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Valuation and analysis on complex equity indexed annuities

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ABSTRACT

Equity-indexed annuities (EIAs) are popular products that eliminate the downside risk while still providing upside potential. Among the three major categories of EIAs, ratchet EIAs are the most popular. Ratchet EIAs with quanto features emerge due to differences in asset returns across countries. The literature covers the pricing of the EIAs that are not quantos, and this paper fills the hole. In deriving pricing formulas, we add an exchange rate model as well as a foreign risk-free rate model to the pricing framework of Black and Scholes. Our formulas cover quanto ratchet EIAs for both compound and simple versions that may have a return cap and employ two types of geometric return averaging. The numerical analyses illustrate how contract features and market parameters affect contract values. The results also highlight the significance of quantos in contract pricing.

1. Introduction

The significance of longevity risk simulates sales of variable annuities. The people aware of the "risk" of living long lives demand annuities. Since longevity risk is un-diversifiable risk to insurance companies, they promote variable annuities that may generate higher returns in the long run than fixed annuities. The average sales of variable annuities have been about 180 billion dollars since 2005 (LIMRA, 2017; LIMRA, 2018; Marrion, 2006). Such products are popular in other regions such as Europe, Japan, Hong Kong, and Taiwan as well. For instance, the average sales of variable annuities in Japan have been about 14 billion dollars since 2007 (LIAJ, 2017) while the average annual growth rate starting from 2002 is 30% with 17 billion dollars of sales in 2018. The sales in Taiwan reach 17 billion US dollars in 2018 with compound annual growth rate (CAGR) of 30% starting from 2002 (TII, 2018).

Facing volatile financial markets, investors demand the products that eliminate the downside risk while keeping upside potential.[3] Equity-indexed annuities (EIAs) are such products. An EIA is a hybrid between a variable and a fixed annuity that allows the policyholder to participate in the potential appreciation of the stock market while eliminating the downside risk by a minimum return guarantee. In US, the sales of EIAs in 2016 are $61 billion dollars, an 11% increase over those in 2015, and the average annual growth

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[2] The author is grateful to the National Science Council of Taiwan for its financial supports (project number MOST 105-2410-H-004 -070 -MY3) and to University of California, Irvine for the kind supports during the visit of the author.
[3] Recent variable annuities pack various forms of guarantees. The guarantees are commonly referred to as GMxBs (i.e., guaranteed minimum benefits of type 'x' such as income benefit and withdraw benefit). Many papers study the valuation and risk management of variable annuities such as Bacinello et al. (2011), Gan and Lin (2015), Xu et al. (2018).

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rate starting from 2000 is 17%.[4]

The product designs of EIAs are diverse but can be divided into three major categories: point-to-point, ratchet, and look-back. The return of the point-to-point EIA is determined by the realized return of the linked index between two time points. Ratchet EIAs are more favorable because returns are credited periodically with a guaranteed minimum and the account value never decreases once the return is credited. A popular design of the look-back EIA is the high-water-mark that earns the highest return on the index attained during the life of the contract.

Ratchet EIAs are one of the most popular designs in the markets (Hardy, 2003). This type of products may vary in contract features such as reset frequency, return accumulation, return cap, and return averaging. Most ratchet EIAs have the annual-reset feature meaning that the return is credited to the contract annually. The annual return may be accumulated in two ways. The simple version of ratchet EIAs add the annual returns up to give the final payout while the compound version accumulate returns compoundedly. To reduce the costs of EIAs, the insurer may place a fixed upper limit, also called ceiling or cap, on the annual return. It may also employ an averaging scheme in calculating the annual return to reduce the volatility of credited returns and thus the costs of guarantees. For instance, an insurer may calculate the geometric average of the index return over several sub-periods as the annual return of the period.

The above descriptions about EIAs indicate that EIAs are sophisticated products to personal buyers. Gerrans and Yap (2014) investigate how sophisticated or naive individual may be when making retirement savings choices. Gerrans et al. (2018) further study individual and peer effects in retirement savings choices. Lin et al. (2017) analyze how financial literacy and information sources affect the demand for life insurance while Grohmann (2018) examine how financial literacy affect savings behaviors using data on the emerging Asian middle class.

The pricing and hedging of EIAs have been studied by researchers. Tiong (2000) derives closed-form solutions for the three major product designs in the standard Black-Scholes (B–S) framework (Black and Scholes, 1973).[5] Gerber and Shiu (2003) provid closed-form formulas for the look-back options and the dynamic guarantees embedded in EIAs. Lee (2003) proposes four designs of EIAs to increase participation rates and derives the associated pricing formulas. Hardy (2004) present a lattice method for valuing ratchet EIAs. Extending the B-S assumption of a constant risk-free rate to stochastic interest rates, Lin and Tan (2003) determine the fair participation rates for the three major designs of EIAs numerically under the Vasicek (1977) short rate model. Jaimungal (2004) assumes that the underlying index followed a geometric Variance-Gamma process and developed closed-form expressions for the prices of point-to-point and ratchet EIAs. Kijima and Wong (2007) adopt the extended Vasicek model and derive pricing formulas for several ratchet EIA products. Chiu et al. (2010) fill a hole left by Kijima and Wong (2007) through deriving the pricing formulas for the capped compound version with two geometric return averaging schemes in the B–S framework. Qian et al. (2010) conduct valuation of equity-indexed annuity under stochastic mortality and interest rates. Yuen and Yang (2010) provide the pricing of Asian-option–related EIAs under a regime switching model by the trinomial tree method. Recently, Cui et al. (2017) render the valuation of EIAs with cliquet-style guarantees in regime-switching and stochastic volatility models with jumps.

Our contribution to the literature in this paper is that we derive the pricing formulas for the ratchet EIAs that have the quanto feature. A contract is a quanto or cross-currency if the linked index is dominated in a different currency (e.g., Baxter and Rennie, 1996; Hull, 2017). For instance, a quanto contract may pay off in Australian dollar with the linked index of S&P 500 that is dominated in US dollar. The quanto feature is common in the derivatives market. The targeted customers include the people interested in international diversification for their portfolios and the people who live in the countries with less-developed capital markets and want to invest in more-developed markets.

Some recently developed variable (also called unit-linked in earlier era) products of life insurance and annuities incorporate this feature. More specifically, quanto EIAs are recently developed products out of the demands from the customers who would like to participate in higher returns offered in foreign countries than those offered in domestic markets. For instance, the annuity buyers in low-interest-rate countries like Japan and Taiwan cast their eyes on Australia and US in which interest rates are significantly higher for a long time.[6] The buyers in highly volatile markets may want to utilize the quanto feature to stabilize their annuity returns.[7] Many of these cases happen in less developed countries, which may be the reason why quanto ratchet EIAs are relatively new to academics and life insurance industries. Mathematically they are more complex than general ratchet EIAs since more stochastic processes and correlations need to be considered. This is another reason for their later development and signifies the difference between quanto and non-quanto products.

To incorporate the quanto feature, we add an exchange rate model as well as a foreign risk-free rate model to the pricing framework of Black and Scholes.[8] Using Girsanov's theorem and the martingale representation theorem, we rewrite the processes of the linked index and the exchange rate so that we may apply the risk-neutral valuation principle to obtain closed-form solutions. Our pricing formulas cover quanto ratchet EIA products with various features including both compound and simple versions that may

[4] Please see online reports on Advantage Compendium (http://www.indexannuity.org).

[5] Under the B–S framework, the linked index follows the geometric Brownian motion while the risk-free rate is assumed to be constant.

[6] The average yields of 10-year government bonds during the period from 2009 to 2018 in Japan, Taiwan, US, and Australia are 0.6%, 1.3%, 2.5%, and 3.7%, respectively (Bloomberg, 2018).

[7] For instance, the standard deviations of stock index returns during the period from 1998 to 2018 in Japan, Taiwan, Hong Kong, China, Australia, and US are 25%, 27%, 29%, 38%, 16%, and 17% respectively (Bloomberg, 2018).

