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ORIGINAL RESEARCH

Imposing Regularity Conditions to Measure Banks' Productivity Changes in Taiwan Using a Stochastic Approach

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Abstract
This paper develops a stochastic approach to impose regularity properties on a directional output distance function (DODF) and an output distance function, which can be estimated by maximum likelihood. We use the resulting parameter estimates to evaluate efficiency and total factor productivity (TFP) growth for Taiwan's commercial banks over the period 2002–2015 and claim that the failure of considering the regularity restrictions and the exclusion of undesirables lead to miscalculated efficiency measures and productivity gains. The outcomes from the regularity-constrained DODF reveal that almost all data-points satisfy the regularity properties, that the managerial abilities of the banks improve after the subprime crisis of 2007, and that the sample banks' TFP grow at an average rate of 1.93% per annum, whereby technical change is the driving force. However, our estimates show downward trends in the growth rate of TFP and technical change.

Keywords Stochastic approach · Regularity properties · Maximum likelihood · TFP growth · Undesirables

JEL Classification C30 · C51 · D24 · G21

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1 Introduction

There are two popular approaches to examine firms' efficiency and productivity change: non-parametric data envelopment analysis (DEA) and parametric stochastic frontier analysis (SFA). Both approaches have their own strengths and weaknesses. DEA is a mathematical programming technique that does not require specifying a production or a cost function form, which is usually unknown to the researchers. However, DEA estimates a deterministic frontier and treats all deviations from the frontier as inefficiency, which can be confounded with the effects of data noise or random shocks. O'Donnell and Coelli (2005) note that this may lead to biased estimates of the shape and position of the frontier surface.

Pioneered by Aigner et al. (1977) and Meeusen and van den Broeck (1977), SFA is an econometric approach that requires one to specify the boundary of the production technology with a composite error term consisting of non-negative inefficiency and a noise component. Production, cost, and profit frontiers are commonly used to measure banks' efficiency and productivity, e.g., Berger et al. (1993) and Berger et al. (1999). Output and directional distance functions are becoming popular recently, due possibly to the fact that they are able to describe the technology relationship between multiple inputs and multiple outputs without requiring information on prices, particularly when prices are not available or inaccurate. See, for example, Färe et al. (2005), Koutsomanoli-Filippaki et al. (2009), Feng and Serletis (2010), Feng and Zhang (2012), Huang et al. (2015), and Huang and Chung (2017).

Distance functions must satisfy theoretical regularity properties, i.e., monotonicity and curvature. Specifically, the directional output distance function (DODF) is non-decreasing in inputs and undesirable outputs, non-increasing in outputs, and jointly concave in desirable and undesirable outputs. Most previous studies in various industries' productivity literature have failed to check or impose those theoretical regularity conditions, except for, e.g., Terrell (1996), Griffiths et al. (2000), Kleit and Terrell (2001), O'Donnell and Coelli (2005), and Feng and Serletis (2010, 2014), who apply a Bayesian approach to impose the required restrictions. Although Geweke (1986), Poirier (1995), and Feng and Serletis (2010) have addressed the relative advantages of the Bayesian approach, it has some disadvantages. Please see, for example, Robert (2007, Chapter 11) for more in-depth comments on Bayesian analysis.

Kuosmanen (2008), Seijo and Sen (2011), and Yagi et al. (2018) suggest the use of convex non-parametric least squares (CNLS) to impose the regularity conditions, but this requires solving the quadratic programming (QP) problem. Kuosmanen and Kortelainen (2012) and Mekaroonreung and Johnson (2012) also rely on employing the QP formulation in the first stage and estimating technical efficiency in the second stage through the modified OLS. Du et al. (2013) and Li et al. (2017) offer a non-parametric regression model that is able to impose monotonicity and curvature.

Failure to consider theoretical regularity conditions into a distance function may lead the estimated unconstrained function to be inconsistent with those conditions, at least for a part of sample points. The subsequent calculations of efficiency scores and productivity changes may be unreliable, if not implausible, and lack any reference. Moreover, the violation of the monotonicity condition results in misleading partial derivatives that are further applied to calculate, e.g., shadow prices, elasticities, and cost shares. For instance, a negative partial derivative of DDF with respect to some input for some data points implies that an increase in that input, while holding other inputs and outputs constant, will decrease (increase) the inefficiency (efficiency) of the corresponding firms, which is economically doubtful.

The current paper examines efficiency and productivity issues for banks in Taiwan over the sample period 2002–2015, by imposing the theoretical regularity conditions on DODF without depending on a Bayesian approach or programming techniques. The Taiwan government passed a financial holding company (FHC) act in 2001, and there are now 16 FHCs operating in the island. A financial holding company is a financial institution engaged in banking-related activities, offering customers a wide range of financial services, such as purchasing insurance products and investment in securities. The commercial banks in Taiwan can be classified into three types. A financial holding bank (FHB) is a subsidiary of an FHC, and its scale is usually larger than non-financial holding banks (Non-FHBs) that do not belong to any financial group. A foreign bank is obligated to follow the regulations of both the home and host countries, and its scale is close to Non-FHBs in Taiwan.

After entering the World Trade Organization in 2002, banks in Taiwan experienced either regulatory change, e.g., the "First Financial Restructuring" from 2002 to 2003 aiming to write off the non-performing loans (NPLs) of financial institutions and encouraging mergers and acquisitions between banks, or financial distresses, e.g., the U.S. subprime crisis starting in August 2007 and the European debt crisis in 2010. Whether the above events have spurred efficiency and productivity in this sector is an interesting and important topic worth studying. Hsiao et al. (2010) apply DDF to explore the efficiency of Taiwan's banking industry using DEA. They find that banks have a lower operating efficiency during the FFR reform period, but have a higher operating efficiency in the post-reform period. Chen (2012) finds that public and private banks in Taiwan have similar efficiency scores and productivity gains, after taking account of the risk input.

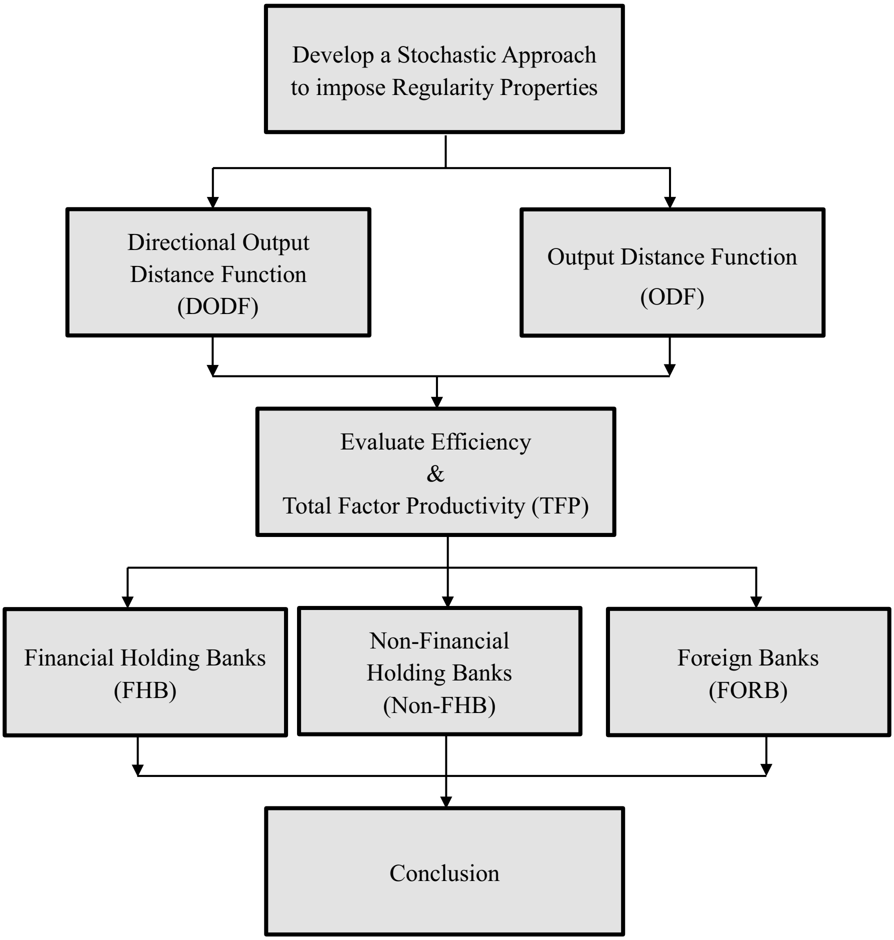
We propose a stochastic approach to deal with the regularity conditions under the SFA framework, which can be implemented by the standard maximum likelihood (ML). The estimators have desirable asymptotic properties and allow for conducting statistical inferences. In contrast, non-parametric programming techniques suffer from the lack of statistical properties. This research appears to be the first one in the literature on the evaluation of banking performance that imposes the regularity conditions using a stochastic approach. Another advantage of our approach is that it will not destroy the flexibility of the translog cost function, as pointed out by, e.g., Diewert and Wales (1987) and Ryan and Wales (2000).[1]

[1] We do not show the relevant results for brevity.

We select DODF for two reasons. First, this function explicitly takes account of undesirables, which are byproducts that are often not marketed, and their disposal is often subject to regulation and hence consumes resources. Since the disposal of undesirables requires firms employing more resources, it is suggested that any performance evaluation model consider their characteristics, e.g., Färe and Grosskopf (2005), Feng and Serletis (2010, 2014), Huang et al. (2015), Huang and Chung (2017), to mention a few. To highlight the importance of including bad outputs we also estimate the output distance function (ODF) that excludes undesirables. Second, DODF, like other forms of distance function and production function, requires quantity information only, which is subject to less degrees of measurement problem. The estimation of a cost or a profit function needs input and/or output price information, which is criticized to be susceptible to measurement errors. Kauko (2009) and Feng and Serletis (2010) have mentioned this problem.

The rest of the paper is arranged as follows. Section 2 introduces the quadratic DODF and translog ODF, proposes the methods of imposing the monotonicity and curvature restrictions on DODF and ODF under a stochastic approach, and illustrates the decomposition of total factor productivity (TFP) changes with respect to DODF and ODF. Section 3 deals with data issues. Section 4 employs our methodology to conduct an empirical study using panel data of 51 commercial banks in Taiwan, discusses the effects of incorporating monotonicity and curvature with and without the inclusion of undesirables, and presents the estimates of TFP growth for both DODF and ODF. The last section concludes the paper.

