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Risk management of deposit insurance corporations with risk-based premiums and credit default swaps

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In this paper, we propose a risk-based model for deposit insurance premiums and provide the closed-form formula for premiums, including early closure, capital forbearance, interest rate risk, and moral hazard. Our numerical analysis confirms the proposed pricing formula and the relative impact of the provisions for deposit insurance premiums. We illustrate how to use credit default swaps (CDSs) to manage the bank's asset risk corresponding to the deposit insurance model. A failed bank, Washington Mutual, is used to demonstrate how to calibrate the model's parameters and calculate fair premiums that are consistent with market risks on the basis of our proposed model and credit derivatives. Finally, a numerical experiment is designed to determine the optimal hedge ratio, which can minimise the variance of cash-flow of the deposit insurance corporations.

Keywords: Deposit insurance; Moral hazard; Capital forbearance; Credit default swaps

JEL Classification: C51, G21, G32

1. Introduction

The management objectives of financial institutions are to protect the rights and benefits of depositors, to maintain financial order, and to promote financial stability and development. Given today's globalised financial environment, the *ex ante* supervision of financial institutions and *ex post* remedies are equally vital. Governments must establish reliable and complete financial management systems and offer deposit insurance such as that provided in the United States by the Federal Deposit Insurance Corporation (FDIC). Deposit insurance provides basic safeguards against bank runs and contagion proliferation that can lead to global financial crises. In addition, the commitments and resulting safety net provided by deposit insurance corporations can strengthen market confidence for investors and prevent panic, particularly during a global recession.

The main goal of deposit insurance corporations is to protect depositors from losses in the event of bank failure so that the stability of the bank system is maintained. As a supervisor of the banking system, the deposit insurer's risk management is vital for maintaining the operations of insurance mechanisms. In this study, we analyse the techniques that deposit insurers can use to manage the risk of bank failure. The first instrument is a risk-based deposit insurance premium formula that incorporates moral hazard, early closure, capital forbearance, and interest rate risk. The second concept involves using credit default swaps (CDS) of security banks to calibrate the bank risk in the security market, credit risk, and interest rate elasticity from the CDS market. On the basis of the proposed premium formula, we develop a market-based method for calibrating the model parameters of future bank assets depending on the security banks' CDSs and comparing the cash flows of deposit insurers to demonstrate the effectiveness of CDS hedging.

Earlier empirical studies show that the CDS spreads have faster and more efficient information than the bond markets, stock market, and credit rating agencies (Blanco *et al.* 2005, Rodríguez-Moreno and Peña 2013). Also, Liu *et al.* (2016) document the CDS is a forward-looking and market-based measure of bank risk and the CDS spread is an appropriate measurement to investigate the impact of deposit insurance schemes on bank's credit risk. They verify the various deposit insurance designs can lessen the adverse impact on bank CDS

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spreads. Kanagaretnam *et al*. (2016) investigate the relations between accounting information and the bank's risk-taking behaviors captured by CDS spread, and they find the bank's asset allocation is associated with bank's CDS spreads. Following the prior literature on the CDS and bank's risk, we attempt to construct a credit market-based deposit insurance premium which can appropriately reflect the bank's risk.

In this study, we adopt a structural model of bank's asset value with derivative pricing techniques to derive fair insurance deposit premiums. Since Merton (1977), deposit insurance has typically been modeled as a European put option; that is, a put contract that is issued by the deposit insurer and written on bank assets that features a strike price equal to the deposit amount and maturity at the audit date. Therefore, the insurance claim is regarded as the option payoff, which is the shortfall between the bank's asset value and the deposit account, and the premium is calculated under the Black–Scholes framework (Ronn and Verma 1986, Thomson 1987, Episcopos 2004). However, this Merton-type setting ignores the possibility of early closure or capital forbearance. Thus, the omission of these possibilities is incongruous with reality.

Banks conforming to the risk-based capital standards of the Basel II regulations can increase their insurance subsidy by concentrating their lending and off-balance-sheet activities (Pennacchi 2006). When banks fail to meet the applicable capital standards, deposit insurers may provide these undercapitalised financial institutions with capital forbearance and require them to take prompt corrective actions to recapitalise during a limited period or close early (Nagarajan and Sealey 1995, Hellmann *et al*. 2000, Kane 2001). Duan and Yu (1994) propose a multiperiod deposit insurance pricing model that simultaneously incorporates these capital standards and the possibility of forbearance. Moreover, Duan and Yu (1994) employ generalised autoregressive heteroscedasticity (GARCH) option pricing techniques to determine the value of deposit insurance. By using a simple model, Lee *et al*. (2005) derive a closed-form solution for calculating deposit insurance premiums under capital forbearance as an option for delaying the resolution of undercapitalised financial institutions.

In addition to capital forbearance, recent studies incorporate early closure policies and stochastic interest rates into the deposit insurance pricing formula using a Merton-type setting. Hwang *et al*. (2009) apply the 'down-and-out' put option formula to determine the regulatory threshold-defined as the lower barrier of the option for deposit insurance-while explicitly considering bankruptcy costs and closure policies. On the basis of the calibration of pricing parameters, Chuang *et al.* (2009) calculate deposit insurance premiums under stochastic interest rates for Taiwan's banks by applying the two-step maximum-likelihood-estimation method. These methods consider the various risks that the deposit scheme faces when providing fair, risk-based deposit insurance premiums.

Hannan and Prager (2006) find that larger banks offer lower deposit interest rates than smaller banks, because small banks need to increase their competitiveness to attract deposit funding progressively. Wagner (2010) analyzes the influence of competition in the deposit market and infers that a highly competitive deposit market increases the deposit rate and thus increases a bank's risk-taking incentives. A study by Hakenes and Schnabel (2011) focus on bank size and bank risk-taking and find that small banks may raise the deposit rate to increase their customer base and accept risky projects. A bank with small size or poor reputations may pay high capital costs to attract depositors. Both decreasing loan rates and increasing deposit rates are coupled with a reduction in the bank interest margin (Saunders and Schumacher 2000), and a low bank margin increases the incentives of the bank to search for yields (Delis and Kouretas 2011). Dell'Ariccia *et al*. (2014) mention that the competition for providing high deposit rate exacerbates agency problems and increases bank's risk-taking activities. To mitigate the risk of high deposit interest rate spread, we considered deposit premiums in our risk-based model and characterised the deposit rate spread as a risk function that is proportional to the credit risk of a bank's loan position.

Although deposit insurance reduces the risk of bank runs, it simultaneously reduces the incentive for depositors to monitor a bank's risk (Demirgüç-Kunt and Detriagache 2002, Laeven 2002, Anginer *et al*. 2014). Wheelock and Kumbhakar (1995) investigate whether security banks are riskier because of moral hazard, adverse selection, or both and determined that the Kansas deposit insurance system may suffer from problems related to both adverse selection and moral hazard. The moral hazard problem is likely to plague deposit insurance schemes because it creates incentives for banks to accept greater risk and engage in risky activities with impunity (Laeven 2002).

If a security bank's asset value cannot meet the capital standard but does not decrease below the forbearance threshold at the time of the audit, the security bank can extend its operations for a certain grace period (Nagarajan and Sealey 1995, Kane 2001, Lee *et al*. 2005). During the grace period, the security bank is asked to increase its capital to satisfy the adequacy requirement. Hakenes and Schnabel (2011) find that the stricter the capital requirement is, the higher is the deposit rate provided by a bank. Moreover, the bank may choose a risky project without conveying this fact to the investors. According to their model, banks react to stricter capital requirements by taking more equity. Our deposit insurance model including both risk-taking activities and moral hazard problem during the grace period into account. And we further specify the mechanism in Section 2.3.

The deposit insurance is a part of the financial system safety net but reduces the incentives of depositors to monitor banks, thus leading to excessive risks.† According to existing literature, the moral hazard problem is related to a bank's risk, so we resort the moral hazard into the bank's asset allocation.

† Demirguc-Kunt and Detragiache (2002) find that deposit insurance exacerbates moral hazard problems in a bank lending scenario and is associated with a high likelihood of a banking crisis. So and Wei (2004) observe that the effect of moral hazard on fair insurance premiums is more significant than the effect of bank's equity and charter values. An insurer should be able to deter banks' risky behavior and close problematic banks when necessary. Therefore, VanHoose (2007) specifies that the fair pricing framework for deposit insurance is crucial to mitigate moral hazard problems. Anginer *et al.* (2014) investigate the relations between deposit insurance and moral hazard for different periods. Their study finds that deposit insurance increases moral hazard and makes financial systems more vulnerable to crises during normal times.

Boyle and Lee (1994) and Duan and Yu (1994) model moral hazard as increasing asset variance. Hooks and Robinson (2002) mention that the structure of a bank's assets measured by portfolio concentrations can be a good proxy for risk and obtained evidence on moral hazard incentives by attempting to quantify shifts in banks' ex-ante risk-taking activities. Mazumdar and Yoon (1996) and So and Wei (2004) reveal that the use of capital requirements to combat moral hazard may be difficult and that the adoption of asset portfolio restrictions as a measure may be more effective.

So and Wei (2004) and Lee *et al*. (2005) consider a moral hazard parameter associated with the equity value of the asset portfolio to amplify a bank's risk. Such an assumption represents that deposit guarantees may exacerbate moral hazard, thus leading to inefficient and risky investments. We resort moral hazard in the deposit insurance to the bank's asset allocation. In our proposed deposit insurance model, as the structure of a bank's asset is specified, we can represent the bank's risk based on the risk at each position of the bank assets.

The study related to this paper is that by Chen (2017), who estimates the deposit insurance premiums from the bank's CDS spread, that is, it proposes insurance deposit premiums under the assumption that the bank's asset value follows a normal firm value diffusion process with a constant volatility which is implied by credit spreads. In contrast to Chen (2017), our bank asset model is more flexible and we involve various deposit insurance policies and moral hazard risk into our model to provide a more comprehensive premium framework.

The remainder of this paper is organised as follows: Section 2 establishes the structures of the bank asset and formulates the deposit insurance schemes as the payoff function of a bank's asset and deposit liabilities. Section 3 derives the risk-based deposit insurance premiums, taking into account closure policies, stochastic interest rate, and moral hazard. On the basis of the proposed premium formula, we use a scenario to demonstrate how risk factors affect premiums. Section 4 employs Washington Mutual (WaMu) as an example to describe the market-based technique that calibrates the volatility of bank assets and the hedging effect of using credit derivatives. Section 5 concludes this paper.

2. Deposit insurance schemes

2.1. Bank asset model

In this section, we first verify the assumptions of the risk-free rate, which can be represented by the Treasury yield. The stochastic risk-free rate $r(t)$ is assumed to adopt the Vasicek (1977) model and leads to the following explicit formula:

$$dr(t) = \kappa(\theta - r(t))dt + \sigma_r dW_r^P(t) \tag{1}$$

where κ represents the mean-reverting force measurement, θ stands for the long-term mean of the risk-free rate, σ_r is the volatility of the risk-free rate, and $W_r^P(t)$ is a Wiener process. Therefore, we define the riskless money market account (MMA) $M(t) = \exp\left\{\int_0^t r(s)ds\right\}$ as the numeraire for the pricing deposit premium.

