x_1 and take life settlement transactions at age x_2. The premiums for the insurance policy are paid at the beginning of each year for 20 years. Since the issuing age of life policies, age, and market condition at life settlement transactions can affect the efficient frontier, we demonstrate with two cases in the following analysis. Case 1 is the base case in the following analysis, assuming that a woman purchased a whole life insurance policy at age 50 in 1995 and took the life settlement transaction at age 65, i.e., in 2010. In contrast, Case 2 considers a younger female policyholder aged 55 taking a life settlement at an older age of 70. The details of these two cases are shown in Table VII.

Table VII
Two Cases of Life Settlement Transactions

The market conditions depend on the year of life settlement transactions. The insurance premium is calculated based on the Illustrative Life Table in 1995 and a constant interest rate of 0.0679, referring to the 10-year Taiwan government bond yield in 1995.

Case	Issuing Age of the Insurance Policy (x_1)	Age at Life Settlement Transaction (x_2)	# of Nonpayment Premiums	Annual Premium (P)
Case 1	50	65	5	1,574.5
Case 2	55	70	5	2,380.5

When the investor takes life settlement transactions, the life settlement company needs to determine the economic value of the life insurance first. The price of the life settlement is calculated according to the pricing framework in Equation (1), and the pricing assumptions are based on the market condition in the transaction year. Under the illustrative two cases, the transactions of life settlements were taken in the year 2010. The details of corresponding yield curves are described in Appendix B. The mortality dynamics follow the LC and CBD models, and the corresponding parameter values of the mortality dynamics are presented in Figures 2 and 3. In addition, we conduct 20,000 simulations.

The prices for the illustrated two cases of life settlement transactions are shown in Table VIII. Comparing the values of the life settlements in Case 1 and Case 2, we find that the value of the life settlement in Case 1 is higher than that in Case 2 because the life expectancy of the policyholder after the life settlement transaction in Case 1 is higher than that in Case 2, which are expected to be 20 and 15 years respectively. Under the same market condition, the longer life expectancy of the insured, the lower return for investing in life settlements. Life expectancy is critical to the price of life settlements.

In addition, we find that the values of life settlements based on the LC model are a bit higher than that based on the CBD model because the life expectancy estimated by the CBD model is longer than that by the LC model. However, according to the goodness fit of the mortality model, the LC model in both in-sample and out-of-sample forecasts performs better than the CBD model does. If we select the CBD model to be the mortality pricing model, it should make life settlements undervalued. Thus, selecting a proper mortality model to capture the longevity risk is very important in pricing life settlements.

Table VIII
Numerical Results of Life Insurance Products

Case 1: $(x_1, x_2) = (50, 65)$; Case 2: $(x_1, x_2) = (55, 70)$.

Mortality Dynamics	Case	Life Expectancy of Policyholders in Transaction Year	Value of Life Settlements
LC Model	Case 1	19.79	66,831
	Case 2	15.09	66,306
CBD Model	Case 1	19.81	66,682
	Case 2	15.11	66,239

B. Efficient Frontier and Sharpe Ratio

We consider a portfolio with three assets, stocks, bonds, and life settlements.[8] To evaluate the portfolio performance, we first simulate the return, risk, and Sharpe ratio for bonds, stocks, and life settlements, and the corresponding results are shown in Table IX and Table X. Intuitively, equity has the highest return and risk. The return and volatility of bonds are around 1.78% and 9.71%, respectively. The return and volatility for life settlements are around 2.02% to 2.75% and 1.84% to 2.61% in different cases. Remarkably, life settlements have a higher Sharpe ratio than stocks and bonds do in all cases. Although our analysis is based on the mortality experience and financial market in Taiwan, it is consistent with the finding in the U.S. market by Braun, Gatzert, and Schmeiser (2012) that the Sharpe ratio of life settlements is higher than that of bonds and stocks.[9] That's why the investor has great interests in life settlements and the market is growing in the capital market.

8 We assume that life settlements are zero correlated with stocks and bonds; the correlation between stocks and bonds is -0.106.