[8] This is consistent with Tiong (2000), Gerber and Shiu (2003), Lee (2003), Hardy (2004), and Chiu et al. (2010). Further reasons for not adopting stochastic interest rates are rendered in footnote 6.

have return caps and employ two types of geometric return averaging. The first, obvious application of our formulas is assessing the values of the ratchet EIA products with the quanto feature. Secondly, these formulas enable actuaries to analyze easily the impacts of various contract features as well as market parameters on the contract value. Thirdly, life insurers can employ these formulas to construct appropriate hedging portfolios for quanto ratchet EIA products.

We in this paper employ the derived formulas to analyze how contract features and market parameters may affect the contract values. Their effects intertwine with each other indeed. We further learn from these numerical demonstrations the importance of the quanto feature in determining the contract value. The price of a quanto ratchet EIA might deviate from that of a non-quanto one by 4.8% under normal market conditions. The deviation could reach 6.7% when the foreign exchange market exhibit high volatilities and high correlations with the linked investment market. Insurance companies therefore should pay close attention to the cost and risk of the quanto feature.

The rest of the paper is organized as follows. In Section 2 we delineate the quanto ratchet EIA contracts under consideration and set up the risk-neutral pricing framework. Closed-form solutions for the considered contracts are derived in Section 3. Section 4 contains numerical demonstrations on how contract features and market parameters affect contract values. Conclusions and remarks are presented in Section 5.

2. Product specification and valuation framework

2.1. Product specification

The underlying variable in pricing ratchet EIAs is the annual return calculated based on the linked index. Let T be the maturity of an EIA contract and $S(t)$ be the linked index at $t \leq T$. Then the annual return of the linked index over the t^{th} year would be:

$$R_t = \frac{S(t)}{S(t-1)}, \qquad t = 1, 2, \cdots, T \tag{1}$$

Insurers often take averages of the index returns over sub-periods of a year when calculating the annual return to reduce the guarantee costs through dampening the return volatility. We analyze two types of geometric averaging in this paper.[9] In the first case (which we refer as G1 hereafter), the annual return of the t^{th} year,$R_{t,\ G1}$, is the geometric average of the indexes sampled with the interval of $1/m$ year. That is,

$$\boldsymbol{R_{t,G1}} = \left[\prod_{i=0}^{m-1} \frac{\boldsymbol{S\left(t-1+\frac{i+1}{m}\right)}}{\boldsymbol{S\left(t-1+\frac{i}{m}\right)}}\right]^{\frac{1}{m}} \tag{2}$$

In the second case (referred as G2 hereafter), the annual return of the t^{th} year denoted by $R_{t,\ G2}$is[10]:

$$\boldsymbol{R_{t,G2}} = \left[\prod_{i=0}^{m-1} \frac{\boldsymbol{S\left(t-1+\frac{i+1}{m}\right)}}{\boldsymbol{S(t-1)}}\right]^{\frac{1}{m}} \tag{3}$$

The next step after calculating the annual return is to calculate the return to be credited to the contract each year. The general formula is as follows:

$$\widetilde{R}_t = 1 + min(max(\alpha(R_{t,\cdot} - 1), f), c) \tag{4}$$

where $R_{t,\cdot}$ denotes the annual return of the t^{th} year with or without geometric averaging, α is the participation rate in the linked index, f represents the minimum guaranteed return rate (also called the floor rate), and c stands for the cap rate. The participation rate is usually less than 100%, which is reasonable in the sense that investors sacrifice some of the upside potential for the downside protection of the minimum guarantee. When $f = 0\%$, the product provides a principal/premium guarantee. The cap rate or ceiling rate c is the maximum rate that can be credited each year. Placing a cap on the credited return is a direct way to reduce the product cost.

The annual return credited to the policy can be accumulated in two ways. For the compound version of ratchet EIAs, the total return at maturity T is calculated as:

$$R^{CR} = \prod_{t=1}^{T} \widetilde{R}_t \tag{5}$$

The version that simply adds up returns, which often referred as simple ratchet EIAs, would pay out

[9] We do not consider arithmetic averaging for two reasons. Firstly, the annual return calculated using the arithmetic averaging scheme is the sum of lognormal random variables. It is well known that the options based on the sum of lognormal random variables have no closed-form pricing formulas under the B–S model (Kemma and Vorst, 1990). Secondly, the closed-form pricing formulas for options based on lognormal random variables can serve as effective control variates in pricing arithmetic-averaging-based options using the Monte Carlo algorithm (Kemma and Vorst, 1990). In other words, the pricing formulas derived later in this paper will be useful in pricing the arithmetic-averaging EIAs.

[10] Note that Eq. (1) can be deemed as the special case of setting $m = 1$ in Eq. (2) and Eq. (3) that means no return averaging.

$$R^{SR} = 1 + \sum_{t=1}^{T} (\widetilde{R}_t - 1) = 1 - T + \sum_{t=1}^{T} \widetilde{R}_t \tag{6}$$

at maturity T for an initial premium of \$1 at time 0.

2.2. Risk-neutral valuation

To take the quanto feature into account, we add an exchange rate model as well as a foreign risk-free rate model to the pricing framework of Black and Scholes. The linked index $S(t)$ and exchange rate $C(t)$ are assumed to follow geometric Brownian motions, and the interest rate r (for local currency) and r_f (for foreign currency) are assumed to be constants.[11] More specifically,[12]

$$\frac{dS(t)}{S(t)} = \mu_S dt + \sigma_S dz_1(t),\ \frac{dC(t)}{C(t)} = \mu_C dt + \sigma_C[\rho dz_1(t) + \bar{\rho} dz_2(t)],\ \frac{dB(t)}{B(t)} = rdt,\ \frac{dD(t)}{D(t)} = r_f dt \tag{7}$$

where σ_S is the volatility of the linked index, σ_C is the volatility of the exchange rate, ρ represents the correlation coefficient of $log(S(t))$ and $log(C(t))$, $\bar{\rho} = \sqrt{1-\rho^2}$ indicates the orthogonal complement of ρ, and $z_i(t)$, $i = 1, 2$ denote independent Brownian motions. $B(t)$and$D(t)$ denote the domestic and foreign money market accounts, respectively.

The model defined in (7) is called the Black-Scholes quanto model (Baxter and Rennie, 1996). To make the model more concrete, we assume a case in which the local currency is Australian dollar and the linked index is denominated in US dollar. The model thus have three tradable assets in terms of Australian dollar: the Australian dollar cash bond$B(t)$, the Australian dollar worth of the US-dollar denominated bond$C(t)D(t)$, and the Australian dollar worth of the linked index$C(t)S(t)$.

Based on Girsanov's theorem and the martingale representation theorem (see, for example, Bjork (2004)), there exists a unique measure Q under which both the discounted processes $\frac{C(t)D(t)}{B(t)}$ and $\frac{C(t)S(t)}{B(t)}$ are martingales. The processes $S(t)$ and $C(t)$ under Q can then be written as:

$$\frac{dS(t)}{S(t)} = (r_f - \rho\sigma_S\sigma_C)dt + \sigma_S d\bar{z}_1(t),\ \frac{dC(t)}{C(t)} = (r - r_f)dt + \sigma_C[\rho d\bar{z}_1(t) + \bar{\rho} d\bar{z}_2(t)] \tag{8}$$

where $\bar{z}_1(t)$and $\bar{z}_2(t)$ are independent Brownian motions under measure Q.

According to the risk-neutral valuation principle (see, for example, Harrison and Kreps (1979) and Harrison and Pliska (1981)), the no-arbitrage price of the EIA contracts can be represented as:

$$V^* = E_Q[e^{-rT} R^*] \tag{9}$$

where $E_Q[\cdot]$denotes the expectation operator under measure Q and the asterisk may be CR or SR.