We use the following diagram to illuminate the research framework of the current paper.



2 Methodology

This section develops a stochastic approach to impose monotonicity and curvature conditions on DODF and ODF and describes the estimation procedures using ML.

2.1 Directional Output Distance Function

Input quantities are defined by $x = \left(x_1, \dots, x_N\right)' \in R_+^N$, and the output vector is defined as $y = \left(y_1, \dots, y_M\right)' \in R_+^M$. In this paper we identify a bad output of non-performing loans, which is jointly produced with the desirable output of various loans and is denoted by $b \in R_+$. The technology set is expressed as:

$T = \{(x, y, b)$: x can be used by banks to produce $(y, b)\}$.

The output set $P(x)$ is defined as:

$$P(x) = \{(y, b)|(x, y, b) \in T\}.$$

A directional vector is written as $g = \left(g_y, -g_b\right)'$, in which $g_y \in R_+^M$ and $g_b \in R_+$. We describe DODF as:

$$\vec{D}_o(x, \mathrm{y}, b;\mathrm{g}) = \sup\{\beta : (y + \beta g_y, b - \beta g_b) \in P(x)\}.$$

It gives the maximum amount by which a desirable (undesirable) output vector can be expanded (contracted) in the direction g, in order to be able to produce on the production frontier with a given input vector. In other words, DODF translates the (y, b) vector in the direction g onto the border of the technology.[2]

Since (y, b) is usually interior to technology $P(x)$, the DODF value is non-negative. A bank having a value of $\vec{D}_o(x, \mathrm{y}, b;\mathrm{g}) = 0$ means that it is already producing at the technology frontier, while a value of $\vec{D}_o(x, \mathrm{y}, b;\mathrm{g}) > 0$ indicates that the bank's actual (x, y, b) is below the frontier. Following Färe et al. (2005), Koutsomanoli-Filippaki et al. (2009), Feng and Serletis (2010, 2014), Huang et al. (2015), Huang and Chung (2017), and many others, we specify the directional vector herein as $g = (1, -1)$, which means that a bank can reach the boundary if it simultaneously reduces its undesirable outputs by β units and increases outputs by β units along with the direction $(1, -1)$.

For notational brevity, we assume that the sample banks hire three inputs to produce two desirable outputs and a single undesirable output. We specify DODF as a flexible quadratic functional form that allows for a non-neutral technological change. After imposing restrictions the symmetry and translation properties and appending a random disturbance term of $v_1 \sim N\left(0, \sigma_{v1}^2\right)$, we express DODF as the following regression equation:

$$y_1 = \overrightarrow{D_o}\left(y_2 - y_1, y_3 + y_1, x, t;1, -1\right) - u_1 + v_1 \tag{1}$$

where $y_3 = b$, t signifies the time trend that is used to capture possible technological advances,

$$\begin{aligned}
&\overrightarrow{D_o}\left(y_2 - y_1, y_3 + y_1, x, t;1, -1\right) \\
&\quad = \beta_0 + \alpha_2\left(y_2 - y_1\right) + \alpha_3\left(y_3 + y_1\right) + \gamma_1 x_1 + \gamma_2 x_2 + \gamma_3 x_3 + \beta_\tau t + \frac{1}{2}\alpha_{22}\left(y_2 - y_1\right)^2 + \frac{1}{2}\alpha_{33}\left(y_3 + y_1\right)^2 \\
&\quad + \alpha_{23}\left(y_2 - y_1\right)\left(y_3 + y_1\right) + \frac{1}{2}\gamma_{11}x_1^2 + \frac{1}{2}\gamma_{22}x_2^2 + \frac{1}{2}\gamma_{33}x_3^2 + \gamma_{12}x_1x_2 + \gamma_{13}x_1x_3 + \gamma_{23}x_2x_3 \\
&\quad + \frac{1}{2}\beta_{\tau\tau}t^2 + \delta_{12}x_1\left(y_2 - y_1\right) + \delta_{13}x_1\left(y_3 + y_1\right) + \delta_{22}x_2\left(y_2 - y_1\right) + \delta_{23}x_2\left(y_3 + y_1\right) + \delta_{32}x_3\left(y_2 - y_1\right) \\
&\quad + \delta_{33}x_3\left(y_3 + y_1\right) + \alpha_{\tau 2}t\left(y_2 - y_1\right) + \alpha_{\tau 3}t\left(y_3 + y_1\right) + \gamma_{\tau 1}tx_1 + \gamma_{\tau 2}tx_2 + \gamma_{\tau 3}tx_3
\end{aligned}$$

[2] We implicitly impose the assumption of null-joint production and weak disposability of good and bad outputs, and strong disposability of the good output y. Please see, e.g., Färe and Grosskopf (2005), for the related definitions.

and $u_1 = \overrightarrow{D_o}(y, b, x, t; 1, -1) \sim \left| N\left(0, \sigma_{u1}^2\right) \right|$ denotes technical inefficiency. Terms v_1 and u_1 are conventionally assumed to be statistically independent.

2.1.1 Monotonicity and Curvature Constraints

When estimating (1), its theoretical properties of monotonicity and curvature conditions have to be considered, as pointed out by, e.g., Chambers (2002), Färe et al. (2005), Färe and Grosskopf (2005), and Feng and Serletis (2010, 2014). Specifically, monotonicity in our case requires that $\overrightarrow{D_o}(y, b, x, t; 1, -1)$ be non-increasing in good outputs and non-decreasing in inputs and bad outputs. The cases of good and bad outputs require:

$$\frac{\partial \overrightarrow{D_o}}{\partial y_1} = -1 - \alpha_2 + \alpha_3 - \alpha_{22}\left(y_2 - y_1\right) + \alpha_{33}\left(y_3 + y_1\right) - \alpha_{23}\left(y_3 + y_1\right) + \alpha_{23}\left(y_2 - y_1\right) - \delta_{12}x_1 + \delta_{13}x_1 - \delta_{22}x_2 + \delta_{23}x_2 - \delta_{32}x_3 + \delta_{33}x_3 + \alpha_{\tau 2}t + \alpha_{\tau 3}t \leq 0 \tag{2}$$

$$\frac{\partial \overrightarrow{D_o}}{\partial y_2} = \alpha_2 + \alpha_{22}\left(y_2 - y_1\right) + \alpha_{23}\left(y_3 + y_1\right) + \delta_{12}x_1 + \delta_{22}x_2 + \delta_{32}x_3 + \alpha_{\tau 2}t \leq 0 \tag{3}$$

$$\frac{\partial \vec{D}_o}{\partial y_3} = \alpha_3 + \alpha_{33}\left(y_3 + y_1\right) + \alpha_{23}\left(y_2 - y_1\right) + \delta_{13}x_1 + \delta_{23}x_2 + \delta_{33}x_3 + \delta_{\tau 3}t \geq 0. \tag{4}$$

Equations (2)–(4) are linearly dependent, and it can be shown that Eq. (2) plus (3) equals Eq. (4) minus unity, which is non-positive, i.e., (2) + (3) = (4) − 1 ≤ 0, or equivalently:

$$0 \leq (4) \leq 1. \tag{5}$$

Equation (5) contains two restrictions, i.e., 0 ≤ (4) and (4) ≤ 1, meaning that the set of restrictions (2)–(4) is the same as (3) and (5) in essence. We therefore choose to impose restrictions (3) and (5) on (1).

The monotonicity restrictions in (2)–(4) ensure that the good and bad outputs have non-negative shadow prices. See, for example, Färe et al. (2005).[3] Specifically, the condition $\partial \overrightarrow{D_o}/\partial y_i \leq 0$, $i = 1, 2$, $(\partial D_o/\partial y_3 \geq 0)$, implies that a bank's technical inefficiency does not increase (decrease) when it can produce more of any output (a bad output) under a given input mix. The property $\partial D_o/\partial x_n \geq 0$, $n = 1, 2, 3$, implies that a bank's technical inefficiency does not decrease when it hires more of an input to produce the same levels of desirables.

Note that (3) and (5) are inequalities that need to be further transformed into equalities, i.e.:

[3] Färe et al. (2005) estimate the stochastic DODF without imposing monotonicity, which violates the monotonicity conditions 57 out of 209 times in 1993 and 20 out of 209 times in 1997.

$$\alpha_2 + \alpha_{22}\left(y_2 - y_1\right) + \alpha_{23}\left(y_3 + y_1\right) + \delta_{12}x_1 + \delta_{22}x_2 + \delta_{32}x_3 + \alpha_{\tau 2}t = v_2 - u_2 \quad (6)$$

$$\alpha_3 + \alpha_{33}\left(y_3 + y_1\right) + \alpha_{23}\left(y_2 - y_1\right) + \delta_{13}x_1 + \delta_{23}x_2 + \delta_{33}x_3 + \alpha_{\tau 3}t = v_3 + u_3 \quad (7)$$

$$\alpha_3 + \alpha_{33}\left(y_3 + y_1\right) + \alpha_{23}\left(y_2 - y_1\right) + \delta_{13}x_1 + \delta_{23}x_2 + \delta_{33}x_3 + \alpha_{\tau 3}t - 1 = v_4 - u_4. \quad (8)$$

Here, we first subtract a non-negative random variable (u) from the right-hand side of the equation with the inequality sign "≤ 0" and add a non-negative random variable (u) to the right-hand side of the equation with the inequality sign "≥ 0". Terms u_j, $j=2, 3, 4$, are three non-negative random variables and assumed to have half-normal distributions for simplicity, i.e., $u_j \sim \left|N\left(0, \sigma_{uj}^2\right)\right|$. They can be viewed like technical inefficiency. Since their presence in (6)–(8) is to assure that the equality signs hold, at least theoretically, it is not suggested to presume that they have some complex distributions, like the truncated normal or Gamma distribution.[4] Next, we add the two-sided error terms of v_2, v_3, and v_4 to (6)–(8), respectively, in order to reflect random shocks and make them become regression equations. Those three error terms are conventionally assumed to be normally distributed with zero means and constant variances, i.e., $v_j \sim N\left(0, \sigma_{vj}^2\right)$, $j=2,\ldots, 4$. They are also assumed to be respectively independent of u_j, $j=2,\ldots, 4$.