9 Braun, Gatzert, and Schmeiser (2012) point out that the annualized return and volatility for the life settlement fund index in the U.S. market during December 2003 to June 2010 is 4.85% and 2.28% respectively. Based on the comparison of our results with Braun, Gatzert, and Schmeiser's (2012), the return of life settlements in Taiwan is lower than that in the U.S., and the risk in both markets are similar. The possible reason for the return gap is that the mortality improvement rate in Taiwan is higher than that in the U.S.

Table IX
Simulated Return, Volatility, and Sharpe Ratio of Stocks and Bonds

Asset	Return	Volatility	Sharpe Ratio
Stock	3.49%	32.07%	0.066
Bond	1.78%	9.71%	0.044

Table X
Simulated Return, Volatility, and Sharpe Ratio of Life Settlements

Mortality Dynamics	Case	Return	Volatility	Sharpe Ratio
LC Model	Case 1	2.02%	2.04%	0.316
	Case 2	2.74%	1.84%	0.744
CBD Model	Case 1	2.03%	2.61%	0.252
	Case 2	2.75%	2.31%	0.597

We evaluate the efficient frontier of the portfolio in Figure 4. To analyze how life settlements influence the efficient frontier, we consider four different investment strategies based on the proportion invested in life settlements. Strategy AA1 is the original portfolio, including only equity and bond, and we count it as our benchmark, i.e., 0% invested in life settlements. Strategy AA2 contains 30% in life settlements and 70% in bonds and stocks; Strategy AA3 contains 50% in life settlements and 50% in bonds and stocks; Strategy AA4 contains 80% in life settlements and 20% in bonds and stocks.[10] In addition, to understand the effect of longevity risk, we present the efficient frontier for both the LC and CBD models. We can compare the efficient frontier for possible investment sets and different investment strategies in Figure 4. The efficient frontiers give the following patterns. First, different investment strategies shift the efficient frontier. The more invested in life settlements, the more upward shifting for the efficient frontier. It implies that the more invested in life settlements, the better the performance of the portfolio. Thus, the efficient frontiers for Strategies AA2, AA3, and AA4 are all better than that for Strategy AA1. For instance, in Case 1 based on the LC model, in order to earn a 2.00% return, Strategy AA1 should take a 9.50% risk, but Strategy AA2 takes a 6.67% risk, Strategy AA3 takes a 4.85% risk, and Strategy AA4 only takes a 2.43% risk. In other words, AA4 investing 80% in life settlements outperforms other investment strategies. This could be explained by the fact shown in Tables IX and X that the Sharpe ratio of life settlements is higher than those of bonds and

[10] The total proportions invested in stocks and bonds are the remaining proportion after being invested in life settlements, i.e., 100%, the proportion of life settlements. We then distribute the remaining proportion across stocks and bonds in all possible mixes on a 10% basis. For instance, Strategy AA3 contains 50% in life settlements and the possible proportion mixes in (Stocks, Bond) of Strategy AA3 are (50%, 0), (40%, 10%), (30%, 20%), (20%, 30%), (10%, 40%), and (0%, 50%).

stocks according the current market condition.[11] In addition, the risk of life settlements resulting from the time of death of the policyholder does not fluctuate with the financial market. The investor can diversify the portfolio risks with life settlements.

Second, the efficient frontiers based on the CBD model in general are a bit more downward than that based on the LC model because the longevity risk measured in the CBD model is greater than that in the LC model. The first finding is in line with the empirical study in Braun, Gatzert, and Schmeiser (2012) that life settlement funds offer attractive returns paired with low volatility. However, the second finding regarding longevity risk cannot be depicted in the empirical analysis.