3. Pricing formulas

3.1 Quanto ratchet EIAs with G1 return averaging

3.1.1 Simple quanto ratchet EIAs

Under risk neutral measure Q, it is well known (e.g., Hull, 2017) that $log(R_{t,\ G1})$ are independent normal random variables with mean $\mu_{G1} = {}^{1}\!/_{m}\left(r_f - \rho\sigma_S\sigma_C - {}^{\sigma_S^2}\!/_{2}\right)$ and ${\sigma_{G1}}^2 = {\sigma_S}^2/m^2$. To compute R^*, a function of $\widetilde{R}_t$, we first rearrange Eq. (4) as:

$$\widetilde{R}_t = (1-\alpha) + \alpha min(max(f_\alpha, R_t), c_\alpha) \tag{10}$$

where $f_\alpha = 1 + f/\alpha$ and $c_\alpha = 1 + c/\alpha$. Then we set

[11] We do not consider stochastic interest rates in this paper for three reasons. Firstly, interest rates probably have little impact on the contract value since the payoffs of ratchet EIAs do not depend on interest rates; this is partly confirmed by the numerical results in Kijima and Wong (2007). Secondly, the models with stochastic short rates and those with constant interest rates give the same pricing formulas when the index return and short rate are driven by independent Brownian motions. More specifically, let $g(S(t): t \le T)$ be the payoff of a ratchet EIA, r_t be the short rate process, and $P(0,T)$ be the price of zero-coupon bond paying a unit amount at time T. The price of a ratchet EIA product V, under stochastic interest rates, is equal to: $E[e^{-\int_0^T r_t dt} g(s(t): t \le T)] = E[e^{-\int_0^T r_t dt}]E[g(s(t): t \le T)] = P(0,T)E[g(s(t): t \le T)]$

On the other hand, V under the constant interest rate assumption is:

$$E[e^{-rT} g(s(t): t \le T)] = e^{-rT} E[g(s(t): t \le T)] = P(0,T)E[g(s(t): t \le T)]$$

when we make the common assumption (see Hull, 2017, for more detail) that both interest rate models calibrate their parameters to fit the current price $P(0,T)$. Thirdly, the pricing formulas are computationally inefficient when interest rates are stochastic. A rule of thumb in high-dimensional integral problems (such as the problem of pricing quanto ratchet EIAs under stochastic interest rates) is to use the Monte Carlo type of algorithms when the maturity of the zero-coupon bond is longer than 3 periods. It is therefore more suitable to use numerical methods instead of pricing formulas for the valuation of quanto ratchet EIAs when interest rates become stochastic. Considering the aforementioned three reasons and that our goal is to provide closed-form formulas for effective contract valuation and contract analysis, we stick to the assumption of constant interest rates.

[12] We are aware of the model risk of using these simple models in pricing EIAs as Chung, Lee, and Wu (2002) demonstrate. This is an inevitable tradeoff between obtaining closed-form pricing formulas and using market-consistent models.

$$X_{t,G1} = min(max(f_\alpha, R_{t,G1}), c_\alpha) \tag{11}$$

We can see that $X_t's$ are independent censored lognormal random variables with censored values f_α and c_α.
Rewriting Eq. (6) using (10) and then substituting into (9), we obtain.

$$V_{G1}^{SR} = e^{-rT} E_Q[R_{G1}^{SR}] = e^{-rT}[1 - \alpha T + \alpha T E_Q(X_{1,G1})] \tag{12}$$

It then remains to compute $E_Q(X_{1,\ G1})$. We first write

$$E_Q(X_{1,G1}) = f_\alpha P(R_{1,G1} \le f_\alpha) + E_Q[R_{1,G1}: f_\alpha \le R_{1,G1} \le c_\alpha] + c_\alpha P(R_{1,G1} \ge c_\alpha) \tag{13}$$

Representing $R_{t,G1}$ as

$$\exp[\mu_{G1} + \sigma_{G1} N(0,1)] \tag{14}$$

and letting

$$d_{1,G1} = \frac{\log f_\alpha - \mu_{G1}}{\sigma_{G1}} \tag{15}$$

$$d_{2,G1} = \frac{\log c_\alpha - \mu_{G1}}{\sigma_{G1}} \tag{16}$$

we obtain.

$$P(R_{1,G1} \le f_\alpha) = P(N(0,1) \le d_{1,G1}) = \Phi(d_{1,G1}),\ P(R_{1,G1} \ge c_\alpha) = P(N(0,1) \ge d_{2,G1}) = \Phi(-d_{2,G1}) \tag{17}$$

And

$$E_Q[R_{1,G1}: f_\alpha \le R_{1,G1} \le c_\alpha] = e^{\mu_{G1} + \frac{1}{2}\sigma_{G1}^2}[\Phi(d_{2,G1} - \sigma_{G1}) - \Phi(d_{1,G1} - \sigma_{G1})] \tag{18}$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal random variable.
Combining Eq. (17) and Eq. (18), we get the explicit formula for $E_Q(X_{1,\ G1})$:

$$E_Q(X_{1,G1}) = f_\alpha \Phi(d_{1,G1}) + c_\alpha \Phi(d_{2,G1}) + e^{\mu_{G1} + \frac{1}{2}\sigma_{G1}^2}[\Phi(d_{2,G1} - \sigma_{G1}) - \Phi(d_{1,G1} - \sigma_{G1})] \tag{19}$$

Substituting Eq. (19) into Eq. (13), we obtain the following proposition.
Proposition 1
The time-0 *price of a T*-year simple quanto ratchet EIA with the G1 averaging scheme is given by:

$$V_{G1}^{SR} = e^{-rT}\left\{1 - \alpha T + \alpha T[f_\alpha \Phi(d_{1,G1}) + c_\alpha \Phi(-d_{2,G1})], + \alpha T e^{\mu_{G1} + \frac{1}{2}\sigma_{G1}^2}[\Phi(d_{2,G1} - \sigma_{G1}) - \Phi(d_{1,G1} - \sigma_{G1})]\right\} \tag{20}$$

where $d_{1,\ G1}$ and $d_{2,\ G1}$ are defined as in Eq. (15) and Eq. (16).

3.1.2 Compound quanto ratchet EIAs

Following the same approach as in the previous section, Eq. (9) can be rewritten as

$$V_{G1}^{CR} = e^{-rT}[1 - \alpha + \alpha E_Q(X_{1,G1})]^T \tag{21}$$

The result below follows by substituting Eq. (19) into (21).
Proposition 2
The time-0 *price of a T*-year compound quanto ratchet EIA with the G1 averaging scheme is given by:

$$V_{G1}^{CR} = e^{-rT}\left\{1 - \alpha + \alpha\left[f_\alpha \Phi(d_{1,G1}) + c_\alpha \Phi(-d_{2,G1}) + e^{\mu_{G1} + \frac{1}{2}\sigma_{G1}^2}(\Phi(d_{2,G1} - \sigma_{G1}) - \Phi(d_{1,G1} - \sigma_{G1}))\right]\right\}^T \tag{22}$$

where $d_{1,\ G1}$ and $d_{2,\ G1}$ are defined as in Eq. (15) and Eq. (16).

Note that the contracts without return averaging are special cases of those that have return averaging schemes with $m = 1$. Furthermore, the product with no cap can be deemed as a special case of the capped product with $c \to \infty$. Our formulas are rather general and applicable to the products with fewer/simpler features.

3.2 Quanto ratchet EIAs with G2 return averaging

We first rewrite the annual return defined in Eq. (3) as:

$$\boldsymbol{R_{t,G2}} = \left[\prod_{i=1}^{m} \frac{S\left(t - 1 + \frac{i}{m}\right)}{S(t-1)}\right]^{\frac{1}{m}} = \left[\frac{S(t-1+1/m)}{S(t-1)} \cdot \frac{S(t-1+2/m)}{S(t-1)} \cdots \frac{S(t)}{S(t-1)}\right]^{\frac{1}{m}} = \left[\prod_{i=1}^{m} Y_i\right]^{\frac{1}{m}} \tag{23}$$

Each Y_i follows the lognormal distribution with mean $\frac{i}{m}\left(r_f - \rho\sigma_S\sigma_C - \frac{\sigma_S^2}{2}\right)$ and variance $\frac{k}{m}\sigma_S^2$.

