考試科目	微積分 81111、81161	系所別	應用數學系	考試時間	2月19日(星期日)第一節

1. Evaluate the limits.

(a) (6%) $\lim_{\theta\to 0}\frac{\tan(2\theta)(1-\cos\theta^2)}{\theta^2\sin^3(3\theta)}$. (b) (6%) $\lim_{x\to\infty}\left(x-x^2\ln\left(\frac{1+x}{x}\right)\right)$.

2. Evaluate the integrals.

(a) (8%) $\int \cos(\ln x)\,dx$.

(b) (8%) $\int \frac{1}{x^2-2x}\,dx$.

(c) (8%) $\int_2^\infty \frac{\sqrt{x^2-4}}{x^3}\,dx$.

(d) (8%) $\int_0^{\frac{\pi}{2}} \frac{1}{1+\sin^2 x}\,dx$.

3. Determine if each series converges or diverges.

(a) (8%) $\sum_{n=1}^{\infty}\frac{(n+1)3^n}{2^{2n+1}}$. (b) (8%) $\sum_{n=1}^{\infty}\ln\left(\frac{n}{3n+1}\right)$.

4. (8%) Find the values of α such that the following integral is convergent.

$$\int_0^\infty \frac{\sin x}{x^\alpha}\,dx.$$

5. (8%) Find the arc length of the curve $r = 1+\sin\theta$ where $\theta\in[0,2\pi]$.

6. (8%) Let $p_k\in[0,1]$, $k=1,2,\ldots,10$ with $\sum_{k=1}^{10}p_k = 1$. Find the maximum of $\sum_{k=1}^{10}(2p_k-3p_k^2)$.

7. (8%) Find the double integral $\int\int_R \frac{x-2y}{3x-y}\,dA$ where R is the parallelogram enclosed by the lines $x-2y=0$, $x-2y=4$, $3x-y=1$ and $3x-y=8$.

8. (8%) Find the line integral $\int_C \tan y\,dx + x\sec^2 y\,dy$ where $C:(t^2,\frac{\pi t}{4})$ from $t=0$ to $t=1$.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考試科目	線性代數 81112、81162	系所別	應用數學系	考試時間	2 月 19 日(日)第二節

Please show all your work.

1. **(15%)** Let $A=[a_{i,j}]$ be a symmetric tridiagonal matrix (i.e. A is symmetric and $a_{i,j}=0$ whenever $1<|i-j|$). Let $M_{i,j}$ denote the matrix obtained from A by deleting the row and column containing $a_{i,j}$ and let B be the matrix obtained from A by deleting the first two rows and columns. Show that

$$\det(A)=a_{1,1}\det(M_{1,1})-a_{1,2}^2\det(B).$$

2. **(15%)** Let $P_2(R)$ denote the space of all polynomials with coefficients in R having degree less than or equal to 2. Let $T:P_2(R)\to P_2(R)$ be defined by $T(f(x))=f(x)+f'(x)+f''(x)$, where $f'(x)$ and $f''(x)$ denote the first and second derivatives of $f(x)$. Either show that T is not invertible or find its inverse.

3. **(15%)** A matrix A is said to be idempotent if $A=A^2$. Show that $I+A$ is nonsingular if A is idempotent.

4. **(15%)** Let $S=\{(1,0,1),(0,1,1),(1,3,3)\}$ be a subset of the inner product space R^3 over the field R. Apple the Gram-Schmidt process to obtain an orthonormal basis β for span(S).

5. **(20%)** Let x,y be nonzero column vectors in R^n with $n\geq 2$ and let $A=xy^T$. Show that (a) $\lambda_1=0$ is an eigenvalue of A with $n-1$ linearly independent eigenvectors and (b) A is diagonalizable if $x^Ty\neq 0$.

