The Valuation of Temperature Derivatives: The Case for Taiwan

Chuang-Chang Chang*
Department of Finance, National Central University

Sharon S. Yang**
Department of Finance, National Central University

Tzu-Yu Huang***
Department of Finance, National Central University

Jr-Wei Huang****
Department of Insurance, Hubei University of Economics

Abstract

This research focuses on the temperature risk and attempts to investigate which distribution is most appropriate for capturing the Taiwan's temperature dynamics. We adopt the Campbell and Diebold (2005) model to describe the temperature characteristics and examine a variety of distributions. We find that the standard Gumbel distribution provides the best fit for both in-sample and out-of-sample performance. Further, we extend Cao and Wei's (2004) approach to obtain the valuation framework for HDD and CDD contracts. Finally, we observe that the effects of different distributions on the value of the temperature derivatives are very significant.

Key words: Temperature derivatives, equilibrium pricing model, HDD, CDD

I. Introduction

Weather is an essential production factor but, at the same time, it is also the most important and least controllable source of risk in agriculture, the power industry, and retail business (Cao and Wei (2004) and Musshoff, Hirschauer, and Odening (2008)). There is scarcely a year without adverse weather events leading to economic losses in production in the different regions of the world (Bates et al. (2008) and Lobell, Ortiz-Monasterio, and Falcon (2007)). For instance, the U.S. suffered economic losses of USD8.4 billion due to severe weather, while flooding

* Chuang-Chang Chang, Professor of Department of Finance, National Central University, Taoyuan, Taiwan.
** Sharon S. Yang, Professor of Department of Finance, National Central University, Taoyuan, Taiwan.
*** Tzu-Yu Huang, Master's Student of Department of Finance, National Central University, Taoyuan, Taiwan.
**** Corresponding author: Jr-Wei Huang, E-mail: jrwei.huang@icloud.com, Associate Professor of Department of Insurance, Hubei University of Economics, No. 8, Yangqiaohu Rd., Jiangxia Dist., Wuhan County, Hubei Province 430205, China, TEL: (86) 155-2715-6007.

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in central Europe in May and June caused losses amounting to USD22 billion in 2013. On the other hand, New Zealand suffered economic losses of over USD20 million in September alone and a drought in China between January and August resulted in losses estimated at USD10 billion in 2013 (Impact Forecasting (2013)).

To hedge against these uncertainties brought about by fickle weather conditions, weather derivatives have been developed on the Chicago Mercantile Exchange (CME) since 1999. Utilities and many other businesses in which weather has a major impact on revenues, such as energy producers and consumers, the leisure industry, the insurance industry, and the agricultural sector, could all be the users of weather derivatives. However, it is primarily the energy sector that has driven the demand for weather derivatives. To date, weather derivative dealing has been structured to cover almost any types of weather variables (temperature, rainfall, snowfall, wind speed, and humidity). However, it is estimated that 98% to 99% of weather derivatives trading is based on temperature-based indices, such as heating degree days (HDD) or cooling degree days (CDD) defined on the basis of average daily temperatures according to the CME[1] (Zapranis and Alexandridis (2009)).

In recent years, the abnormal climate has caused the market for weather derivatives to grow steadily. Yang, Li, and Wen (2011) observe that the notional value of weather derivatives contracts in 2006 and 2007 was USD19 billion, and this amount grew to USD32 billion in 2007 and 2008. It is shown that the weather derivatives have gained increasing popularity in recent years as a tool to manage weather risk. Although weather derivatives have become more and more important, there is not yet a standardized and effective valuation model for weather derivatives. For example, in pricing the most traded volume of temperature derivatives, a number of considerations make pricing temperature derivatives more difficult than pricing traditional derivatives (Alaton, Djehiche, and Stillberger (2002), Brody, Syroka, and Zervos (2002), Cao and Wei (2004), Campbell and Diebold (2005), and Huang, Shiu, and Lin (2008)). First, the underlying temperature indices are not tradable; we cannot use the traditional arbitrage-free pricing method to value temperature derivatives. Second, although the liquidity of temperature derivative markets has improved, the degrees of temperature derivative markets completeness are less than those of traditional derivative markets completeness, which is also why the classic Black-Scholes-Merton methodology cannot be directly applied. Therefore, prior researchers have confronted the challenge of developing a forecasting model that can be integrated into the options pricing framework, while providing accurate estimates and forecasts.

1 An HDD is the number of degrees by which the daily temperature is below the base temperature, while a CDD is the number of degrees by which the daily temperature is above the base temperature. The base temperature is usually 65°F in the U.S. and 18°F in Europe and Japan. HDDs and CDDs are usually accumulated over a month or over a season.

There is a variety of temperature models proposed in the literature. Dischel (1998) uses a simple stochastic Brownian motion for forecasting temperature. Considine (2000) fits the distribution of HDD historical values and evaluate options by multiplying the payout of the option by the product of the probability distribution. Alaton, Djehiche, and Stillberger (2002) construct a temperature model, which takes into account temperature trends, seasonality, and mean reversion. Furthermore, they derive the closed-form pricing formula for the weather derivative using the Gaussianity property of the underlying distribution of the temperature process. Huang, Shiu, and Lin (2008) study Alaton, Djehiche, and Stillberger (2002) to consider the clustering of volatility by incorporating a GARCH process into the temperature. They show that for HDD and CDD the call price is higher under the ARCH effects variance than under the fixed variance. Brody, Syroka, and Zervos (2002) observe that temperature dynamics exhibit long-range temporal dependencies and apply a fractional Brownian motion to drive an Ornstein-Uhlenbeck process. Cao and Wei (2004) point out some important features that should be considered in modeling the future dynamics of temperature, which include (i) seasonal variation patterns, (ii) mean reversion characteristics, (iii) the autoregressive property in temperature change, (iv) a larger variation in daily temperature in the winter than in the summer, and (v) global warming. Furthermore, Campbell and Diebold (2005) employ a time-series approach to capture and forecast daily average temperature features in line with those of Cao and Wei (2004) and they further consider not only the conditional mean but also the conditional variance in daily temperature behavior in constructing the future distribution of the underlying indices. The above studies on the temperature models are based on the general assumption of the normal or lognormal distribution in terms of capturing the weather uncertainty for obtaining weather derivative pricing formulas. However, it is a fact that not all weather uncertainty can be captured by the Gaussian distribution. For instance, Stern and Coe (1984) analyze rainfall data from more than 15 countries and use non-stationary Markov chains to describe the occurrence of rain and gamma distributions to depict the rainfall amounts. Yue, Ourada, and Bobee (2001) show that a bivariate Gamma distribution constructed from a specified Gamma marginal distribution is useful for representing the joint probabilistic properties of multivariate hydrological events, such as floods and storms. Loukas et al. (2001) use a Gumbel theoretical distribution and provide a best fit to the maximum annual rainfall using data for Greece. Therefore, the temperature variable may be based on different distributions.

We extend the existing literature to examine the setting of the distributions for temperature data for different temperature models. We consider the distribution based on the Gamma transformation and do not restrict the temperature disturbance to following a normal distribution. Our model is built on the well-developed temperature model in Campbell and Diebold (2005) that allows for conditional mean dynamics with a contribution from global warming and seasonal and cyclical components, and that also considers conditional volatility dynamics with contributions coming from both seasonal and cyclical components. Thus, the purpose of this study is threefold. First, we examine the distribution of temperature disturbances by means of a Gamma transformation based on the empirical data. Second, we extend Campbell and Diebold's (2005) temperature model with an adjustment to the disturbance by considering the Gamma transformed distributions. The normal distribution assumed in Campbell and Diebold (2005) is compared with this distribution. The out-of-sample forecast performance of the temperature based on different distributions is examined. Finally, we use an equilibrium option pricing method, which is an extension of the approach adopted by Cao and Wei (2004). They apply an extended version of Lucas's (1978) equilibrium pricing model where the direct estimation of the market price of weather risk is avoided. Instead, pricing is based on the stochastic process of the weather index, an aggregated dividend, and an assumption regarding the utility function of a representative investor. In other words, they consider an equilibrium pricing model with a joint process of the temperature and the aggregated dividend. Furthermore, we analyze the market prices of the temperature risk to establish if it is significant in the valuation of weather derivatives.

An empirical analysis is conducted to investigate the distribution setting on the temperature model. We demonstrate this by using a unique data set for Taiwan's daily temperature. Taiwan is located in the East Asian coastal area, and the ocean climate patterns affect Taiwan's climate. With its special climate and geographical location, the aquaculture in Taiwan is particularly sensitive to temperature variations. Natural disasters, such as typhoons, floods, coldness, hail, droughts, and earthquakes, often result in huge losses to the farmers. Furthermore, Taiwan's Council of Agriculture has pointed out that frost damage has caused more than USD2 billion in agricultural and fishery crop losses in Taiwan during the period from 1949 to 2009. How to deal with the weather risk is thus very important for both the farmers and the government. To date, the government has used the indemnity method in managing the weather risk in Taiwan when the farmers have faced serious losses. However, the farmers may not be covered by an indemnity when incurring serious losses because they did not meet the indemnity criteria. In addition, the government may experience serious financial problems when indemnifying its customers due to catastrophic events. Thus, how to use alternative instruments to manage the weather risk is needed in Taiwan. Weather derivatives are introduced to demonstrate that it is necessary to price weather derivatives based on the case of Taiwan. Therefore, we focus on our analysis using Taiwan's temperature data.