Banks provide a personal saving rate (deposit rate) for drawing funds on the basis of their own financial condition. Therefore, the dynamics process of deposit liabilities $D(t)$ should increase with the deposit rate $r(t) + \varepsilon$, where ε is the difference between the deposit rate and the risk-free rate; this measure is denoted as the deposit rate spread. Specifically,

$$dD(t) = (r(t) + \varepsilon)D(t)dt \tag{2}$$

where the dynamics of deposit liabilities have no uncertainty risk except for the risk-free rate risks.

As documented in the literature, deposit insurance schemes generate a moral hazard problem (Grossman 1992, Wheelock and Kumbhakar 1995, Gropp and Versala 2004, Cull *et al*. 2005, Beck *et al*. 2006). When a moral hazard exists, a bank may engage in excessive risk-taking to realise an additional profit in order to cover the extra deposit interest. Thus, the bank's risk should include the risk of moral hazard caused by the deposit rate spread. In addition, the deposit may be subjected to a default risk; as we discuss below, that risk cannot be completely covered through the deposit insurance scheme designed in our model. On the basis of the specification of riskless money market accounts $M(t)$ and a bank's outstanding deposit liabilities, $D(t)$, deposit liabilities as can be expressed as

$$D(t) = D(0)\exp\left\{\int_0^t r(s)ds + \varepsilon t\right\} = D(0)M(t)\exp\{\varepsilon t\} \tag{3}$$

Table 1 presents the statistical reports of the financial statements of FDIC-insured institutions from the FDIC website† and presents the asset allocation of all commercial banks insured by the FDIC from 1999 to 2018. Table 1 demonstrates that the total percentage of bank assets in reserve, securities, and loans‡ is greater than 90%; that is, the risk of banks' assets derives mostly from these three components.

Because bank assets fluctuate closely and stably around a given financial strategy, we assume in this paper that the allocation of bank assets consists of the following: (1) the amount held in reserves and cash§, $R(t)$; (2) the loan position, with price process $L(t)$; and (3) the investment position, with price process $S(t)$. The investment position includes securities and trading account assets, hereinafter referred to as securities. The reserve position is allocated among a fixed proportion, γ, of bank assets, where ω is the fraction of bank assets invested in securities, and the remaining fraction $1 - \gamma - \omega$ comprises

† https://www5.fdic.gov/sdi/main.asp?formname = compare
‡ In the table, reserves are defined as 'cash and due from depository institutions' of a banking report, and the term securities represents 'securities', 'federal funds sold and reverse repurchase agreements', and the 'trading asset account' of the balance sheets from the banking report at the FDIC.
§ We assumed that the reserve position enhances with the risk-free interest rate although cash has no interest because cash accounts for a small percentage of this subject. The reserve position includes "Cash and Balances Due' in the bank's balance sheet. Based on the statistics in the deposit institutions' balance sheet provided by FDIC, the 'total noninterest bearing balances' account for 11.7% and 13.3% of 'Cash and Balances Due' in 2017/12/31 and 2018/12/31, respectively. Moreover, the reserve is less than approximately 10% of the bank's assets. For simplicity, we assumed that the reserve position enhances with the risk-free rate in a bank's asset model.

Table 1. Asset Allocation of FDIC-insured Commercial Banks from 1999 to 2018.

	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008
Total assets	5,735	6,246	6,552	7,077	7,602	8,416	9,041	10,092	11,176	12,309
Reserves	366	370	390	384	387	388	400	433	482	1,042
	(6.39)	(5.92)	(5.96)	(5.42)	(5.10)	(4.61)	(4.43)	(4.29)	(4.31)	(8.46)
Securities	1,530	1,663	1,793	2,044	2,237	2,441	2,515	2,815	3,104	3,374
	(26.68)	(26.63)	(27.37)	(28.88)	(29.42)	(29.00)	(27.81)	(27.90)	(27.78)	(27.41)
Loans	3,430	3,751	3,812	4,079	4,352	4,833	5,313	5,913	6,537	6,682
	(59.81)	(60.06)	(58.18)	(57.64)	(57.25)	(57.43)	(58.77)	(58.59)	(58.49)	(54.28)
	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
Total assets	11,823	12,065	12,649	13,391	13,673	14,475	14,893	15,628	16,218	16,728
Reserves	977	923	1,196	1,334	1,633	1,854	1,686	1,738	1,833	1,617
	(8.26)	(7.65)	(9.45)	(9.96)	(11.94)	(12.81)	(11.32)	(11.12)	(11.30)	(9.67)
Securities	3,308	3,530	3,713	3,978	3,760	3,945	3,966	4,174	4,271	4,511
	(27.98)	(29.25)	(29.36)	(29.71)	(27.50)	(27.25)	(26.63)	(26.71)	(26.33)	(26.96)
Loans	6281	6,377	6,540	6,896	7,115	7,518	8,061	8,492	8,893	9,346
	(53.13)	(52.85)	(51.71)	(51.50)	(52.03)	(51.94)	(54.12)	(54.34)	(54.83)	(55.87)

Table 1 presents the asset allocation of all commercial banks that the FDIC insured from 1999 to 2018 and indicates that the total percentage of bank assets in reserves, securities, and loans is greater than 90%. The data source is the statistical reports of the financial statements of FDIC-insured institutions from the FDIC website, and the dollar figures are in billions. The numbers in parentheses represent the proportion of the correspondent items to total assets. 'Reserves' represents the 'cash and due from depository institutions' from the banking report, and 'Securities' includes the 'securities', 'Federal funds sold and reverse repurchase agreements', and 'trading asset account' of the balance sheet from the FDIC banking report.

outstanding loans. The dynamics of bank assets, $A(t)$, evolve in accordance with the following†:

$$\frac{dR(t)}{R(t)} = r(t)dt \tag{4}$$

$$\frac{dL(t)}{L(t)} = (r(t) + \varepsilon + \lambda - \phi\kappa(\theta - r(t)))dt + \phi dr(t) + \sigma_c(1+\varepsilon)dW_L^P(t) \tag{5}$$

$$\frac{dS(t)}{S(t)} = \mu dt + \sigma_S dW_S^P(t) \tag{6}$$

$$\frac{dA(t)}{A(t)} = \gamma\frac{dR(t)}{R(t)} + \omega\frac{dS(t)}{S(t)} + (1-\gamma-\omega)\frac{dL(t)}{L(t)} \tag{7}$$

where ϕ denotes the instantaneous interest rate elasticity parameter of the loans. $\phi dr(t)$ represents the variation in the interest rate $dr(t)$ that has an influence on the variation in the bank's loan position, and the interest rate elasticity parameter ϕ amplifies the degree of the influence. $\phi dr(t)$ is termed as the interest rate elasticity. λ is the interest rate spread, which is the difference between loan interest rates and the personal savings rate. $W_L^P(t)$ is the Wiener process under the physical probability measure and represents the uncertainty of the loan position. We assume $W_L^P(t)$ is independent of $W_r^P(t)$. σ_c denotes the constant credit risk, orthogonal to the interest rate risk, and we assume that the credit risk of banks is proportional to deposit rate spreads. $\sigma_c(1+\varepsilon)dW_L^P(t)$ denotes that the credit risk is not only controlled by the constant credit risk σ_c but also associated with the deposit rate spread $(1+\varepsilon)$. According to Duan *et al.* (1995) and Chuang *et al.* (2009), the total loan risk can be expressed as $\sigma_L = \sqrt{\phi^2\sigma_r^2 + \sigma_c^2(1+\varepsilon)^2}$. The security dynamics follow Black–Scholes dynamics, with an instantaneous rate of return, $\mu > 0$; the volatility parameter for the securities market is σ_S. The term $W_S^P(t)$ is the Wiener process under the physical probability measure representing the securities market risk, which is independent of $W_L^P(t)$ and $W_r^P(t)$. The dynamics of reserves and cash positions increase with the risk-free interest rate and are held at a constant reserves-to-assets ratio, γ.

The bank's asset dynamics result from Equations (4)–(6), which express that a bank's assets are weighted by reserves and cash, investments, and loan positions. The asset dynamics can be derived as follows:

$$\frac{dA(t)}{A(t)} = (\omega\mu + (1-\omega)r(t) + (1-\gamma-\omega)(\varepsilon+\lambda))dt + \sigma dW_A^P(t) \tag{8}$$

where $\sigma = \sqrt{\omega^2\sigma_S^2 + (1-\gamma-\omega)^2(\phi^2\sigma_r^2 + \sigma_c^2(1+\varepsilon)^2)}$ refers to the total risk of bank assets, and $W_A^P(t)$ is a Wiener process for bank assets.

2.2. *Early closure and capital forbearance policies*

Valuation models such as those of Merton (1977) and Ronn and Verma (1986) assume that the regulatory authorities can monitor a bank's assets only at the maturity of an insurance contract; in other words, at the time of an audit. According to Brockman and Turtle (2003) and Episcopos (2008),

† The details of how to obtain Equation (5) can be found in the Appendix in Duan *et al.* (1995), where they describe the dynamics of a bank's asset as containing credit risk and interest rate risk. They project the bank's asset, which shows the Wiener process credit risk on the interest rate variable to yield $dV_t/V_t = \mu dt + \phi_v dr_t + \psi dW_t$. A given interest rate process is dr_t: represents the interest rate risk by stochastic variation against time, and it can adjust the size of the variation of interest risk by multiplying ϕ_v, which is interpreted as the instantaneous interest rate elasticity. Lee and Yu (2002) and Lo *et al.* (2013) also adopted the model; they assume that the dynamic of the insurer's assets and insurer's liability follow the Wiener process of interest rate elasticity to characterize an asset that is sensitive to interest rates risks. Based on Duan *et al.* (1995), we further consider the interest rate spread and the deposit rate spread on the bank's liability dynamic as control variables for credit risks.

bank creditors and depositors do not wait for the maturity date of debts or deposits. A bank can go bankrupt before the audit date if its asset value is lower than a specific threshold, which causes debt holders and depositors to withdraw their money. Therefore, the bankruptcy during the audit window period should be considered when evaluating the risk of early closure.

To explicitly model regulations for an early closure, we assume that the maintenance ratio, $\eta \in (0, 1)$, represents the minimum asset-to-debt ratio required to keep the bank functioning. In other words, if the bank's asset value is lower than its maintenance working capital level, $\eta D(t)$, during the contract period, then the bank is considered bankrupt. We follow Hwang *et al.* (2009) who use a barrier option approach with the first passage time model to incorporate capital forbearance and bankruptcy costs in the deposit insurance premiums. Accordingly, we define the first passage time τ, which is where the bank assets fall enough to breach the maintenance working capital level. We define the first hitting time as follows:

$$\tau = \inf\{t > 0 | A(t) \leq \eta D(t)\} \tag{9}$$

In our model, if τ is no later than the auditing time, the banks will be taken over or become bankrupt immediately when their assets fall and hit the early closure threshold, $\eta D(\tau)$, which is even lower than the capital forbearance threshold. That is, if the bank defaults before the time of the audit, in the case of premature closure, it will not be able to meet the conditions for capital forbearance and a grace period.

Similar approaches are adopted in a previous study for deposit insurance premiums and firm values of insurance companies. Grosen and Jørgensen (2002) use the barrier option framework to analyze the market value of life insurance companies: they introduce the risk of a premature default to the valuation of a life insurance contract with a simple knock-out barrier option. Chen and Suchanecki (2007) illustrate the price of the issued life insurance contract in the standard Parisian down-and-out option framework and consider the first passage time for the premature closure of the insurance company. Episcopos (2008) argues that the insurer owns a down-and-in call option on the bank assets, so that the barrier option theory can be applied to the contingent claims of a regulated bank. This action addresses the problem of early bank closure.