For the cases of inputs, monotonicity requires that DODF is non-decreasing in inputs. Following the same rule as (6)–(8), we transform the three inequalities into the following three regression equations:

$$\gamma_1 + \gamma_{11}x_1 + \gamma_{12}x_2 + \gamma_{13}x_3 + \delta_{12}\left(y_2 - y_1\right) + \delta_{13}\left(y_3 + y_1\right) + \gamma_{\tau 1}t = v_5 + u_5 \quad (9)$$

$$\gamma_2 + \gamma_{22}x_2 + \gamma_{12}x_1 + \gamma_{23}x_3 + \delta_{22}\left(y_2 - y_1\right) + \delta_{23}\left(y_3 + y_1\right) + \gamma_{\tau 2}t = v_6 + u_6 \quad (10)$$

$$\gamma_3 + \gamma_{33}x_3 + \gamma_{13}x_1 + \gamma_{23}x_2 + \delta_{32}\left(y_2 - y_1\right) + \delta_{33}\left(y_3 + y_1\right) + \gamma_{23}t = v_7 + u_7 \quad (11)$$

The foregoing assumptions on v and u are also applicable to (9)–(11) and hence ignored here. It is worth stressing that the presence of u_j, $j=2,\ldots, 7$, is the core of the current paper, because their existence in (6)–(11) ensures those inequalities (monotonicity property) intact in essence, on the one hand, and transforms the inequality signs into equalities, on the other hand.

Curvature requires $\overrightarrow{D_o}(y, b, x, t; 1, -1)$ be jointly concave in desirable and undesirable outputs. Let F be the Hessian matrix of DODF with respect to good and bad outputs, i.e.:

[4] It is noteworthy that if different distributions are assumed, then the resulting parameter estimates and the number of constraint violations may change somewhat.

$$F_1 = \begin{bmatrix} \alpha_{22} & \alpha_{23} \\ \alpha_{23} & \alpha_{33} \end{bmatrix}.$$

According to Morey (1986), concavity in outputs will be confirmed if and only if all the principal minors of F that are of odd-numbered order are non-positive and all the principal minors that are of even-numbered order are non-negative. The curvature conditions imply that:

1. $\alpha_{22} \leq 0$ and $\alpha_{33} \leq 0$, (12)

2. $|F_1| = \begin{vmatrix} \alpha_{22} & \alpha_{23} \\ \alpha_{23} & \alpha_{33} \end{vmatrix} = \alpha_{22}\alpha_{33} - \alpha_{23}^2 \geq 0.$ (13)

[5]The above two sets of inequality restrictions involve unknown parameters only.

Redefine α_{22} and α_{33} as $\alpha_{22} = -a_{22}^2$ and $\alpha_{33} = -a_{33}^2$, respectively. We then estimate a_{22} and a_{33}, instead of the original α_{22} and α_{33}. This technique ensures that the concavity condition holds and is in the same spirit as Wiley, Schmidt, and Bramble (1973) and Diewert and Wales (1987). Equation (13) is difficult to directly impose on (1). However, the imposition of (12) is usually sufficient to ensure (13) to hold. If the restriction of (13) fails to hold by chance, then we suggest re-estimating (1) by giving a different set of initial values for the unknown parameters until all of the inequality restrictions are met.[6] For the case of ODF the inequality restrictions, derived by the curvature conditions, contain the unknown parameters and input and output variables, which can be transformed into equalities. Please see Eqs. (36)–(39) in the "Appendix" for details.

2.1.2 Estimation Procedure

Equations (1) and (6)–(11) construct a system of regressions with composed errors. Since Eqs. (6)–(8) are linearly dependent, we arbitrarily drop (7) from the system of equations and propose estimating the remaining six equations simultaneously by ML. In this manner, we can similarly impose monotonicity on the DODF of (1) to the usage of a Bayesian approach. To simplify the derivation of the likelihood function, we assume that all of the seven composed errors, either $v+u$ or $v-u$, in those equations are statistically independent; otherwise, the copula methods used by, e.g., Lai and Huang (2013), Amsler et al. (2014), and Huang et al. (2018), or the closed skew normal family of distributions developed by Chen et al. (2014) have to be utilized to get the required joint distributions. The resulting likelihood function will be complicated and quite difficult to converge during the estimation process.

It is widely known that the probability density function (pdf) of the composed error of $\varepsilon = v \pm u$ can be shown to be:

[5] It can be shown that $|F_2| = \begin{vmatrix} \alpha_{22} & \alpha_{23} \\ \alpha_{23} & \alpha_{33} \end{vmatrix} = \alpha_{22}\alpha_{33} - \alpha_{23}^2 = |F_3| = \begin{vmatrix} \alpha_{11} & \alpha_{13} \\ \alpha_{13} & \alpha_{33} \end{vmatrix} = |F_1| \geq 0.$

[6] Since our likelihood function may have multiple local maxima, a change in initial conditions tends to result in distinct parameter estimates.

$$f(\varepsilon) = \frac{2}{\sigma}\phi\left(\frac{\varepsilon}{\sigma}\right)\Phi\left(\frac{\pm\lambda\varepsilon}{\sigma}\right), \tag{14}$$

where $\lambda = \sigma_u/\sigma_v$ and $\sigma = \sqrt{\sigma_u^2 + \sigma_v^2}$. The joint pdf for a bank is simply the multiplicative of (14) over the seven composed errors, and the corresponding likelihood function for the entire sample can be obtained by multiplying the above joint pdf over each bank and the sample period.

The independence assumption among those composed errors appears to be strong, but in fact harmless, since the parameter estimators are still consistent and the inclusion of (6)–(11) into the likelihood function is mainly for taking monotonicity into account such that the estimated parameters of (1) are in line with production theories at each sample point, instead of enhancing the efficiency of the estimators.[7] It is noteworthy that the presence of Eqs. (6)–(11) in the likelihood function does not increase the number of parameters to be estimated, except for the variances of v_j and u_j, $j=2,\ldots, 7$, in those equations.

Recall that the concavity conditions of (12) and (13) involve only the unknown parameters and omit all input and output variables. We thus do not translate them from inequality signs into equality signs by appending composed errors like (6)–(11). Rather, we recommend jointly estimating (1), (6), and (8)–(11) by ML and examine whether the estimated coefficients satisfy (12) and (13). If the conditions are violated, then we adjust the starting values of the set of parameters and re-run the program until the conditions are met.

Once the coefficient estimates are obtained, we can use them to calculate the level of technical inefficiency (*TI*) according to the following formula:

$$TI = E\left(u_1|\varepsilon_1\right) = \mu_{*1} + \sigma_{*1}\frac{\phi\left(\frac{-\mu_{*1}}{\sigma_{*1}}\right)}{1 - \Phi\left(\frac{-\mu_{*1}}{\sigma_{*1}}\right)}, \tag{15}$$

where $\varepsilon_1 = v_1 - u_1$, $\sigma_1^2 = \sigma_{v1}^2 + \sigma_{u1}^2$, $\mu_{*1} = -\varepsilon_1\sigma_{u1}^2/\sigma_1^2$, and $\sigma_{*1}^2 = \sigma_{u1}^2\sigma_{v1}^2/\sigma_1^2$. Since the scale of our sample banks varies considerably, we further transform the *TI* level into the measure of technical efficiency (*TE*) to eliminate the scale differences, i.e.,

$$\overrightarrow{TE} = 1 - \hat{TI} \,/\, \hat{\overrightarrow{D_o}}, \tag{16}$$

where "^" over a variable means the predicted value of that variable. Note that the technical efficiency score of $\overrightarrow{TE}$ must lie between 0 and 1.

A caveat worth mentioning is that the resulting set of slope coefficient estimates may not be able to insure that all monotonicity conditions are satisfied for all data points. This can be ascribable to the presence of the disturbances v_j's, $j=2,\ldots, 7$, inherent in Eqs. (6)–(11). Perhaps there are some observations undergoing sizeable

[7] Note that our simultaneous estimation procedure with respect to (2) and (7)–(12) can still raise the efficiency of the estimators somewhat due to the imposition of cross-equation restrictions on the coefficients under study.

and favorable or unfavorable impacts, such that $v_j - u_j > 0$ or $v_j + u_j < 0, j=2,\ldots, 7$, leading them to be outliers or extreme input or output mixes and accordingly conflicting with monotonicity. Viewed from this angle, this may not be regarded as a disadvantage, as opposed to the use of a Bayesian approach, by which the resulting parameter estimates guarantee the restrictions to be observed for all data points irrespective of the occurrence of heavy shocks.

The "Appendix" describes how to impose monotonicity and curvature on an ODF, under the framework of a stochastic approach, and addresses the estimation procedure.

2.2 TFP Changes

We can use the parameter estimates of DODF and ODF to calculate TFP changes for the sample banks due to the availability of panel data. According to Trivedi (1981) and Diewert and Fox (2008), continuous time Divisia indices can be approximated by discrete-time Törnqvist formulae. Under the case of DODF, Feng and Serletis (2014) approximate the continuous-time technological change component of the Divisia–Luenberger productivity index (TC^{L}) between periods t and $t+1$ by:

$$TC^{L}(t,t+1) \equiv \frac{1}{2}\left\{\ln\left[\frac{1+\vec{D}_o(Z(t),t+1;g)}{1+\vec{D}_o(Z(t),t;g)}\right]+\ln\left[\frac{1+\vec{D}_o(Z(t+1),t+1;g)}{1+\vec{D}_o(Z(t+1),t;g)}\right]\right\}, \tag{17}$$

and the efficiency change (EC^{L}) component by:

$$EC^{L}(t,t+1) \equiv -\ln\left[\frac{1+\vec{D}_o(Z(t+1),t+1;g)}{1+\vec{D}_o(Z(t),t;g)}\right], \tag{18}$$

where $Z(t) \equiv (y(t), x(t))$. The sum of (17) and (18) yields the discrete-time Divisia–Luenberger productivity index (PG^{L}):

$$PG^{L}(t,t+1) \equiv \frac{1}{2}\left\{\ln\left[\frac{1+\vec{D}_o(Z(t),t;g)}{1+\vec{D}_o(Z(t+1),t;g)}\right]+\ln\left[\frac{1+\vec{D}_o(Z(t),t+1;g)}{1+\vec{D}_o(Z(t+1),t+1;g)}\right]\right\}. \tag{19}$$

A positive (negative) value of TC^{L} means that the efficient frontier shifts upward (downward) over time, corresponding to technological progress (regress). A positive (negative) value of EC^{L} implies that the actual production level of a bank is moving toward (away from) the efficient frontier. Taking the natural exponent of (19), one obtains a Malmquist-type index, akin to the Malmquist–Luenberger productivity index proposed by Chung et al. (1997). The difference between them is the chosen directional vector, where (19) is constant, as opposed to $y(t)$ used by the Malmquist–Luenberger productivity index.