The data set contains a sample of 30,660 daily high/low temperatures over the period from 1970 through 2011, obtained from the Central Weather Bureau of Taiwan. After examining different Gamma transformed distributions, the empirical analysis shows that the Gumbel distribution assumption for the temperature disturbance provides more accuracy in terms of forecasting ability than that under the normal distribution assumption. If we ignore the Gumbel distribution, and instead assume that a normal distribution exists, it would cause the risk premiums to be underestimated. In addition, the risk premium represents a significant part of the determination of the prices of temperature derivatives that are evaluated based on risk aversion and aggregate dividend process parameters that conform to the empirical reality. Finally, the market price of weather risk is more pronounced for option prices than for forward prices due to the non-linearity in the option's payoff. Based on the numerical findings, we can sum up the contributions of this article regarding the literature on weather derivative pricing in the following ways. First, we use a reasonable distribution for Taiwan's temperature data. We discover that the distribution of temperatures could be Gumbel distributed and that our results appear to reject the assumption of a normal distribution according to Taiwan's temperature data. Thus, it is important to obtain a proper distribution to capture the temperature uncertainty. The Gumbel distribution has been used to model the distribution of the maximum (or the minimum) of a number of samples for various distributions, for example, the folds, earthquakes, athletic speed records, and maximal records, such as the hottest day or the wettest month (Vitiello and Poon (2010)). To the best of our knowledge, we first investigate the Gumbel distribution using temperature data. Second, we extend Campbell and Diebold's (2005) temperature model with different distribution settings, including the proposed Gumbel distribution for temperature dynamics, in order to value the HDD forward and HDD option contracts. Based on the equilibrium option pricing method, which is an extension of Cao and Wei's (2004) approach, the corresponding valuation frameworks for pricing temperature derivatives are established. Thus, we are able to analyze the market prices of temperature risk numerically, as the risk premium can affect the prices of temperature risk significantly.

The remainder of this paper is organized as follows. In Section II, we describe the settings of different temperature models and the equilibrium option pricing method, including the daily temperature behavior, the aggregate dividends process, and the utility function of the representative investor. In Section III, we provide the valuation formulas of the extended Campbell and Diebold (2005) model followed by the Gumbel distribution transformed by the Gamma distribution. In Section IV, we present the Monte Carlo simulation results. Finally, in the last section we draw our conclusions.

II. Model Specification

The martingale method and the equilibrium asset pricing model are the two main categories of pricing approaches for weather derivatives. The martingale method typically applies the principle of non-arbitrage to price the derivatives based on the tradable underlying asset. This method is further adapted to evaluate the weather derivatives by obtaining the market price of temperature risk while the temperature is non-tradable. Alaton, Djehiche, and Stillberger (2002) evaluate the weather derivative by using the martingale estimation function proposed by Bibby and Sørensen (1995). Cao and Wei (2004) first use an equilibrium asset pricing model for pricing weather derivatives that is an extension of Lucas's (1978) model. In general, when using the equilibrium asset pricing model, we need to know the utility function of the representative investor. According to Lucas's (1978) pure exchange economy, the fundamental uncertainties are driven by two state variables: the aggregate dividend and a state variable representing the weather condition. Aggregate dividend variables can be regarded as the aggregate output or dividends of the market portfolio and the weather conditions could be temperature, rainfall, snowfall, or the number of typhoons. We use the equilibrium asset pricing model based on Cao and Wei (2004).

Let W denote the joint dynamic combined with temperature, and δ be the aggregate dividend which is an exogenous process on a given probability space (Ω, F, P). The filtration, $F_t \equiv \sigma\left(\delta_\tau, W_\tau; \tau \in \{0,1,2,\ldots,t\}\right)$, assembles the infinitely representative investor's information at time t. Under the standard equilibrium condition that the total consumption is equal to the aggregate dividend, the contingent claim at time t with a payoff q_T at a future time T is given by

$$X(t,T) = \frac{1}{U'(\delta_t, t)} E_t\left[U'(\delta_T, T) q_T\right] \qquad \forall t \in (0,T), \tag{1}$$

where $U'(\bullet)$ is the marginal utility function of the representative investor.

The contingent claim at time t could be evaluated by Equation (1) as long as the temperature factors, dividend process, and the agent's preference determined by the relationship with the total consumption are specified first. We describe the settings of these variables as follows.

A. The Temperature Variable Model

A.1. Temperature Model Setting

To capture the main characteristics of temperature behavior in modeling temperature dynamics, we extend the temperature model proposed by Campbell and Diebold (2005) and further consider different distribution settings in the disturbance term to examine the abnormality of variation in temperature. Let W_t denote the daily average temperature. The daily average temperature dynamic is modeled by trend, seasonal, and cyclical components as follows.

$$W_t = Trend_t + Seasonal_t + \sum_{l=1}^{L} \theta_{t-l} W_{t-l} + \sigma_t \xi_t, \tag{2}$$

in which

$$Trend_t = \beta_0 + \beta_1 t,$$

$$Seasonal_t = \sum_{p=1}^{P}\left[\chi_{c,p}\cos\left(2\pi p\cdot\frac{d(t)}{365}\right)+\chi_{s,p}\sin\left(2\pi p\cdot\frac{d(t)}{365}\right)\right],$$

$$\sigma_t^2 = \sum_{q=1}^{Q}\left(\gamma_{c,q}\cos\left(2\pi q\cdot\frac{d(t)}{365}\right)+\gamma_{s,q}\sin\left(2\pi q\cdot\frac{d(t)}{365}\right)\right)+\sum_{r=1}^{R}\alpha_r\sigma_{t-r}^2\xi_{t-r}^2+\sum_{s=1}^{S}\lambda_s\sigma_{t-s}^2,$$

where the conditional mean dynamics of W_t with contributions comes from global warming, seasonal components, and cyclical components, and also allows for conditional volatility dynamics (σ_t^2) with contributions coming from both seasonal and cyclical components. $d(t)$ represents the cycles for each year, i.e., $d(t)$=1, 2, ..., 365. ξ_t is the disturbance term at time t and is assumed to be identical to and independently normally distributed with N(0,1) in Campbell and Diebold (2005). Instead, we extend their study to further examine the abnormality of variation in temperature and consider different Gamma transformed distributions in the disturbance term.

A.2. Statistical Analysis and Model Fitting with Taiwan Average Temperature Time Series

To capture the future dynamics of weather uncertainty on temperature, an empirical study is conducted to examine the abnormality of variations in the proposed temperature model. We demonstrate this with the temperature risk in Taiwan and employ a unique dataset, obtained from the Central Weather Bureau in Taiwan, of daily highs/lows of temperature measured in degrees Celsius over the 1970 to 2011 period. The data consist of a sample of 30,660 daily highs/lows.[2] We use the daily average temperature (W), which is widely reported and followed. Figure 1 depicts the daily average temperatures series for the last 40 years in Taiwan. The graphs show that the temperature has been oscillating and has increased over time, although the trend is not remarkably noticeable; in Taiwan, the daily average temperature moves repeatedly and regularly between periods of high temperature (summer) and low temperature (winter). These characteristics justify the inclusion of seasonality and the trend of temperature in the model. Furthermore, the daily average temperature varies within a range from 8°C to 32°C over the year. On the other hand, we also investigate the volatility of Taiwan daily temperature in Figure 2. The trend of the volatility shows a clear seasonal pattern. That is, the temperature variation in the winter is larger than that in the summer. The seasonal effect is consistent with what Cao and Wei (2004) discover for the U.S. market.

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[2] We remove February 29 from every leap year in our sample to keep to 365 days per year.