At the first hitting time for premature closure, the bank's asset value equals the maintenance working capital. Accordingly, the deposit insurance payoff at the time τ can be expressed as follows:

$$P(\tau) = (1 - \eta)D(\tau) \tag{10}$$

which is a threshold on the underlying asset whose price breaches the maintenance working capital level, resulting in default. The insurer pays deposit insurer compensation $(1 - \eta)D(\tau)$ to depositors when the bank defaults. Merton's (1977) original deposit insurance pricing model and other deposit insurance pricing models (Ronn and Verma 1986, Lee *et al.* 2005) do not allow for premature default because default can occur only when a claim matures. In our scenario, we first investigate passage time structural models corresponding to various specifications of the basic components of a credit risk model.

So and Wei (2004) specify that capital forbearance and the capital ratio play key roles in determining the deposit insurance premium. In our model, if the bank can operate until audit time T_1, then the regulatory authority examines the value of a bank's assets at that time. The assumptions of capital forbearance are similar to those of Duan and Yu (1994): the regulator offers the security bank capital forbearance for a grace period, Δ, if its asset value cannot meet the capital standard, $\alpha D(T_1)$, but does not fall below the capital forbearance threshold $\beta D(T_1)$, where β is greater than maintenance ratio η. The financially distressed bank can extend its operations until the time of the next audit, T_2, which is $T_1 + \Delta$ if the insuring agent promises to restore the asset value to a level higher than the bank's outstanding deposit liabilities, $D(T_2)$. Once the bank's value drops below the capital forbearance threshold, $\beta D(T_1)$, at time T_1, or $D(T_2)$ at time T_2, the regulator takes over the depository institution.

The early closure policy is designed for bankruptcy before the auditing time. Before the auditing time, a reasonable maintenance ratio η must be less than the capital forbearance threshold parameter $(\eta < \beta < 1)$† in our model. The bank asset should always be higher than the minimum maintenance capital, that is, $A(\tau) > \eta D(\tau)$, to avoid the bank from being overtaken. Here, $\eta < \beta < 1$ implies that if a bank keeps operating until the auditing time T_1, the condition $(1 - \eta)D(\tau) > D(\tau) - A(\tau)$ for $\tau < T_1$ must be held. When the system reaches the auditing time, this condition will not be a large shock for the bank asset over time because we adopted Ito's process, which is a continuous-time model as the bank asset model.

In the context of deposit insurance, regardless of whether the security bank closes or is taken over, the liquidation of insured deposits must restore the asset value. The payoffs of the deposit insurance contract at time T_1 without default before the audit can be expressed as follows:

$$P(T_1) = \begin{cases} 0 & \text{if } A(T_1) > \alpha D(T_1) \\ F(T_1, T_2) & \text{if } \alpha D(T_1) \geq A(T_1) > \beta D(T_1) \\ D(T_1) - A(T_1) & \text{if otherwise} \end{cases} \tag{11}$$

where $F(T_1, T_2)$ denotes the value of the capital forbearance and the grace period with maturity T_2 at audit time T_1. Equation (11) can be regarded as the payoff for a compound option that is a generalised writer-extendible put option. A pricing model of retractable and extendible bonds is presented by Brennan and Schwartz (1977) and Ananthanarayanan and Schwartz (1980). Longstaff (1990) extends the work of those authors to develop holder- and writer-extendible options and applied those options to evaluate real estate options, warrants, extendible bonds, and American options. In this paper, we model the deposit insurance scheme as a general writer-extendible option when the bank works until audit time T_1. If the bank's assets are greater than the capital standard, the deposit insurance payoff equals zero. However, if the asset

† In our scenario analysis in section 3.2, the basic parameter setting is $\beta = 0.97$ and $\eta = 0.8$.

value cannot satisfy the capital standard, the deposit insurance payoff is a writer-extendible put option in which the underlying asset is the bank asset value and the strike price is the capital forbearance threshold $\beta D(T_1)$. If the asset value is lower than $\beta D(T_1)$ at time T_1, the deposit insurer pays the difference between the deposit liabilities and the bank's assets to cover the deposit losses; otherwise, the results lead to another put option with maturity T_2. Longstaff (1990) demonstrates that a deposit insurance payoff can degenerate into a writer-extendible put option with a time-variant strike price if parameter α tends to infinity and $\beta = 1$.

On the basis of the capital forbearance for deposit insurance, the general extendible put option is in force because the bank's assets are lower than $\alpha D(T_1)$ but do not fall below the capital forbearance threshold, $\beta D(T_1)$. The financially distressed bank can extend its operations until the time of the next audit, T_2, if the insuring agent promises to restore the asset value of the bank's outstanding deposit liabilities, $D(T_2)$. The option is extended with time to maturity, Δ, and the strike price, which is the bank's outstanding deposit liabilities at T_2. Therefore, the payoff at time T_2 can be written as follows:

$$F(T_2, T_2) = \begin{cases} 0 & \text{if } A(T_2) \geq D(T_2) \\ D(T_2) - A(T_2) & \text{if otherwise} \end{cases} \tag{12}$$

When the asset value cannot increase above the deposit liabilities at audit time T_2, the claim amount is the difference between the bank's deposit liabilities and its asset value. According to the Basel Accord, banks are required to maintain 8% of their risk-weighted assets as their minimum equity capital standard. For simplicity, we assume the risk-weighted asset value at the current time is $A(t)$ and the total deposit debt is $D(t)$. Because $A(t) - D(t)$ over $A(t)$ must be greater than 8% to meet the capital adequacy ratio, the capital standard $A(t)/D(t)$ is required to be greater than 1.087. Therefore, in this paper, the capital standard parameter α is set at 1.087 (see also Lee *et al.* 2005).

Note that we do not need to consider the capital forbearance for early closure, because the maintenance ratio is less than the capital forbearance threshold parameter. If a bank's capital adequacy ratio is less than its maintenance ratio, it certainly fails to satisfy the forbearance threshold. Hence, the regulator should take over the bank directly. Therefore, the early closure policy does not conflict with the forbearance and grace period.

2.3. *Bank risk-taking and moral hazard*

To illustrate the risk-taking behavior of a bank which provides a high deposit rate, we considered deposit premiums in our risk-based model and characterised the deposit rate spread as a risk function that is proportional to the credit risk of a bank's loan position. It may be assumed that when a bank provides a higher deposit rate spread than other banks, it broadens the lending conditions and accepts a greater credit risk for a higher return to compensate for the additional payment. Thus, higher the probability of risk-taking operations, the higher should be the insurance premiums charged by the deposit insurance corporations.

We further consider moral hazards in our premium pricing model. In an earlier study on the valuation of deposit insurance, So and Wei (2004) and Lee *et al.* (2005) incorporate the moral hazard problem and deposit insurance premium under forbearance through a risk-shifting portfolio strategy. That is, a moral hazard operation may occur in the forbearance period. The security bank may adjust the underlying holdings of its security positions by increasing the number of high-yielding securities it holds or by increasing the weight of investment positions, which typically exhibit high yields and offer profits quicker.

By contrast, some studies documented that banks are asked to increase their capital and decrease their risks under the forbearance policy. Kahn and Santos (2005) find that the regulatory authority has supervisory powers to force the bank to shut down when it is in a poor financial condition, which improves the bank's investment decisions in the forbearance problem. Altunbas *et al.* (2007) document that the capital regulation in banking is effective for increasing capital ratios without substantially shifting their portfolio toward riskier assets. It is reasonable that the supervision authority pays more attention to banks whose asset value cannot meet the capital standard. Delis and Kouretas (2011) point out that the effective regulatory and supervisory power over these banks may hold the key to a more prudent bank behavior.

Anginer *et al.* (2014) find that a good supervision can enhance the positive effects of deposit insurance during turbulent periods and reduce the negative effects due to moral hazard during normal times. To construct a flexible deposit insurance model including the moral hazard, our model was developed using a framework similar to that proposed by Lee *et al.* (2005). This framework sets a moral hazard intensity to increase the risk-taking activities in the grace period. We considered another portfolio share $\tilde{\omega}$ of a high-risk asset (securities) during the grace period. If the portfolio share is higher than that in the regular case, that is, $\tilde{\omega} > \omega$, the moral hazard problem in the deposit insurance is represented. The opposite case, that is, $\tilde{\omega} < \omega$, represents that an enhanced supervision reduces bank's risk-taking activities. Depending on banks' asset dynamics in Equation (8), the portfolio share gives a weight to securities and leads to a different volatility of banks' asset risk $\tilde{\sigma} = \sqrt{\tilde{\omega}^2\sigma_S^2 + (1-\gamma-\tilde{\omega})^2(\phi^2\sigma_r^2 + \sigma_c^2(1+\varepsilon)^2)}$ in the grace period.

3. Risk-based deposit insurance premiums

3.1. *Closed-form formula for deposit insurance premiums*

On the basis of the asset dynamics under the physical measure in Equation (8), we adopt the standard practice of changing the probability measure to prevent arbitrage opportunities in the risk-neutral probability measure. The dynamics of the relative bank asset with respect to the money market account should be a martingale under the risk-neutral measure (Q); thus, the dynamics of bank assets are as follows:

$$dA(t) = r(t)A(t)dt + \sigma A(t)dW_A^Q(t) \tag{13}$$

where $\sigma = \sqrt{\omega^2\sigma_S^2 + (1-\gamma-\omega)^2(\phi^2\sigma_r^2 + \sigma_c^2(1+\varepsilon)^2)}$ is referred to as the total risk of bank assets, and $W_A^Q(t)$ represents the Wiener processes under the risk-neutral measure.

The bank's assets and liability are essential criteria for determining whether a bank has failed. To simplify the formula, we use the asset-to-debt ratio as a variable for the insurance payment. The relative values of assets and debts eliminate the effect on if risk-free rates on returns, and the influence of risk-free rates is observed only in the volatility of bank assets. By applying Itô's lemma, we can express the relative dynamics of a bank's assets as follows:

$$d\frac{A(t)}{D(t)} = \frac{A(t)}{D(t)}(-\varepsilon dt + \sigma dW_A^Q(t)) \tag{14}$$

In line with the deposit insurance schemes mentioned in Section 2, the risk-based premium that the security bank pays to the deposit insurer can be divided into the following three components:

$$\begin{aligned} P(0) = {} & E^Q\left[\frac{P(\tau)}{M(\tau)} I_{\{\tau<T_1\}}\right] + E^Q\left[\frac{P(T_1)}{M(T_1)} I_{\left\{\tau\geq T_1, \frac{A(T_1)}{D(T_1)}\leq\beta\right\}}\right] \\ & + E^Q\left[\frac{P(T_1)}{M(T_1)} I_{\left\{\tau\geq T_1, \beta<\frac{A(T_1)}{D(T_1)}<\alpha\right\}}\right] \end{aligned} \tag{15}$$