Since Y_i's are not independent of each other, we need to make transformations. Set $Z_1 \equiv log(Y_1)$, $Z_2 \equiv log(Y_2) - log(Y_1)$, $\cdots$, $Z_m = log(Y_m) - log(Y_{m-1})$. We observe that Z_i's are non-overlapping Brownian motion increments and are thus independent and normally distributed with mean $\frac{1}{m}\left(r_f - \rho\sigma_S\sigma_C - \frac{\sigma_S^2}{2}\right)$ and variance $\frac{1}{m}\sigma_S^2$. Taking log on both sides of Eq. (23), we obtain:

$$log\,R_{t,G2} = \frac{1}{m}\sum_{i=1}^{m} log\,Y_i = \frac{1}{m}[Z_1 + (Z_1 + Z_2)+\cdots+(Z_1 + Z_2+\cdots+Z_m)] \tag{24}$$

It then follows that $logR_{t,\ G2}$ are independent normal random variables with mean $\mu_{G2} = \frac{m+1}{2m}\left(r_f - \rho\sigma_S\sigma_C - \frac{\sigma_S^2}{2}\right)$ and variance $\sigma_{G2}^2 = \frac{(m+1)(2m+1)}{6m^2}\sigma_S^2$.

Defining $X_{t,\ G2} = min\,(max(f_\alpha, R_{t,\ G2}), c_\alpha)$ and then employing the same logics in deriving the previous propositions, we obtain the pricing formulas for quanto EIA contracts with the G2 return averaging as follows.

Proposition 3

The pricing formula for the simple quanto ratchet EIAs with the G2 averaging scheme is:

$$V_{G2}^{SR} = e^{-rT}\begin{Bmatrix} 1 - \alpha T + \alpha T\,[f_\alpha\Phi(d_{1,G2}) + c_\alpha\Phi(-d_{2,G2})] \\ + \alpha T e^{\mu_{G2}+\frac{1}{2}\sigma_{G2}^2}[\Phi(d_{2,G2} - \sigma_{G2}) - \Phi(d_{1,G2} - \sigma_{G2})] \end{Bmatrix} \tag{25}$$

where

$$d_{1,G2} = \frac{log f_\alpha - \mu_{G2}}{\sigma_{G2}} \tag{26}$$

$$d_{2,G2} = \frac{log\,c_\alpha - \mu_{G2}}{\sigma_{G2}} \tag{27}$$

$$\mu_{G2} = \frac{m+1}{2m}\left(r_f - \rho\sigma_S\sigma_C - \frac{\sigma_S^2}{2}\right) \tag{28}$$

$$\sigma_{G2}^2 = \frac{(m+1)(2m+1)}{6m}\sigma_S^2 \tag{29}$$

Proposition 4

The pricing formula for the compound quanto ratchet EIAs with the G2 averaging scheme is:

$$\boldsymbol{V_{G2}^{CR} = e^{-rT}\left\{1 - \alpha + \alpha\left[f_\alpha\Phi(d_{1,G2}) + c_\alpha\Phi(-d_{2,G2}) + e^{\mu_{G2}+\frac{1}{2}\sigma_{G2}^2}(\Phi(d_{2,G2} - \sigma_{G2}) - \Phi(d_{1,G2} - \sigma_{G2}))\right]\right\}^T} \tag{30}$$

where $d_{1,\ G2}$, $d_{2,\ G2}$, μ_{G2} and ${\sigma_{G2}}^2$ are defined as in Eq. (26) to Eq. (29).

The formulas for G2 and G1 averaging schemes do look alike. This is reasonable since they differ from each other only in how to average intra-year returns. However, the derivations for propositions 3 and 4 need an additional insight/trick to make dependent intra-year returns independent. We thus maintain separate derivations for both sets of propositions to better illustrate the differences between G1 and G2 averaging schemes.

4. Numerical illustrations

4.1 Valuation example

Consider a ratchet EIA contract denominated in Australian dollars and sold in Taiwan. A typical contract usually has maturity 3 to 7 years. We thus select $T = 5$ years. We set annual ceiling rate $c = 30\%$, annual floor rate $f = 0\%$, participate rate $\alpha = 100\%$. We also specify the number of averaging in a year $m = 4$ when applicable.

Assume that the linked index of the contract is S&P 500. This contract thus has the quanto feature because it pays off in Australian dollar and the linked index is S&P 500 that is dominated in US dollar. Using the monthly data from January 2000 to June 2010, we estimate the annual volatility and correlation parameters as: $\sigma_S = 16.47\%$ (the volatility of S&P 500), $\sigma_C = 13.84\%$ (the volatility of the exchange rate USD/AUS), and $\rho = -0.52$ (the correlation coefficient of $log(S(t))$ and $log(C(t))$. We use the 5-year Australian treasury rate on June 30, 2010 as the proxy of the risk free rate r. It was 4.78%, and the 5-year risk free rate of US dollar r_f was 1.83%.

We will use the above settings to illustrate how contract features and market parameters may affect the value of the contract. For each set of parameters we examine six product specifications: simple version with no averaging (SR), compound version with no averaging (CR), simple version with the G1 averaging scheme (SR_G1), compound version with the G1 averaging scheme (CR_G1), simple version with the G2 averaging scheme (SR_G2), and compound version with the G2 averaging scheme (CR_G2). The contract values with these six specifications under the above settings (i.e., $T = 5$ years, $c = 30\%$, $f = 0\%$, $\alpha = 100\%$, $m = 4$ (when applicable), $\sigma_S = 16.47\%$, $\sigma_C = 13.84\%$, $\rho = -0.52$, $r = 4.78\%$, and $r_f = 1.83\%$) are reported in Table 1.

Table 1 shows that CR has the largest contract value of $113.69 while SR_G1 has the lowest one ($86.26). It further shows that compound versions of contracts have higher values than simple versions, ceteris paribus: $113.69 vs. $108.75, $86.55 vs. $86.26, and $102.23 vs. $99.84. This implies that the compound returns generated under the current settings are higher than simple returns. We

Table 1
Contract values for six product designs

	Contract value		
Averaging scheme / accumulation method	None	G1	G2
Simple return (SR)	108.75	86.26	99.84
Compounded return (CR)	113.69	86.55	102.23

also observe from Table 1 that return averaging decreases the contract value: \$113.69 vs. \$86.55 and \$102.23 for the compound version contracts and \$108.75 vs. \$86.26 and \$99.84 for the simple version. This is as expected because return averaging reduces the volatility of the annual return and thus reduces the value of the embedded options. The first type of averaging scheme produces a lower contract value than the second type, ceteris paribus, because the former averages over non-overlapping sub-periods and has a stronger averaging effect as a result.

4.1.1. Return cap c.

The values of the contract with various return caps were shown in Fig. 1 and Table 2. The contract value increases with the return cap, as expected, because imposing an upper bound on the creditable return truncates the upside potential. The value increases at a diminishing rate (i.e., all curves are concave). This is reasonable because the probability of hitting the return cap decreases at an increasing rate when the cap rises, as long as the probability density of positive returns is a decreasing function of returns.

We observe from Table 2 that the return cap has the greatest impact on the contract with no return averaging while has the least impact on the contract with the G1 averaging scheme. More specifically, the percentage change of the contract value given a change in the return cap is the largest when there is no return averaging and is the smallest when returns are averaged by the first type of scheme. The underlying reason is that the probability of hitting the upper bound is lower with a stronger return averaging scheme when the cap is raised. Stronger return averaging thus produces a smaller value increase when raising the cap.

Table 2 also shows that the return cap has more impact on the compound version than on the simple version. The percentage change in the value of the compound version contract given a change in the return cap is larger than that of the simple version, ceteris paribus. This is comprehensible since the compound version generates higher returns in our current settings and is thus bounded more by return caps.