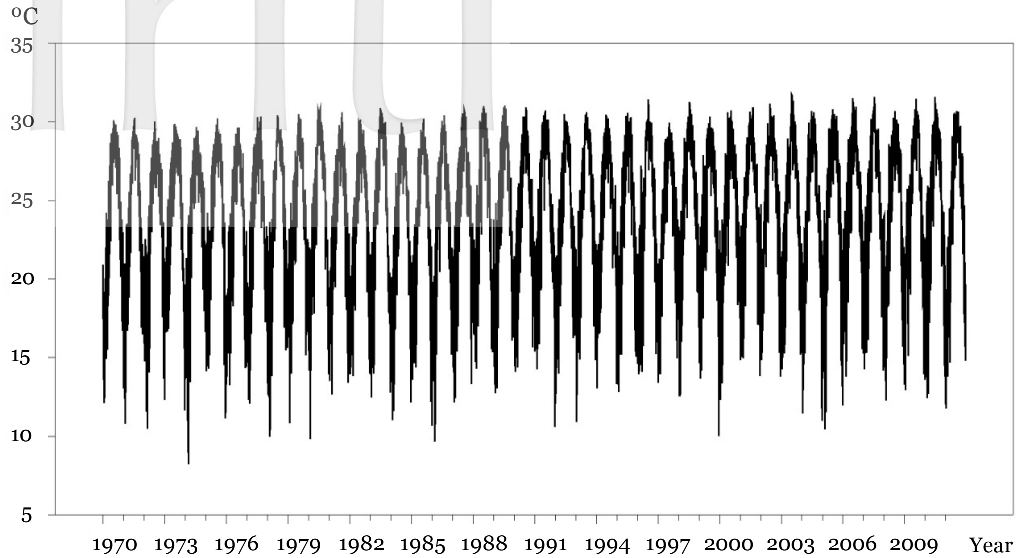

Figure 1. Time Series Plot of Daily Average Temperatures (1970 to 2011)

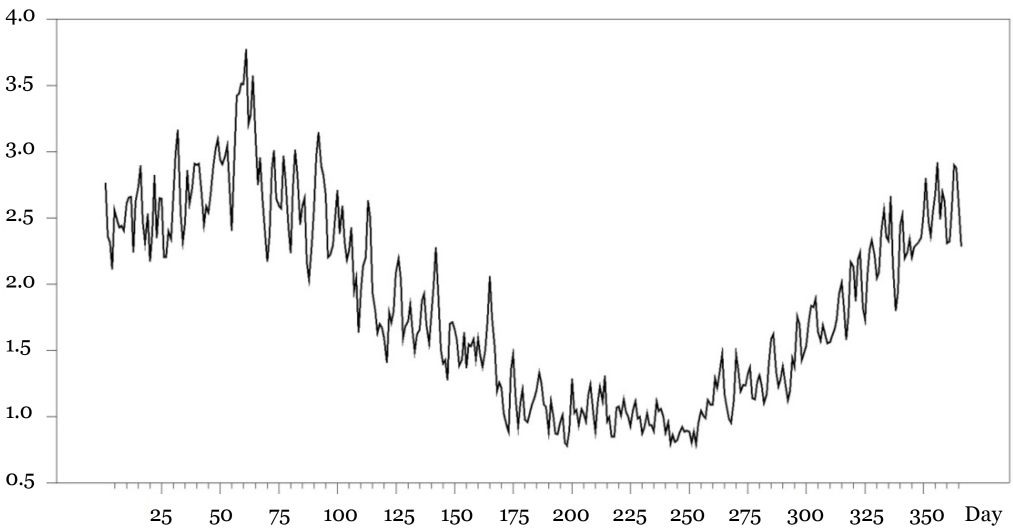

Figure 2. Volatility of Taiwan Daily Average Temperautres

The graph shows the standard deviation for each of the 365 calendar days. For each calendar day, the standard deviation is calculated from the corresponding 42 observations in the sample (January 1, 1970 to December 31, 2011).

To examine whether the normality is appropriate to capture Taiwan's temperature uncertainty, we first estimate the temperature model in Equation (2) based on Taiwan's temperature data and carry out the residual analysis. The results of the parameter estimates are shown in Table I. The estimation results support the following characteristics with the conditional mean equation (W_t) and the conditional variance (σ_t^2) of Taiwan's temperature. Firstly, Taiwan's average temperature displays a statistically significant trend that may be associated with

Table I
Parameter Estimates for Daily Average Temperature Model

The parameter estimates are based on the optimal lags of P=2, Q=2, R=1, S=1, and L=3. The optimal lag structures (P, Q, R, S, and L) are selected based on the criteria of Akaike information criterion (AIC) and Bayesian information criterion (BIC).[3] Thus, ** and *** denote significance at the 5% and 1% levels, respectively. Standard errors are in parentheses.

Parameter	Parameter Value
β_0	7.1062*** (0.1553)
β_1	2.51×10^{-5}*** (2.74×10^{-6})
$\chi_{c,1}$	-2.0074*** (0.0450)
$\chi_{s,1}$	-0.8919*** (0.0306)
$\chi_{c,2}$	-0.0482*** (0.0193)
$\chi_{s,2}$	-0.0594*** (0.0196)
θ_1	0.8437*** (0.0085)
θ_2	-0.2211*** (0.0108)
θ_3	0.0645*** (0.0082)
$\gamma_{c,1}$	0.5120*** (0.0859)
$\gamma_{s,1}$	0.5827*** (0.1076)
$\gamma_{c,2}$	-0.1604*** (0.0331)
$\gamma_{s,2}$	0.0410*** (0.0184)
α_1	0.0575** (0.0085)
λ_1	0.6219*** (0.0594)
Log-Likelihood	-22,825.62
AIC	0.364
BIC	0.371

urbanization and global warming effects. Secondly, the conditional mean dynamics displays both statistically and economically important seasonality. Thirdly, the conditional mean dynamics also displays strong cyclical persistence. Fourthly, we find that the volatility exhibits a seasonal effect, as the amplitude of

3 AIC = − 2/T ln (likelihood) + 2/T × (number of parameters), Akaike (1973). BIC = −2/T ln (likelihood) + ((number of parameters) × ln (T)) / T. The numerical results regarding the selection of the optimal lags are available upon request.

the residual fluctuation varies over the course of each year, becoming wider in the winter and narrower in the summer. Finally, the conditional variance also exhibits strong cyclical persistence.

After obtaining the parameter estimates, we calculate the residuals of Taiwan's daily average temperature. Table II and Figure 3 present the descriptive statistics and a graph of residuals. In Table II, the skewness and kurtosis coefficients suggest a leptokurtic distribution with negative skewed residuals for Taiwan's average temperature. The Jarque-Bera statistics show that the hypothesis of the normal distribution is rejected. A typical shape of the Gumbel distribution is presented in Figure 4. Furthermore, we use the Kolmogorov-Smirnov test to identify that the residuals of Taiwan's daily average temperature is to follow the Gumbel distribution. Here, our purpose is to present an appropriate distribution for capturing Taiwan's temperature uncertainty. Thus, we evaluate the forecasting performance for the out-of-sample period to investigate the importance of the distribution to the temperature process. Recall that the in-sample period refers to the estimation period from 1970 to 2010, while the out-of-sample period covers the forecasting horizon, which is a one-year period from January 1, 2011 to December 31, 2011.

Table II
Descriptive Statistics for Residuals of Taiwan's Average Temperatures from January 1, 1970 to December 31, 2011.

The skewness and excess kurtosis statistics include a test of the null hypotheses where each is zero (the population values if the series is i.i.d. normal). The Jarque-Bera statistic is used to test for normality based on the skewness and kurtosis measures combined. *** denotes significance at the 1% level.

Mean	Media	Max.	Min.	Std. Dev.	Skewness	Excess Kurtosis	Jarque-Bera
-0.0199	0.0742	3.1751	-5.8574	1.0023	-0.5498***	0.7896***	1170.6***

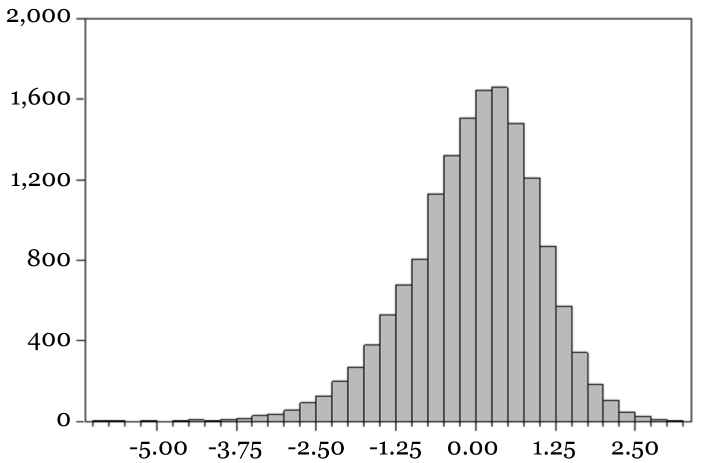

Figure 3. Histograms and Descriptive Statistics for Residuals of Taiwan's Average Temperatures

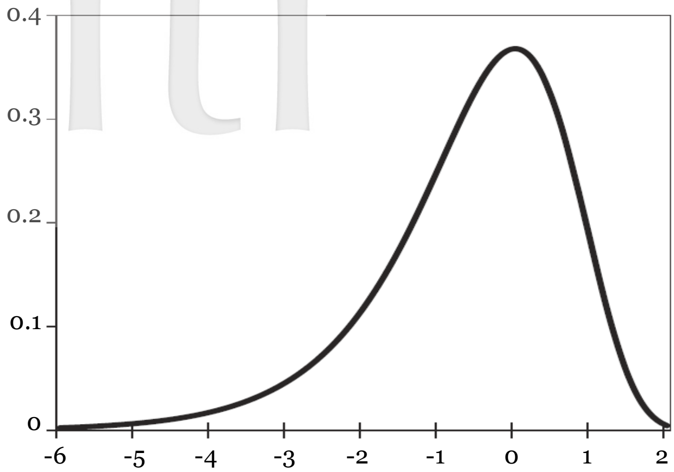

Figure 4. The Standard Gumbel Density Function

Table III presents the performance measured by the root mean square error (RMSE), the mean absolute error (MAE), and the mean absolute percentage error (MAPE) under different distribution assumptions. In particular, we compare the forecasting performance based on both the normal distribution and Gumbel distribution assumptions. According to the forecasting ability, the Gumbel distribution transformed by a Gamma distribution is better than the normal one. This is very reasonable and convincing since the Gumbel distribution is a special case of an extreme value distribution and is more capable of capturing the abnormality of variation in temperature.