The closed-form solution of the risk-based deposit insurance premium comprises an audit window component (the first term in Equation (15), denoted as P^a), capital forbearance component (the second term in Equation (15), P^c), and a grace period component (P^Δ, the third term in Equation (15)). These are expressed as follows:

$$P^a = (1-\eta)D(0)e^{\frac{v-u}{\sigma^2}B(\eta)}\left[\Phi(c_1(\eta,u)) + e^{\frac{2u}{\sigma^2}B(\eta)}\Phi(c_1(\eta,-u))\right] \tag{16}$$

$$\begin{aligned} P^c = {} & D(0)e^{\varepsilon T_1}\left[\begin{matrix} (\Phi(c_1(\beta,v))) - \Phi(c_1(\eta,v)) \\ -e^{\frac{2v}{\sigma^2}B(\eta)}(\Phi(c_2(\beta,\eta,v)) - \Phi(-c_1(\eta,-v))) \end{matrix}\right] \\ & - A(0)e^{\varepsilon T_1}\left[\begin{matrix} (\Phi(c_1(\beta,\tilde{v})) - \Phi(c_1(\eta,\tilde{v}))) \\ -e^{\left(\frac{2v}{\sigma^2}+2\right)B(\eta)}(\Phi(c_2(\beta,\eta,\tilde{v})) - \Phi(-c_1(\eta,-\tilde{v}))) \end{matrix}\right] \end{aligned} \tag{17}$$

$$\begin{aligned} P^\Delta = e^{\varepsilon T_1}\Bigg(& D(0)\left\{\begin{matrix} N(c_1(\alpha,v),e_1(v),\delta) - N(c_1(\beta,v),e_1(v),\delta) \\ -e^{\frac{2vB(\eta)}{\sigma^2}}[N(c_2(\alpha,\eta,v),e_2(v),\delta) - N(c_2(\beta,\eta,v),e_2(v),\delta)] \end{matrix}\right\} \\ & +A(0)\left\{\begin{matrix} N(c_1(\alpha,\tilde{v}),e_1(\tilde{v}),\delta) - N(c_1(\beta,\tilde{v}),e_1(\tilde{v}),\delta) \\ -e^{\left(\frac{2v}{\sigma^2}+2\right)B(\eta)}[N(c_2(\alpha,\eta,\tilde{v}),e_2(\tilde{v}),\delta) - N(c_2(\beta,\eta,\tilde{v}),e_2(\tilde{v}),\delta)] \end{matrix}\right\}\Bigg) \end{aligned} \tag{18}$$

where

$$v = -\varepsilon - \frac{\sigma^2}{2},$$

$$\sigma = \sqrt{\omega^2\sigma_S^2 + (1-\gamma-\omega)^2(\phi^2\sigma_r^2 + \sigma_c^2(1+\varepsilon)^2)},$$

$$u = \sqrt{v^2 - 2\sigma^2\varepsilon\tilde{v}} = v + \sigma^2, \quad c_1(x,z) = \frac{B(x) - zT_1}{\sigma\sqrt{T_1}},$$

$$c_2(x,y,z) = \frac{B(x) - 2B(y) - zT_1}{\sigma\sqrt{T_1}}, \quad e_1(v) = \frac{B(1) - vT_2}{\sigma\sqrt{T_2}},$$

$$e_2(v) = \frac{B(1) - 2B(\eta) - vT_2}{\sigma\sqrt{T_2}}, \quad B(x) = \ln\frac{xD(0)}{A(0)},$$

$$\delta = \sqrt{\frac{T_1}{T_2}}, \quad N(c,e,\delta) = \int_{-\infty}^{c}\Phi\left(\frac{e-\delta Z}{\sqrt{1-\delta^2}}\right)\varphi(Z)dZ.$$

where $\varphi(\cdot)$ and $\Phi(\cdot)$ are the probability density and cumulative distribution functions of the standard normal distribution, respectively. Additional details are provided in the appendix.

P^a denotes the early closure component, which is used to evaluate the present value of the payment for banks that reach the default threshold before audit time T_1 such that the regulator must implement the bankruptcy process. Similarly, P^c is presented as the capital forbearance component because its value is similar to that of a down-and-out put option whose strike price is the capital forbearance threshold $\beta D(T_1)$ at maturity time T_1. Although the bank may not go bankrupt, if it reaches the regulatory closure point, $\beta D(T_1)$, then the regulator takes over the bank. The depositor can still obtain a rebate on the basis of the difference between the regulatory closure point and the deposit insurance amount. The rebate should be reflected in the premium as P^c. P^Δ is regarded as a grace period component because its value depends on a regulatory delay. An undercapitalised institution can improve its financial position by continuing to work during the grace period.

As shown in Equation (15), the risk-based deposit insurance premium consists of the following components: the audit window, the capital forbearance, and the grace period, $P^a + P^c + P^\Delta$. In our pricing model, the moral hazard is considered in the grace period component, and we specify the moral hazard through another weight in the portfolio share $\tilde{\omega}$ to adjust for the volatility of the bank's asset. As shown in section 2.3, if we consider the moral hazard in the risk-based premiums, the total risk of the bank's asset becomes $\tilde{\sigma} = \sqrt{\tilde{\omega}^2\sigma_S^2 + (1-\gamma-\tilde{\omega})^2(\phi^2\sigma_r^2 + \sigma_c^2(1+\varepsilon)^2)}$. We define the grace period component of premium with moral hazard as $\tilde{P}^\Delta$ which is the same as $\tilde{P}^\Delta$ defined in Equation (18), except the total bank's risk is changed from $\sigma = \sqrt{\omega^2\sigma_S^2 + (1-\gamma-\omega)^2(\phi^2\sigma_r^2 + \sigma_c^2(1+\varepsilon)^2)}$ to $\tilde{\sigma}$. Therefore, the premium with moral hazard can be expressed as $P^a + P^c + \tilde{P}^\Delta$.

3.2. *Scenario analysis of premiums*

In this section, we numerically investigate the proposed model to determine how the value of deposit insurance under closure policies that consider early closure and forbearance varies with respect to critical parameters such as the capital forbearance threshold, the length of the grace period, and the cross effects of the two. Moreover, we analyse how asset allocation and moral hazard affect deposit insurance premiums. On the basis of the parameters used by Lee *et al.* (2005) and Chuang *et al.* (2009) unless otherwise specified, the following parameters are used throughout: $A(0) = 100$, $D(0) = 90$, $\gamma = 0.1$, $\omega = 0.25$, $\sigma_S = 0.3$, $\sigma_c = 0.1$, $\sigma_r = 0.01$, $\varepsilon = 0$, $\kappa = 0.1$, $\theta = 0.05$, $\phi = -0.5$, $T_1 = 1$, $\Delta = 0.5$, $\alpha = 1.087$, $\beta = 0.97$, and $\eta = 0.8$.

Table 2 depicts the relationship between the deposit insurance premium and closure policies crossed with the debt-to-asset ratio. The closure policies comprise the early closure and capital forbearance provisions. A regulator cannot examine a bank's business operations until the audit period ends, unless the bank cannot function. Therefore, the probability and cost of bankruptcy depend on maintaining the ratio η. As the sensitivity analysis demonstrates in Table 2, the deposit insurance premium decreases with the maintenance ratio but increases with the debt-to-asset ratio. A low maintenance ratio reduces the opportunity of early closure but increases future risk. Therefore, low maintenance lowers the premium of the early closure component but increases the premium of the capital forbearance and grace components. Integrating these three components, we observe that premiums and maintenance ratios are adversely related. However, the premium is dominated by the debt-to-asset ratio because that ratio most directly reflects the risks of operations and bankruptcy. A high debt-to-asset ratio represents a high risk of bankruptcy and thereby requires a high premium per deposit.

The forbearance provision is interpreted through β and Δ, which are the capital forbearance threshold and the grace period length, respectively. According to the analysis in Table 2, the premium formula reveals that when the value of β is low, the risk of bankruptcy at the audit time is low, but the risk at the end of the grace period is high. Combining these two components, we determine that the premium decreases with high capital forbearance thresholds. Conversely, due to the grace period component, the higher the Δ is, the higher the premium is. This case can be considered a European put with a long time to maturity and high costs. Furthermore, if moral hazard in the grace period is considered, the premium rises because it increases the weight of bank assets allocated in investment positions and raises banks' operational risks during the grace period.

If the moral hazard is considered in the deposit insurance, the premium rises because we take another weighting of the portfolio share $\tilde{\omega} > \omega$ to adjust the total risk of the bank's asset in the grace period. The bank increases the weight of bank assets allocated in investment positions and increases its operational risks during the grace period. Hence, the premiums with moral hazards are $P^a + P^c + \tilde{P}^{\Delta}$ as per the pricing formula in Equation (15). The premiums P^a and P^c are defined in Equation (16) and Equation (17). $\tilde{P}^{\Delta}$ is defined in equation (18), except the total risk of the bank's asset is $\tilde{\sigma} = \sqrt{\tilde{\omega}^2\sigma_S^2 + (1-\gamma-\tilde{\omega})^2(\phi^2\sigma_r^2 + \sigma_c^2(1+\varepsilon)^2)}$, which has a more substantial weight allocated in the investment positions.

Deposit premiums for the credit market (loan position) and the securities market (investment position) that cross alternative portfolio shares for bank asset allocation ω are reported in Table 3. An increase in credit risk or security market risk volatility is reflected in high deposit premiums. If the weight of securities decreases and most assets are allocated in loan positions, credit risk has a significant effect on premiums. As expected, security market risk appears to have an increasing influence as the weight of securities increases. This finding reveals that asset allocation is a key determinant of the deposit premium in the case of fixed market risk.

3.3. *Extreme case analysis*

Most banks pursue growth and are currently in 'too big to fail' states; thus, not only can bank managers obtain high salaries and enhance their personal prestige but banks can also gain benefits from low funding costs and enjoy substantial safety-net benefits (Benston *et al*. 1995, Carbó-Valverde

Table 2. Scenario Analysis of Deposit Insurance Premium with Closure Policies.

	Maintain ratio (η)			Forbearance threshold (β)			Grace period (Δ)		
	0.85	0.9	0.95	0.9	0.95	1	0.25	0.5	1
Debt-to-asset ratio = 0.88									
DI premium	88.52	88.23	80.52	92.75	91.15	78.45	70.72	88.52	117.26
Early closure component	5.99	21.17	38.93	0.97	0.97	0.97	0.97	0.97	0.97
Capital forbearance component	36.94	21.79	2.48	12.97	33.45	48.49	41.95	41.95	41.95
Grace period component	45.60	45.28	39.11	78.80	56.73	28.99	27.80	45.60	74.34
DI with moral hazard	127.73	127.40	117.66	136.09	132.64	111.31	111.53	127.73	157.21
Debt-to-asset ratio = 0.90									
DI premium	125.49	124.88	111.54	131.61	129.21	112.13	103.96	125.50	159.98
Early closure component	11.96	37.56	61.95	2.21	2.21	2.21	2.21	2.21	2.21
Capital forbearance component	56.69	31.12	3.14	22.94	54.39	75.26	66.44	66.44	66.44
Grace period component	56.84	56.20	46.45	106.46	72.60	34.65	35.31	56.85	91.33
DI with moral hazard	172.79	172.07	155.13	185.41	179.95	150.58	153.23	172.80	208.01
Debt-to-asset ratio = 0.92									
DI premium	172.03	170.80	148.92	180.42	177.00	155.26	147.24	172.05	211.47
Early closure component	22.49	62.96	93.54	4.71	4.71	4.71	4.71	4.71	4.71
Capital forbearance component	82.15	41.67	3.68	38.20	83.81	111.18	99.93	99.93	99.93
Grace period component	67.39	66.18	51.70	137.50	88.48	39.37	42.60	67.41	106.83
DI with moral hazard	226.16	224.67	196.87	244.12	235.98	197.84	203.68	226.18	266.29

The early closure provision is determined on the basis of the maintain ratio, η, and the forbearance provision is interpreted on the basis of β and Δ, which are the capital forbearance thresholds. DI refers to the insurance premium per deposit in basis points. The basic setting of the deposit insurance contract's time to maturity is assumed to be 1 year, the minimum capital requirement α is set at 1.087, the capital forbearance threshold $\beta = 0.97$, maintain ratio $\eta = 0.8$, and grace period $\Delta = 0.5$. The bank's asset allocation in its investment position is the proportion of $\omega = 0.25$ and its reserve asset ratio $\gamma = 0.1$. The volatility of the security market, credit market, and interest rate are $\sigma_s = 0.3$, $\sigma_c = 0.1$, and $\sigma_r = 0.01$, respectively. The interest rate elasticity is $\varphi = -0.5$. Ignore the impact of the deposit rate spread; in other words, $\epsilon = 0$. In the case of a moral hazard, the weight of the bank's assets on securities is replaced by $\tilde{\omega} = 0.35$ during the grace period component: the premiums are $P^a + P^c + \tilde{P}^{\Delta}$ in section 3.1.