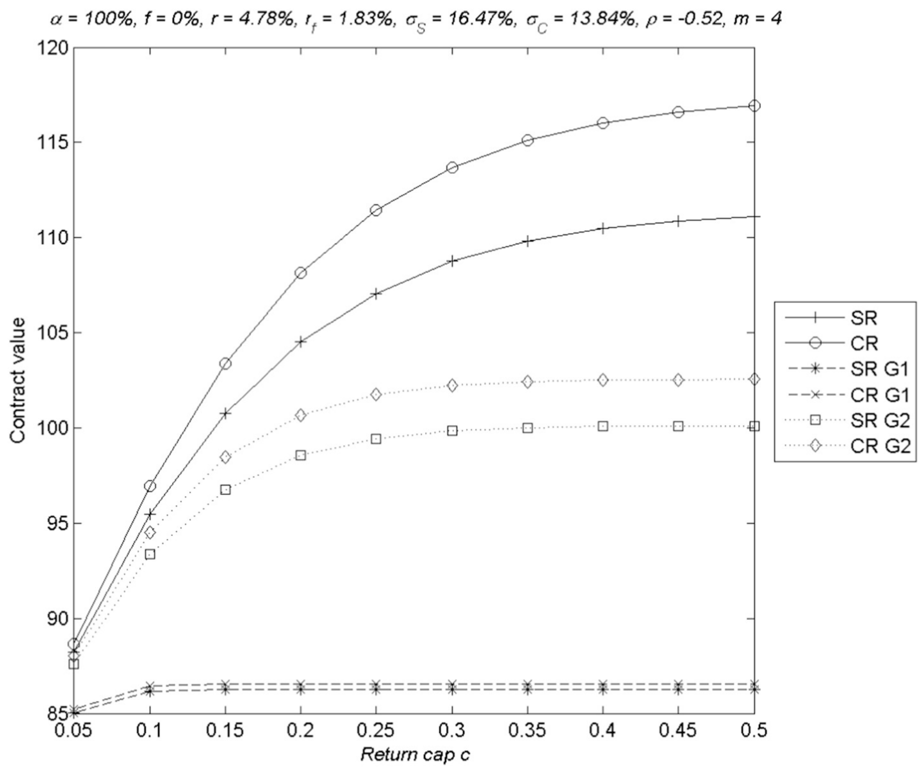

Fig. 1. Impact of return cap on the contract value *V*.

Table 2
Impact of return cap on the contract value V

Return accumulation and averaging scheme	Return cap (c)							
	0.1	0.2	$\frac{V(0.2)-V(0.1)}{V(0.1)}$	0.3	$\frac{V(0.3)-V(0.2)}{V(0.2)}$	0.4	$\frac{V(0.4)-V(0.3)}{V(0.3)}$	No Cap
SR	95.45	104.52	10%	108.75	4%	110.49	2%	111.44
CR	96.93	108.13	12%	113.69	5%	116.04	2%	117.34
SR_G1	86.17	86.26	0%	86.26	0%	86.26	0%	86.26
CR_G1	86.46	86.55	0%	86.55	0%	86.55	0%	86.55
SR_G2	93.37	98.57	6%	99.84	1%	100.07	0%	100.11
CR_G2	94.50	100.67	7%	102.23	2%	102.51	0%	102.56

4.2.2 Return floor rate f

The value of the contract increases with the return floor as Fig. 2 and Table 3 show. This is understandable since the return floor provides protection for the downside risk. Furthermore, the value increases at an increasing rate. This is because the probability of hitting the lower bound increases at an increasing rate when the lower bound rises, provided that the probability density of the returns is an increasing function for the returns that are lower than the floors.

We observe from Table 3 that return floor has the least impact on the contract without return averaging and has the greatest impact on the contract with the G1 averaging, given the same way of return accumulation. More specifically, the percentage change of the contract value given a change in the return floor is the smallest when there is no return averaging and is the largest when returns are averaged by the first type of scheme. The underlying reason is that the value contributed by the return volatility decreases with the floor rate. The reduction in the contract value due to the volatility dampening of return averaging thus decreases with the floor rate as well. Therefore, we observe that the value increases the fastest/slowest with the floor rate for the contract with the strongest/weakest return averaging scheme.

Table 3 also shows that return floor has more impact on the compound version than on the simple version. The percentage change in the value of the compound version contract is larger than that of the simple version. The rationale is probably that the compound version suffers more from low returns than the simple version does under our parameter settings. The return floor truncates some of the low returns and thus creates more values for the compound version. The higher the floor is, the more benefit the compound version enjoys.

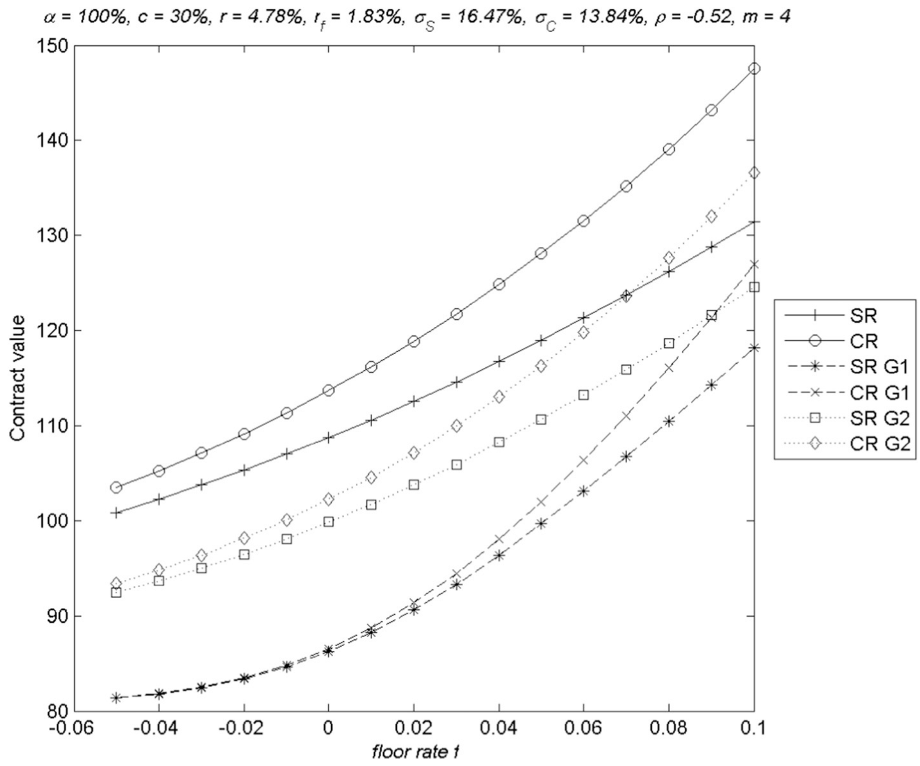

Fig. 2. Impact of return floor rate on the contract value.

Table 3
Impact of return floor on the contract value V

Return accumulation and averaging scheme	Return floor (f)						
	−0.02	0	$\frac{V(0)-V(-0.02)}{V(-0.02)}$	0.02	$\frac{V(0.02)-V(0)}{V(0)}$	0.04	$\frac{V(0.04)-V(0.02)}{V(0.02)}$
SR	105.32	108.75	3.26%	112.56	3.50%	116.75	3.72%
CR	109.16	113.69	4.15%	118.90	4.58%	124.83	4.99%
SR_G1	83.37	86.26	3.46%	90.63	5.07%	96.37	6.34%
CR_G1	83.48	86.55	3.67%	91.37	5.57%	98.03	7.28%
SR_G2	96.47	99.84	3.49%	103.77	3.93%	108.24	4.31%
CR_G2	98.14	102.23	4.17%	107.16	4.82%	113.00	5.45%

4.2.3 Participating rate α

Fig. 3 and Table 4 show that the value of the contract increases with the participation rate, as expected. It is interesting to see that the contract value is nearly linear to the participation rate especially when return averaging is present. The moderate concavity shown by the curves of SR_N and CR_N is because higher participation rates bring out more effects of the return cap and we see from Section 4.2.1 that the return cap has the greatest impact on the contract with no return averaging.

Table 4 demonstrates that participation has the greatest impact on the contract with no return averaging but has the least impact on the contract with the G1 averaging scheme. The rationale is that the participating rate amplifies/condenses the effect of return averaging since it is the multiplier to the annual return in Eq. (4). The reduction in the contract value due to return averaging thus increases with the participating rate.

Table 4 also demonstrates that the impact of participation is more significant on the compound version than on the simple version. This is because the compound version enjoys higher returns in our settings that is manifested by participation.

4.2.4 Return averaging frequency *m*.

Fig. 4 and Table 5 illustrate that the contract value decreases with the frequency of return averaging. This is because higher frequencies produce stronger averaging effects and reduce the volatilities of annual returns to a larger extent. The reduced volatilities then decrease the value of the options embedded in the ratchet EIA products.