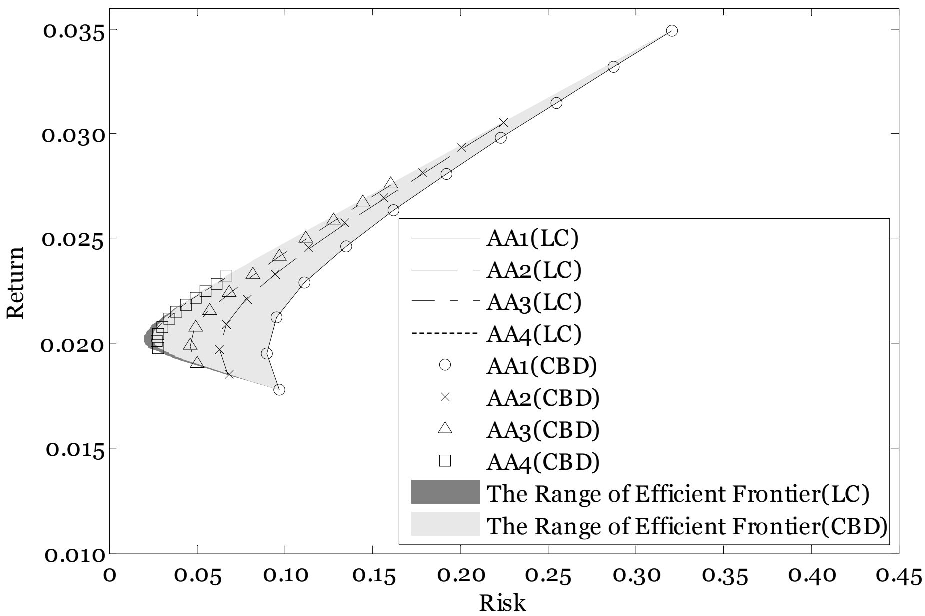

Figure 4. Ranges of Efficient Frontiers and Efficient Frontiers of Four Investment Strategies: The LC vs. CBD Models (Case 1)[12]

Strategy AA1 has no life settlements. Strategy AA2 contains 30% in life settlements. Strategy AA3 contains 50% in life settlements. Strategy AA4 contains 80% in life settlements.

We carry out the sensitivity analysis in the following. To simplify our analysis, we illustrate with Strategy AA4 based on the LC model only. We first investigate the impacts of the age effect of life settlement transactions on investment performance by comparing the efficient frontiers for the life settlement

---

[11] The calculation of the Sharpe ratio for life settlements depends on the mortality experience and the financial market. Such an effect compared with bonds and stocks could be changed a bit if the market condition is different.

[12] We only present the efficient frontiers for Case 1 in Figure 4 because the results of Case 2 are similar to those of Case 1. Thus, we do not repeat the analysis.

transaction in Case 1 and in Case 2 as shown in Figure 5. The efficient frontier has a more upward shape in Case 2. For instance, given the volatility is equal to 5%, the return is 2.22% for Case 1 and 2.80% for Case 2. In other words, the Sharpe ratio for Case 2 is greater than that for Case 1. Note that the life settlement transaction in Case 2 is expected to have a shorter duration, i.e., life expectancy is shorter. Therefore, the investor in life settlements is expected to receive the payment of death benefits earlier in Case 2 and benefit from the early death of the policyholder.

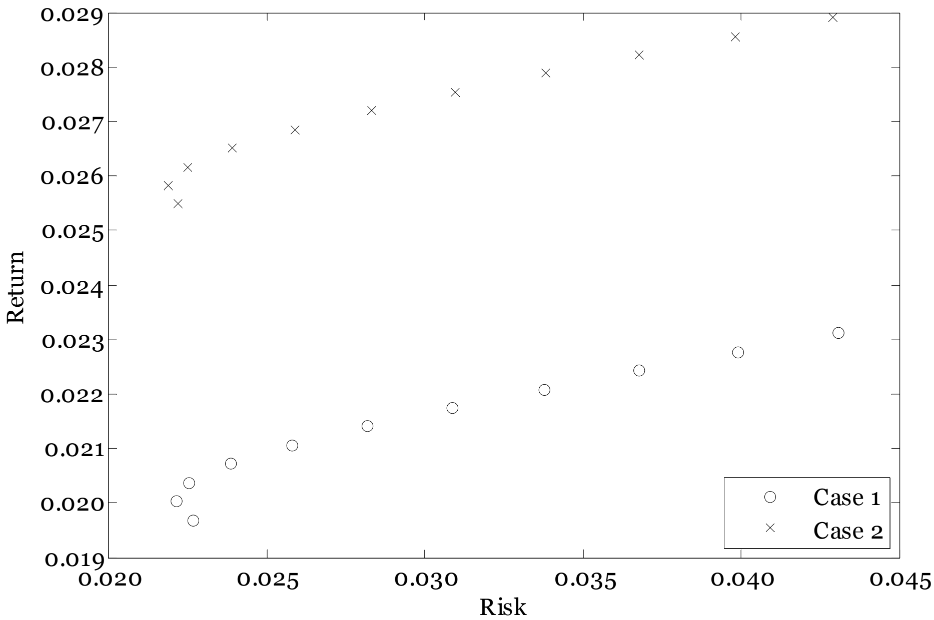

Figure 5. Efficient Frontiers for Different Ages at Insurance and Life Settlements