Table 3. Deposit Insurance Premium with Asset Allocation.

Credit risk (σ_c)	0.1	0.1	0.1	0.05	0.1	0.2
Security market risk (σ_s)	0.05	0.1	0.2	0.3	0.3	0.3
Weights on securities $\omega = 0.1$						
DI premium	70.65	71.87	76.77	11.24	84.91	205.92
Early closure component	0.10	0.11	0.16	0.00	0.29	18.70
Capital forbearance component	30.19	30.95	34.03	1.63	39.28	114.86
Grace period component	40.36	40.81	42.57	9.61	45.35	72.37
Weights on securities $\omega = 0.3$						
DI premium	29.04	39.42	83.17	112.11	153.78	218.24
Early closure component	0.00	0.00	0.26	1.27	5.65	23.23
Capital forbearance component	7.79	12.57	38.15	57.42	84.97	120.82
Grace period component	21.25	26.85	44.77	53.42	63.16	74.19
Weights on securities $\omega = 0.5$						
DI premium	8.13	33.13	152.21	304.17	318.51	341.77
Early closure component	0.00	0.00	5.40	70.46	80.78	98.82
Capital forbearance component	0.94	9.58	83.97	149.45	152.15	155.41
Grace period component	7.19	23.54	62.84	84.26	85.58	87.53

The deposit premiums for the credit market and security market risk cross alternative portfolio shares, ω, for the bank assets in the investment position. DI refers to the insurance premium per deposit in basis points. The contract time to maturity is assumed to be 1 year, and the other parameters are set as follows: $A(0) = 100$, $D(0) = 90$, and the initial debt-to-asset ratio is 0.9. The minimum capital requirement, α, is set at 1.087, $\beta = 0.97$, the length of the grace period is $\Delta = 0.5$, and the maintain ratio is $\eta = 0.8$. The reserve asset ratio is $\gamma = 0.1$. The volatility of the interest rate is $\sigma_r = 0.01$, and the interest rate elasticity is $\varphi = -0.5$. The deposit rate spread is $\epsilon = 0$.

et al. 2013, Demirgüç-Kunt and Huizinga 2013). However, bank size is correlated with risk. Large banks can suffer substantially and become risky because of international capital markets, especially during financial recessions.

To stabilise the global financial environment and avoid bank panics that could lead to severe financial crises, governments and financial supervisory authorities provide these large banks with financial support and special supervisory mechanisms, considering the potential costs of their failure to the economy (Mishkin *et al.* 2006, Moshirian 2011). Being too big to fail can be considered an extreme type of capital forbearance; thus, we estimate the risk to deposit insurers for these large banks on the basis of extreme forbearance provisions by reducing the maintenance ratio and forbearance threshold to close to zero and extending the forbearance period to several years.

Table 4 shows the deposit premiums of specific large banks with extreme closure policies. Compared with the standard

Table 4. Deposit Insurance Premium with Extreme Case.

	Standard case			Extreme case		
	ω			ω		
	0.15	0.25	0.35	0.15	0.25	0.35
Debt-to-asset ratio = 80/100						
DI premium	7.66	15.44	36.06	39.85	67.51	129.70
Early closure component	0.00	0.03	0.46	0.00	0.00	0.00
Capital forbearance component	1.45	4.06	13.29	0.00	0.00	0.00
Grace period component	6.20	11.35	22.33	39.85	67.51	129.70
Debt-to-asset ratio = 85/100						
DI premium	32.19	51.19	91.32	133.35	184.94	282.59
Early closure component	0.04	0.31	2.98	0.00	0.00	0.00
Capital forbearance component	10.42	20.25	43.33	0.00	0.00	0.00
Grace period component	21.73	30.64	45.01	133.35	184.94	282.59
Debt-to-asset ratio = 90/100						
DI premium	96.01	129.44	190.77	308.98	379.43	501.91
Early closure component	0.57	2.57	13.90	0.00	0.00	0.00
Capital forbearance component	46.60	69.08	106.91	0.00	0.00	0.00
Grace period component	48.84	57.80	69.96	308.98	379.43	501.91
Debt-to-asset ratio = 95/100						
DI premium	221.28	266.51	343.39	549.02	629.89	765.09
Early closure component	4.94	14.57	49.16	0.00	0.00	0.00
Capital forbearance component	141.20	171.60	207.08	0.00	0.00	0.00
Grace period component	75.14	80.34	87.15	549.02	629.89	765.09

This table presents the deposit premiums of specific large banks with extreme closure policies. Compared with the standard case, the deposit premium is higher in the extreme case. DI refers to the insurance premium per deposit in basis points. The parameters in the standard case are $\alpha = 1.087$, $\beta = 0.97$, $\Delta = 0.5$, $\eta = 0.8$, $\gamma = 0.1$, $\sigma_r = 0.05$, $\varphi = -0.5$, $\sigma_s = 0.3$, $\varepsilon = 0$, and $\sigma_c = 0.1$. The parameters in the extreme case are the same as those of the standard case except that $\Delta = 5$, $\beta = 0.1$, and $\eta = 0.1$.

case, the deposit premiums are higher in the extreme case. This finding reveals that the deposit insurer assumes considerably higher risks to cover the deposits for large banks. The most influential factors for insurance premium deposits are the debt-to-asset ratio, followed by the weight of asset allocation. The lower the debt-to-asset ratio is, the higher the impact of the weights of security investments is. This relationship reveals that bank asset allocation and total risk result directly from the insurance premium decision.

4. Risk management with CDS

On September 25, 2008, WaMu bank, the 118-year-old banking giant failed; consequently, the FDIC assumed control of the bank. It was among the largest bankruptcies in history. Although the FDIC quickly sold the banking subsidiaries to JPMorgan Chase Bank, this case provides an example for examining the risks that the FDIC faced. In this section, we use WaMu's financial statement and its CDS' market price to calibrate the volatility parameter for the security market σ_S, credit risk σ_c, and interest rate elasticity ϕ. Subsequently, we evaluate the theoretical deposit insurance premium for WaMu by using the proposed risk-based model. Finally, we compare the differences in cash flows to determine whether the deposit insurance corporation used CDSs to hedge the credit risk of the insured bank.

Because bank risk is derived mainly from bank asset volatility, which greatly influences deposit insurance premiums, an appropriate estimation of the volatility of bank risk is critical in pricing the insurance premium deposit. Calibrating the volatility of bank assets through market trading information, such as that of the CDS market is a considerable method for use before pricing the insurance premium deposit. CDSs are described by referring to the cash flows of the premium leg and default leg. The present value of the premium leg is obtained on the basis of the present value of all payments made by the protection buyer:

$$CDS_{premium} = K \sum_{i=1}^{n} B(0, t_i) Q(t_i) \tag{19}$$

where K is the fixed insurance payment dependent on the period, $B(0,t_i)$ represents the present value of a zero coupon bond with maturity t_i, and $Q(t_i)$ denotes the probability of survival.

In the premium leg, the recovery rate and default probability must be determined first. We can obtain the recovery rate by using Moody's Default & Recovery Database, and we set the recovery rate $R = 0.57$ for WaMu. The default probability curve can be calculated using the bootstrap procedure to ascertain the default probability curve from market CDS spreads; subsequently, we apply the cubic spline to determine the survival probability at each CDS payment date. Thus, we acquire the value of the premium leg by discounting the expected cash flows.

The default leg is a contingent payoff made by the protection seller in case of default, and it can be formulated as the expected present value of the par value minus the recovery rate as follows:

$$CDS_{default} = \int_0^T (1 - R) B(0, t)(1 - Q(t)) dt \tag{20}$$

where T denotes the maturity period of the CDS, and R reflects the recovery rate in case of a credit event. The fair spread for the CDS is determined by letting two legs be equal.

To determine market-based deposit insurance premiums, we attempt to calibrate the volatility parameter for the security market, credit risk, and interest rate elasticity on the basis of the implied information in the CDS market. The alternative expression of the default leg can be derived using the first passage time theory under the debt-to-asset ratio dynamics described in Equation (14). We assume that the discount rate for the CDS payment when a default event occurs is a constant number, which is the initial risk-free rate, r_0.

$$CDS_{default} = (1 - R)\left(\frac{D(0)}{A(0)}\right)^{\frac{v-\tilde{u}}{\sigma^2}} \left[\Phi\left(\frac{\ln(\frac{D(0)}{A(0)}) - \tilde{u}T}{\sigma\sqrt{T}}\right) + \left(\frac{D(0)}{A(0)}\right)^{\frac{2\tilde{u}}{\sigma^2}} \Phi\left(\frac{\ln(\frac{D(0)}{A(0)}) + \tilde{u}T}{\sigma\sqrt{T}}\right)\right] \tag{21}$$

where $\tilde{u} = \sqrt{v^2 + 2\sigma^2 r_0}$. Subsequently, we use Equation (21) to evaluate the value of the CDS default leg rather than Equation (20). Given the bank's initial assets and debts, we can use a series of CDS spreads to calibrate the volatility of a bank's assets by setting the premium leg, Equation (19), to be equal to the default leg to Equation (21).

The objective function to calibrate the volatility parameters is defined as the difference between Equation (19) and Equation (21)

$$\underset{\phi,\sigma_S,\sigma_c}{\arg\min} \left\| \begin{array}{l} K \sum_{t_i \le T} B(0, t_i) Q(t_i) - \\ (1 - R)\left(\frac{D(0)}{A(0)}\right)^{\frac{v-\tilde{u}}{\sigma^2}} \left[\Phi\left(\frac{\ln(\frac{D(0)}{A(0)}) - \tilde{u}T}{\sigma\sqrt{T}}\right) \right. \\ \left. \quad + \left(\frac{D(0)}{A(0)}\right)^{\frac{2\tilde{u}}{\sigma^2}} \Phi\left(\frac{\ln(\frac{D(0)}{A(0)}) + \tilde{u}T}{\sigma\sqrt{T}}\right)\right] \end{array} \right\| \tag{22}$$

where the bank's asset risk

$$\sigma = \sqrt{\omega^2\sigma_S^2 + (1 - \gamma - \omega)^2(\phi^2\sigma_r^2 + \sigma_c^2(1 + \varepsilon)^2)}$$

is the function of ϕ, σ_S, and σ_c. The quarterly coupon payment time from now to maturity is denoted by t_i. T is the time to maturity of the CDS, and we may take more than three CDSs with different maturities to calibrate ϕ, σ_S, and σ_c. For instance, if we take the CDSs with a maturity of 1-, 2-, 3-, 4-, 5-year, then we have a five-valued vector for T from 1–5 as an objective function with the same control variable ϕ, σ_S, and σ_c. We applied the nonlinear least-square solver to calibrate the parameters by minimising the objective function in Equation (22). The other notations in Equation (22) referring Equation (16), (19), and (21).