Table 5 shows that averaging frequency has more impact on the G1 averaging scheme than on G2. Remember that G1 averages returns over non-overlapping sub-periods while G2 averages on cumulative returns of sub-periods. The marginal effect of increasing

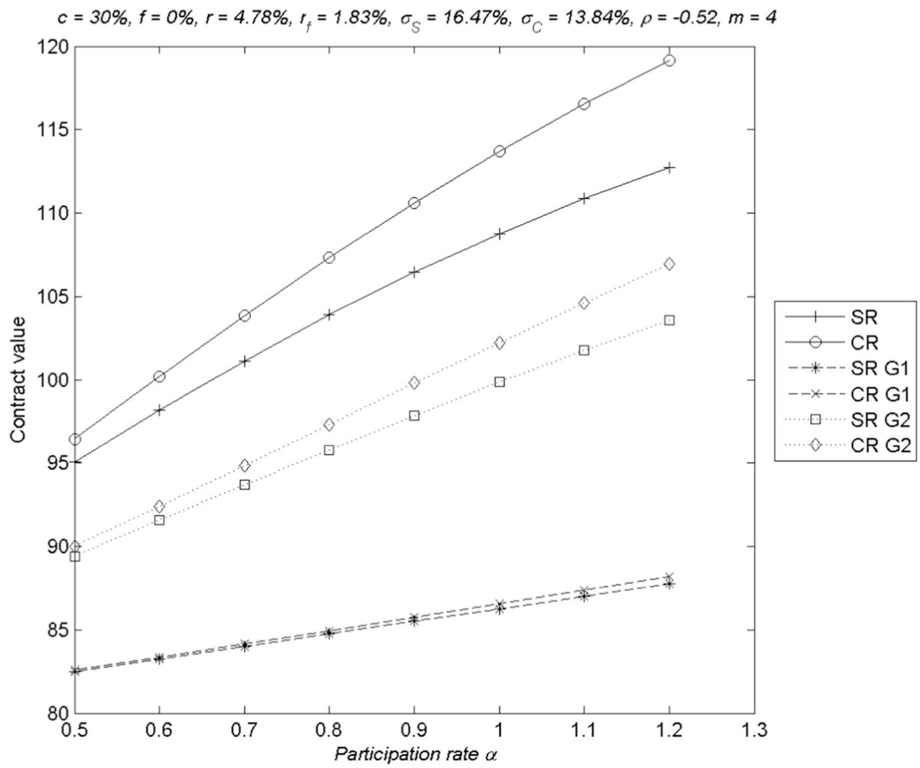

Fig. 3. The impact of participation rate on the contract value.

Table 4
Impact of participation rate on the contract value V

Return accumulation and averaging scheme	Participation rate (α)						
	0.6	0.8	$\frac{V(0.8)-V(0.6)}{V(0.6)}$	1	$\frac{V(1)-V(0.8)}{V(0.8)}$	1.2	$\frac{V(1.2)-V(1)}{V(1)}$
SR	98.17	103.91	5.84%	108.75	4.67%	112.74	3.67%
CR	100.19	107.33	7.13%	113.69	5.92%	119.15	4.80%
SR_G1	83.25	84.75	1.81%	86.26	1.77%	87.76	1.74%
CR_G1	83.36	84.94	1.90%	86.55	1.89%	88.18	1.89%
SR_G2	91.56	95.79	4.62%	99.84	4.24%	103.59	3.75%
CR_G2	92.42	97.33	5.31%	102.23	5.04%	106.93	4.60%

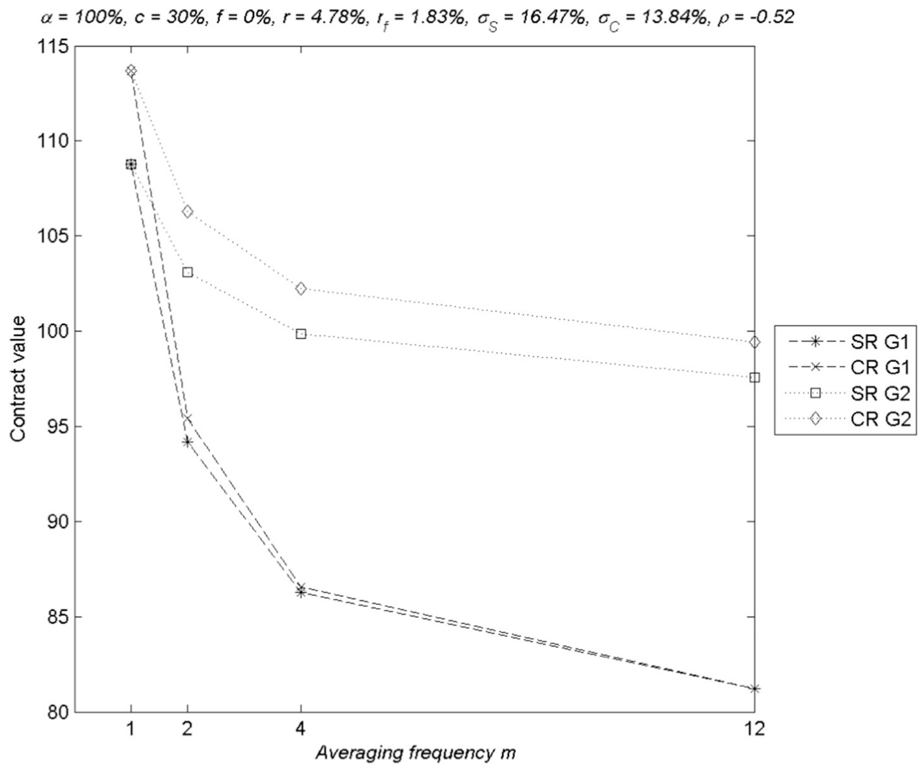

Fig. 4. Impact of return averaging frequency on the contract value.

Table 5
Impact of return averaging frequency on the contract value V

Return accumulation and averaging scheme	Averaging frequency (m)						
	1	2	$\frac{V(2)-V(1)}{V(1)}$	4	$\frac{V(4)-V(1)}{V(1)}$	12	$\frac{V(12)-V(1)}{V(1)}$
SR_G1	108.75	94.18	−13.40%	86.26	−20.69%	81.20	−25.33%
CR_G1	113.69	95.44	−16.05%	86.55	−23.87%	81.23	−28.55%
SR_G2	108.75	103.09	−5.21%	99.84	−8.19%	97.56	−10.30%
CR_G2	113.69	106.29	−6.51%	102.23	−10.08%	99.44	−12.53%

the number of sub-periods is thus larger for G1.

Table 5 also shows that averaging frequency reduces the differences in contract values between the compound and simple versions. The underlying reason is that more frequent averaging produces more stable annual returns around the mean return. Since the mean return is small and the maturity is short, the compound version and the simple versions result in similar values with high averaging frequencies. Another observation from Table 5 is that the impact of averaging frequency on the contract value is smaller for the simple version than for the compound version.

Table 6
Impact of maturity on the contract value

Return accumulation and averaging scheme	Maturity (T)		
	3	5	7
SR	106.45	108.75	109.75
CR	108.00	113.69	119.68
SR_G1	91.60	86.26	81.13
CR_G1	91.70	86.55	81.69
SR_G2	100.57	99.84	98.41
CR_G2	101.33	102.23	103.14

4.2.5 Contract maturity T.

Table 6 shows that the contract value increases with maturity when there are no return averaging. This is reasonable because the value of the embedded option increases with maturity. On the other hand, the first type of geometric averaging on the index returns reduces the contract value and causes the value decreases with maturity. The second type of return averaging is almost neutral and leaves the return accumulation method (simple versus compound) the role of determining the contract. Since compounding is more favorable to contract owners, we observe from Table 6 that the contract value of CR_G2 increases with maturity while SR_G2 makes contract value decrease with maturity.

4.3 Market parameter analyses

4.3.1 Volatility of the linked index

Fig. 5 shows that the value of the contract increases with the volatility of the linked index, which is as expected. The contract value increases at a diminishing rate due to the two constraints from the return cap and the return floor.

The volatility enlarges the differences in the contract values between the compound and the simple versions. This is probably because a higher volatility coupled with a return floor produces more high returns that benefit the compound version more.