Under the case of ODF, Feng and Serletis (2014) show the respective components of technological change (TC^S) and efficiency change (EC^S) as:

$$TC^S(t, t+1) \equiv \frac{1}{2}\left\{ \ln\left[\frac{D_o(Z(t), t+1)}{D_o(Z(t), t)}\right] + \ln\left[\frac{D_o(Z(t+1), t+1)}{D_o(Z(t+1), t)}\right] \right\}, \tag{20}$$

$$EC^S(t, t+1) \equiv \ln\left[\frac{D_o(Z(t+1), t+1)}{D_o(Z(t), t)}\right]. \tag{21}$$

The positive (negative) values of TC^S and EC^S have similar implications to TC^L and EC^L, respectively. The sum of the above two equations yields the TFP index (PG^S):

$$PG^S = TC^S + EC^S \tag{22}$$

Finally, taking the natural exponent of (22), we obtain the Malmquist (output-oriented) TFP change index between period t and period $t+1$, as suggested by Färe et al. (1994).[8]

3 The Data

We extract a total of 51 commercial banks in Taiwan from the Taiwan Economic Journal database, spanning 2002–2015, i.e., after the country's entrance into WTO. Five out of the 51 banks are foreign banks (FORB), fourteen of them are financial holding banks (FHB), and the remaining are non-financial holding banks (Non-FHB).[9] The resulting unbalanced panel data contain 526 bank-year observations. We identify three inputs, two good outputs, and an undesirable according to the intermediation approach, proposed by Sealey and Lindley (1977). Specifically, banks are treated as financial intermediaries employing deposits (x_1), capital (x_2), and labor (x_3) to produce two desirable outputs, i.e., investments (y_1) and loans (y_2), and a single undesirable, i.e. non-performing loans (NPL, y_3). Input x_1 includes various deposits and borrowed money, x_2 is defined as the value of net fixed assets, and x_3 is defined as the number of full-time equivalent employees. Output y_1 is defined as other earning assets, including government and corporate securities, y_2 is the sum of short- and long-term loans, and the bad output of y_3 is defined as NPLs that are jointly produced with loans. All of the above variables, except for labor, are

8 Although the use of ODF allows one to decompose the Malmquist TFP change index into three items, i.e., TC^S, EC^S, and SC (a scale component), DODF is unable to address the term of SC. Since this paper mainly focuses on the use of DODF, we follow Feng and Serletis (2014) who ignore this scale component.

9 A financial holding company (FHC) is a financial institution engaged in banking-related activities, offering customers a wide range of financial services, such as purchasing insurance products and investment in securities. An FHB is a subsidiary of an FHC, and its scale is usually larger than Non-FHBs, which do not belong to any financial group. Here, a foreign bank is obligated to follow the regulations of both the home and host countries, and its scale is close to Non-FHBs in Taiwan.

Table 1 Sample statistics

	Mean	SD	Minimum	Maximum
investments (y_1)*	154,173.12	191,484.50	333.55	1,394,612.12
loans (y_2)*	504,108.06	512,893.02	7786.71	2,298,267.25
NPL (y_3)*	7732.08	12,979.60	0.00	105,588.77
deposits (x_1)*	671,655.43	679,729.70	1018.73	3,718,464.50
net fixed assets (x_2)*	12,718.39	16,421.74	535.88	105,981.63
employees (x_3)**	3541.50	2437.54	382.00	10,708.00
Number of observations	526			

*Measured by millions of NTD and deflated by Taiwan's CPI with the base year of 2011

**Measured by number of persons

measured in terms of millions of New Taiwan dollars (NTD) and are deflated by the consumer price index (CPI) of Taiwan with the base year of 2011. Table 1 summarizes the descriptive statistics for the above input and output variables.

During the sample period, the mean values of y_1 and y_2 grow with time until 2007. After then, the growth rates slow down due to the occurrence of the U.S. subprime financial crisis. The average NPL decreases in the sample period, indicating that the degrees of financial soundness of the sample banks improve over time. As for the three inputs, the average values of deposits and labor increase steadily over time, while average fixed assets initially go up until 2008 and stay stable after then.

Recall that there are three forms of banks, i.e., FHB, Non-FHB, and FORB. Generally speaking, FHB has the largest scale in terms of loans, deposits, and fixed assets, followed by Non-FHB and FORB. As far as the undesirable output is concerned, Non-FHB has the highest average y_3 until 2005, and FHB experiences the highest NPLs thereafter, followed by Non-FHB and FORB. Figure 1 depicts the trends of the six variables.

4 Empirical Results

4.1 Parameter Estimates

Table 2 shows the coefficient estimates of DODF with and without the imposition of regularity properties, denoted by restricted and unrestricted models, respectively. As expected, the log-likelihood value of the restricted model is far less than that of the unrestricted model. The vast majority of the parameter estimates for both models are significant at least at the 5% level. Although most of their slope parameters (except two) have the same signs, the standard errors of the restricted model are much smaller than those of the corresponding unrestricted model. This can be attributed to the fact that the former takes the regularity conditions into account and adopts a simultaneous estimation procedure, which, in fact, imposes the cross-equation restriction. This leads the coefficient estimates to be more efficient, as stressed by footnote 3.

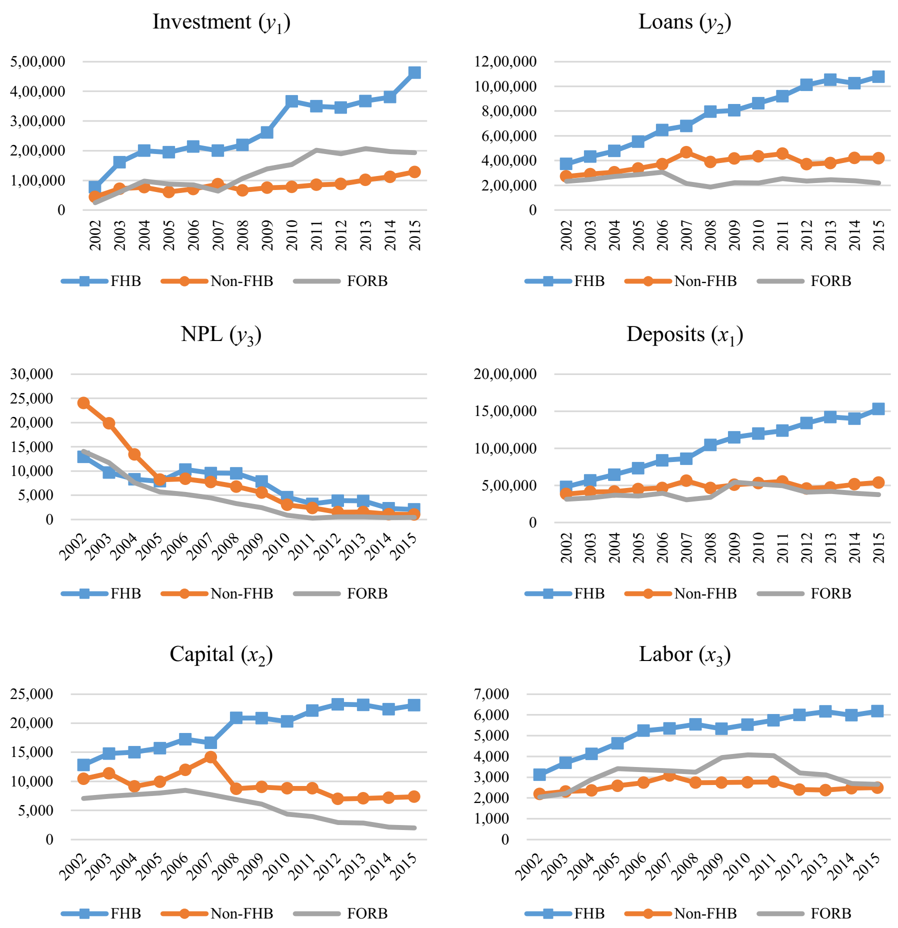


Fig. 1 Trends of the six variables

Table 3 presents the coefficient estimates of ODF with and without the considerations of monotonicity and curvature constraints, also denoted by restricted and unrestricted models, respectively. Only 14 out of 41 parameter estimates from the restricted model attain significance at least at the 10% level, due possibly to the exclusion of undesirables from the model such that ODF is unable to appropriately characterize the production technology for banks. In addition, the restricted model estimates 18 additional distribution parameters for σ and λ, which largely reduce the degrees of freedom of the restricted model. The unrestricted model is found to have 15 out of 23 significant estimates at least at the 10% level, and the log-likelihood value of the restricted model is far less than that of the unrestricted model.

On the basis of the foregoing discussion, we argue that the imposition of the theoretical properties causes substantial differentiation of the coefficient estimates among the restricted and unrestricted models for both DODF and ODF, in terms of their