Corresponding to the trading period in which WaMu's CDS was actively traded in the market, we calibrate the

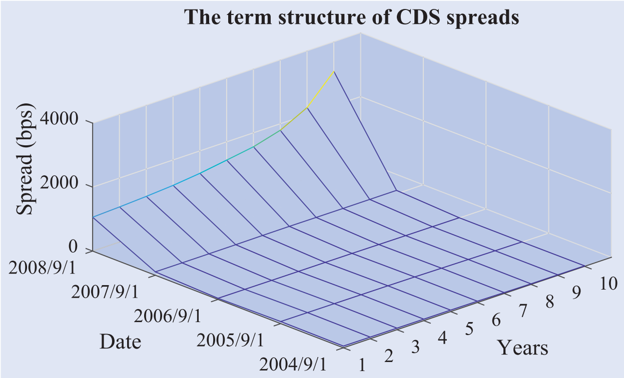

Figure 1. Term structure of WaMu's CDS spreads.

model parameters at the following dates: September 1, 2004; September 2, 2005; September 1, 2006; September 4, 2007; and September 2, 2008, and evaluate the 1-year deposit insurance premiums for these dates. The term structure of WaMu's CDS spreads is presented in Figure 1. Based on these spreads, the default probabilities and model parameters that we calibrate are presented in Table 5.

Table 5 shows that the default probability significantly increased on September 2, 2008, and WaMu indeed failed in 2008. The default probability, volatility of the security market, credit risk, and interest rate elasticity in the model dramatically increased in value in 2008.

Given the calibrated parameters from the CDS market in the proposed model, we use the information in the bank's financial statements to evaluate the annul deposit insurance premiums. The total assets, total liability, reserve position (γ) ratio, and investment position (ω) of WaMu, and the annual premiums on one deposit from 2004 to 2008 are listed in Table 6.

The annual deposit insurance premiums in 2004 and 2005 were 0.04 and 0.06 bps, respectively, for each dollar of deposit, which decreased to 0.006 bps in 2006. Because the CDS spread increased in 2007, the credit risk is also reflected in the premiums, which increased to 0.6 bps. In the year WaMu failed, 2008, the premiums increased significantly to 301 bps, which represents the FDIC should have charged 3% of WaMu's deposit as premiums.

Next, a numerical experiment is designed to determine the optimal hedge ratio, h, which represents the percentage of premiums used to purchase the CDSs can minimise the variance of cash-flow of deposit insurers. The FDIC was sued by WaMu for US\$13 billion after the sale of its banking operations to JPMorgan; thus, we may assume that this amount represents the underwriting losses of the FDIC for covering WaMu.

We may assume a portion of annual premiums are used to purchase 1-year CDSs and no capital forbearance case for simplicity. By using historical market data, we simulate WaMu's assets and deposit liabilities by setting $A(0) = 301947$ million, $D(0) = 276689$ million, $\gamma = 0.0227$, $\omega = 0.1036$, $\sigma_S = 0.0768$, $\sigma_c = 0.0461$, and $\phi = 0.0673$, which are the averages of the values in Tables 5 and 6. The interest rate parameters are calibrated from daily Treasury yield curve rates: $\kappa = 0.2723$, $\theta = 0.0549$, $\sigma_r = 0.0079$, and the initial interest rate is given as 0.0340. The remaining contract parameters are as follows: $T = 1$, $\Delta = 0$, $\varepsilon = 0$ $\alpha = \beta = 1$, and $\eta = 0.8$.

We generate 100,000 sample paths to simulate the payments as the secured bank becomes insolvent, and we solve the optimal hedge ratio by minimising the variance in the

Table 5. Default Probabilities and Model Parameters Calibrated from WaMu's CDSs.

	2004/9/1	2005/9/1	2006/9/1	2007/9/4	2008/9/2
Default probabilities within selected years					
1-year	0.0027	0.0013	0.0013	0.0179	0.4047
2-year	0.0059	0.0048	0.0037	0.0355	0.4953
3-year	0.0098	0.0126	0.0071	0.0528	0.5416
4-year	0.0198	0.0238	0.0128	0.0716	0.5815
5-year	0.0357	0.0385	0.0208	0.0921	0.6218
6-year	0.0477	0.0522	0.0273	0.1072	0.6374
7-year	0.0557	0.0652	0.0332	0.1173	0.6538
8-year	0.0661	0.0795	0.0413	0.1302	0.6767
9-year	0.0783	0.0950	0.0515	0.1461	0.6975
10-year	0.0927	0.1117	0.0633	0.1640	0.7168
Calibrated parameters					
σ_s	0.0357	0.0329	0.0330	0.0465	0.2356
σ_c	0.0215	0.0198	0.0198	0.0279	0.1414
ϕ	0.0471	− 0.1442	0.0340	0.0290	0.3702

Table 6. Financial Ratios and Premiums of WaMu.

	2003/12/31	2004/12/31	2005/12/31	2006/12/31	2007/12/31
Total Asset (in million)	234680	272927	330706	345610	325808
Total Liability (in million)	215927	251795	300694	315612	299415
Reserve Position (γ)	0.0378	0.0172	0.0193	0.0231	0.0159
Security Position (ω)	0.1653	0.0732	0.0862	0.0815	0.1120
Loan Position	0.7969	0.9096	0.8945	0.8954	0.8721
Premiums per dollar	4.08e-06	6.36e-06	5.80e-07	6.64e-05	0.0301

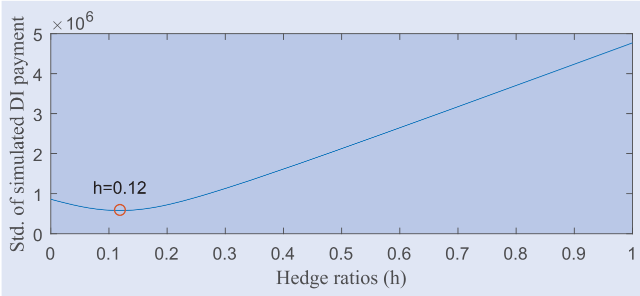

Figure 2. Standard deviations of deposit insurance payments under various hedge ratios.

Beside the studies mentioned above, the CDS is a very concentrated market and is contracting at a fast pace in recent years. Deposit insurance corporations provide deposit insurance that guarantees the safety of depositor accounts and charges insurance premiums from depository institutions that maintain the deposit insurance funds. If the deposit insurer involved too much in the CDS market, the financial safety net system may be affected by risk contagion and undermine the effect of the risk isolation. The CDS market allows us to calibrate the bank's asset risk. But, if the deposit insurer intends to hedge through the CDSs, it needs to consider the consequences in term of financial stability carefully.

present value of the payment function:

$$\min_h \operatorname{Var}(f(T)e^{-rT} \cdot 1_{\{\tau \geq T\}} + f(\tau)e^{-r\tau} \cdot 1_{\{\tau < T\}})$$

$$\text{where } f(t) = D(t) - A(t) - h \cdot \frac{P^a(0) + P^c(0)}{CDS_{\text{premium}}(0)} \cdot 10000000 \tag{23}$$

In Equation (23), τ is the first time that bank assets drop to breach the maintenance working capital level. $D(t) - A(t)$ represents the amount of deposit insurance that should be paid by the insurer when the bank fails. $P^a(0) + P^c(0)$ is the premiums without capital forbearance. $(P^a(0) + P^c(0))/CDS_{\text{premium}}$ stands for units of the CDSs which the FDIC uses all received premiums to buy, and the CDS_{premium} is the cost for one CDS contract at a notional amount of \$10 million. The relationship between the variance of the objective function and the hedging ratio is displayed in Figure 2, and we choose the optimal hedge ratio, $h = 0.12$. That is, the FDIC has minimum variance of cash-flow by using 12% of the annual premiums of WaMu to buy WaMu's 1-year CDS.†

However, there are some concerns about the CDS market. Allen and Carletti (2006) show the credit risk transfer can be beneficial when banks face uniform demand for liquidity, but it can also induce contagion and increase the risk of crises. Heyde and Heyer (2010) point out the CDS creates a channel of contagion because it allows the banks to have contingent claims on each other. The criticism of the CDS, including that CDS is largely harmful to firms because it decreases the efficiency of the bond market and experiences no improvement in liquidity (Das *et al.* 2014). The CDS price is inconsistency derived from the private price's providers because it is an Over-The-Counter (OTC) market (Mayordomo *et al.* 2014). And the CDS market more effectively contribute to price discover than stock market only during the tranquil times (Lovreta and Forte 2015).

† The deposit insurance is an insurance contract between a bank and an insurer, and a CDS links the insurer and investors in the credit market. If an insurer takes part of the deposit insurance premium to buy the CDS, then a part of the bank's risk that the insurer bears is essentially transferred to the CDS market. In this manner, the bank's total asset risk remains at the same level, but the cash-flow variation of the insurer decreases because the insurer receives less premium and bears less risk of the bank. The CDS sellers (protection seller) receive CDS premiums from insurers and provide protection to insurers on the bank deposits.

5. Conclusion

Analysing premiums on the basis of the necessary conditions required by deposit insurance—including minimum capital requirements, capital forbearance thresholds, grace periods, early closure regularity, and the prevention of moral hazard—is a common concern in the literature. This paper constructs an explicit deposit insurance scheme and derives a closed-form pricing formula for fair premiums that comprises three components: early closure, capital forbearance, and a grace period.

To determine the importance of risk factors and insurance policies, we use scenario analysis to ascertain how the debt-to-asset ratio, policy instruments, weights of asset allocation, credit and security market risks, and moral hazard affect premiums. We extend a long grace period to demonstrate how much risk the deposit insurance corporation assumes in the case of banks that are too big to fail. The numerical results reveal that insurance premiums increase quickly with the debt-to-asset ratio and the additional moral hazard in the grace period, and the ratio of asset allocations has a substantial influence on premiums because of the various risks in the security and loan markets. Moreover, the premiums increase as the capital forbearance threshold decreases when the increment of the grace period component is larger than the decrement of the capital forbearance component.

The main advantage of the closed-form solutions of the proposed model is that calibrating the model's parameters from the market is easy. Rather than using historical deposit insurance premiums, we propose calibrating the parameters on the basis of credit derivatives and evaluating the risk-based premiums according to the expected equilibrium of credit market participants on the future credit risk of the bank.

The deposit insurance corporations act as the bank's ultimate credit risk taker. We suggest that deposit insurers charge more premiums from risky banks and determine proper premiums for its insurance on the basis of the active credit derivatives market. Risk-based premiums depend on ascertaining the bank's risk through the active credit market and subsequently evaluating the premiums under the option pricing framework, including the comprehensive deposit insurance scheme.

By applying the proposed premiums pricing formula, we use WaMu as an example to illustrate how to calibrate the model parameters from the credit market and derive the risk-based premiums. In the year WaMu failed, the premiums increased significantly to 301 bps, which represents the FDIC should have charged 3% of WaMu's deposit as premiums.

