| 考 試 科 目 | 近代物理 | 系所別 | 應用物理研究所 | 考試時間 | 2月 18日（一）第二節 |

1. [45 points] **Short answer questions**

5
(a) When a certain light source illuminates a metal surface, electrons are emitted from the metal with kinetic energies up to the value K. The light source is replaced with one that has the same wavelength but less intensity. With this new light source, does the upper limit of the electron kinetic energies increase, decrease, or remain the same? EXPLAIN YOUR ANSWER.

5
(b) A particle of mass M at rest decays into two identical particles each of mass $m = 0.100M$ that travel in opposite directions. What is the speed of these particles?

5
(c) At temperature T a body emits its most intense radiation at a wavelength of $5.6\,\mu m$. What is the wavelength of the most intense radiation emitted by the same body at temperature $4T$?

5
(d) A particle of mass m in an infinite square well with walls at $x = 0$ and $x = L$ is in a state with energy $E = 9\hbar^2\pi^2/(2mL^2)$. What is the probability that the particle is between $x = 0$ and $x = L/6$?

5
(e) A particle of mass m and spin zero is in a three-dimensional isotropic well described by a potential energy function $V(r) = \frac{1}{2}m\omega^2 r^2$, where $r^2 = x^2 + y^2 + z^2$. How many states have enegy $\frac{7}{2}\hbar\omega$?

5
(f) Including the electron spin, how many possible states are there for the $n = 4$ energy level of hydrogen? For the the $n = 5$ level?

5
(g) In a Stern-Gerlach type of experiment on an atom (such as boron) with a single $2p$ electron, into how many components would the beam be split?

5
(h) Explain how a p-n junction diode operates as a light-emitting diode (LED).

5
(i) The Fourier series expansion of a function $f(x)$ that is periodic with period 2π is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nx) + \sum_{n=1}^{\infty} b_n \sin(nx).$$

Consider the square wave given by the graph below. Write down all coefficients in the Fourier series expansion that are equal to zero.
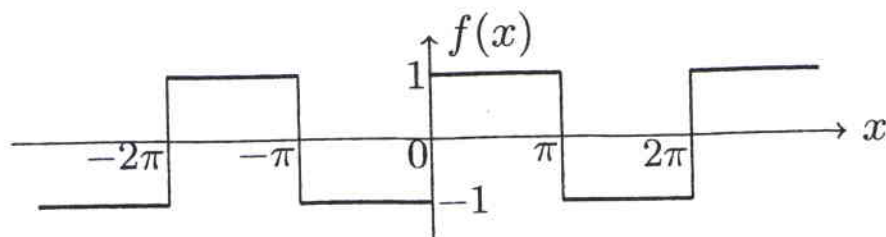
考 試 科 目	近代物理	系 所 別	應用物理研究所	考 試 時 間	2月 18日（一）第二 節

2. [*10 points*] **Relativity**

How fast and in what direction must Galaxy A be moving if an absorption line found at 550 nm (green) for a stationary galaxy is shifted to 450 nm (blue) for A? How fast and in what direction is Galaxy B moving if it shows the same line shifted to 750 nm (red)?

3. [*20 points*] **The Schrödinger equation**

In a certain region of space, a particle is described by the wave function

$$\psi(x) = Cxe^{-bx},$$

where C and b are real constants. By substituting into the Schrödinger equation, find the potential energy in this region and also find the energy of the particle.

4. [*25 points*] **Nonstationary states**

Consider a particle in a one-dimensional infinite square well of length L described initially ($t = 0$) by a wavefunction that is a superposition of the ground state ($\varphi_1(x)$) and the first excited state ($\varphi_2(x)$) of the well:

$$\Psi(x,0) = C\big[\varphi_1(x) + \varphi_2(x)\big],$$

where $\varphi_1(x)$ and $\varphi_2(x)$ are normalized.

(a) Find the value of C that normalizes $\Psi(x,0)$.