B. The Aggregate Dividend Process

Cao and Wei (2004) extend Lucas's (1978) equilibrium asset pricing model under the pure exchange economy where the fundamental uncertainties are driven by the aggregate dividend and the weather conditions. Furthermore, their model considers the mean-reversion in the rate of change in the aggregate dividend

Table III
Forecasting Abilities under Different Temperature Disturbance Assumptions

$$\mathrm{RMSE} = \sqrt{\frac{1}{\mathrm{N}}\sum_{t=T+1}^{T+N}\left(W_t - \hat{W}_t\right)^2},\ \mathrm{MAE} = \frac{1}{\mathrm{N}}\sum_{t=T+1}^{T+N}\left|W_t - \hat{W}_t\right|,\ \mathrm{MAPE} = \frac{1}{\mathrm{N}}\sum_{t=T+1}^{T+N}\left|\frac{W_t - \hat{W}_t}{W_t}\right|,$$

where W_t is the actual value of the daily average temperature, $\hat{W}_t$ is the forecast value of the daily average temperature, and N is the number of observations.

	RMSE	MAE	MAPE
Normal	2.6554	2.0217	0.1098
Gumbel Distribution	2.5396	1.9958	0.1014

suggested by Marsh and Merton (1987). The derivative prices are normally within 1% of the "risk-neutral" prices if the contemporaneous correlation of the temperature process and the aggregate dividend process only is considered. However, we also take into account the lagged correlation, which makes the market price of risk become more important. Therefore, the residual of their aggregate dividend model takes into account the lagged correlations of the temperature residual. The aggregate dividend model is shown as follows:

$$\ln\delta_t = \alpha + \mu \ln \delta_{t-1} + v_t, \qquad \forall\ \mu \le 1 \tag{3}$$

$$v_t = \sigma\varepsilon_t + \sigma\left[\frac{\varphi}{\sqrt{1-\varphi^2}}\xi_t + \eta_1\xi_{t-1} + \eta_2\xi_{t-2} + \eta_3\xi_{t-3} + \ldots + \eta_m\xi_{t-m}\right], \qquad 0 \le m \le +\infty$$

$$\varepsilon_t \overset{iid}{\sim} N(0,1) \amalg \xi_i \qquad \forall\ i = t, t-1, \ldots, t-m,$$

where $1-\mu$ is the speed of the measured mean reversion, ε_t describes the randomness due to all factors except for the temperature, and ξ_i are innovations of the temperature process as defined in Equation (2). In light of this construction, the contemporaneous correlation between temperature and aggregate dividend is φ, and η_j is the coefficient of temperature-related lagged terms used to capture the lagged effects on the aggregate dividend. Based on the inevitability and assumption, $\sum_{j=1}^{m}\eta_j^2(\forall m)$ is bounded. When t represents a future time, the conditional variance of v_t is $\sigma^2\left[1+\frac{\varphi}{1-\varphi^2}+\sum_{j=1}^{m}\eta_j^2\right]$, which can be explained in three parts: in the first part, all factors except for the temperature contribute to σ^2; in the second part, the contemporaneous temperature contributes to $\sigma^2\frac{\varphi^2}{1-\varphi^2}$; in the final part, the lagged terms of the temperature contribute to $\sigma^2\sum_{j=1}^{m}\eta_j^2$. If $\varphi = 0, \eta_j = 0, \forall j$, then the aggregate dividend is independent of the temperature process.

C. The Utility Function of the Representative Investor

Following the literature generated by Cao and Wei (2004), we consider the utility function with constant relative aversion as:

$$U(\delta_t, t) = e^{-\rho t}\frac{\delta_t^{\kappa+1}}{\kappa+1}, \qquad \rho > 0, \kappa \le 0, \tag{4}$$

where ρ is the rate of time preference, and κ is the risk parameter.

According to the models we described above, in the next section we use these models to illustrate the valuation framework for HDD/CDD derivatives.

III. The Valuation of Temperature Derivatives

The valuation of the HDD/CDD derivatives, an appropriate assumption for modeling the temperature uncertainty should be considered. According to the empirical analysis in Section II.A. based on the Taiwan temperature data, we find that the temperature residuals are not normally distributed. The transformation of the standard Gumbel distribution provides the best fit for the Taiwan temperature data. Therefore, we assume that the transformed temperature disturbances follow the standard Gumbel distribution, which is

$$f(z) = \exp\left\{z - e^{z}\right\}, \qquad z \in \Re. \tag{5}$$

By adopting Equation (4), we can derive the marginal utility function as

$$U'(\delta_t, t) = e^{-\rho t}\delta_t^{\kappa}. \tag{6}$$

Then, a contingent claim X_T can be represented by

$$X(t,T) = e^{-\rho(T-t)}\delta_t^{-\kappa}E_t\left[\delta_T^{\kappa}q_T\right]. \tag{7}$$

Next, we use Equation (7) with the model introduced in Section II to price the pure discount bond and other HDD/CDD derivatives under the Gumbel distribution transformed by the Gamma distribution.

A. Discount Factor

Let $q_T = 1$, the contingent claim at time *t*, be regarded as the value of a pure discount bond at time *t* with maturity *T*. We denoted it by $D(t, T)$ and derive the following closed-form formula:

$$\begin{aligned} D(t,T) &= e^{-\rho(T-t)}\delta_t^{-\kappa}E_t[\delta_T^{\kappa}] \\ &= e^{-\rho(T-t)}\delta_t^{-\kappa}A(t,T)^{\kappa}e^{\frac{1}{2}\sigma^2\kappa^2\sum_{i=t+1}^{T}\mu^{2(T-i)}}\beta^{B(t,T)}C(t,T), \qquad \beta > 0, \end{aligned} \tag{8}$$

where

for $T - t \geq m+1$,

$$A(t,T) = \delta_t^{\mu^{T-t}}\exp\left\{\alpha\sum_{i=t+1}^{T}\mu^{T-i} + \sigma\sum_{i=1}^{m}\left[\sum_{j=1}^{i}\eta_{m+1-j}\mu^{T-t-(i+1-j)}\right]\hat{\xi}_{t+i-m}\right\}, \tag{9}$$

$$B(t,T)=\kappa\sigma\left\{\sum_{i=t+1}^{T-m}\left[\sum_{j=i}^{m+i}\eta_{j-i}\mu^{T-j}\right]+\sum_{i=T-m+1}^{T}\left[\sum_{j=0}^{T-i}\eta_j\mu^{T-i-j}\right]\right\}, \tag{10}$$

$$C(t,T)=\prod_{i=t+1}^{T-m}\Gamma\left(\kappa\sigma\sum_{j=i}^{m+i}\eta_{j-i}\mu^{T-j}+1\right)\prod_{i=T-m+1}^{T}\Gamma\left(\kappa\sigma\sum_{j=0}^{T-i}\eta_j\mu^{T-j-i}+1\right), \tag{11}$$

and

for $T-t\leq m$,

$$A(t,T)=\delta_t^{\mu^{T-t}}\exp\left\{\alpha\sum_{i=t+1}^{T}\mu^{T-i}+\sigma\sum_{i=1}^{T-t}\left[\sum_{j=1}^{i}\eta_{m+1-j}\mu^{T-t-(i+1-j)}\right]\hat{\xi}_{t+i-m}\right\}, \tag{12}$$

$$B(t,T)=\kappa\sigma\sum_{i=t+1}^{T}\left[\sum_{j=0}^{T-i}\eta_j\mu^{T-i-j}\right], \tag{13}$$

$$C(t,T)=\prod_{i=t+1}^{T}\Gamma\left(\kappa\sigma\sum_{j=0}^{T-i}\eta_j\mu^{T-j-i}+1\right), \tag{14}$$

where $\eta_0=\dfrac{\varphi}{\sqrt{1-\varphi^2}}$ and $\hat{\xi}_{t-l}(0\leq l\leq m-1)$ are the realized error terms for the temperature variable.