Disclosure statement

No potential conflict of interest was reported by the author(s).

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Appendix: The Closed-Form Solution for Deposit Insurance Premium

The risk-based premium of deposit insurance paid by the deposit insurer to the security bank is the expected discounted insurance payoff under a risk-neutral probability measure and can be decomposed into three parts, as shown in Equation (14). The first part is the default premium of the security bank before the time of the audit. Second, the default premium of the security bank will be made when the bank's value drops below the capital forbearance threshold at the time of the audit. Finally, the deposit insurance must pay the default premium when the regulator takes over the depository institution during or at the end of the grace period:

$$\begin{aligned}P(0) &= E^Q\left[\frac{P(\tau)}{M(\tau)}I_{\{\tau<T_1\}}\right]\\ &\quad + E^Q\left[\frac{P(T_1)}{M(T_1)}I_{\left\{\tau\geq T_1,\frac{A(T_1)}{D(T_1)}<\beta\right\}}\right]\\ &\quad + E^Q\left[\frac{P(T_1)}{M(T_1)}I_{\left\{\tau\geq T_1,\beta<\frac{A(T_1)}{D(T_1)}<\alpha\right\}}\right].\end{aligned}$$

To price the risk-based premium of deposit insurance, we show the asset value and the asset-debt ratio of the discounted bank, according to the model assumptions:

$$\frac{A(t)}{M(t)} \stackrel{d}{=} A(0)\exp\left\{-\frac{1}{2}\sigma^2 t+\sigma W^Q(t))\right\} \tag{A1}$$

and

$$\begin{aligned}\frac{A(t)}{D(t)} &\stackrel{d}{=} \frac{A(0)}{D(0)}\exp\left\{\left(-\frac{\sigma^2}{2}-\varepsilon\right)t+\sigma W_A^Q(t)\right\}\\ &= \frac{A(0)}{D(0)}\exp\{vt+\sigma W_A^Q(t)\},\end{aligned}$$

where $D(t)=D(0)e^{\varepsilon t}M(t)$ denotes the deposit liability that increases with the money market account, $v=-\varepsilon-\sigma^2/2$, $\sigma=\sqrt{\omega^2\sigma_S^2+(1-\gamma-\omega)^2\sigma_L^2+g(\varepsilon)}$ represents the total risk of the bank's assets as weighted by loan position and investing position, $\sigma_L=\sqrt{\phi^2\sigma_r^2+\sigma_c^2}$ represents the risk of the loan position that incorporates interest rate risk, σ_S is the secondary market risk, and $\stackrel{d}{=}$ indicates equal in distribution. The three components of the deposit insurance premium are derived in the following lemmas.

Lemma 1 *The premium of the audit window component is given by the following:*

$$\begin{aligned}&E^Q\left[\frac{P(\tau)}{M(\tau)}I_{\{\tau<T_1\}}\right]\\ &\quad=\xi_\varepsilon(1-\eta)D(0)e^{\frac{v-u}{\sigma^2}\ln\left(\frac{\eta D(0)}{A(0)}\right)}\left[\Phi\left(\frac{\ln\frac{\eta D(0)}{A(0)}-uT_1}{\sigma\sqrt{T_1}}\right)\right.\\ &\quad\left.+e^{\frac{2u}{\sigma^2}\ln\left(\frac{\eta D(0)}{A(0)}\right)}\Phi\left(\frac{\ln\frac{\eta D(0)}{A(0)}+uT_1}{\sigma\sqrt{T_1}}\right)\right]\end{aligned}$$

where $v=-\varepsilon-\sigma^2/2$ and $u=\sqrt{v^2-2\sigma^2\varepsilon}$.

Proof According to the definition of early closure that $\tau=\inf\{t|A(t)\leq\eta D(t)\}$ comes before T_1 and the theorem of the first passage time, we obtain the probability of default before the time of the audit as follows:

$$\begin{aligned}&\Pr^Q\left(\tau<T_1|\frac{A(0)}{D(0)}>\eta\right)\\ &\quad=\Pr^Q\left(\min_{0\leq s\leq T_1}\frac{A(s)}{D(s)}<\eta\,\middle|\,\frac{A(0)}{D(0)}>\eta\right)\\ &\quad=\Pr^Q\left(\min_{0\leq s\leq T_1}(vs+\sigma W^Q(s))<\ln(\frac{\eta D(0)}{A(0)})\,\middle|\,\frac{A(0)}{D(0)}>\eta\right)\\ &\quad=\Phi\left(\frac{\ln(\frac{\eta D(0)}{A(0)})-vT_1}{\sigma\sqrt{T_1}}\right)+e^{\frac{2v}{\sigma^2}\ln\left(\frac{\eta D(0)}{A(0)}\right)}\Phi\left(\frac{\ln(\frac{\eta D(0)}{A(0)})+vT_1}{\sigma\sqrt{T_1}}\right)\end{aligned}$$

Let $f_\tau(t)=\Pr^Q[\tau\in dt]=\frac{1}{\partial T_1}\partial\Pr^Q\left(\tau<T_1|\frac{A(0)}{D(0)}>\eta\right)\Big|_{\tau=T_1}$ be the density function of τ that occurs instantaneously. A further straightforward calculation yields $f_\tau(t)=\frac{|y|}{\sqrt{2\pi\sigma^2t^3}}e^{\frac{(y-vt)^2}{-2\sigma^2t}}$, where $y=\ln(\frac{\eta D(0)}{A(0)})$.

$$\begin{aligned}&E^Q\left[\frac{P(\tau)}{M(\tau)}I_{\{\tau<T_1\}}\right]\\ &\quad=\xi_\varepsilon(1-\eta)D(0)E^Q[e^{\varepsilon\tau}I_{\{\tau<T_1\}}]\\ &\quad=\xi_\varepsilon(1-\eta)D(0)\int_0^{T_1}e^{\varepsilon t}\frac{|y|}{\sqrt{2\pi\sigma^2t^3}}e^{\frac{(y-vt)^2}{-2\sigma^2t}}dt\\ &\quad=\xi_\varepsilon(1-\eta)D(0)e^{\frac{v-u}{\sigma^2}\ln\left(\frac{\eta D(0)}{A(0)}\right)}\left[\Phi\left(\frac{\ln\frac{\eta D(0)}{A(0)}-uT_1}{\sigma\sqrt{T_1}}\right)\right.\\ &\qquad\left.+e^{\frac{2u}{\sigma^2}\ln\left(\frac{\eta D(0)}{A(0)}\right)}\Phi\left(\frac{\ln\frac{\eta D(0)}{A(0)}+uT_1}{\sigma\sqrt{T_1}}\right)\right]\end{aligned}$$

■

Lemma 2 *The deposit insurance premium of a capital forbearance component is calculated as follows:*

$$E^Q\left[\frac{P(T_1)}{M(T_1)}I_{\left\{\tau\geq T_1,\frac{A(T_1)}{D(T_1)}<\beta\right\}}\right]$$

$$= \xi_\varepsilon D(0)e^{\varepsilon T_1}\begin{bmatrix}(\Phi(c_1(\beta,v))-\Phi(c_1(\eta,v)))\\ -e^{\frac{2vB(\eta)}{\sigma^2}}(\Phi(c_2(\beta,\eta,v))-\Phi(-c_1(\eta,-v)))\end{bmatrix}$$
$$-A(0)\begin{bmatrix}(\Phi(c_1(\beta,\tilde{v}))-\Phi(c_1(\eta,\tilde{v})))\\ -e^{\left(\frac{2v}{\sigma^2}+2\right)B(\eta)}(\Phi(c_2(\beta,\eta,\tilde{v}))-\Phi(-c_1(\eta,\tilde{v})))\end{bmatrix}$$

where $B(x)=\ln\left(\frac{xD(0)}{A(0)}\right)$, $c_1(x,z)=\frac{B(x)-zT_1}{\sigma\sqrt{T_1}}$, $c_2(x,y,z)=\frac{B(x)-2B(y)-zT_1}{\sigma\sqrt{T_1}}$, $v=-\varepsilon-\sigma^2/2$, *and* $\tilde{v}=v+\sigma^2$.

Proof To compute the joint probability of the no-default event before the time of the audit and the default event at the time of the audit, we can also evaluate the joint probability of the asset-debt ratio $A(T_1)/D(T_1)$ lower than b_1 at maturity and the minimum of the ratio $\min\limits_{0\le s\le T_1}(A(s)/D(s))$ higher than b_2 before maturity under the risk-neutral measure as follows:

$$\Pr^Q\left(\frac{A(T_1)}{D(T_1)}<b_1,\ \min_{0\le s\le T_1}\frac{A(s)}{D(s)}>b_2\right)$$
$$=\Pr^Q\left(vT_1+\sigma W^Q(T_1)<\ln\left(\frac{b_1D(0)}{A(0)}\right),\ \min_{0\le s\le T_1}(vs+\sigma W^Q(s))>\ln\left(\frac{b_2D(0)}{A(0)}\right)\right)$$
$$=\left[\Phi\left(\frac{\ln\left(\frac{b_1D(0)}{A(0)}\right)-vT_1}{\sigma\sqrt{T_1}}\right)-\Phi\left(\frac{\ln(\frac{b_2D(0)}{A(0)})-vT_1}{\sigma\sqrt{T_1}}\right)\right]$$
$$-e^{\frac{2v}{\sigma^2}\ln\left(\frac{b_2D(0)}{A(0)}\right)}\left[\Phi\left(\frac{\ln\left(\frac{b_1D(0)}{A(0)}\right)-2\ln\left(\frac{b_2D(0)}{A(0)}\right)-vT_1}{\sigma\sqrt{T_1}}\right)-\Phi\left(\frac{-\ln\left(\frac{b_2D(0)}{A(0)}\right)-vT_1}{\sigma\sqrt{T_1}}\right)\right] \tag{A2}$$

where $\Phi(\cdot)$ denotes the cumulative function of standard normal distribution.