Table 2 Parameter estimates of DODF

Variable	Restricted model		Un-restricted model	
	Estimate	Standard error	Estimate	Standard error
Intercept	− 7.28E+08***	1.73E−01	− 7.28E+08***	3.521E+01
x_1	− 3.45***	3.79E−09	− 3.5030***	1.30E−06
x_2	9.23E+01***	4.21E−08	9.2192 E+01***	1.51E−05
x_3	1.67E+01***	2.57E−07	1.4556 E+01***	8.29E−05
$y_2 - y_1$	1.34E+01***	3.84E−09	1.3455 E+01***	1.33E−06
$y_3 + y_1$	1.87***	6.06E−09	1.8963***	1.96E−06
$0.5x_1^2$	6.69E−08***	4.95E−17	6.70E−08***	2.03E−14
$0.5x_2^2$	1.20E−05***	6.72E−15	1.20E−05***	3.51E−12
$0.5x_3^2$	1.10E−03***	5.73E−13	1.10E−03***	1.65E−10
$0.5(y_2 - y_1)^2$	− 1.25E−07***	4.59E−17	1.75E−07***	1.66E−14
$0.5(y_3 + y_1)^2$	− 1.09E−07***	1.73E−16	3.91E−07***	7.59E−14
x_1x_2	3.12E−06***	6.04E−16	2.64E−07***	2.52E−13
x_1x_3	1.86E−06***	4.13E−15	1.86E−06***	1.71E−12
x_2x_3	7.46E−05***	6.15E−14	7.47E−05***	2.31E−11
$x_1(y_2 - y_1)$	− 1.14E−07***	4.78E−17	− 1.15E−07***	1.75E−14
$x_1(y_3 + y_1)$	− 1.22E−07***	7.27E−17	− 1.22E−07***	3.13E−14
$x_2(y_2 - y_1)$	2.60E−08***	5.85E−16	2.90E−08***	2.13E−13
$x_2(y_3 + y_1)$	− 1.80E−06***	7.16E−16	− 1.80E−06***	3.26E−13
$x_3(y_2 - y_1)$	− 6.58E−06***	3.90E−15	− 6.58E−06***	1.71E−12
$x_3(y_3 + y_1)$	− 1.54E−05***	6.94E−15	− 1.54E−05***	2.79E−12
$(y_2 - y_1)(y_3 + y_1)$	1.05E−07***	7.31E−17	2.05E−07***	2.94E−14
t	− 7.30E+05***	1.73E−04	− 7.30E+05***	3.53 E−02
$0.5t^2$	− 3.66E+02***	8.68E−08	− 3.6656 E+02***	1.77E−05
x_1t	− 1.74E−03***	1.89E−12	− 1.75E−03***	6.51E−10
x_2t	4.63E−02***	2.10E−11	4.64 E−02***	7.57E−09
x_3t	7.41E−03***	1.29E−10	7.84E−03***	4.15E−08
$(y_2 - y_1)t$	6.74E−03***	1.92E−12	6.75E−03***	6.65E−10
$(y_3 + y_1)t$	4.06E−04***	3.03E−12	4.22E−04***	9.83E−10
σ_1	7.53E−01***	4.50E−11	6.9893***	8.34E−08
λ_1	1.01***	7.44E−11	7.6798	9.5622E+03
σ_2	2.10E−01***	5.53E−02		
λ_2	2.99	3.73		
σ_4	1.97**	9.28E−01		
λ_4	4.00E+01	4.99E+07		
σ_5	5.68E−02***	1.32E−02		
λ_5	1.34	1.59		
σ_6	4.52E−01***	1.74E−03		
λ_6	3.33E+02***	1.28		
σ_7	1.17***	2.06E−02		
λ_7	1.18***	4.06E−01		
Log-likelihood	− 4.35E+12		− 2.38E+09	
# of observations	526		526	

Table 2 (continued)

*, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively. $\lambda_j = \sigma_{uj}/\sigma_{vj}$ and $\sigma_j = \sqrt{\sigma_{uj}^2 + \sigma_{vj}^2}, j=1, 2, 4, \ldots, 7$

signs, magnitudes, and standard errors. We now exploit the coefficient estimates in Tables 2 and 3 to check whether the regularity constraints are convincing or not, and the outcomes are shown in Table 4. Obviously, the regularity-constrained models of both DODF and ODF outperform the respective unrestricted models, in terms of the number of observations that contradict monotonicity and curvature conditions. Feng and Serletis (2010, 2014) obtain similar outcomes using a Bayesian approach. With regard to DODF, all sample points satisfy the monotonicity condition, except for 10 and 18 observations of $\partial \vec{D}_o/\partial x_2$ and $\partial \vec{D}_o/\partial x_3$, respectively. Eight of the 10 observations come from four banks that are close to failure, and the remaining two observations correspond to the first-year data from two newly established banks. The 18 observations relate to three large commercial banks granting huge amounts of loans, along with sizeable NPLs. Those 28 observations may be caused by either bad luck or extreme values. Using the parameter estimates in Table 2, we can readily show that the curvature conditions of (12) and (13) are proved. Our stochastic approach seems successful in the imposition of the regularity properties at each data-point, while it tolerates a few exceptional sample points arising from, e.g., outlier and/or bad luck. Diewert and Wales (1987) and Feng and Serletis (2015) obtain similar results.

Turning to ODF, Table 4 uncovers that merely four out of the nine restrictions have quite small numbers of observations inconsistent with the required restrictions. In effect, the total number of such observations is only 11, caused by similar reasons to the case of DODF after inspecting the individual sample points. Conversely, 8 out of the 9 restrictions are violated by the unrestricted ODF with such tremendous observations. Although most coefficient estimates of the unrestricted ODF attain statistical significance, it suffers from the serious problem of inconsistency with theoretical regularity conditions. We thus do not recommend it to examine such issues as shadow prices of pollutants, economies of scale, efficiency scores, and TFP changes.

4.2 Technical Efficiency

Given that the regularity-constrained models are preferable to the unconstrained models, we only compute and present average efficiency scores and TFP change indices over the sample period with the parameter estimates from the constrained models of DODF and ODF. Table 5 reports the average efficiency scores for both DODF and ODF. In the context of DODF, the average efficiency score of all banks is equal to 0.797, which is lower than that for ODF at 0.862. Although a direct comparison of the estimated efficiency measures between DODF and ODF is invalid, because they are evaluated along different directions, i.e., $g=(1,-1)$ and $g(t)=(y(t), x(t))$, respectively, this result is still in line with expectation due to the omission of undesirables by ODF, i.e., NPLs in our case. Undesirables are usually not freely disposable and are subject to regulations. Banks have to hire more resources to "clean

Table 3 Parameter estimates of ODF

Variable	Restricted model		Un-restricted model	
	Estimate	Standard error	Estimate	Standard error
Intercept	214.6690	1827.2200	1793.1300	2143.4900
$\log(x_1)$	46.5233***	10.3725	65.0475***	12.3403
$\log(x_2)$	−5.1635	7.0618	5.0384	7.6752
$\log(x_3)$	−58.7325***	14.6404	−92.2874***	14.3351
$\log(y2/y1)$	1.9978	4.4754	4.0877	6.2115
$0.5[\log(x_1)]^2$	−0.4778***	0.0928	−0.5491***	0.0269
$0.5[\log(x_2)]^2$	0.0163	0.0439	0.0656*	0.0367
$0.5[\log(x_3)]^2$	−0.6695***	0.1234	−0.8582***	0.1120
$0.5[\log(y2/y1)]^2$	0.1871***	0.0521	0.1853***	0.0183
$\log(x_1)\log(x_2)$	0.1354***	0.0422	0.0570	0.0435
$\log(x_1)\log(x_3)$	0.5085***	0.0990	0.7689***	0.0607
$\log(x_2)\log(x_3)$	−0.2180***	0.0585	−0.1809***	0.0512
$\log(x_1)\log(y2/y1)$	0.0704***	0.0171	0.1167***	0.0252
$\log(x_2)\log(y2/y1)$	−0.0171*	9.74E−03	−0.0537***	0.0201
$\log(x_3)\log(y2/y1)$	−0.0605**	0.0237	−0.0589*	0.0354
t	0.2785	1.8364	1.8851	2.1480
t^2	1.85E−04	9.23E−04	9.91E−04	1.08E−03
$\log(x_1)t$	0.02384***	5.09E−03	0.0331***	6.18E−03
$\log(x_2)t$	−2.42E−03	3.60E−03	2.41E−03	3.84E−03
$\log(x_3)t$	−0.0288***	7.11E−03	−0.0457***	7.20E−03
$\log(y2/y1)t$	9.79E−04	2.26E−03	2.05E−03	3.12E−03
σ_1	1.1356	1.2183	0.1900***	6.84E−03
λ_1	1.1087	2.1530	2.8422***	0.3184
σ_2	0.8893	8.5168		
λ_2	2.0203	45.4962		
σ_4	1.9255	66.2167		
λ_4	7.7808	660215.00		
σ_5	0.1782	0.5874		
λ_5	0.9826	19.3614		
σ_6	1.3600	24.6488		
λ_6	126,623.00	315,287.00		
σ_7	0.6198	1.6987		
λ_7	2.2400	35.8449		
σ_8	0.6744***	0.1793		
λ_8	1.0433	95.2595		
σ_9	0.6821	0.7162		
λ_9	0.2741	16.1188		
σ_{10}	3.7953	95.5812		
λ_{10}	3.8592	102.0010		

Table 3 (continued)

Variable	Restricted model		Un-restricted model	
	Estimate	Standard error	Estimate	Standard error
σ_{11}	0.2327	0.4680		
λ_{11}	0.1897	5.3894		
Log-likelihood	−49,909.90		3.5568	
# of observations	526		526	

*, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively. $\lambda_j = \sigma_{uj}/\sigma_{vj}$ and $\sigma_j = \sqrt{\sigma_{uj}^2 + \sigma_{vj}^2}, j = 1, 2, 4, \ldots, 11$

Table 4 Number of observations contradicts with monotonicity and curvature

	$\frac{\partial \vec{D}_o}{\partial y_1} \le 0$	$\frac{\partial \vec{D}_o}{\partial y_2} \le 0$	$\frac{\partial \vec{D}_o}{\partial y_3} \ge 0$	$\frac{\partial \vec{D}_o}{\partial x_1} \ge 0$	$\frac{\partial \vec{D}_o}{\partial x_2} \ge 0$	$\frac{\partial \vec{D}_o}{\partial x_3} \ge 0$
DODF						
Restricted model	0	0	0	0	10	18
Un-restricted model	0	205	0	508	358	240
	$\frac{\partial \ln D_o}{\partial \ln y_1} \ge 0$	$\frac{\partial \ln D_o}{\partial \ln y_2} \ge 0$		$\frac{\partial \ln D_o}{\partial \ln x_1} \le 0$	$\frac{\partial \ln D_o}{\partial \ln x_2} \le 0$	$\frac{\partial \ln D_o}{\partial \ln x_3} \le 0$
ODF						
Restricted model	6	0		0	0	0
Un-restricted model	96	0		1	389	399
	F_{312}	F_{313}	F_{323}	F		
Restricted model	1	1	0	3		
Un-restricted model	145	526	291	416		

Table 5 Average efficiency scores for DODF and ODF across bank groups

	Mean	SD	Minimum	Maximum
DODF				
All Banks	0.7967	0.1491	0.4982	1.0000
Non-FHB	0.7332	0.1493	0.4982	1.0000
FHB	0.8853	0.1016	0.5834	1.0000
FORB	0.8659	0.0665	0.6500	0.9650
ODF				
All Banks	0.8619	0.0776	0.5895	0.9539
Non-FHB	0.8879	0.0634	0.6258	0.9539
FHB	0.8153	0.0828	0.5895	0.9369
FORB	0.8894	0.0253	0.8423	0.9274

up" NPLs, which tend to pull down banks' managerial abilities. These figures imply that the ratios of the actual level of output to the potential level of output are 79.7% and 86.2%, respectively. To achieve the production frontier the sample banks have to increase their output quantities by 20.3% and 13.8%, respectively.