Case 1: $(x_1, x_2) = (50, 65)$; Case 2: $(x_1, x_2) = (55, 70)$.

Since mortality plays an important role in determining the return and risk for life settlements, we further study the impact of mortality improvement on the investment performance by conducting the sensitive analysis on the trend of mortality improvement. Recall that the mortality time trend is captured by κ_t in the LC model. To reflect the impact of mortality improvement, we set the mortality improvement in the range from $0.5\kappa_t$ to $1.5\kappa_t$. The corresponding values of return, volatility, and the Sharpe ratio of life settlements are shown in Table XI. It is very intuitive that the shorter life expectancy of the policyholders in the transaction year causes a higher value of life settlements, a bit lower return, and lower volatility on life settlements. Moreover, the impact of mortality improvement on life settlements is significant because it decreases the Sharpe ratio obviously when the life expectancy increases. Therefore, if investors ignores future mortality improvement on life settlements, they could overvalue the return and underestimate the risk on life settlement transactions.

Table XI
Impact of Mortality Improvement on Life Settlements (Case 1)

Mortality Time Trend	Life Expectancy of Policyholders in Transaction Year	Value of Life Settlements	Return	Volatility	Sharpe Ratio
$0.5\kappa_t$	17.23	71,602	1.97%	0.76%	0.786
$0.8\kappa_t$	18.81	68,866	1.97%	1.45%	0.410
$\boldsymbol{\kappa_t}$	**19.81**	**68,976**	**2.02%**	**2.04%**	**0.316**
$1.3\kappa_t$	21.22	63,464	2.17%	3.09%	0.258
$1.5\kappa_t$	22.09	61,080	2.24%	3.95%	0.221

We further investigate the effect of mortality improvement on the portfolio. Figure 6 shows the efficient frontiers of the portfolio for different κ_t. It gives an interesting effect that efficient frontiers do not shift upward in a consistent pattern as the mortality improvement increases. For instance, when the risk of the portfolio is greater than 4.00%, the efficient frontier in the time trend of $1.5\kappa_t$ gives the most upward pattern than those in other situations. However, it does not apply when the risk of the portfolio is smaller, such as 2.50%, because the increases in risk and return of life settlements are not at the same extent. Thus, the effects of mortality improvement on risk and return of the portfolio may not be consistent, depending on the proportion of life settlements in the portfolio. Taking more longevity risks can in general result in a better investment performance for more aggressive investors.