The default premium of the security bank when the bank's value drops below the capital forbearance threshold at the time of the audit can be derived as follows:

$$E^Q\left[\frac{P(T_1)}{M(T_1)}I_{\left\{\tau\ge T_1,\frac{A(T_1)}{D(T_1)}<\beta\right\}}\right]$$
$$=E^Q\left[\xi_\varepsilon D(0)e^{\varepsilon T_1}I_{\left\{\tau>T_1,\frac{A(T_1)}{D(T_1)}<\beta\right\}}\right]$$
$$-E^Q\left[\xi_\varepsilon A(0)e^{vT_1+\sigma W^Q(T_1)}I_{\left\{\tau>T_1,\frac{A(T_1)}{D(T_1)}<\beta\right\}}\right]$$
$$=\xi_\varepsilon e^{\varepsilon T_1}\left(D(0)\Pr^Q\left(\frac{A(T_1)}{D(T_1)}<\beta,\ \min_{0\le s\le T_1}\frac{A(s)}{D(s)}>\eta\right)-A(0)\Pr^{\tilde{Q}}\left(\frac{A(T_1)}{D(T_1)}<\beta,\ \min_{0\le s\le T_1}\frac{A(s)}{D(s)}>\eta\right)\right)$$

According to the Girsanov theorem, $\tilde{Q}$ is another measure related to the Q measure, and the Brownian motion under $\tilde{Q}$ will be $dW^Q_{A,t}=dW^{\tilde{Q}}_{A,t}+\sigma dt$. Appling the results of Equation (A2), we have the following:

$$E^Q\left[\frac{P(T_1)}{M(T_1)}I_{\left\{\tau\ge T_1,\frac{A(T_1)}{D(T_1)}<\beta\right\}}\right]$$
$$=\xi_\varepsilon D(0)e^{\varepsilon T_1}\left[(\Phi(c_1(\beta,v)))-\Phi(c_1(\eta,v)))-e^{\frac{2vB(\eta)}{\sigma^2}}(\Phi(c_2(\beta,\eta,v))-\Phi(-c_1(\eta,-v)))\right]$$
$$-\xi_\varepsilon A(0)e^{\varepsilon T_1}\left[(\Phi(c_1(\beta,\tilde{v}))-\Phi(c_1(\eta,\tilde{v})))-e^{\left(\frac{2v}{\sigma^2}+2\right)B(\eta)}(\Phi(c_2(\beta,\eta,\tilde{v}))-\Phi(-c_1(\eta,-\tilde{v})))\right]$$

Hence, we complete the proof of lemma 2. ■

LEMMA 3 *The deposit insurance premium of the grace period component is represented as the following:*

$$E^Q\left[\frac{P(T_1)}{M(T_1)}I_{\left\{\tau>T_1,\beta<\frac{A(T_1)}{D(T_1)}<\alpha\right\}}\right]$$
$$=\xi_\varepsilon e^{\varepsilon T_1}\left(D(0)\begin{Bmatrix}N(c_1(\alpha,v),e_1(v),\delta)-N(c_1(\beta,v),e_1(v),\delta)\\ -e^{\frac{2vB(\eta)}{\sigma^2}}[N(c_2(\alpha,\eta,v),e_2(v),\delta)-N(c_2(\beta,\eta,v),e_2(v),\delta)]\end{Bmatrix}\right.$$
$$\left.+A(0)\begin{Bmatrix}N(c_1(\alpha,\tilde{v}),e_1(\tilde{v}),\delta)-N(c_1(\beta,\tilde{v}),e_1(\tilde{v}),\delta)\\ -e^{\left(\frac{2v}{\sigma^2}+2\right)B(\eta)}[N(c_2(\alpha,\eta,\tilde{v}),e_2(\tilde{v}),\delta)-N(c_2(\beta,\eta,\tilde{v}),e_2(\tilde{v}),\delta)]\end{Bmatrix}\right)$$

where $N(c,e,\delta)=\int_{-\infty}^{c}\Phi(\frac{e-\delta Z}{\sqrt{1-\delta^2}})\varphi(Z)dZ$, $\varphi(Z)$ *represents the probability density function of standard normal distribution,* $\delta=\sqrt{T_1/T_2}$, $v=-\varepsilon-\sigma^2/2$, $\tilde{v}=v+\sigma^2$, $B(x)=\ln\frac{xD(0)}{A(0)}$, $c_1(x,z)=\frac{B(x)-zT_1}{\sigma\sqrt{T_1}}$, $c_2(x,y,z)=\frac{B(x)-2B(y)-zT_1}{\sigma\sqrt{T_1}}$, $e_1(v)=\frac{B(1)-vT_2}{\sigma\sqrt{T_2}}$, *and* $e_2(v)=\frac{B(1)-2B(\eta)-vT_2}{\sigma\sqrt{T_2}}$.

Proof

$$E^Q\left[\frac{P(T_1)}{M(T_1)}I_{\left\{\tau>T_1,\beta<\frac{A(T_1)}{D(T_1)}<\alpha\right\}}\right]$$
$$=E^Q\left[\frac{\xi_\varepsilon}{M(T_1)}E^Q\left[\frac{M(T_1)}{M(T_2)}\max\{D(T_2)-A(T_2),0\}\ \middle|\ \beta<\frac{A(T_1)}{D(T_1)}<\alpha,\ \min_{0\le s\le T_1}\frac{A(s)}{D(s)}>\eta\right]\right]$$
$$=E^Q\left[\xi_\varepsilon D(0)e^{\varepsilon T_1}\Phi(b_2)I_{\left\{\tau>T_1,\beta<\frac{A(T_1)}{D(T_1)}<\alpha\right\}}\right]$$
$$-E^Q\left[\xi_\varepsilon A(0)e^{vt+\sigma W^Q(t)}\Phi(b_1)I_{\left\{\tau>T_1,\beta<\frac{A(T_1)}{D(T_1)}<\alpha\right\}}\right]$$
$$=\xi_\varepsilon D(0)e^{\varepsilon T_1}E^Q\left[\Phi(b_2)I_{\left\{\tau>T_1,\beta<\frac{A(T_1)}{D(T_1)}<\alpha\right\}}\right]$$
$$-\xi_\varepsilon A(0)e^{\varepsilon T_1}E^{\tilde{Q}}\left[\Phi(b_1)I_{\left\{\tau>T_1,\beta<\frac{A(T_1)}{D(T_1)}<\alpha\right\}}\right]$$
$$=\xi_\varepsilon e^{\varepsilon T_1}\left(D(0)\int_{B(\beta)}^{B(\alpha)}\int_{B(\eta)}^{x}\Phi(b_2)f(x,y)dydx-A(0)\int_{B(\beta)}^{B(\alpha)}\int_{B(\eta)}^{x}\Phi(b_1)f'(x,y)dydx\right) \tag{A3}$$

where $b_1=-\frac{\ln\frac{A(T_1)}{D(T_1)}-v\varDelta}{\sigma\sqrt{\varDelta}}$, $b_2=b_1-\sigma\sqrt{\varDelta}$, and $B(z)=\ln\frac{zD(0)}{A(0)}$.

The $f(x,y)$ takes the first derivative of Equation (A2) with respect to $a^*=\ln\left(\frac{b_1D(0)}{A(0)}\right)$ and $b^*=\ln(\frac{b_2D(0)}{A(0)})$, and then we obtain the joint probability density as follows:

$$f(x,y) = \left.\frac{\partial^2 \Pr\left(vT_1 + \sigma W^Q(T_1) < a^*, \min_{0\le s\le T_1}(vs + \sigma W^Q(s)) > b^*\right)}{\partial a^* \partial b^*}\right|_{\substack{a^*=x,\\ b^*=y}}$$

$$= \frac{2(x-2y)}{\sigma^2 T_1\sqrt{2\pi\sigma^2 T_1}} e^{-\frac{(x-2y)^2}{2\sigma^2 T_1}+\left(\frac{vx}{\sigma^2}-\frac{v^2 T_1}{2\sigma^2}\right)}$$

$f'(x,y)$ is the joint probability density function of Equation (A2) under another measure $\tilde{Q}$, where the relative asset dynamic under the $\tilde{Q}$ measure is $\frac{A(t)}{D(t)} = \frac{A(0)}{D(0)}\exp\{(\nu+\sigma^2)t + \sigma W^{\tilde{Q}}(t))\}$. Thus, as with $f(x,y)$, we can verify the following:

$$f'(x,y) = \frac{2(x-2y)}{\sigma^2 T_1\sqrt{2\pi\sigma^2 T_1}} \times \exp\left\{-\frac{(x-2y)^2}{2\sigma^2 T_1} + \left(\frac{(\nu+\sigma^2)x}{\sigma^2} - \frac{(\nu+\sigma^2)^2 T_1}{2\sigma^2}\right)\right\}.$$

The first term of Equation (A3) can be computed as follows:

$$\xi_\varepsilon D(0)e^{\varepsilon T_1}\int_{B(\beta)}^{B(\alpha)}\int_{B(\eta)}^{x}\Phi(b_2)f(x,y)dydx$$

$$= \xi_\varepsilon D(0)e^{\varepsilon T_1}\int_{B(\beta)}^{B(\alpha)}\int_{B(\eta)}^{x}\Phi\left(-\frac{\ln\frac{A(T_1)}{D(T_1)}+\nu\Delta}{\sigma\sqrt{\Delta}}\right) \times \left(\frac{2(x-2y)}{\sigma^2 T_1\sqrt{2\pi\sigma^2 T_1}} e^{-\frac{(x-2y)^2}{2\sigma^2 T_1}+\left(\frac{vx}{\sigma^2}-\frac{v^2 T_1}{2\sigma^2}\right)}\right)dydx$$

$$= \xi_\varepsilon D(0)e^{\varepsilon T_1}\int_{B(\beta)}^{B(\alpha)}\Phi\left(\frac{B(1)-x-\nu\Delta}{\sigma\sqrt{\Delta}}\right) \times \frac{1}{\sqrt{2\pi\sigma^2 T_1}} e^{\frac{vx}{\sigma^2}-\frac{v^2 T_1}{2\sigma^2}}\left(\int_{B(\eta)}^{x}\frac{2(x-2y)}{\sigma^2 T_1}e^{-\frac{(x-2y)^2}{2\sigma^2 T_1}}dy\right)dx$$

$$= \xi_\varepsilon D(0)e^{\varepsilon T_1}\left\{\begin{array}{c}\int_{c_1(\beta,\nu)}^{c_1(\alpha,\nu)}\Phi\left(\frac{B(1)-\sigma\sqrt{T_1}Z-\nu T_2}{\sigma\sqrt{\Delta}}\right)\frac{e^{-\frac{Z^2}{2}}}{\sqrt{2\pi}}dZ \\ -e^{\frac{2\nu B(\eta)}{\sigma^2}}\int_{c_2(\beta,\eta,\nu)}^{c_2(\alpha,\eta,\nu)}\Phi\left(\frac{B(1)-2B(\eta)-\sigma\sqrt{T_1}\tilde{Z}-\nu T_2}{\sigma\sqrt{\Delta}}\right) \\ \frac{e^{-\frac{\tilde{Z}^2}{2}}}{\sqrt{2\pi}}d\tilde{Z}\end{array}\right\}$$

$$= \xi_\varepsilon D(0)e^{\varepsilon T_1}\left\{\begin{array}{c}[N(c_1(\alpha,\nu),e_1(\nu),\delta) - N(c_1(\beta,\nu),e_1(\nu),\delta)] \\ -e^{\frac{2\nu B(\eta)}{\sigma^2}}[N(c_2(\alpha,\eta,\nu),e_2(\nu),\delta) \\ -N(c_2(\beta,\eta,\nu),e_2(\nu),\delta)]\end{array}\right\}$$

where $Z = (x-\nu T_1)/\sigma\sqrt{T_1}$ and $\tilde{Z} = (x-2B(\eta)-\nu T_1)/\sigma\sqrt{T_1}$ are changing variables to simplify the integrations. Similarly, the second term in Equation (A3) can be derived as the following:

$$\xi_\varepsilon A(0)e^{\varepsilon T_1}\int_{B(\beta)}^{B(\alpha)}\int_{B(\eta)}^{x}\Phi(b_1)f'(x,y)dydx$$

$$= \xi_\varepsilon A(0)e^{\varepsilon T_1}\left\{\begin{array}{c}N(c_1(\alpha,\tilde{\nu}),e_1(\tilde{\nu}),\delta) - N(c_1(\beta,\tilde{\nu}),e_1(\tilde{\nu}),\delta) \\ -e^{\left(\frac{2\nu}{\sigma^2}+2\right)B(\eta)}(N(c_2(\alpha,\eta,\tilde{\nu}),e_2(\tilde{\nu}),\delta) \\ -N(c_2(\beta,\eta,\tilde{\nu}),e_2(\tilde{\nu}),\delta))\end{array}\right\}$$

Hence, we complete the calculation of lemma 3. ∎