Under the framework of DODF, FHBs lead the remaining two forms of banks, followed by FORBs and then Non-FHBs. The difference between FHBs and Non-FHBs and the difference between FORBs and Non-FHBs are significant at the 1% level, but the difference between FHBs and FORBs is insignificant. Conversely, the ODF results reveal that FORBs and Non-FHBs outperform FHBs, and the differences attain the 1% level of significance, while the difference between FORBs and Non-FHBs is insignificant.

An interesting issue worth studying is whether the efficiency measure from ODF, which is unable to consider the bad output, NPLs, leads to suspicious outcomes about technical efficiency. If ODF produces valid efficiency scores for situations where undesirables exist, then the rankings should be roughly in line with those from the DODF for individual banks.

To formally inspect whether the efficiency scores measured against DODF and ODF generate similar rankings, we calculate the Spearman rank correlation coefficient (ρ) between the constrained DODF and ODF for each sample year, as suggested by Feng and Serletis (2010, 2014). The formula is:

$$\rho = 1 - \frac{6\sum_{i=1}^{n}\left(Rank_{i1} - Rank_{i2}\right)^2}{n(n^2-1)}, \tag{23}$$

where $Rank_{i1}$ is the rank of bank i (=1,…, n) based on DODF, and $Rank_{i2}$ is the rank of the same bank based on ODF. A value of $\rho = -1$ (+ 1) indicates a perfect negative (positive) correlation, while a value of $\rho = 0$ implies no correlation. We also calculate 95% confidence intervals for ρ using SAS software.

Column 1 of Table 6 presents the results. The values of ρ across the sample period range between −0.55 and −0.85, indicating that the ranking of the efficiency measures yielded from DODF is negatively correlated with that from ODF. The 95% confidence intervals in the table reveal that such a highly negative correlation is significantly different from zero at the 5% level. This result verifies that those two distance functions tend to give contradictory efficiency rankings, and that the preclusion of bad outputs is apt to invert the ranking of individual banks. Thus, the radial ODF is not appropriate for conditions where undesirables are present.

The U.S. subprime mortgage crisis, occurring between 2007 and 2010, has heavily impacted the economic activities of the U.S., Europe, and other economies, including Taiwan, for years. We thus split the sample period into two sub-periods: Pre-2007 (2002–2007) and Post-2007 (2008–2015). Table 7 presents the average technical efficiency measures in the two sub-periods across two types of banks, i.e., FHBs and Non-FHBs.[10] The results from DODF prove that the performance of both types of banks in the second sub-period surpasses the performance in the first sub-period. The last column of Table 7 shows the p values of testing for the null hypothesis that the mean efficiencies of the Pre- and Post-2007 sub-periods are equal. All tests are decisively rejected at the 1% level of significance, which may be ascribable

[10] Four out of the five foreign banks started their businesses in Taiwan after 2007. Hence, there are no observations for the four banks prior to 2007.

Table 6 Spearman rank correlation coefficients of efficiency and TFP indices

Year	ρ of efficiency (95% confidence interval)	ρ of EC (95% confidence interval)	ρ of TC (95% confidence interval)	ρ of PG (95% confidence interval)
2003	−0.5506 (−0.7406, −0.2791)	0.0631 (−0.2421, 0.3570)	−0.1472 (−0.4300, 0.1618)	0.0627 (−0.2425, 0.3566)
2004	−0.6818 (−0.8264, −0.4530)	0.2872 (−0.0207, 0.5454)	−0.2588 (−0.5226, 0.0501)	0.2368 (−0.0727, 0.5047)
2005	−0.7606 (−0.8264, −0.4530)	−0.0922 (−0.3894, 0.2224)	−0.2305 (−0.5056, 0.0872)	−0.0422 (−0.3455, 0.2691)
2006	−0.7266 (−0.8645, −0.4869)	−0.0062 (−0.3387, 0.3278)	0.1011 (−0.2411, 0.4209)	0.0115 (−0.3230, 0.3434)
2007	−0.8118 (−0.9179, −0.5974)	0.0305 (−0.3398, 0.3927)	0.4015 (0.0258, 0.6778)	0.0877 (−0.2888, 0.4406)
2008	−0.7851 (−0.9019, −0.5614)	−0.0694 (−0.4139, 0.2926)	0.4363 (0.0798, 0.6938)	−0.1972 (−0.5181, 0.1724)
2009	−0.8117 (−0.9153, −0.6075)	0.2847 (−0.0849, 0.5854)	0.3593 (−0.0061, 0.6400)	0.1718 (−0.1970, 0.4980)
2010	−0.8238 (−0.9212, −0.6290)	−0.5024 (−0.7378, −0.158)	0.3056 (−0.0631, 0.6010)	−0.5238 (−0.7516, −0.184)
2011	−0.8491 (−0.9335, −0.6755)	0.2907 (−0.0786, 0.5899)	0.0343 (−0.3241, 0.3840)	0.1935 (−0.1760, 0.5153)
2012	−0.8335 (−0.9249, −0.6511)	−0.0414 (−0.3847, 0.3119)	0.0378 (−0.3152, 0.3815)	−0.1059 (−0.4393, 0.2530)
2013	−0.8464 (−0.9312, −0.6745)	0.1584 (−0.2036, 0.4823)	0.2207 (−0.1430, 0.5319)	0.0502 (−0.3040 0.3922)
2014	−0.8039 (−0.9089, −0.6030)	0.0866 (−0.2652, 0.4180)	0.0943 (−0.2581, 0.4245)	−0.0424 (−0.3804, 0.3055)
2015	−0.7814 (−0.8935, −0.5778)	0.1184 (−0.2198 0.4312)	−0.0172 (−0.3439, 0.3131)	0.1328 (−0.2061, 0.4433)
2003–2015	−0.7599 (−0.8010, −0.7116)	0.0603 (−0.0326,0.1522)	0.1041 (0.0113, 0.1951)	−0.0363 (−0.1285, 0.0566)

to the fact that, facing such severe impacts, the sample banks devoted efforts to enhance production efficiency and reduce NPLs in order to weather the crisis.

The finding that the technical efficiency score of the banks in Taiwan improves with time is consistent with Hsiao et al. (2010), who apply DEA to evaluate 40 commercial banks in Taiwan over the period 2000–2005. On the contrary, ODF tells a reverse story. The sample banks' efficiency worsens after the crisis due potentially to the preclusion of undesirables from the model.

4.3 TFP Change Indices

We calculate the individual components of TFP change indices, based on (18)–(23), with respect to DODF and ODF for the sample banks, with the results in Table 8. On the basis of DODF, element TC dominates TFP change indices, since its average value (2.32% per annum) exceeds that of EC (−0.39% per annum). More specifically, item TC is found to be the main driver of productivity gains for Non-FHBs and FHBs, while FORBs experience technological regress, along with the greatest efficiency improvements among the three forms of banks. This implies that even when the undesirable is taken into account, innovation acts as a crucial role in stimulating productivity growth in Non-FHBs and FHBs. The failure of undertaking new innovations by FORBs may be justified by the fact that four out of the five FORBs are newly established, such that the benefit of learning-by-doing does not have enough time to be accumulated. We find a downward trend in the growth rate of TFP, governed by TC, since the absolute average values of PG^L, TC^L, and EC^L in the Pre-2007 period (4.95%, 6.61%, −1.66%) exceed those in the Post-2007 period (−1.05%, −1.92%, 0.86%).

Our findings are congruent with Feng and Serletis (2010) and Feng and Zhang (2012). The former estimate ODF subject to theoretical regularity under a Bayesian framework in an attempt to examine the TFP change of large U.S. banks spanning 2000–2005, while the latter estimate a true random effects stochastic ODF model also under a Bayesian framework and compare the productivity and efficiency of large banks and community banks in the U.S. over the period 1997–2006. Both papers verify a clear descending trend of TFP change for the sample banks, and the technical change component dominates this trend. Feng and Serletis (2014) take undesirables into account and estimate DODF and ODF, using aggregate data on 15 OECD countries over the period 1981–2000. Both papers support that technical change is the driving force of TFP change. Using DEA, Chang et al. (2012) probe the sources of productivity growth for 19 Chinese commercial banks over the period 2002–2009. Based on the measure of the Luenberger productivity index derived from a directional distance function, they conclude that technological gains outweigh the efficiency regressions and constitute the driving force of TFP growth. The forgoing statements lead us to infer that the component of technical change is apt to be the primary source of productivity gains.

Although our ODF suggests that the element EC is the dominant determinant of TFP indices, it deteriorates over time, resulting in productivity losses for the three types of banks, even though technological advance prevails. FHBs in Taiwan are

Table 7 Average efficiency scores before and after the 2007 financial crisis

	Mean	SD	Minimum	Maximum	p value[a]
DODF					
All Banks					
Pre-2007	0.7495	0.1421	0.4982	1.0000	5.49E−13
Post-2007	0.8391	0.1426	0.5220	1.0000	
Non-FHB					
Pre-2007	0.6988	0.1352	0.4982	1.0000	9.16E−06
Post-2007	0.7740	0.1554	0.5220	1.0000	
FHB					
Pre-2007	0.8523	0.0976	0.6230	1.0000	1.00E−04
Post-2007	0.9084	0.0983	0.5834	1.0000	
ODF					
All Banks					
Pre-2007	0.8786	0.0631	0.6587	0.9539	1.22E−06
Post-2007	0.8469	0.0860	0.5895	0.9482	
Non-FHB					
Pre-2007	0.8891	0.0646	0.6587	0.9539	7.17E−01
Post-2007	0.8865	0.0622	0.6258	0.9482	
FHB					
Pre-2007	0.8552	0.0564	0.7353	0.9369	7.39E−11
Post-2007	0.7873	0.0870	0.5895	0.9216	

[a]Testing for the hypothesis that the mean values of the Pre-2007 (2002–2007) and Post-2007 (2008–2015) sub-periods are equal

specifically undergoing quite fast rates of efficiency regression (−9.99% per annum) and productivity reduction (−7.90% per annum). The dominance of efficiency improvement denotes that when good outputs alone are incorporated, a retreat in efficiency, instead of a technological advance, plays a pivotal role in the determination of TFP indices. This finding differs from that of our DODF and is also inconsistent with, e.g., Feng and Serletis (2010, 2014) and Feng and Zhang (2012). We find a declining trend in the growth rate of TFP, since the average values of PG^S, TC^S, and EC^S in the Pre-2007 period (−2.85%, 1.07%, −3.64%) exceed those in the Post-2007 period (−3.22%, 1.10%, −4.32%). However, the driving force comes from EC, instead of TC, thus conflicting with the outcomes of DODF.