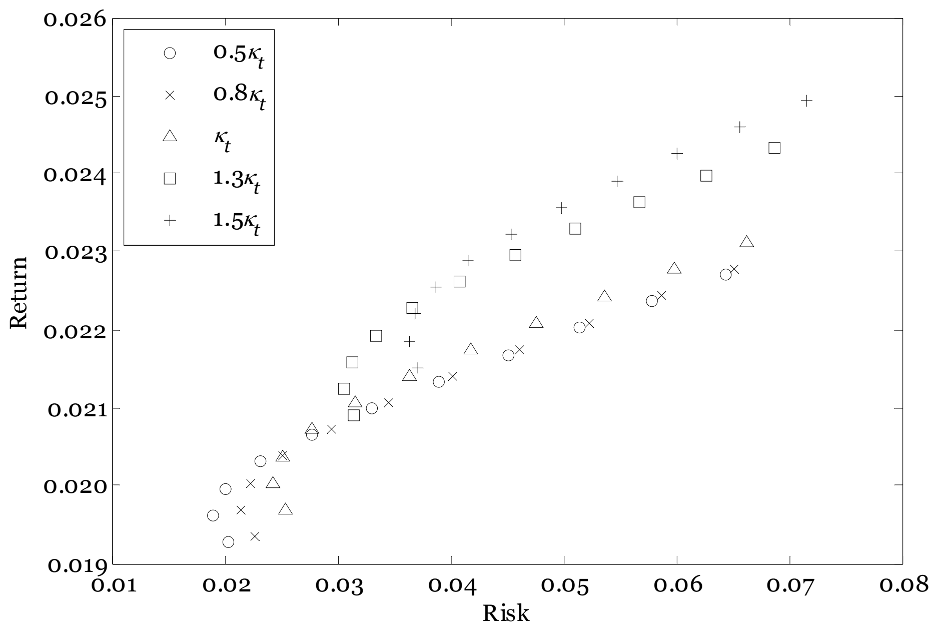

Figure 6. Efficient Frontiers for Different κ_t (Case 1)

Next, we consider the impacts of transaction costs on life settlements and investment performance. We assume the transaction costs to be 0.30% and 0.60%, respectively. The impacts of transaction costs on the return, volatility, and Sharpe ratio of life settlements are shown in Table XII. Taking into account transaction costs would reduce the returns on life settlements. As a result, a larger transaction cost causes a lower Sharpe ratio on life settlements.

Moreover, Figure 7 shows the efficient frontier of the portfolio when transaction costs of life settlements are taken into consideration. Higher transaction costs of life settlements provide a worse efficient frontier. The transaction cost is a factor to influence the performance of the portfolio. Ignoring the transaction would overestimate the investment performance of the portfolio.

Table XII
Impacts of Transaction Costs on Life Settlements (Case 1)

Transaction Cost	Return	Volatility	Sharpe Ratio
0.00%	2.02%	2.04%	0.316
0.30%	1.72%	2.04%	0.169
0.60%	1.42%	2.04%	0.026

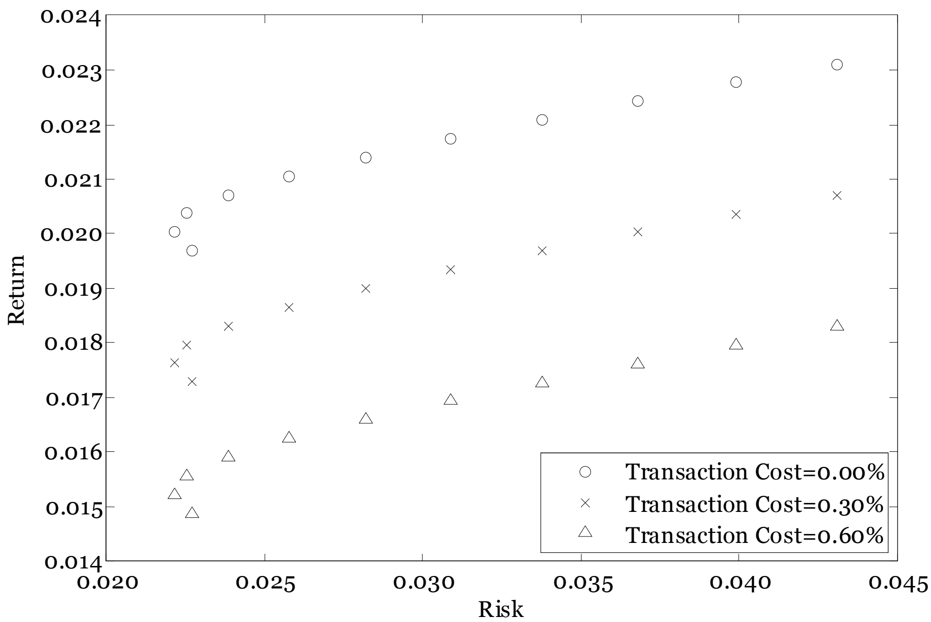

Figure 7. Efficient Frontiers for Different Transaction Costs (Case 1)

VI. Conclusion

Life settlements have been developed in the financial market in recent years. Some studies have shown that life settlements offer attractive returns paired with low volatility and are virtually uncorrelated with other asset classes. Therefore, life settlements have attracted the investor's attention, especially after the financial crisis in 2008. However, longevity risk can affect the investment performance of life settlements. As life expectancy is increasing dramatically, longevity risk shall be taken into account when the investment performance of life settlements is analyzed. This research provides a pricing framework for life settlements considering the stochastic mortality model. For comparison purposes, we employ both the LC and CBD models to capture the future dynamics of mortality rates and use the empirical mortality experience to access the goodness fit of these two models. To the best of our knowledge, this paper is the first to conduct a simulation study to evaluate the investment performance of life settlements in the presence of longevity risk.