Proof: See Appendix

B. Valuation of HDD and CDD Forward and Option Contracts

Assume that there is an HDD forward contract with a tick size of NTD1, a strike price of K and the accumulation of heating degree days between dates T_1 and T_2. Then, based on Equations (4) and (7), we can obtain the value at time t of the HDD forward contract as follows:

$$\begin{aligned} f_{HDD}(t,T_1,T_2,K)&=E_t\left(\frac{U'(\delta_{T_2},T_2)}{U'(\delta_t,t)}[HDD(T_1,T_2)-K]\right)\\ &=e^{-\rho(T_2-t)}E_t\left(\frac{\delta_{T_2}^{\kappa}}{\delta_t^{\kappa}}[HDD(T_1,T_2)-K]\right). \end{aligned} \tag{15}$$

Take the forward price at time t, $F_{HDD}(t,T_1,T_2)$, to equal the strike price K such that $f_{HDD}(t,T_1,T_2,K)=0$, that is,

$$E_t\left(\frac{U'(\delta_{T_2},T_2)}{U'(\delta_t,t)}\left[HDD(T_1,T_2)-K\right]\right)=0. \tag{16}$$

We can obtain

$$F_{HDD}(t,T_1,T_2)=K=\frac{E_t\left[U'\left(\delta_{T_2},T_2\right)\left(\sum_{i=T_1}^{T_2}HDD_i\right)\right]}{E_t\left[U'\left(\delta_{T_2},T_2\right)\right]}=\frac{E_t\left[\delta_{T_2}^{\kappa}\left(\sum_{i=T_1}^{T_2}HDD_i\right)\right]}{E_t\left[\delta_{T_2}^{\kappa}\right]}. \tag{17}$$

Next, we consider a European HDD option with strike price X and an accumulation period from T_1 to T_2. For an HDD call, the expired payoff is $\max\left(\sum_{i=T_1}^{T_2}HDD_i-X,0\right)$, and the call price can be expressed as

$$C_{HDD}(t,T_1,T_2,X)=e^{-\rho(T_2-t)}\delta_t^{-\kappa}E_t\left(\delta_{T_2}^{\kappa}\max\left(\sum_{i=T_1}^{T_2}HDD_i-X,0\right)\right). \tag{18}$$

Similarly, an HDD put price can be expressed by

$$P_{HDD}(t,T_1,T_2,X)=e^{-\rho(T_2-t)}\delta_t^{-\kappa}E_t\left(\delta_{T_2}^{\kappa}\max\left(X-\sum_{i=T_1}^{T_2}HDD_i,0\right)\right). \tag{19}$$

The CDD derivatives are analogously expressed by replacing the above notation "HDD" with "CDD." To solve the above pricing formula for different temperature derivatives, it is difficult to obtain closed-form solutions without further restrictions for the dividend and temperature variables. Therefore, Monte Carlo simulations are conducted to calculate the price for temperature derivatives and the corresponding numerical results are analyzed in Section IV.

C. Market Price of Risk

We refer to the perspective of Cao and Wei (2004) for decomposing the derivatives price into two parts: the expected future spot value and the market price of risk, i.e., the risk premium. The expected future spot value is the value of the future payoff discounted by the discount factor. On the other hand, the risk premium implies the unpredictable temperature risk. That is, the correlation between the aggregate dividend and the temperature determines the risk premium in terms of the value of the temperature derivatives. With the HDD contract as an example, we decompose the valuation to clarify the relationship between the value and the risk premium as follows.

$$
\begin{aligned}
F_{HDD}(t,T_1,T_2) &= \frac{E_t\left[\delta_{T_2}^{\kappa}\left(\sum_{i=T_1}^{T_2} HDD_i\right)\right]}{E_t\left[\delta_{T_2}^{\kappa}\right]} \\
&= \frac{Cov_t\left(\delta_{T_2}^{\kappa},\left(\sum_{i=T_1}^{T_2} HDD_i\right)\right)+E_t\left[\delta_{T_2}^{\kappa}\right]E_t\left[\sum_{i=T_1}^{T_2} HDD_i\right]}{E_t\left[\delta_{T_2}^{\kappa}\right]} \\
&= E_t\left[\sum_{i=T_1}^{T_2} HDD_i\right]+\frac{Cov_t\left(\delta_{T_2}^{\kappa},\left(\sum_{i=T_1}^{T_2} HDD_i\right)\right)}{E_t\left[\delta_{T_2}^{\kappa}\right]} \\
&= E_{tt}\left[\sum_{i=T_1}^{T_2} HDD_i\right]+\Psi_{F,HDD}(T_2)\,, \qquad (20)
\end{aligned}
$$

$$
\begin{aligned}
C_{HDD}(t,T_1,T_2,X) &= D(t,T_2)\left\{E_t\left[\max\left(\sum_{i=T_1}^{T_2} HDD_i - X, 0\right)\right]+\frac{Cov_t\left[\delta_{T_2}^{\kappa},\max\left(\sum_{i=T_1}^{T_2} HDD_i - X, 0\right)\right]}{E_t\left[\delta_{T_2}^{\kappa}\right]}\right\} \\
&= D(t,T_2)\left\{E_t\left[\max\left(\sum_{i=T_1}^{T_2} HDD_i - X, 0\right)\right]+\Psi_{C,HDD}(T_2)\right\}, \qquad (21)
\end{aligned}
$$

$$
\begin{aligned}
P_{HDD}(t,T_1,T_2,X) &= D(t,T_2)\left\{E_t\left[\max\left(X-\sum_{i=T_1}^{T_2} HDD_i, 0\right)\right]+\frac{Cov_t\left[\delta_{T_2}^{\kappa},\max\left(X-\sum_{i=T_1}^{T_2} HDD_i, 0\right)\right]}{E_t\left[\delta_{T_2}^{\kappa}\right]}\right\} \\
&= D(t,T_2)\left\{E_t\left[\max\left(X-\sum_{i=T_1}^{T_2} HDD_i, 0\right)\right]+\Psi_{P,HDD}(T_2)\right\}, \qquad (22)
\end{aligned}
$$

where $Cov(\cdot,\cdot)$ is the covariance and Ψ is the risk premium of each derivative.

Through the above equations, we can observe that the weather conditions affect the discount factor and the risk premium, which are the major elements for temperature derivative pricing. Similarly, the formula for CDD derivatives can be obtained by using the same method. Owing to the negative correlation between HDD and the temperature, the risk premiums in Equations (20) and (21) are negative, but those in Equation (22) are positive when $\varphi < 0$ and $\eta_i \le 0,\ \forall i.$ The CDD derivatives are reversed. A summary of the different cases is provided in Table IV.

Table IV
The Relation between Correlations and Risk Premium

Element	$\varphi < 0, \eta_i \le 0, \forall i$	$\varphi > 0, \eta_i \ge 0, \forall i$
$\Psi_{F,HDD}, \Psi_{C,HDD}, \Psi_{P,CDD}$	Negative	Positive
$\Psi_{F,CDD}, \Psi_{C,CDD}, \Psi_{P,HDD}$	Positive	Negative

Furthermore, we can observe that once the temperature and the aggregate dividend are completely independent, the derivatives price will be equal to the risk neutral value. In other words, if the joint processes are completely independent, i.e., $\varphi = 0,\ \eta_i = 0,\ \forall i,$ then the value at time *t* of the temperature derivatives is the future payoff discounted by the risk-free rate.

IV. Simulation and Analysis

We study the impact of the distribution assumption regarding the temperature disturbance on the price of the HDD and CDD forward and option contracts. However, since the HDD contracts are a mirror image of the CDD contracts by nature, to maintain brevity, we only report the results for HDD contracts. Table V contains the base parameter values used to calculate the prices of temperature derivatives. For comparison purposes, we use the assumptions outlined in Cao and Wei (2004).

Table V
Base Assumption of Parameter Values for Pricing Temperature Derivatives

[a] The risk-free rate is based on the return on a one-year deposit with the Bank of Taiwan. [b] The mean reversion rate is $1-\mu$. [c] TAIEX stands for the Taiwan Stock Exchange Capitalization Weighted Stock Index.

Parameter	Notion	Value
Rate of Time Preference[a]	ρ	1.24%
Mean Reversion[b]	μ	0.9
TAIEX Daily Variance[c]	σ	0.0154%
Aggregate Dividend Lags	η	0, 15, 30
Risk Aversion	κ	-2, -10, -40
Contemporaneous Correlation	φ	-0.25, -0.15, 0.15, 0.25

We first investigate the risk premiums for HDD contracts. In particular, we study the effects of different lagged correlated numbers, correlation levels, and risk aversion on the risk premiums for HDD contracts. For each derivative, the risk premium ψ, which is defined in Section III, is exhibited by the percentage differences between the derivative values (excluding the discount factor) and the risk-neutral values. Table VI shows the simulated results of the HDD forward price under both the Gumbel distribution and the normal distribution, where the values in parentheses are the simulated results under the normal distribution.

Through Table VI, we can obtain the following observations: (i) it is consistent with the theoretical result in Table IV that the sign of the correlation coefficient determines the sign of the risk premium; (ii) given a fixed correlation, a higher risk aversion brings a larger risk premium, and thus it makes sense that those investors ask for higher returns; (iii) under the same risk aversion level, a higher correlation level gives rise to a larger risk premium, which also makes intuitive sense; (iv) the largest risk premium under the Gumbel (normal) distribution is

Table VI
Risk Premium in HDD Forward Prices: Lagged Correlations

The risk-neutral forward is calculated by $\varphi = 0, \eta_i = 0, \forall i$. The forward price is calculated by excluding the discount factor. The risk premiums are the percentage differences between the risk-neutral value and the derivative price. For example, under $\kappa = -2, \varphi = 0.15$, and 15-lagged correlations, the price is 0.171% (0.154%), which is higher than the risk-neutral value under the Gumbel (normal) distribution.