The G1 averaging scheme enjoys the most value increase from the volatility increase while no return averaging benefits the least. The rationale might be that a stronger return averaging makes the return cap and floor be weaker boundary conditions and thus leave more room for the contract value to grow as the return volatility increases. Look at the examples of SR and CR. The return cap and

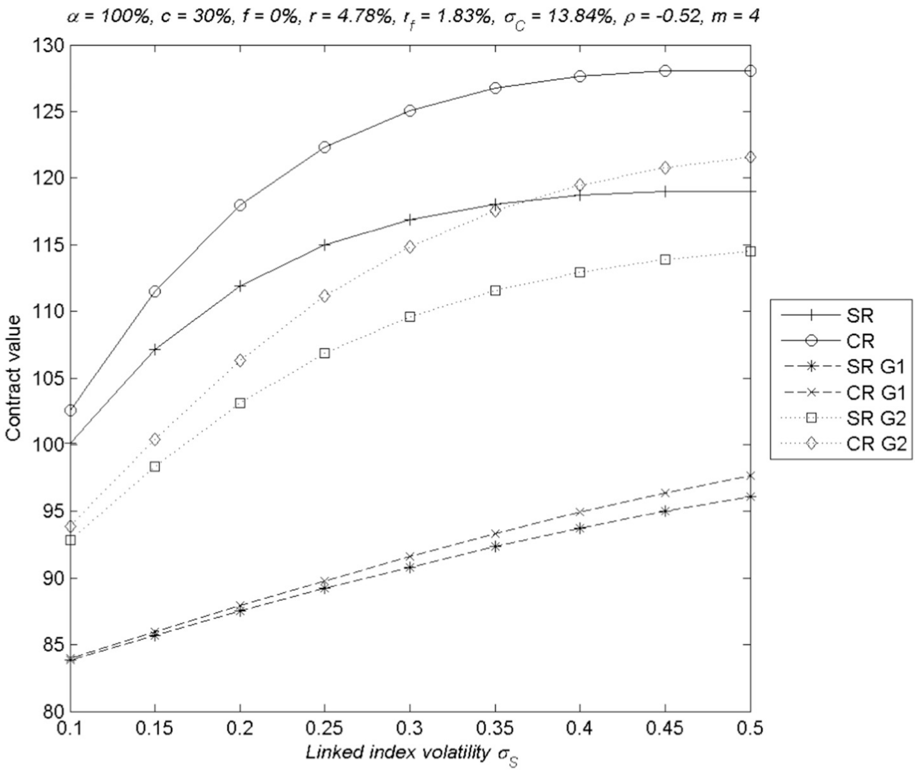

Fig. 5. Impact of the volatility of the linked index on the contract value.

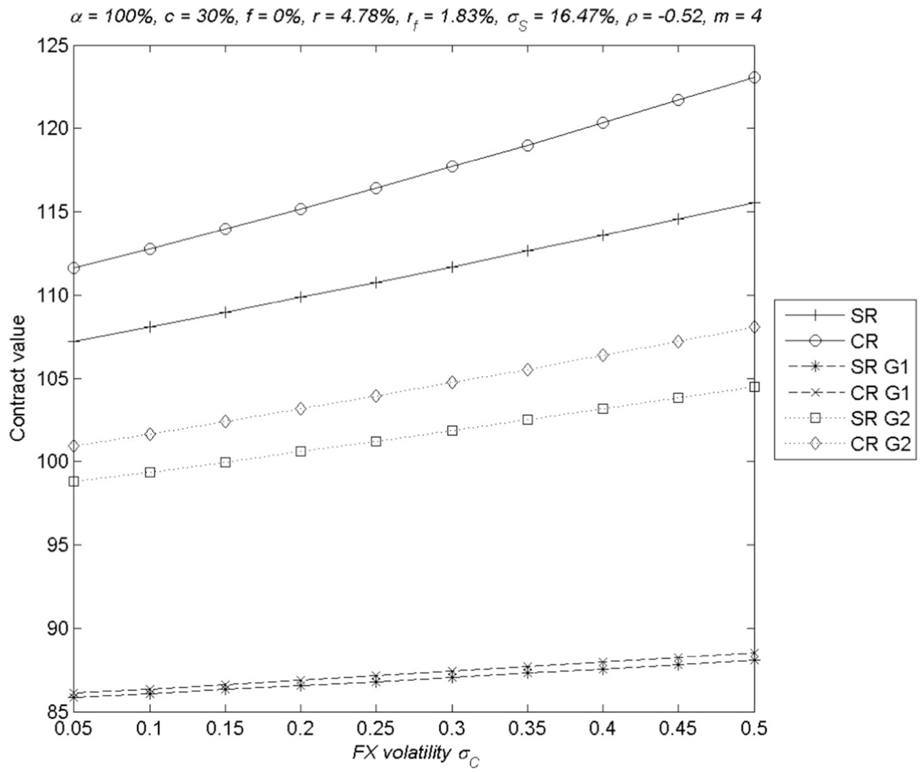

Fig. 6. Impact of the volatility of exchange rate σ_C on the contract value V.

floor soon become binding constraints when the volatility of the linked index reaches 30%. The contract values increase at minor rates thereafter.

4.3.2 Volatility of the exchange rate

The value of the contract increases with the volatility of exchange rate as Fig. 6 shows. It is interesting to see that the contract value looks like a linear function of the exchange rate volatility. The lack of non-linearity may be because the exchange rate volatility does not affect the variance of $log(R_t)$.[13] Fig. 6 further shows that the volatility of exchange rate seems to produce similar impacts on the contract values across return accumulation methods and return averaging schemes.

4.3.3 Correlation coefficient

The value of the contract decrease with the correlation coefficient of $log(S(t))$ and $log(C(t))$ as Fig. 7 illustrates. From the pricing formulas derived in section 3, we see that this correlation coefficient always appears together with the exchange rate volatility. We thus expect to see similar impacts on the contract value from both the correlation coefficient and the exchange rate volatility. Fig. 6 does show that the contract value looks like a linear function of the correlation coefficient and the impact of the coefficient are similar across return accumulation methods and averaging schemes, as in the cases of the exchange rate volatility.

Fig. 7 also demonstrates the importance of incorporating the quanto feature correctly. It depicts the possible magnitude of mis-pricing should the quanto feature be ignored, given the exchange rate volatility of 13.84%. The mis-pricing is more than 4.8% when ρ is at the historical level of −0.52. The magnitude of mis-pricing will increase further with the exchange rate volatility.

4.3.4 Domestic risk-free rate

Fig. 8 shows that the value of the contract decreases with the domestic risk-free rate *r*. This is reasonable because *r* acts as the discount rate in calculating the present value of the cash flow at maturity. The curves show little convexity since the contract maturity is merely 5 years. The impacts of *r* on the contract values look to be similar across return accumulation methods and return averaging schemes.

4.3.5 Foreign risk-free rate

Fig. 9 shows that the value of the contract increases with the foreign risk-free rate at a moderately increasing speed. This effect is the most appearing when there is no return averaging and is the least significant with the G1 return averaging. Fig. 9 further shows that the differences in the contract values between the compound and simple versions increase with the foreign risk-free rate.

[13] On the other hand, the linked index volatility affects the variance as well as the mean of $\log(R_t)$.