We utilize the efficient frontier to assess the investment performance of a portfolio that includes three assets, namely, stocks, bonds, and life settlements. Our results show that portfolios with life settlements have better investment performance. In addition, we also investigate the factors, such as the transaction cost and the trend of mortality improvement that could reduce the investment performance of life settlements. A larger transaction cost and higher mortality improvement cause a lower Sharpe ratio on life settlements. Therefore, if we ignore such factors while pricing life settlements, we would overvalue the investment performance of life settlements. However, taking more longevity risks could result in an upward efficient frontier, especially for aggressive investors. The findings of this research can fill the gap that the empirical study cannot deal with.

In our analysis, we assume that there is no correlation between life settlements and financial assets, such as bonds and stocks. However, the actual correlation between life settlements and bonds needs to be further examined because the principal effect for bond price is the interest rate, and the price of life settlements can also be influenced by the interest rate. It is worth finding more empirical evidence for life settlements.

Appendix A. Parameter Estimation

Brillinger (1986) hypothesizes that the number of death complies with the Poisson distribution $D_{x,t} \sim \text{Poisson}(\lambda_{x,t})$, and the mean is equal to the number of death with the parameter as

$$\lambda_{x,t} = m_{x,t} E_{x,t},$$

where $E_{x,t}$ denotes the exposure-to-risk at age x at time t.

In this research, we use the MLE method to estimate the parameters. The parameters estimation method of the log-likelihood function is described as follows,

$$L(\theta) = \sum_{x,t} [D_{x,t} \ln(\lambda_{x,t}) - \lambda_{x,t} - \ln(D_{x,t}!)], \tag{A1}$$

where θ is the parameters of the model. In the LC model, $\theta \equiv (a_x, b_x, \kappa_t)$; In the CBD model, $\theta \equiv (A_t^1, A_t^2)$.

The parameters of the log-linear model in Equation (A1) can be solved by the Newton method.

Appendix B. Market Condition

The first big change is the interest rate. After the financial crash, the interest rate is no longer high, and the yield curve by the CIR model in 2010 (initial short rate is equal to 1.37%) is showed in Figure B1. Moreover, the yield is an invert yield curve in 2010.