Risk-Neutral Forward 814.9691 (669.8511)		$\kappa = -2$		$\kappa = -10$		$\kappa = -40$	
		$\varphi = -a$	$\varphi = a$	$\varphi = -a$	$\varphi = a$	$\varphi = -a$	$\varphi = a$
				$a = 0.15$			
Zero-Lagged Correlation	Forward Price	814.399 (669.688)	815.679 (670.226)	813.152 (668.796)	816.735 (671.020)	809.223 (666.646)	820.493 (675.038)
	Risk Premium	-0.070% (-0.024%)	0.087% (0.056%)	-0.223% (-0.157%)	0.217% (0.174%)	-0.705% (-0.478%)	0.678% (0.774%)
15-Lagged Correlations	Forward Price	812.784 (669.231)	816.361 (670.883)	807.211 (664.901)	823.451 (674.348)	786.407 (652.723)	853.501 (687.807)
	Risk Premium	-0.268% (-0.093%)	0.171% (0.154%)	-0.952% (-0.739%)	1.041% (0.671%)	-3.505% (-2.557%)	4.728% (2.681%)
30-Lagged Correlations	Forward Price	812.825 (669.044)	816.973 (671.138)	807.486 (669.285)	824.191 (674.478)	784.208 (652.174)	854.976 (688.399)
	Risk Premium	-0.263% (-0.120%)	0.246% (0.192%)	-0.918% (-0.845%)	1.132% (0.691%)	-3.775% (-2.639%)	4.909% (2.769%)
				$a = 0.25$			
Zero-Lagged Correlation	Forward Price	814.166 (669.506)	815.987 (670.390)	812.501 (668.605)	818.301 (671.502)	803.876 (664.456)	826.596 (675.587)
	Risk Premium	-0.099% (-0.051%)	0.125% (0.080%)	-0.303% (-0.186%)	0.409% (0.247%)	-1.361% (-0.805%)	1.427% (0.856%)
15-Lagged Correlations	Forward Price	812.469 (668.366)	817.744 (671.391)	803.637 (663.306)	827.454 (677.037)	775.592 (645.576)	870.021 (696.742)
	Risk Premium	-0.307% (-0.222%)	0.340% (0.230%)	-1.390% (-0.977%)	1.532% (1.073%)	-4.832% (-3.624%)	6.755% (4.015%)
30-Lagged Correlations	Forward Price	812.328 (667.911)	817.974 (671.730)	802.628 (663.233)	827.735 (677.182)	774.166 (644.537)	871.990 (698.350)
	Risk Premium	-0.324% (-0.290%)	0.369% (0.280%)	-1.514% (-0.988%)	1.566% (1.094%)	-5.007% (-3.779%)	6.997% (4.255%)

6.997% (4.255%) for 30-lagged (30-lagged) correlation when $\kappa = $-40 and $\varphi = 0.25$; (v) not only the forward prices but also the risk premiums under the Gumbel distribution are higher than those under the normal distribution; and (vi) by comparing three different cases of lagged correlated numbers, the risk premium is higher if we consider more lagged correlations, but the risk premiums between the 15-lagged correlations and 30-lagged correlations are almost the same. This implies that the effect converges when the lagged number of correlations is higher.

We further analyze the effect for HDD call and put options. For the purpose of letting the risk-neutral call and put options be at-the-money and characterized by equality, under the Gumbel (normal) distribution we set the strike price equal to the risk-neutral forward price, namely, 814.9691 (669.8511). Other setups are the same as those for the forward price. The results of the HDD call and put options are illustrated in Tables VII and VIII, respectively.

Table VII
Risk Premium in HDD Call Prices: Lagged Correlations

Risk-Neutral Call Price 38.0556 (27.3056)		κ = -2		κ = -10		κ = -40	
		$\varphi = -a$	$\varphi = a$	$\varphi = -a$	$\varphi = a$	$\varphi = -a$	$\varphi = a$
				a = 0.15			
Zero-Lagged Correlation	Call Price	37.694	38.694	35.680	39.019	35.024	42.752
		(27.139)	(27.472)	(26.457)	(27.673)	(25.739)	(29.891)
	Risk Premium	-0.949%	1.679%	-6.240%	2.533%	-7.965%	12.341%
		(-0.610%)	(0.610%)	(-3.107%)	(1.346%)	(-5.737%)	(9.470%)
15-Lagged Correlations	Call Price	37.200	38.715	33.906	42.632	25.000	61.693
		(27.000)	(27.588)	(24.771)	(29.200)	(19.240)	(37.160)
	Risk Premium	-2.248%	1.732%	-10.903%	12.025%	-34.307%	62.114%
		(-1.119%)	(1.035%)	(-9.281%)	(6.938%)	(-29.538%)	(36.090%)
30-Lagged Correlations	Call Price	36.778	38.806	33.882	43.514	23.929	62.275
		(26.806)	(27.727)	(24.686)	(29.371)	(19.174)	(37.587)
	Risk Premium	-3.358%	1.971%	-10.966%	14.344%	-37.122%	63.641%
		(-1.831%)	(1.543%)	(-9.595%)	(7.566%)	(-29.780%)	(37.653%)
				a = 0.25			
Zero-Lagged Correlation	Call Price	37.333	38.944	35.373	40.278	32.811	44.750
		(27.085)	(27.472)	(26.143)	(28.471)	(24.267)	(30.156)
	Risk Premium	-1.898%	2.336%	-7.048%	5.839%	-13.782%	17.591%
		(-0.807%)	(0.610%)	(-4.257%)	(4.267%)	(-11.129%)	(10.437%)
15-Lagged Correlations	Call Price	37.057	39.649	32.161	44.339	20.618	71.874
		(26.611)	(27.833)	(23.800)	(31.000)	(16.404)	(43.308)
	Risk Premium	-2.624%	4.186%	-15.489%	16.511%	-45.822%	88.865%
		(-2.543%)	(1.933%)	(-12.838%)	(13.530%)	(-39.925%)	(58.604%)
30-Lagged Correlations	Call Price	36.389	39.750	32.150	44.667	20.289	73.154
		(26.333)	(28.278)	(23.563)	(31.676)	(16.385)	(43.955)
	Risk Premium	-4.380%	4.453%	-15.516%	17.372%	-46.685%	92.229%
		(-3.561%)	(3.561%)	(-13.706%)	(16.007%)	(-39.996%)	(60.973%)

Table VIII
Risk Premium in HDD Put Prices: Lagged Correlations

The option price is calculated by excluding the discount factor. The values in parentheses are the results under the normal distribution.

Risk-Neutral Put Price 38.0556 (27.3056)		κ = -2		κ = -10		κ = -40	
		$\varphi = -a$	$\varphi = a$	$\varphi = -a$	$\varphi = a$	$\varphi = -a$	$\varphi = a$
				a = 0.15			
Zero-Lagged Correlation	Put Price	38.250 (27.306)	37.591 (27.111)	39.667 (27.486)	36.508 (26.257)	40.756 (28.957)	34.217 (24.696)
	Risk Premium	0.511% (0.000%)	-1.219% (-0.712%)	4.234% (0.660%)	-4.066% (-3.840%)	7.096% (6.046%)	-10.087% (-9.558%)
15-Lagged Correlations	Put Price	39.400 (27.611)	37.108 (26.639)	41.656 (29.686)	34.132 (24.743)	53.917 (36.220)	22.907 (19.360)
	Risk Premium	3.533% (1.119%)	-2.490% (-2.442%)	9.462% (8.717%)	-10.311% (-9.385%)	41.679% (32.647%)	-39.807% (-29.099%)
30-Lagged Correlations	Put Price	38.917 (27.611)	36.850 (26.694)	41.324 (28.971)	34.171 (24.829)	54.524 (36.783)	22.392 (19.109)
	Risk Premium	2.263% (1.119%)	-3.168% (-2.238%)	8.587% (6.101%)	-10.206% (-9.071%)	43.274% (34.707%)	-41.159% (-30.019%)
				a = 0.25			
Zero-Lagged Correlation	Put Price	38.520 (27.556)	37.156 (26.944)	39.731 (28.176)	36.156 (26.248)	43.973 (29.622)	33.036 (24.444)
	Risk Premium	1.219% (0.916%)	-2.365% (-1.322%)	4.402% (3.190%)	-4.993% (-3.873%)	15.549% (8.484%)	-13.191% (-10.478%)
15-Lagged Correlations	Put Price	39.571 (28.111)	36.484 (26.306)	43.387 (30.314)	32.200 (23.857)	60.059 (40.788)	16.641 (16.288)
	Risk Premium	3.983% (2.950%)	-4.129% (-3.662%)	14.010% (11.019%)	-15.387% (-12.629%)	57.819% (49.378%)	-56.272% (-40.347%)
30-Lagged Correlations	Put Price	39.647 (28.278)	36.722 (26.389)	44.727 (30.588)	32.000 (24.235)	61.632 (41.705)	16.577 (15.455)
	Risk Premium	4.182% (3.561%)	-3.504% (-3.357%)	17.532% (12.022%)	-15.912% (-11.244%)	61.952% (52.733%)	-56.440% (-43.401%)

For the call price, the result is the same as for the forward price. As expected, the result for the put price is inversed. It is worth noting that the risk premiums for options are larger than those for the forward contract due to the payoff types between them, namely, linear and non-linear. Under the Gumbel (normal) distribution, the largest risk premium, 92.229% (60.973%), is for a call with $\kappa = -40$ and $\varphi = 0.25$, and that of 61.952% (52.733%) is for a put with $\kappa = -40$ and $\varphi = -0.25$ for 30-lagged correlations. According to the statements in Table VI through Table VIII, the risk premium is small if we only consider the contemporaneous correlation between the dividend and temperature processes. Furthermore, the difference in the results between the 15-lagged correlations and 30-lagged correlations cannot be explained. On the other hand, both the 15-lagged correlations and 30-lagged correlations have identical portions of the variance contributed by the temperature variations. Therefore, we suggest that the number of lagged terms of the correlation should be 15.[4] On the other hand, the results reveal that in the case of the normal distribution assumption, the risk premium is lower for HDD forward prices and options, compared with the Gumbel distribution assumption; thus, these risk premiums would tend to be underestimated if we were to ignore the important properties of abnormal variations in Taiwan's temperature.