To examine whether the Divisia–Luenberger productivity index produces an analogous ranking to the conventional Feng and Serletis (2010) productivity index, we compute the Spearman rank correlation coefficient between the two indices for each of the sample years, as suggested by Feng and Serletis (2010, 2014). The formula is the same as (23), and the results are shown in Table 6, columns 2–4. According to the 95% confidence intervals, the vast majority of the correlation coefficients are insignificantly different from zero, implying that there is little correlation between the two rankings in these years. We assert that the exclusion of bad outputs is likely to substantially alter the ranking of individual banks, and thus the conventional Feng

Table 8 Average TFP change indices for both DODF and ODF

DODF	TC^L	EC^L	PG^L
All Banks	0.0231 (0.0967)	−0.0039 (0.2854)	0.0193 (0.3211)
Non-FHB	0.0194 (0.0769)	0.0006 (0.2153)	0.0200 (0.2414)
FHB	0.0376 (0.1269)	−0.0156 (0.3920)	0.0219 (0.4439)
FORB	−0.0178 (0.0486)	0.0169 (0.1407)	−0.0010 (0.1247)
ODF	TC^S	EC^S	PG^S
All Banks	0.0108 (0.0585)	−0.0399 (0.2535)	−0.0290 (0.2229)
Non-FHB	0.0051 (0.0276)	−0.0068 (0.2285)	−0.0017 (0.2143)
FHB	0.0209 (0.0918)	−0.0999 (0.2942)	−0.0790 (0.2384)
FORB	0.0086 (0.0035)	−0.0179 (0.1656)	−0.0093 (0.1655)

Numbers in parentheses are standard deviations

and Serletis (2010) productivity index is invalid for conditions where undesirables exist.

5 Conclusion

Almost all past works that consider regularity conditions count on the use of a Bayesian approach. This article proposes a stochastic approach that is capable of imposing the regularity conditions on DODF and ODF under the framework of a system regression model, where each constituent equation contains composed error terms. The existence of the one-sided error in the composed errors allows one to transform the inequality constraints, required by either monotonicity or curvature conditions, into equalities such that those equalities can be treated as regression equations with error components, after attaching statistical noises to those equalities. In the context of our stochastic approach, we estimate a quadratic DODF and a translog radial ODF. Our stochastic approach appears to be successful, because the resulting coefficient estimates satisfy the regularity constraints for most sample points. The quite small number of observations failing to meet the regularity constraints can be classified as either outliers or extreme values.

The unrestricted DODF and ODF are found to be inferior to the restricted counterparts due to the fact that their parameter estimates result in a large number of observations being inconsistent with the production theory. Moreover, the constrained DODF tends to outperform the constrained ODF due to the exclusion of undesirables from the latter. Although traditional Divisia-type productivity indices have drawn much attention from academic researchers, they overlook bad outputs. The Spearman rank correlation coefficients of the efficiency scores and productivity change indices obtained from the above two functions are found to be either negatively correlated or insignificantly correlated, supporting the superiority of the constrained DODF. Following Feng and Serletis (2014), this paper applies the so-called Divisia–Luenberger productivity index to examine the TFP changes of Taiwan's commercial banks. Our empirical results verify that by precluding bad outputs, the conventional productivity index yielded from ODF not only results in misleading conclusions regarding productivity growth and technological change, but also results in wrong conclusions concerning efficiency change.

Based on DODF, the average efficiency score of FHBs exceeds the remaining two forms of banks, implying that public policies encourages, e.g., merger and acquisitions among banks and/or that the formation of a large financial conglomerate may improve managerial abilities for the financial sector. Moreover, a larger bank is able to undertake new innovations that help stimulate its technology. Since element TC is found to dominate TFP change indices, it is worthwhile for banks to adopt innovations swiftly in an attempt to speed up productivity growth

Feng and Serletis (2014) point out three potential problems about the use of DODF in the last section. The first two include that there are alternative choices for the directional vector, other than $g=(1, -1)$, and that other translation variables, than $\phi = -y_1$ in (2), can be equivalently selected. The third problem they propose relates to the endogeneity inherent in Shephard input/output distance functions and DODF, due to the presence of $\left(y_1, y_2, y_3\right)'$ on the right-hand side of (1). Here, we further claim that the input variables in (1) may also be subject to the endogeneity problem in distance and production functions, arising from the correlation between unobserved productivity and input demands, as emphasized by Olley and Pakes (1996).

Finally, according to Amsler et al. (2014), our setting of u_1 in (1) may be preferable to those who assume it is time invariant, or it has “the scaling property” of Wang and Ho (2010), except for overlooking the time dependence of u_1. To address this problem, Amsler et al. (2014) suggest the use of copula methods. Recall that the main theme of the current paper is to develop a stochastic approach to impose monotonicity and curvature on DODF and ODF. The resolution of the foregoing debatable issues, particularly the last two, is non-trivial and out of this paper's scope. Those problems are worth a thorough investigation by future research works.

Appendix

Imposition of Monotonicity and Curvature on an ODF

We re-define the vector of y as $y=\left(y_1, y_2\right)' \in R_+^2$, because undesirables are excluded from ODF. The vector x is intact. We define the output set $S(x)$ as:

$$S(x) = \{y|y \text{ can be produced by } x\}.$$

The output distance function $D_o(y,x,t)$ can be written as:

$$D_o(y,x,t) = min\{\delta|0 < \delta \leq 1, y/\delta \in S(x)\}. \tag{24}$$

A value of $D_o(y,x,t)$ equaling unity means that the bank is already producing on the efficient frontier, while a value of $D_o(y,x,t)$ less than unity reveals that the bank is technically inefficient due to managerial inability. Following Orea (2002), O'Donnell and Coelli (2005), and Feng and Serletis (2010, 2014), we specify the translog functional form for ODF:

$$\ln D_o(y,x,t) = a_0 + \sum_{i=1}^{2} a_i \ln y_i + \frac{1}{2}\sum_{i=1}^{2}\sum_{k=1}^{2} a_{ik} \ln y_i \ln y_k + \sum_{n=1}^{3} b_n \ln x_n + \frac{1}{2}\sum_{n=1}^{3}\sum_{j=1}^{3} b_{nj} \ln x_n \ln x_j + \delta_\tau t + \frac{1}{2}\delta_{\tau\tau} t^2 + \sum_{n=1}^{3}\sum_{i=1}^{2} g_{ni} \ln x_n \ln y_i + \sum_{i=1}^{2} \delta_{\tau i} t \ln y_i + \sum_{n=1}^{3} \delta_{\tau n} t \ln x_n \tag{25}$$

After imposing the homogeneity and symmetrical properties and appending a statistical noise term, we transform ODF into a regression equation:

$$\begin{aligned} -\ln y_1 = {} & a_0 + a_2 \ln\left(\frac{y_2}{y_1}\right) + \frac{1}{2}a_{22}\left[\ln\left(\frac{y_2}{y_1}\right)\right]^2 + b_1 \ln x_1 + b_2 \ln x_2 + b_3 \ln x_3 + \frac{1}{2}b_{11}\left(\ln x_1\right)^2 \\ & + \frac{1}{2}b_{22}\left(\ln x_2\right)^2 + \frac{1}{2}b_{33}\left(\ln x_3\right)^2 + b_{12}\ln x_1 \ln x_2 + b_{13}\ln x_1 \ln x_3 + b_{23}\ln x_2 \ln x_3 \\ & + \delta_\tau t + \frac{1}{2}\delta_{\tau\tau} t^2 + g_{12}\ln x_1 \ln\left(\frac{y_2}{y_1}\right) + g_{22}\ln x_2 \ln\left(\frac{y_2}{y_1}\right) + g_{32}\ln x_3 \ln\left(\frac{y_2}{y_1}\right) \\ & + \delta_{\tau 2} t \ln\left(\frac{y_2}{y_1}\right) + \delta_{\tau 1} t \ln x_1 + \delta_{\tau 2} t \ln x_2 + \delta_{\tau 3} t \ln x_3 + V_1 + U_1 \end{aligned} \tag{26}$$

where $U_1 = -\ln D_o(y,x,t) \sim \left|N\left(0,\sigma_{U1}^2\right)\right|$ is a one-sided error signifying technical inefficiency, $V_1 \sim N\left(0,\sigma_{V1}^2\right)$ is the random disturbance, and U_1 and V_1 are assumed to be statistically independent.

Monotonicity and Curvature Constraints

Monotonicity requires that $D_o(y,x,t)$ be non-decreasing in outputs and non-increasing in inputs—that is:

$$\frac{\partial D_o}{\partial y_2} = \frac{\partial \ln D_o}{\partial \ln y_2}\frac{D_o}{y_2} \geq 0 \text{ and } \frac{\partial D_o}{\partial x_n} = \frac{\partial \ln D_o}{\partial \ln x_n}\frac{D_o}{x_n} \leq 0, \quad n = 1,2,3, \tag{27}$$

or, equivalently:

$$\frac{\partial \ln D_o}{\partial \ln y_2} \geq 0 \text{ and } \frac{\partial \ln D_o}{\partial \ln x_n} \leq 0, \quad n = 1,2,3, \tag{28}$$

since D_o/y_2 and D_o/x_n are positive. The property $\partial D_o/\partial y_2 \geq 0$ implies that a bank's technical efficiency does not diminish when it can produce more of an output, say y_2, after employing a given input mix. The property $\partial D_o/\partial x_n \leq 0$ implies that a bank's technical efficiency does not rise when it hires more of an input, say x_n, to produce the same output levels.