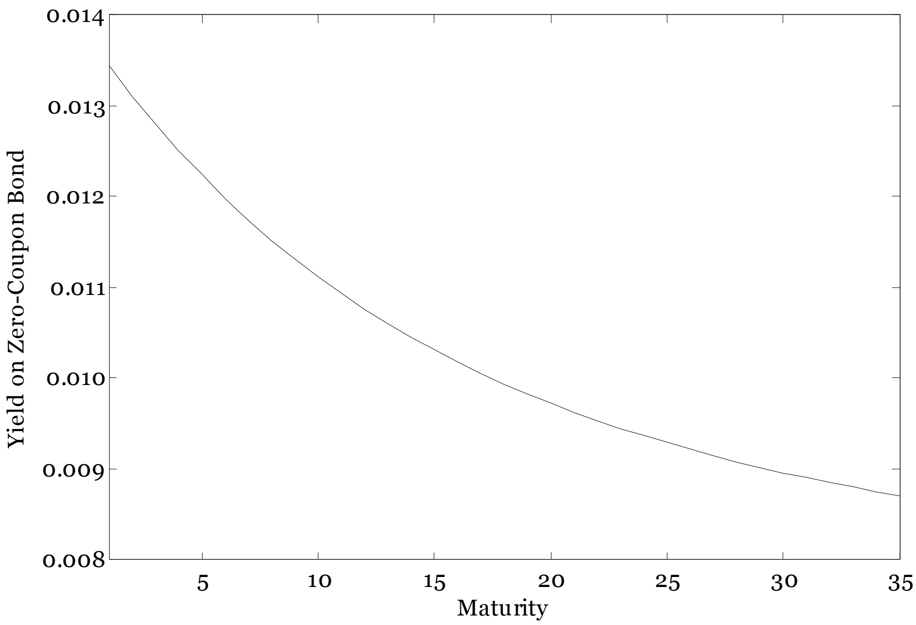

Figure B1. Yield Curve

REFERENCES

American Council of Life Insurers, 2008, *Life Insurers Fact Book 2008 No. GR08-108*.

Berketi, Alexandra K., 1999, Insolvency risk and its impact on the policyholders' investment choices: A mean-variance approach for participating life insurance business in U.K., *Insurance: Mathematics and Economics* 25, 349-372.

Braun, Alexander, Nadine Gatzert, and Hato Schmeiser, 2012, Performance and risks of open-end life settlement funds, *Journal of Risk and Insurance* 79, 193-230.

Brillinger, David R., 1986, A biometrics invited paper with discussion: The natural variability of vital rates and associated statistics, *Biometrics* 42, 693-734.

Cairns, Andrew J. G., David Blake, and Kevin Dowd, 2006, A two-factor model for stochastic mortality with parameter uncertainty: Theory and calibration, *Journal of Risk and Insurance* 73, 687-718.

Cairns, Andrew J. G., David Blake, Kevin Dowd, Guy D. Coughlan, David Epstein, Alen Ong, and Igor Balevich, 2009, A quantitative comparison of stochastic mortality models using data from England and Wales and the United States, *North American Actuarial Journal* 13, 1-35.

Cox, John C., Jonathan E. Ingersoll, and Stephen A. Ross, 1985, A theory of the term structure of interest rates, *Econometrica* 53, 385-407.

Denuit, Michel, Pierre Devolder, and Anne-Cécile Goderniaux, 2007, Securitization of longevity risk: Pricing survivor bonds with Wang transform in the Lee-Carter framework, *Journal of Risk and Insurance* 74, 87-113.

Doherty, Neil A., and Hal J. Singer, 2003a, The benefits of a secondary market for life insurance policies, *Real Property, Probate, and Trusts Journal* 38, 449-478.

Doherty, Neil A., and Hal J. Singer, 2003b, Regulating the secondary market for life insurance policies, *Journal of Insurance Regulation* 21, 63-99.

Dorr, David C., 2008, Extending the efficient frontier through life settlements, *Journal of Structured Finance* 14, 59-63.

Dowd, Kevin, David Blake, Andrew J. G. Cairns, and Paul Dawson, 2006, Survivor swaps, *Journal of Risk and Insurance* 73, 1-17.

Erkmen, Bilkan, 2011, On life settlement pricing, *Michigan Journal of Business* 4, 71-106.

Gardner, Randy, Julie Welch, and Neil Covert, 2009, Tax aspects of life settlement arrangements, *Journal of Financial Planning* 22, 32-34.

Gatzert, Nadine, 2010, The secondary market for life insurance in the United Kingdom, Germany, and the United States: Comparison and overview, *Risk Management and Insurance Review* 13, 279-301.

Gatzert, Nadine, Gudrun Hoermann, and Hato Schmeiser, 2009, The impact of the secondary market on life insurers' surrender profits, *Journal of Risk and Insurance* 76, 887-908.

Giacalone, Joseph A., 2001, Analyzing an emerging industry: Viatical transactions and the secondary market for life insurance policies, *Southern Business Review* 27, 1-7.

Hsieh, Ming-Hua, Jin-Lung Peng, Chenghsien Tsai, and Jennifer L. Wang, 2012, Explaining the rate spreads of life settlements, Working paper.

Kamath, Suneet, and Timothy Sledge, 2005, Life insurance long view - Life settlements need not be unsettling, *Bernstein Research Call* (Sanford C. Bernstein and Co., LLC, New York).

Katt, Peter C., 2008, Insurance: A life settlement mosaic, *Journal of Financial Planning* 21, 34-35.