Finally, we set $\mu = 0.9$ which corresponds to a mean reversion rate of 0.1. To see how sensitive the results are to this mean reversion parameter, the calculations in Table VI through Table VIII are repeated by assuming four other levels of $\mu = 0.80$, 0.85, 0.90, 0.95, and 0.99. Note that $\mu = 0.99$ roughly corresponds to a random walk. In Table IX, it is seen that a higher value of μ or a lower mean reversion speed, leads to a bigger risk premium in forward and option values. This makes intuitive sense since a higher μ means a bigger variation in the aggregate dividends. Furthermore, the risk premiums for options are larger than for the forward contracts due to the payoff types between them, which may be linear and non-linear. To illustrate, for options, when μ increases from 0.9 to 0.99, the risk premium increases by more than 10-fold. With a near-random walk, the risk premium is more than 10% for all option values. An obvious conclusion is that in determining the significance of the market price of risk for the temperature variable, the degree of mean reversion in the aggregate dividend process must be carefully determined.

V. Conclusions

The Council of Agriculture in Taiwan has pointed out that frost damage has caused more than NTD60 billion in agricultural and fishery crop losses in Taiwan over the period from 1949 to 2009. In view of the temperature dynamics and high amounts of such losses, the development of temperature derivatives is very important in Taiwan in developing hedging instruments to mitigate weather risk. In this paper, we address the characteristics of the temperature distribution for capturing the weather uncertainty in Taiwan and propose a Gumbel distribution incorporating Campbell and Diebold's (2005) temperature model for pricing temperature derivatives that is based on temperature data in Taiwan. We discover that in terms of forecasting ability, the temperature model under the Gumbel distribution is more accurate than that under a normal distribution. We also utilize the equilibrium approach proposed by Cao and Wei (2004) to deal with the extended temperature model of Campbell and Diebold (2005) by using a Gumbel distribution to price HDD and CDD forward and option contracts. Under the

4 Cao and Wei (2004) point out that the lagged term of the aggregate dividend is used to fully assess the significance of the risk premium. In order to understand how many lagged terms are specified, it is necessary to examine the lagged term of the empirical results.

Table IX

Impact of Mean Reversion in the Dividend Process for HDD Contracts

Forward and option prices are all for HDD contracts for Taiwan. Except for the mean reversion parameter, all other aspects of the calculation are the same as in Tables VI though Table VIII. The assumption of parameters is $a = 0.15$, 30-lagged and Gumbel distribution.

Panel A. Forward Prices

Mean Reversion	Risk Neutral Forward	κ = -2		κ = -10		κ = -40	
		$\varphi = -a$	$\varphi = a$	$\varphi = -a$	$\varphi = a$	$\varphi = -a$	$\varphi = a$
0.80	814.9690	-0.043%	0.031%	-0.473%	0.540%	-1.596%	2.016%
0.85	814.9690	-0.156%	0.107%	-0.513%	0.593%	-2.287%	2.815%
0.90	814.9690	-0.263%	0.246%	-0.918%	1.132%	-3.775%	4.909%
0.95	814.9690	-0.492%	0.374%	-1.973%	2.183%	-6.692%	8.808%
0.99	814.9690	-1.554%	1.575%	-7.483%	8.047%	-22.368%	25.674%

Panel B. Option Prices

Mean Reversion	Risk Neutral Call (Put) Price	κ = -2		κ = -10		κ = -40	
		$\varphi = -a$	$\varphi = a$	$\varphi = -a$	$\varphi = a$	$\varphi = -a$	$\varphi = a$
0.80	Call (Put)	-0.949%	0.671%	-4.802%	6.642%	-18.271%	24.114%
	38.0556	(1.094%)	(-1.898%)	(5.335%)	(-4.964%)	(15.553%)	(-19.335%)
0.85	Call (Put)	-2.409%	1.678%	-6.152%	10.365%	-23.728%	35.388%
	38.0556	(1.941%)	(-2.744%)	(6.423%)	(-6.642%)	(25.525%)	(-25.468%)
0.90	Call (Put)	-3.358%	1.971%	-10.966%	14.344%	-37.122%	63.641%
	38.0556	(2.263%)	(-3.168%)	(8.587%)	(-10.206%)	(43.274%)	(-41.159%)
0.95	Call (Put)	-6.602%	4.087%	-21.606%	26.732%	-58.453%	122.166%
	38.0556	(3.833%)	(-3.943%)	(20.175%)	(-20.342%)	(91.895%)	(-66.225%)
0.99	Call (Put)	-16.753%	18.248%	-62.277%	111.574%	-99.592%	450.141%
	38.0556	(16.356%)	(-15.428%)	(97.793%)	(-61.898%)	(388.758%)	(-99.616%)

equilibrium approach, the pricing framework relies on the fundamental uncertainties, which are set to be driven by the aggregate dividend and the temperature. The effects of the model settings with the pricing framework for temperature derivatives are investigated and compared numerically. According to the simulation results, we find that ignoring the abnormal variations in temperature would cause an underestimation of the risk premium when we use the normal distribution assumption. Furthermore, we demonstrate that the effects of different distributions on the value of the temperature derivatives are very significant. Therefore, the justifiability of the distribution is critical for pricing the temperature derivatives.

Since the data sets for macroeconomic variables in Taiwan are smaller and less frequent, we do not estimate the dividend model. For the purposes of our analysis, we set the parameters of the dividend process by resorting to Cao and Wei's (2004) dividend model. There is room for future research to examine the dividend process in different economic environments and to make the study more accurately match the reality.

Appendix. Proof of the Value of the Pure Discount Bond

First, we illustrate the distribution of the temperature residual. According to Figures 3 and 4, the patterns of residuals are similar to the Gumbel distribution; hence, we use the Gamma transformed method proposed by Vitiello and Poon (2010) to explain the density function of the temperature residual.

THEOREM: Let $h(z)$ be some transformation of z. If x in $h(z)=ax$ has a Gamma density and $h(\bullet)$ is a monotonic differentiable function, then the density function of z is given by:

$$f(z)=\frac{1}{\Gamma(p)}|h'(z)|\,h(z)^{p-1}e^{-h(z)}, \tag{A1}$$

where $h'(z)$ is the first derivative of $h(z)$, which is gamma distributed, and $f(z)$ is a transformed Gamma density.

For $p=1$ and by setting $h(z)=\exp(z)$, the variable z follows the standard Gumbel density function. Based on the above theorem, we can obtain the density function of the temperature residual. By the theorem, since the temperature residual is similar to the Gumbel distribution, we can set $h(\xi)=\exp(\xi)=\beta x$, where $x\sim Exp(\lambda=1)$. Then we obtain the density function of ξ as follows.

$$f(\xi)=\frac{1}{\beta}e^{\xi-\frac{e^{\xi}}{\beta}}, \tag{A2}$$

and the moment generating function of ξ is

$$M_{\xi}(t)=E(e^{\xi t})=E\left[(\beta X)^{t}\right]=\beta^{t}\Gamma(t+1). \tag{A3}$$

Second, in Equation (3), we iterate the process and obtain:

$$\begin{aligned}\ln\delta_T&=\alpha\frac{1-\mu^{T-t}}{1-\mu}+\mu^{T-t}\ln\delta_t+\sum_{j=t+1}^{T}\nu_j\mu^{T-j}\\&=\alpha\frac{1-\mu^{T-t}}{1-\mu}+\mu^{T-t}\ln\delta_t+\sum_{j=t+1}^{T}\sigma\varepsilon_j\mu^{T-j}+\sum_{j=t+1}^{T}\sigma\left(\sum_{i=0}^{m}\eta_i\xi_{j-i}\right)\mu^{T-j},\end{aligned} \tag{A4}$$

where $\eta_0=\frac{\varphi}{\sqrt{1-\varphi^2}}$, and we suppose that ν_j has m lagged error terms without loss of generality.