(b) Find $\Psi(x,t)$ at any later time t.

(c) Find the average energy $\langle E \rangle$ for $\Psi(x,t)$.

(d) Determine the uncertainty ΔE of energy for $\Psi(x,t)$.

備 註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考 試 科 目	普通物理	系 所 別	應用物理研究所	考 試 時 間	2 月 18 日（一）第一節

1. **(16%)** A block of mass 0.50 kg is pushed against a horizontal spring of negligible mass until the spring is compressed a distance x. The force constant of the spring is 800 N/m. When it is released, the block travels along a frictionless, horizontal surface to the bottom of a vertical circular track of radius R = 0.50 m, and continues to move up the track. The block's speed at the bottom of the track is v_{bot} = 10.0 m/s, and the block experiences an average friction force of 5.00 N while sliding up the track. (a) What is x? (b) If the block were to reach the top of the track, what would be its speed at that point? (c) Does the block actually reach the top of the track, or does it fall off before reaching the top? (d) What is the normal force exerted by the track on the block when the block reach top?
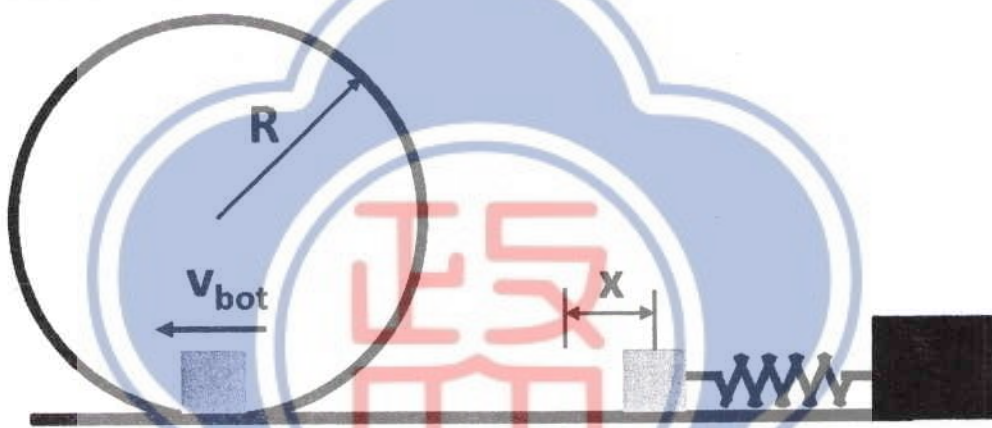