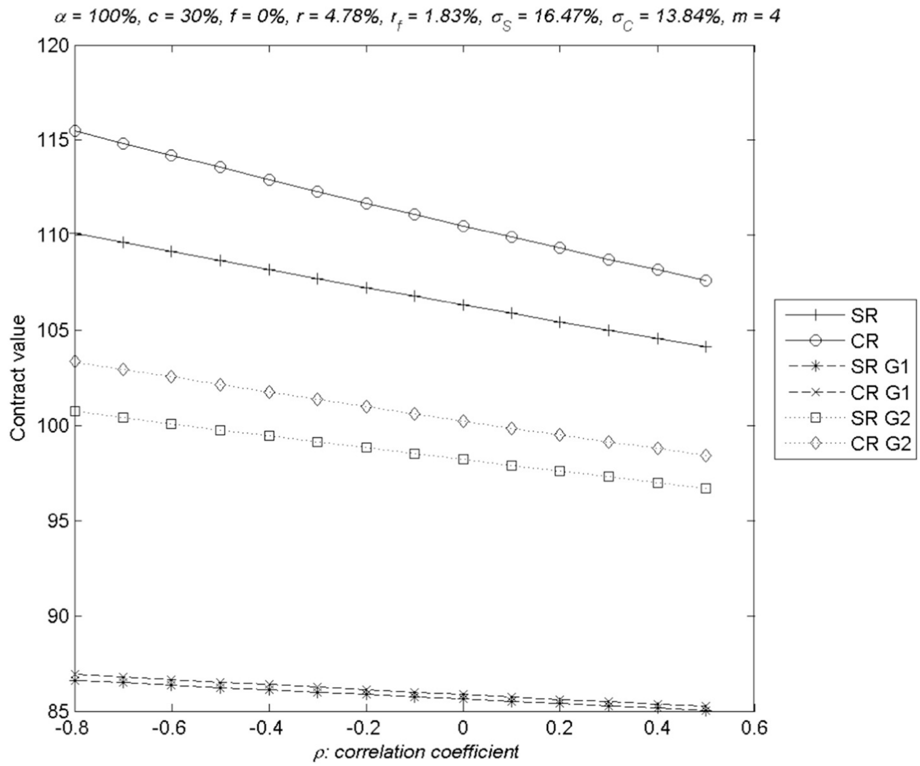

Fig. 7. Impact of the correlation coefficient ρ on the contract value V.

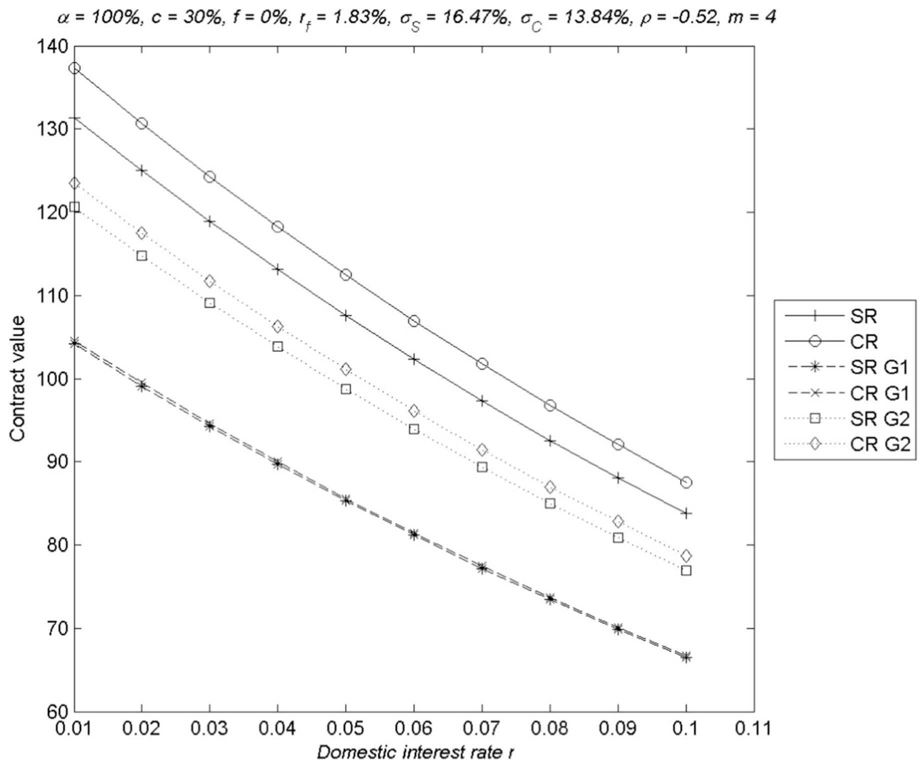

Fig. 8. Impact of the domestic risk free rate on the contract value.

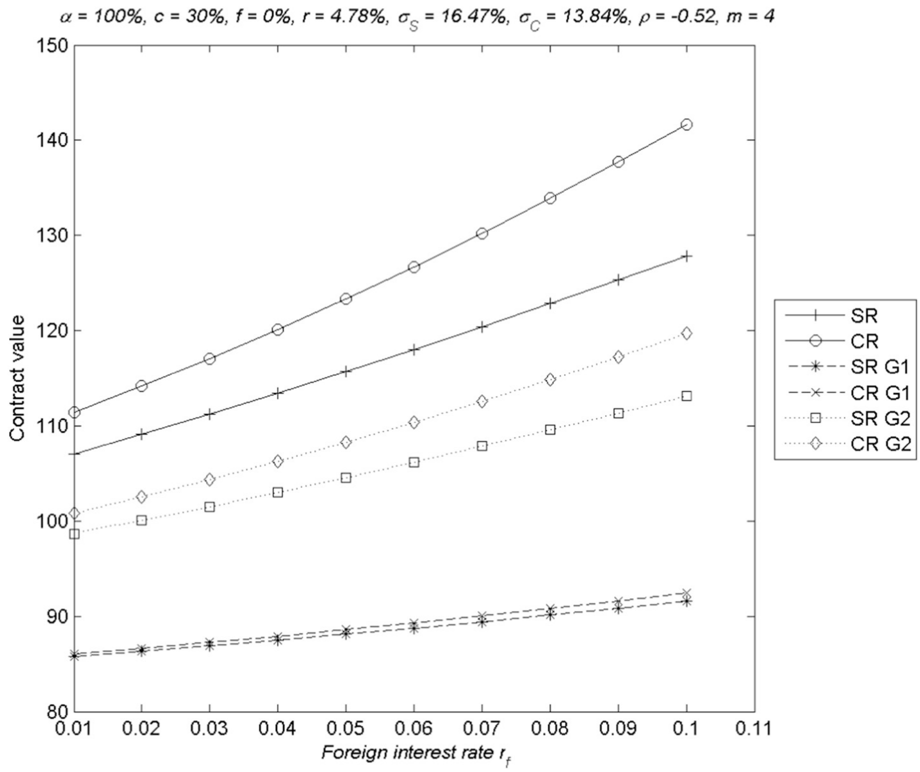

Fig. 9. Impact of the foreign risk free on the contract value

5. Conclusions

Annuities are essential to modern people who are exposed to longevity risk. Variable annuities may mitigate longevity risk by providing higher returns than fixed annuities, with the costs of larger return uncertainties. Quanto Ratchet EIAs render good features for the consumers who desire downside protection and international diversification while retain some upside potential. The ratchet design provides for the options-like properties and the quanto feature links to a foreign investment. The pricing and hedging of EIAs attract some research attention, but the studies have not covered the quanto feature yet. This paper fills the hole.

To take the quanto feature into account, we add an exchange rate model as well as a foreign risk-free rate model to the pricing framework of Black and Scholes. We derive pricing formulas that cover quanto ratchet EIA products with various features including both compound and simple versions that may have a return cap and employ two types of geometric return averaging. In addition to setting premiums, actuaries can employ these formulas to analyze the impacts of various contract features as well as market parameters on the contract value and construct appropriate hedging portfolios for the product.

We employed these formulas to conduct numerical analyses. The results showed that the value of a quanto ratchet EIA increases with the return cap at a diminishing rate, increases with the return floor at a increasing rate, increasing with the participating rate almost linearly, and decreases with the return averaging frequency if there is any. The impacts of these contract features/parameters on the contract value often differ across return accumulation methods and return averaging schemes.

We also analyze how market parameters may affect the contract value. The results demonstrated that the contract value increases with the return volatility at a diminishing rate, increases with the exchange rate volatility almost linearly, decreases with the correlation coefficient between the log returns of the linked index and exchange rate in a linear-like way, decreases with the domestic risk-free rate almost linearly, and increases with the foreign risk-free rate with a modest convexity. Alternative return accumulation methods and averaging schemes produce different impacts on the contract value when the market parameters change.

We observed from the numerical analyses the importance of the quanto feature in determining the contract value. The price of a quanto ratchet EIA might deviate from that of a non-quanto one by 4.8% under normal market conditions. The deviation could reach 6.7% when the foreign exchange market exhibit high volatilities and/or high correlations with the linked investment market. Actuaries therefore should carefully evaluate the cost and risk of the quanto feature.

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