When

for $T-t\geq m+1$,

$$\sum_{j=t+1}^{T}\sigma\left(\sum_{i=0}^{m}\eta_i\xi_{j-i}\right)\mu^{T-j}=\sigma\left\{\sum_{i=1}^{m}\left[\sum_{j=1}^{i}\eta_{m+1-j}\mu^{T-t-(i+1-j)}\right]\hat{\xi}_{t+i-m}\right.$$
$$\left.+\sum_{i=t+1}^{T-m}\left[\sum_{j=i}^{m+i}\eta_{j-i}\mu^{T-j}\right]\xi_i+\sum_{i=T-m+1}^{T}\left[\sum_{j=0}^{T-i}\eta_j\mu^{T-i-j}\right]\xi_i\right\}, \qquad \text{(A5)}$$

and

for $T-t\le m$,

$$\sum_{j=t+1}^{T}\sigma\left(\sum_{i=0}^{m}\eta_i\xi_{j-i}\right)\mu^{T-j}$$
$$=\sigma\left\{\sum_{i=1}^{T-t}\left[\sum_{j=1}^{i}\eta_{m+1-j}\mu^{T-t-(i+1-j)}\right]\hat{\xi}_{t+i-m}+\sum_{i=t+1}^{T}\left[\sum_{j=0}^{T-i}\eta_j\mu^{T-i-j}\right]\xi_i\right\}. \qquad \text{(A6)}$$

Here, the $\hat{\xi}_{t-l}(0\le l\le m-1)$ are the realized error terms for the temperature variable.

Third, we use the moment generating function to obtain the conditional expectation of the aggregate dividend at time T. Because ε_j follows an i.i.d. normal distribution and the moment generating function of ξ is given by Equation (A3), we can use the moment generating function to obtain the conditional expectation of the aggregate dividend at time T. Assume that $\varepsilon_j \amalg \xi_i,\ \forall i,j$ and that F_t is the infinitely representative investor's information at time t.

For $T-t\ge m+1$,

$$E(\delta_T\mid F_t)=A(t,T)E\left[\exp\left\{\sigma\sum_{i=t+1}^{T}\mu^{T-i}\varepsilon_i+\sigma\left(\sum_{i=t+1}^{T-m}\left[\sum_{j=i}^{m+i}\eta_{j-i}\mu^{T-j}\right]\xi_i\right.\right.\right.$$
$$\left.\left.\left.+\sum_{i=T-m+1}^{T}\left[\sum_{j=0}^{T-i}\eta_j\mu^{T-i-j}\right]\xi_i\right)\right\}\mid F_t\right]$$
$$=A(t,T)\exp\left\{\frac{1}{2}\sigma^2\sum_{i=t+1}^{T}\mu^{2(T-i)}\right\}\beta^{B(t,T)}C(t,T),$$

where

$$A(t,T)=\delta_t^{\mu^{T-t}}\exp\left\{\alpha\sum_{i=t+1}^{T}\mu^{T-i}+\sigma\sum_{i=1}^{m}\left[\sum_{j=1}^{i}\eta_{m+1-j}\mu^{T-t-(i+1-j)}\right]\hat{\xi}_{t+i-m}\right\}$$

$$B(t,T)=\sigma\left\{\sum_{i=t+1}^{T-m}\left[\sum_{j=i}^{m+i}\eta_{j-i}\mu^{T-j}\right]+\sum_{i=T-m+1}^{T}\left[\sum_{j=0}^{T-i}\eta_j\mu^{T-i-j}\right]\right\}$$

$$C(t,T)=\prod_{i=t+1}^{T-m}\Gamma\left(\sigma\sum_{j=i}^{m+i}\eta_{j-i}\mu^{T-j}+1\right)\cdot\prod_{i=T-m+1}^{T}\Gamma\left(\sigma\sum_{j=0}^{T-i}\eta_j\mu^{T-i-j}+1\right).$$

For $T-t \le m$,

$$E(\delta_T \mid F_t) = A(t,T) E\left[\exp\left\{ \sigma \sum_{i=t+1}^{T} \mu^{T-i}\varepsilon_i + \sigma \sum_{i=t+1}^{T}\left[\sum_{j=0}^{T-i} \eta_j \mu^{T-i-j} \right]\xi_i \right\} \mid F_t \right]$$
$$= A(t,T)\exp\left\{ \frac{1}{2}\sigma^2 \sum_{i=t+1}^{T} \mu^{2(T-i)} \right\} \beta^{B(t,T)} C(t,T),$$

where

$$A(t,T) = \delta_t^{\mu^{T-t}} \exp\left\{ \alpha \sum_{i=t+1}^{T} \mu^{T-i} + \sigma \sum_{i=1}^{T-t}\left[\sum_{j=1}^{i} \eta_{m+1-j}\mu^{T-t-(i+1-j)} \right]\hat{\xi}_{t+i-m} \right\}$$

$$B(t,T) = \sigma \sum_{i=t+1}^{T}\left[\sum_{j=0}^{T-i} \eta_j \mu^{T-i-j} \right]$$

$$C(t,T) = \prod_{i=t+1}^{T} \Gamma\left(\sigma \sum_{j=0}^{T-i} \eta_j \mu^{T-i-j} + 1 \right).$$

Finally, we use the above result to derive the value of the pure discount bond. By Equation (7) and assuming that $q_T = 1$, then

$$D(t,T) = e^{\rho t} \cdot \delta_t^{-\kappa} E_t(e^{-\rho T}\delta_T^{\kappa}) = e^{-\rho(T-t)} \delta_t^{-\kappa} E_t(\delta_T^{\kappa})$$
$$= e^{-\rho(T-t)} \delta_t^{-\kappa} A(t,T)^{\kappa} e^{\frac{1}{2}\sigma^2\kappa^2 \sum_{i=t+1}^{T} \mu^{2(T-i)}} \beta^{B(t,T)} C(t,T),$$

where

for $T-t \ge m+1$,

$$A(t,T) = \delta_t^{\mu^{T-t}} \exp\left\{ \alpha \sum_{i=t+1}^{T} \mu^{T-i} + \sigma \sum_{i=1}^{m}\left[\sum_{j=1}^{i} \eta_{m+1-j}\mu^{T-t-(i+1-j)} \right]\hat{\xi}_{t+i-m} \right\}$$

$$\mathrm{B}(t,T) = \kappa\sigma \left\{ \sum_{i=t+1}^{T-m}\left[\sum_{j=i}^{m+i} \eta_{j-i}\mu^{T-j} \right] + \sum_{i=T-m+1}^{T}\left[\sum_{j=0}^{T-i} \eta_j \mu^{T-i-j} \right] \right\}$$

$$C(t,T) = \prod_{i=t+1}^{T-m} \Gamma\left(\kappa\sigma \sum_{j=i}^{m+i} \eta_{j-i}\mu^{T-j} + 1 \right) \prod_{i=T-m+1}^{T} \Gamma\left(\kappa\sigma \sum_{j=0}^{T-i} \eta_j \mu^{T-j-i} + 1 \right),$$

and

for $T-t \le m$,

$$A(t,T) = \delta_t^{\mu^{T-t}} \exp\left\{ \alpha \sum_{i=t+1}^{T} \mu^{T-i} + \sigma \sum_{i=1}^{T-t} \left[\sum_{j=1}^{i} \eta_{m+1-j} \mu^{T-t-(i+1-j)} \right] \hat{\xi}_{t+i-m} \right\},$$

$$B(t,T) = \kappa\sigma \sum_{i=t+1}^{T} \left[\sum_{j=0}^{T-i} \eta_j \mu^{T-i-j} \right],$$

$$C(t,T) = \prod_{i=t+1}^{T} \Gamma\left(\kappa\sigma \sum_{j=0}^{T-i} \eta_j \mu^{T-j-i} + 1 \right).$$

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評價溫度衍生性商品 — 以臺灣為例

張傳章*
國立中央大學財務金融學系

楊曉文**
國立中央大學財務金融學系

黃子毓***
國立中央大學財務金融學系

黃志偉****
湖北經濟學院保險學系

摘 要

本文著重於溫度風險及探討何種分配最能捕捉臺灣溫度之動態行為。我們採用 Campbell 與 Diebold (2005) 模型捕捉臺灣溫度之特性及探討在不同機率分配之影響。我們發現標準 Gumbel 分配在樣本內外皆提供良好的配適與預測能力。此外，我們延伸 Cao 與 Wei (2004) 之評價方法並求得 HDD 與 CDD 之價格。最後，我們發現在不同機率分配假設下其對溫度衍生性商品影響十分顯著。

關鍵詞：溫度衍生性商品、均衡定價模型、日高溫度指數、日低溫度指數

* 張傳章，國立中央大學財務金融學系教授。
** 楊曉文，國立中央大學財務金融學系教授。
*** 黃子毓，國立中央大學財務金融學系碩士生。
**** 通訊作者：黃志偉，E-mail: jrwei.huang@icloud.com，湖北經濟學院保險學系副教授，430205 中國湖北省武漢市江夏區楊橋湖大道 8 號，電話：(86) 155-2715-6007。