2. **(16%)** A uniform solid disk of radius R and mass M is free to rotate on a frictionless pivot through a point on its rim. If the disk is released from rest in the position shown by the copper-colored circle, (a) what is the moment of inertia of the disk about the pivot point? (if we know the moment of inertia of a disk is $MR^2/2$) (b) What is the angular speed when the disk reaches the position indicated by the dashed circle? (c) What is the speed of the lowest point on the disk in the dashed position? (d) If the disk is replaced by a uniform hoop, what is the angular speed when the hoop reaches the position indicated by the dashed circle? (Answer the question in terms of g, M, and R)
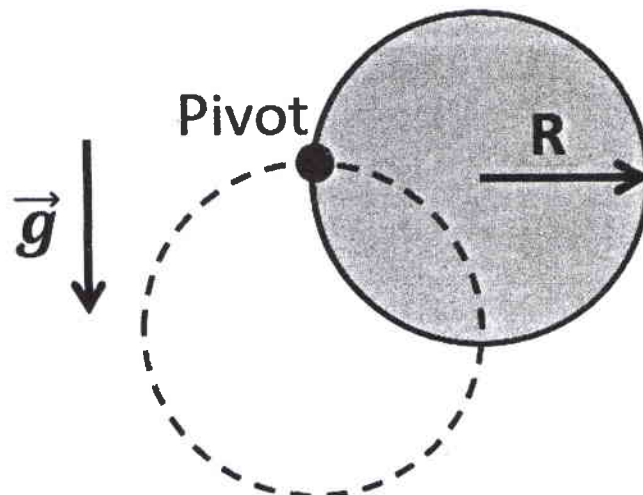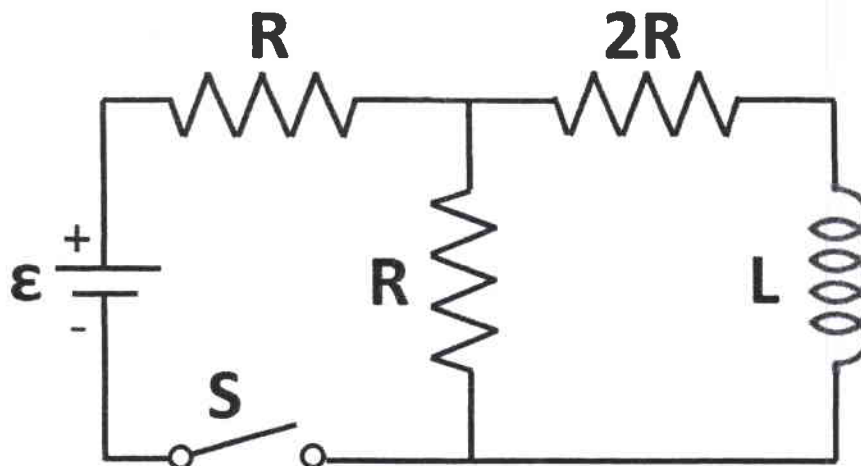
考 試 科 目	普通物理	系 所 別	應用物理研究所	考 試 時 間	2 月 18 日（一）第 一 節