Kohli, Sachin, 2006, Pricing death: Analyzing the secondary market for life insurance policies and its regulatory environment, *Buffalo Law Review* 54, 279-320.

Koissi, Marie-Claire, Arnold F. Shapiro, and Göran Högnäs, 2006, Evaluating and extending the Lee-Carter model for mortality forecasting: Bootstrap confidence interval, *Insurance: Mathematics and Economics* 38, 1-20.

Lee, Ronald D., and Lawrence R. Carter, 1992, Modeling and forecasting U.S. mortality, *Journal of the American Statistical Association* 87, 659-671.

Liao, Hsien-Hsing, Sharon S. Yang, and Yi-Hsin Huang, 2007, The design of securitization for longevity risk: Pricing under stochastic mortality model with tranche technique, Working paper.

Lin, Yijia, and Samuel H. Cox, 2005, Securitization of mortality risks in life annuities, *Journal of Risk and Insurance* 72, 227-252.

Lubovich, Jandra, Jon Sabes, and Paul Siegert, 2008, Introduction to methodologies used to price life insurance policies in life settlement transactions, Working paper.

Markowitz, Harry, 1952, Portfolio selection, *Journal of Finance* 7, 77-91.

Mason, Joseph R., and Hal J. Singer, 2008, A real options approach to valuing life settlements transactions, *Journal of Financial Transformation* 23, 61-68.

National Conference of Insurance Legislators, 2007, *Life Settlements Model Act*.

Perera, Nemo, and Brian Reeves, 2006, Risk mitigation for life settlements, *Journal of Structured Finance* 12, 55-60.

Renshaw, Arthur E., and Steven Haberman, 2003, Lee-Carter mortality forecasting with age-specific enhancement, *Insurance: Mathematics and Economics* 33, 255-272.

Rosenfeld, Sam, 2009, Life settlements: Signposts to a principal asset class, Working paper.

Securities Industry and Financial Markets Association, 2009, U.S. bond market outstanding, http://www.sifma.org/research/research.aspx?ID=10806 (last visited on December 13, 2009).

Seitel, Craig L., 2006, Inside the life settlement industry: An institutional investor's perspective, *Journal of Structured Finance* 12, 38-40.

Seitel, Craig L., 2007, Inside the life settlement industry: A provider's reflections on the challenges and opportunities, *Journal of Structured Finance* 13, 70-75.

Smith, Brian B., and Stephen L. Washington, 2006, Acquiring life insurance portfolios: Diversifying and minimizing risk, *Journal of Structured Finance* 12, 41-45.

Stone, Charles A., and Anne Zissu, 2007, The return on a pool of senior life settlements, *Journal of Structured Finance* 13, 62-69.

Wang, Jennifer L., Ming-Hua Hsieh, and Chenghsien Tsai, 2012, Using life settlements to hedge the mortality risk of life insurers: An asset-liability management approach, Working paper.

Wang, Shaun S., 2000, A class of distortion operators for pricing financial and insurance risks, *Journal of Risk and Insurance* 67, 15-36.

Yang, Sharon S., and Chou-Wen Wang, 2013, Pricing and securitization of multi-country longevity risk with mortality dependence, *Insurance: Mathematics and Economics* 52, 157-169.

考慮長壽風險下保單貼現商品對效率前緣之影響

楊曉文*
國立中央大學財務金融學系

葉俞昀
國立中央大學財務金融學系

摘要

保單貼現商品具有與傳統投資工具相關性低的特性，為近年來資本市場相當重視的投資工具。本研究以效率前緣來分析投資組合中加入保單貼現商品的投資效益。由於老年保單貼現商品價格主要受死亡率影響，故本研究藉由隨機死亡率模型來建構保單貼現商品的定價，亦探討年齡、死亡率的改善及交易成本對投資組合表現的影響。

關鍵詞：保單貼現、效率前緣

* 通訊作者：楊曉文，E-mail: syang@ncu.edu.tw，國立中央大學財務金融學系教授，32001 桃園縣中壢市中大路 300 號，電話：(886) 3-4227151，傳真：(886) 3-4226062。
作者感謝科技部之研究補助 (NSC 99-2410-H-008-019-MY3)。