6. **(20%)** Let T be a linear operator on a vector space V, and suppose that V is a $T-$cyclic subspace of itself. Prove that if U is a linear operator on V, then $UT=TU$ if and only if $U=g(T)$ for some polynomial $g(t)$.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

國立政治大學 106 學年度 碩士班 招生考試試題

考試科目	微積分 81111、81161	系所別	應用數學系	考試時間	2月19日(星期日)第一節

1. Evaluate the limits.

(a) (6%) $\lim_{\theta\to 0}\dfrac{\tan(2\theta)(1-\cos\theta^2)}{\theta^2\sin^3(3\theta)}$. (b) (6%) $\lim_{x\to\infty}\left(x-x^2\ln\left(\dfrac{1+x}{x}\right)\right)$.

2. Evaluate the integrals.

(a) (8%) $\int \cos(\ln x)\,d\,x$.

(b) (8%) $\int \dfrac{1}{x^2-2x}\,d\,x$.

(c) (8%) $\int_2^\infty \dfrac{\sqrt{x^2-4}}{x^3}\,d\,x$.

(d) (8%) $\int_0^{\frac{\pi}{2}} \dfrac{1}{1+\sin^2 x}\,d\,x$.

3. Determine if each series converges or diverges.

(a) (8%) $\sum_{n=1}^{\infty}\dfrac{(n+1)3^n}{2^{2n+1}}$. (b) (8%) $\sum_{n=1}^{\infty}\ln\left(\dfrac{n}{3n+1}\right)$.

4. (8%) Find the values of α such that the following integral is convergent.

$$\int_0^\infty \frac{\sin x}{x^\alpha}\,d\,x.$$

5. (8%) Find the arc length of the curve $r=1+\sin\theta$ where $\theta\in[0,2\pi]$.

6. (8%) Let $p_k\in[0,1]$, $k=1,2,...,10$ with $\sum_{k=1}^{10}p_k=1$. Find the maximum of $\sum_{k=1}^{10}(2p_k-3p_k^2)$.

7. (8%) Find the double integral $\int\int_R \frac{x-2y}{3x-y}\,d\,A$ where R is the parallelogram enclosed by the lines $x-2y=0$, $x-2y=4$, $3x-y=1$ and $3x-y=8$.

8. (8%) Find the line integral $\int_C \tan y\,d\,x + x\sec^2 y\,d\,y$ where $C:(t^2,\frac{\pi t}{4})$ from $t=0$ to $t=1$.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考試科目	線性代數 81112、81162	系所別	應用數學系	考試時間	2 月 19 日(日) 第二節

Please show all your work.

1. **(15%)** Let $A=[a_{i,j}]$ be a symmetric tridiagonal matrix (i.e. A is symmetric and $a_{i,j}=0$ whenever $1<|i-j|$). Let $M_{i,j}$ denote the matrix obtained from A by deleting the row and column containing $a_{i,j}$ and let B be the matrix obtained from A by deleting the first two rows and columns. Show that

$$\det(A)=a_{1,1}\det(M_{1,1})-a_{1,2}^2\det(B).$$

2. **(15%)** Let $P_2(R)$ denote the space of all polynomials with coefficients in R having degree less than or equal to 2. Let $T:P_2(R)\to P_2(R)$ be defined by $T(f(x))=f(x)+f'(x)+f''(x)$, where $f'(x)$ and $f''(x)$ denote the first and second derivatives of $f(x)$. Either show that T is not invertible or find its inverse.

3. **(15%)** A matrix A is said to be idempotent if $A=A^2$. Show that $I+A$ is nonsingular if A is idempotent.

4. **(15%)** Let $S=\{(1,0,1),(0,1,1),(1,3,3)\}$ be a subset of the inner product space R^3 over the field R. Apple the Gram-Schmidt process to obtain an orthonormal basis β for span(S).

5. **(20%)** Let x,y be nonzero column vectors in R^n with $n\geq 2$ and let $A=xy^T$. Show that (a) $\lambda_1=0$ is an eigenvalue of A with $n-1$ linearly independent eigenvectors and (b) A is diagonalizable if $x^Ty\neq 0$.

6. **(20%)** Let T be a linear operator on a vector space V, and suppose that V is a $T-$cyclic subspace of itself. Prove that if U is a linear operator on V, then $UT=TU$ if and only if $U=g(T)$ for some polynomial $g(t)$.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。