3. **(16%)** In the figure, the long, straight wire carried a time varying t current $I_1(t) = 5t$ (A). The wire lies in the plane of the rectangular loop, which is made of metal with resistance per meter of 0.0693×10^{-6} (Ω/m). The dimensions in the figure are a = 2 m, b = 3 m, and c = 2 m. (a) Determine the magnetic flux through the loop due to the current I_1. (b) Find the current in the loop. (c) What is the direction of current in the loop? (d) Find magnitude and direction of the net force exerted on the loop by the magnetic field created by the wire at t=2s. Note ln(2)=0.693.
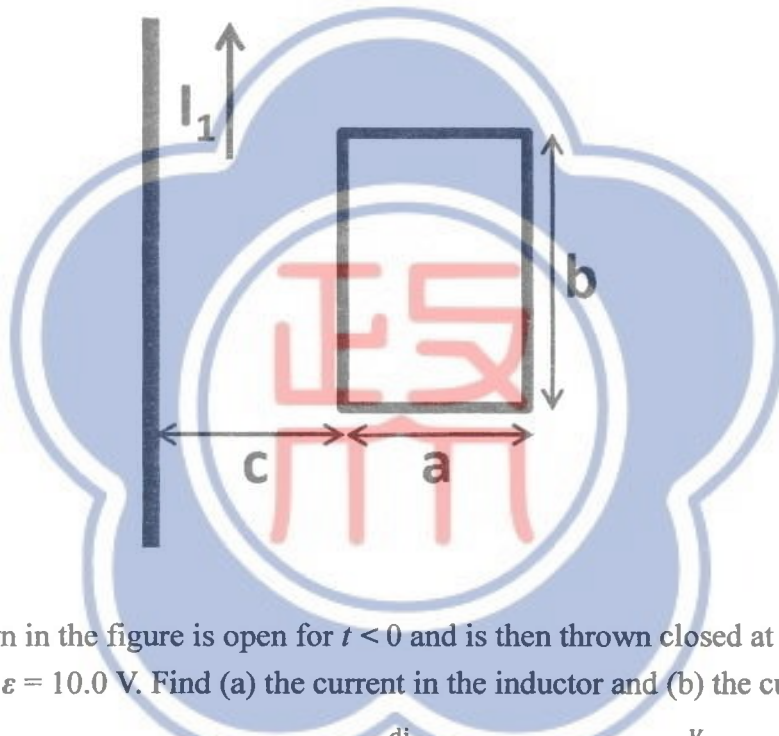
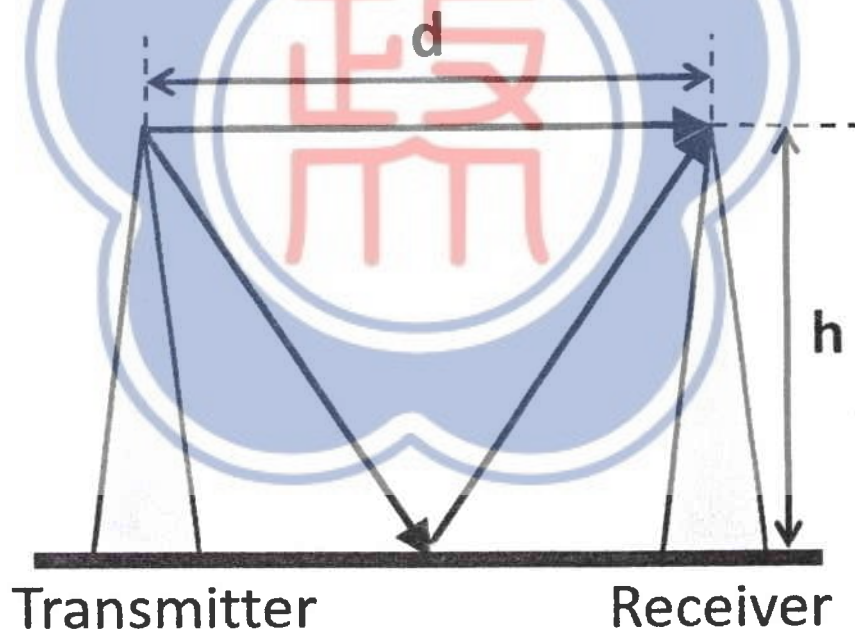4. **(20%)** The switch shown in the figure is open for $t < 0$ and is then thrown closed at time $t = 0$. Assume $R = 4.00\ \Omega$, $L = 1.00$ H, and $\varepsilon = 10.0$ V. Find (a) the current in the inductor and (b) the current in the switch as functions of time thereafter. (RL circuit: $V - iR_{eq} - L\frac{di}{dt} = 0 \rightarrow i(t) = \frac{V}{R_{eq}}(1 - e^{-R_{eq}t/L})$)

考 試 科 目	普通物理	系 所 別	應用物理研究所	考 試 時 間	2 月 18 日(一) 第一節

5. **(16%)** A 1.00-m-radius circular mirror focuses the Sun's rays onto a circular absorbing plate 2.70 cm in radius, which holds a can containing 1.00 kg of water at 20.0°C. (a) If the solar intensity is 1370 W/m², what is the intensity on the absorbing plate? At the plate, what are the maximum magnitudes of the fields (b) $\vec{E}$ and (c) **B** (d) If 40.0% of the energy is absorbed, what time interval is required to bring the water to its boiling point? Note the specific heat capacity of water is 4186 J/kg°C, the permeability in free space $\mu_0 = 4\pi \times 10^{-7}$ Tm/A, the speed of light in vacuum is 3×10^8 m/s

6. **(16%)** Figure shows a radio-wave transmitter and a receiver separated by a distance d = 96.0 m and both a distance h = 36.0 m above the ground. The receiver can receive signals both directly from the transmitter and indirectly from signals that reflect from the ground. Assume the ground is level between the transmitter and receiver and a 180° phase shift occurs upon reflection. Determine the longest wavelengths that interfere (a) constructively and (b) destructively

Transmitter   Receiver

備 註	一、作答於試題上者,不予計分。 二、試題請隨卷繳交。