We re-write (28) for output y_2 as:

$$\frac{\partial \ln D_o}{\partial \ln y_2} = a_2 + a_{22} \ln\left(\frac{y_2}{y_1}\right) + g_{12} \ln x_1 + g_{22} \ln x_2 + g_{32} \ln x_3 + \delta_{\tau 2} t \ \geq 0.$$

Since the ODF must be homogeneous in y_1 and y_2, we have:

$$\frac{\partial \ln D_o}{\partial \ln y_1} = 1 - \frac{\partial \ln D_o}{\partial \ln y_2} \geq 0 \text{ and } \frac{\partial \ln D_o}{\partial \ln y_2} \leq 1.$$

Translating the above inequalities into equalities, we obtain:

$$a_2 + a_{22} \ln\left(\frac{y_2}{y_1}\right) + g_{12} \ln x_1 + g_{22} \ln x_2 + g_{32} \ln x_3 + \delta_{\tau 2} t \ = V_2 + U_2 \tag{29}$$

$$a_2 - 1 + a_{22} \ln\left(\frac{y_2}{y_1}\right) + g_{12} \ln x_1 + g_{22} \ln x_2 + g_{32} \ln x_3 + \delta_{\tau 2} t \ = V_3 - U_3. \tag{30}$$

Similarly, the monotonicity conditions for the three inputs can be expressed as:

$$b_1 + b_{11} \ln x_1 + b_{12} \ln x_2 + b_{13} \ln x_3 + g_{12} \ln\left(\frac{y_2}{y_1}\right) + \delta_{\tau 1} t = V_4 - U_4, \tag{31}$$

$$b_2 + b_{22} \ln x_2 + b_{12} \ln x_1 + b_{23} \ln x_3 + g_{22} \ln\left(\frac{y_2}{y_1}\right) + \delta_{\tau 2} t = V_5 - U_5, \tag{32}$$

$$b_3 + b_{33} \ln x_3 + b_{13} \ln x_1 + b_{23} \ln x_2 + g_{32} \ln\left(\frac{y_2}{y_1}\right) + \delta_{\tau 3} t = V_6 - U_6. \tag{33}$$

Here, $U_n \sim \left|N\left(0, \sigma_{Un}^2\right)\right|$ and $V_n \sim N\left(0, \sigma_{Vn}^2\right)$, n=2,…, 6, are respectively one-sided and two-sided errors and are statistically independent. The presence of V_n in (29)–(33) makes the individual equations be regression equations and captures random shocks.

Curvature requires that ODF be convex in outputs and quasi-convex in inputs. See, for example, Färe and Grosskopf (1994) and O'Donnell and Coelli (2005). For ODF to be convex in outputs it is sufficient that all the principal minors of the Hessian matrix are non-negative. In our two-output case this requires that:

$$\frac{\partial^2 D_o}{\partial y_2^2} = \frac{\partial^2 \ln D_o}{\partial \left(\ln y_2\right)^2} \frac{1}{y_2} \frac{D_o}{y_2} + \left(\frac{\partial \ln D_o}{\partial \ln y_2}\right)^2 \frac{D_o}{y_2^2} - \frac{\partial \ln D_o}{\partial \ln y_2} \frac{D_o}{y_2^2} \geq 0,$$

or, equivalently:

$$a_{22} + \frac{\partial \ln D_o}{\partial \ln y_2}\left(\frac{\partial \ln D_o}{\partial \ln y_2} - 1\right) \geq \ 0, \tag{34}$$

since $D_o / y_2^2 \geq 0$. The necessary condition of (34) is $a_{22} \geq 0$, because:

$$\frac{\partial \ln D_o}{\partial \ln y_2}\left(\frac{\partial \ln D_o}{\partial \ln y_2} - 1\right) \le 0.$$

Equation (28) can be transformed into an equality:

$$a_{22} + \frac{\partial \ln D_o}{\partial \ln y_2}\left(\frac{\partial \ln D_o}{\partial \ln y_2} - 1\right) = U_7 + V_7. \tag{35}$$

Quasi-convexity in inputs will be achieved if and only if all the principal minors of the following bordered Hessian matrix are non-positive:

$$F = \begin{pmatrix} 0 & f_1 & f_2 & f_3 \\ f_1 & f_{11} & f_{12} & f_{13} \\ f_2 & f_{12} & f_{22} & f_{23} \\ f_3 & f_{13} & f_{23} & f_{33} \end{pmatrix},$$

where $f_n = \partial D_o/\partial x_n = \left(D_o/x_n\right)\partial \ln D_o/\partial \ln x_n$, $n=1,\ 2,\ 3$; $f_{mn} = \partial^2 D_o/\partial x_m \partial x_n$, m, $n=1$, 2, 3. There are seven principal minors in total to be considered. Among them, the following three conditions obviously hold for certainty and hence can be ignored, i.e.:

$$F_{11} = \left|\begin{pmatrix} 0 & f_1 \\ f_1 & f_{11} \end{pmatrix}\right| = -\left(f_1\right)^2 \le 0,$$

$$F_{22} = \left|\begin{pmatrix} 0 & f_2 \\ f_2 & f_{22} \end{pmatrix}\right| = -\left(f_2\right)^2 \le 0,$$

$$F_{33} = \left|\begin{pmatrix} 0 & f_3 \\ f_3 & f_{33} \end{pmatrix}\right| = -\left(f_3\right)^2 \le 0.$$

However, the remaining four conditions must be considered:

$$F_{312} = \left|\begin{pmatrix} 0 & f_1 & f_2 \\ f_1 & f_{11} & f_{12} \\ f_2 & f_{12} & f_{22} \end{pmatrix}\right| = 2f_1 f_{12} f_2 - \left(f_2\right)^2 f_{11} - \left(f_1\right)^2 f_{22} \le 0,$$

$$F_{313} = \left|\begin{pmatrix} 0 & f_1 & f_3 \\ f_1 & f_{11} & f_{13} \\ f_3 & f_{13} & f_{33} \end{pmatrix}\right| = 2f_1 f_{13} f_3 - \left(f_3\right)^2 f_{11} - \left(f_1\right)^2 f_{33} \le 0,$$

$$F_{323} = \left|\begin{pmatrix} 0 & f_2 & f_3 \\ f_2 & f_{22} & f_{23} \\ f_3 & f_{23} & f_{33} \end{pmatrix}\right| = 2f_2 f_{23} f_3 - \left(f_3\right)^2 f_{22} - \left(f_2\right)^2 f_{33} \le 0,$$

$$F = \left| \begin{pmatrix} 0 & f_1 & f_2 & f_3 \\ f_1 & f_{11} & f_{12} & f_{13} \\ f_2 & f_{12} & f_{22} & f_{23} \\ f_3 & f_{13} & f_{23} & f_{33} \end{pmatrix} \right| = -f_1 \begin{vmatrix} f_1 & f_{12} & f_{13} \\ f_2 & f_{22} & f_{23} \\ f_3 & f_{23} & f_{33} \end{vmatrix} + f_2 \begin{vmatrix} f_1 & f_{11} & f_{13} \\ f_2 & f_{12} & f_{23} \\ f_3 & f_{13} & f_{33} \end{vmatrix} - f_3 \begin{vmatrix} f_1 & f_{11} & f_{12} \\ f_2 & f_{12} & f_{22} \\ f_3 & f_{13} & f_{23} \end{vmatrix} \le 0.$$

Let $S_n = \partial \ln D_o / \partial \ln x_n$, $n=1, 2, 3$. We simplify and re-formulate the foregoing four inequalities into equalities:

$$2S_1S_2h_{12} - S_2^2h_{11} - S_1^2h_{22} = V_8 - U_8, \tag{36}$$

$$2S_1S_3h_{13} - S_3^2h_{11} - S_1^2h_{33} = V_9 - U_9, \tag{37}$$

$$2S_2S_3h_{23} - S_3^2h_{22} - S_2^2h_{33} = V_{10} - U_{10}, \tag{38}$$

$$\begin{aligned} &- S_1^2h_{22}h_{33} - 2h_{12}h_{23}S_1S_3 - 2S_1S_2h_{13}h_{23} + 2h_{13}h_{22}S_1S_3 + 2h_{12}h_{33}S_1S_2 + h_{23}^2S_1^2 \\ &+ 2h_{11}h_{23}S_2S_3 + h_{13}^2S_2^2 - 2h_{12}h_{13}S_2S_3 - h_{11}h_{33}S_2^2 - h_{11}h_{22}S_3^2 + h_{12}^2S_3^2 = V_{11} - U_{11}. \end{aligned} \tag{39}$$

Here, $h_{11} = S_1^2 + b_{11} - S_1$, $h_{22} = S_2^2 + b_{22} - S_2$, $h_{33} = S_3^2 + b_{33} - S_3$, $h_{12} = S_1S_2 + b_{12}$, $h_{13} = S_1S_3 + b_{13}$, and $h_{23} = S_2S_3 + b_{23}$.

As for the error components in (35)–(39) we assume that $U_n \sim \left| N\left(0, \sigma_{Un}^2\right) \right|$ and $V_n \sim N\left(0, \sigma_{Vn}^2\right)$, $n=7,\ldots, 11$, are respectively one-sided and two-sided errors and are statistically independent. The presence of V_n in the five equations makes the individual equations be regression equations and captures statistical noise.

Estimation Procedure

Equations (26), (29)–(33), and (35)–(39) form a system of regression equations with composed errors. Since (29) and (30) are linearly dependent we arbitrarily delete (30) from the system of equations and suggest estimating the remaining ten equations simultaneously by ML. In this manner, we can similarly impose monotonicity and curvature on the ODF of (26) to the usage of a Bayesian approach. It is noteworthy that here we impose five more curvature restrictions on ODF compared to DODF, whose curvature conditions involve merely unknown parameters.

Following the case of DODF, we assume that all of the eleven composed errors, either $v+u$ or $v-u$, in those equations are statistically independent. Their individual pdf's are similar to (14) in the text and the corresponding likelihood function can be readily derived. We therefore omit their derivation. After getting the coefficient estimates, we apply the following formula, proposed by Battese and Coelli (1988), to directly calculate the measure of technical efficiency:

$$TE_o = E\left(e^{-U_1}|\omega_1\right) = \frac{\Phi\left(\frac{\mu_{*1}-\sigma_{*1}^2}{\sigma_*}\right)}{\Phi\left(\frac{\mu_{*1}}{\sigma_*}\right)} \exp\left(0.5\sigma_{*1}^2 - \mu_{*1}\right), \tag{40}$$

where $\omega_1 = V_1 + U_1$, $\sigma_1^2 = \sigma_{U1}^2 + \sigma_{V1}^2$, $\mu_{*1} = -\omega_1\sigma_{U1}^2/\sigma_1^2$, and $\sigma_{*1}^2 = \sigma_{U1}^2\sigma_{V1}^2/\sigma_1^2$. Term TE_o must lie between 0 and 1.

An analogous caveat to the case of DODF is worth citing, i.e., the resulting set of slope coefficient estimates may not be able to ensure that all monotonicity and curvature conditions are satisfied for all data points. This is attributed to the addition of the disturbances V_j's, $j=2,\ldots, 11$, to Eqs. (29)–(33) and (35)–(39) and appears to be an advantage over the Bayesian approach. The number of violating observations is expected to be relatively small. We shall discuss this in the empirical study section below.

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